Appl. Math. Inf. Sci. 6 No. 1S pp. 35S-39S (2012)

Applied Mathematics & Information Sciences
An International Journal

A New Method for Modification Consistency of the Judgment Matrix Based on Genetic Ant Algorithm

Jinghui Gao[1] and Rui Shan[2]
[1]College of Science, Yanshan University, Qinhuangdao, China
[2]College of Science, Yanshan University, Qinhuangdao, China
Email Address: gjhlove123@163.com srlhy@hotmail.com

Received May 02, 2011; Revised July 25, 2011; Accepted September 12, 2011
Published online: 1 January 2012

Abstract: Article proposed a modification method on consistency of judgment matrix based on genetic ant algorithm, which considered the inconsistency of judgment matrix causing by its disturbance and we founded a nonlinear programming problem to make it the least modification mount under the premise of consistency. The existence of global optimal solution was proved, finally we designed a corresponding genetic ant algorithm to solve the model and compared the method with existing approach, even applied it to the decision-making. Numerical examples proved that the nonlinear programming model is effective; to solve the model is reasonable and feasible in genetic ant algorithm.

Keywords: Genetic ant algorithm; Judgment matrix; Consistency; Nonlinear programming model

1 Introduction

T.L.Saaty had presented a practical method to deal with decision-making issues in 1970, called Analytic Hierarchy Process, signed as AHP, which was a determined property and quantifying method. It's has applied to the analysis, the forecast and the decision-making of complex system in [1-2]. The main problem of AHP in practical application is how to examine and revise judgment matrix's consistency issues, hotspot on AHP theory research as well. The main ways are: experience estimate method, the optimal transitive matrix method in [5], vector cosine method, pattern recognition method, induced matrix method in [6] and so on. The methods were presented in [5-6] were similar, modified the maximum deviation judgment, which were greedy algorithm thoughts, and not the overall optimal modification methods. It bought about deviation of original judgment matrix, and finally deviated from experts original ideals. It had proposed the judgment method in [4], it used pretreatment of maximum deviation value and the average square error of judgment information to screen expert's consistency of judgment matrix discrepancies among expert advice, which increased the comprehensive judgment matrix consistency, but it provided good ideas for modification judgment matrix consistency. This paper proposes a new method for modification consistency of the judgment matrix based on genetic ant algorithm, which considers inconformity of judgment matrix as the matrix after been disturbed. We designed a nonlinear programming model based on the minimum deviation of judgment matrix; genetic ant algorithm is the best way to solve it by examples.

2 New Method of Modification Judgment Matrix Consistency

Suppose A is an n steps reciprocal judgment matrix, w is sort vector of A, $W=(w_{ij})=\left|\frac{w_i}{w_j}\right|$ is characteristic matrix of A. If considers A as the matrix W is disturbed, namely

$$a_{ij}=\frac{w_i}{w_j}\varepsilon_{ij},\varepsilon_{ij}>0,\varepsilon_{ji}=\frac{1}{\varepsilon_{ij}} \tag{2.1}$$

so

$$\varepsilon_{ij}=\frac{w_j}{w_i}a_{ij} \tag{2.2}$$

when $\varepsilon_{ij}=1(i,j=1,2,\Lambda,n)$, A is consistent fully.

Correction methods presented in [4-6] were similar: each deviation was β, and $0.2\le\beta\le0.5$, then found the largest deviation elements in judgment matrix A to revise.

$$\varepsilon_{ed}=\max_{i,j}\left(\varepsilon_{ji}\right),\varepsilon_{ed}>1 \tag{2.3}$$

when $\bar{\varepsilon}_{ed}=\varepsilon_{ed}-\beta$, if $\bar{\varepsilon}_{ed}<1$, then $\bar{\varepsilon}_{ed}=1$ and it satisfies the nature of reciprocal matrix:

$$\bar{\varepsilon}_{ed}=\frac{1}{\bar{\varepsilon}_{de}},\bar{\varepsilon}_{ij}=\varepsilon_{ij};i\neq e,j\neq d \tag{2.4}$$

then gets matrix after revise,

$$\bar{A}=(\bar{a}_{ij})=\left|\frac{w_i}{w_j}\bar{\varepsilon}_{ij}\right| \tag{2.5}$$

In the same way, continuing to revise $\bar{A}$ and update continuously through iteration method till reaching satisfaction consistency. It designs a kind of modification judgment matrix method in [7], which isn't an iteration method. It makes deviation variable ε_{ij} modify α_{ij} and solve a nonlinear programming problem for getting satisfactory consistency to make the amount of total modification least.

The paper proposed a judging matrix modification method based on existing literature research, and proved the existence of the optimal solution on the nonlinear programming problem and applied to decision-making system, proving its validity finally.

2.1 New Method

To make each deviation variable ε_{ij}, modify α_{ij} for $\bar{A}$, the amount of total modification least, namely

$$\min\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left|\alpha_{ij}\right| \tag{2.6}$$

Generally,

$$CI=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2) \tag{2.7}$$

It is consistency index of judgment matrix; we should set threshold value $\overline{CI}$ according to analytic hierarchy process, the deviation value after revising satisfied

$$CI=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)\le\overline{CI} \tag{2.8}$$

Matrix after revise should have satisfied the properties of inter-contrary, $\varepsilon_{ij}-\alpha_{ij}>0$. So we can obtain a nonlinear programming problem, which satisfied consistency to make the least deviation value of judgment matrix after modification as follows

$$\begin{cases}\min\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left|\alpha_{ij}\right| \\ s.t.\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)\le\overline{CI} \\ \varepsilon_{ij}-\alpha_{ij}>0\end{cases} \tag{2.9}$$

We can obtain $\varepsilon_{ij}-\alpha_{ij}(i<j)$ by solving it; it satisfies the nature of reciprocal matrix

$$\bar{\varepsilon}_{ij}=\begin{cases}\varepsilon_{ij}-\alpha_{ij},i<j \\ \frac{1}{\varepsilon_{ij}-\alpha_{ij}},i>j\end{cases} \tag{2.10}$$

Thus, $\bar{A}=(\bar{a}_{ij})=\left|\frac{w_i}{w_j}\bar{\varepsilon}_{ij}\right|$ it is the judgment matrix after modification A. $\bar{A}=(\bar{a}_{ij})=\left|\frac{w_i}{w_j}\bar{\varepsilon}_{ij}\right|$, It satisfies consistency, where $CI\le\overline{CI}$, proven process can be seen in [7].

3 Existence Theorem of Nonlinear Programming Problem (2.9) Global Optimal Solutions

Define 1. Suppose $S=\left\{\alpha_{ij}\left|g_l(\alpha_{ij})\le0\right.\right\}$,

$g_l(\alpha_{ij})=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)-\overline{CI}$, E is considered as a important gather adjacent to α_{ij}^*, so

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left|\varepsilon_{ij}-\alpha_{ij}\right|,\quad g_l(\alpha_{ij}),l\in E \tag{3.1}$$

It is differential adjacent to α_{ij}^*. $g_l(\alpha_{ij}),l\notin E$ is continuous within α_{ij}^*.

Define 2. If exists global optimal point α_{ij}^*, vector gather $g_l(\alpha_{ij}^*)$ have nothing to linear and have α_{ij}^* in the decrease direction.

$$\{g_l(\alpha_{ij}^*)=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}^*+\frac{1}{\varepsilon_{ij}-\alpha_{ij}^*}-2)-\overline{CI},l\in E\} \tag{3.2}$$

Theorem 1 the sufficient necessary condition of α_{ij}^* exists, if α_{ij}^* is feasible in the decrease direction.

Prove: Exists a bulge gather within $\alpha_{ij}^*\in E$ in [9], as we know from (3.1). In the decline direction of feasible region

$$g_l(\alpha_{ij})=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)-\overline{CI} \quad (3.3)$$

there have a function s to make equation below true

$$g_l(\alpha_{ij}^*)=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}^*+\frac{1}{\varepsilon_{ij}-\alpha_{ij}^*}-2)-\overline{CI}$$
$$\le s\cdot[\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)-\overline{CI}] \quad (3.4)$$

because

$$x^*=\overline{\alpha_{ij}^*}\frac{x_i}{x_j}>0,\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\overline{\alpha_{ij}^*}\frac{x_i}{x_j}+\frac{1}{\overline{\alpha_{ij}^*}\frac{x_i}{x_j}})-2\ge 0 \quad (3.5)$$

so

$$F(w^*)=\min F(w)=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\alpha_{ij}^*\frac{w_i}{w_j}+\frac{1}{\alpha_{ij}^*\frac{w_i}{w_j}})-2\ge 0 \quad (3.6)$$

$$F(w)=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\overline{\alpha_{ij}}\frac{w_i}{w_j}+\frac{1}{\overline{\alpha_{ij}}\frac{w_i}{w_j}})-2\ge 0 \quad (3.7)$$

Thus, there exists a function s to make equation (3.4) true. Theorem 1 has been proved, which means that nonlinear programming problem (2.9) existing global optimal point α_{ij}^*.

4 Nonlinear Programming Problem(2.9) Solution

Nonlinear programming problem(2.9) is difficult to deal in common way. Genetic ant search algorithm combines genetic algorithm with ant algorithm. The local search of genetic algorithm is quite feeble to the feedback information from system, which has a low efficiency in precise solution result of information redundancy, tardiness speed, however, ant search algorithm converges to optimal tail through information accumulate, which has distributed search performance. Two algorithms complement each other's from falling into local optimization and overcome the problem of premature convergence in genetic algorithm, so it enhances algorithm's performance after coalescence.

The paper proposes a genetic ant search algorithm to solve nonlinear programming problem above.

It showed that a fitness function method of restraint optimization problem in [8]. We choose it as fitness function in the article

$$F(\alpha)=\frac{1}{|1+(1.1)^{e_1}|}\times\frac{1}{|(1.1)^{e_1}|}$$
$$e_1=\sum_{i=1}^{n}\sum_{j=i+1}^{n}|\alpha_{ij}| \quad (4.1)$$
$$e_2=|\overline{CI}-\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\varepsilon_{ij}-\alpha_{ij}+\frac{1}{\varepsilon_{ij}-\alpha_{ij}}-2)|+|\varepsilon_{ij}-\alpha_{ij}|$$

When $F(\alpha)$ is fitness function of algorithm, the larger it is, the better is $F(\alpha)$.

To achieve the minimum of total modification and satisfactory consistency requirements for nonlinear programming problem, we set

$$\alpha_{ij}\in(0,\varepsilon_{ij})\cap(0,2+\overline{CI}).$$

considering the influence of judgment matrix accuracy, we take float-point encoding.

$$L=\alpha_{12},\Lambda,\alpha_{1n}\alpha_{23},\Lambda,\alpha_{2n},\Lambda,\alpha_{nn},$$
$$\alpha_{ij}\in(0,\varepsilon_{ij})\cup(\varepsilon_{ij},2+\overline{CI}),$$

then combine genetic algorithm with ant search algorithm.

5. Example Test

1) Example 1. suppose judgment matrix $A_1=(a_{ij})$,

$$A_1=\begin{vmatrix}1 & 1/9 & 3 & 1/5\\ 9 & 1 & 5 & 2\\ 1/3 & 1/5 & 1 & 1/2\\ 5 & 1/2 & 2 & 1\end{vmatrix}$$

Sort weight vector is (0.1133, 0.5336, 0.0896, 0.2636), according to addition-multiplication method, revised matrix by induced matrix method in [6] is

$$A_1=\begin{vmatrix}1 & 1/9 & 1 & 1/5\\ 9 & 1 & 5 & 2\\ 1 & 1/5 & 1 & 1/2\\ 5 & 1/2 & 2 & 1\end{vmatrix}$$

Sort weight vector of A_1 is (0.0723, 0.5527, 0.1048, 0.2688), the consistency index is 0.1937; gain the revised matrix as follow, by 20 times in genetic ant search algorithm.

$$A_1'=\begin{vmatrix}1 & 0.1111 & 2.9999 & 0.2000\\ 9 & 1 & 4.9998 & 1.9999\\ 0.3333 & 0.2000 & 1 & 0.5000\\ 5.0000 & 0.5000 & 2.0000 & 1\end{vmatrix}$$

Sort weight vector is (0.0609, 0.5495, 0.1020, 0.2799), the consistency index is 0.0310; the modification deviation with the method in this article is less than the method in [6], and sort weight vector is coherent as well.

2) Example 2. A certain enterprise will promotes a cadre as higher leader from cadres C1, C2, C3 because of management needs. The main indexes for assessment cadres include: health condition B1, operation lever B2, writing lever B3, eloquence B4, work style B5, policy lever B6, mutual communication B7, team consciousness B8, total target. It means that the enterprise will choose A as leader. Given everyone a mark, full total is 100. So found hierarchy model as follows.

We obtain judgment matrix A_2 on the basis of figure 1 and analytic hierarchy process in [9] as follow

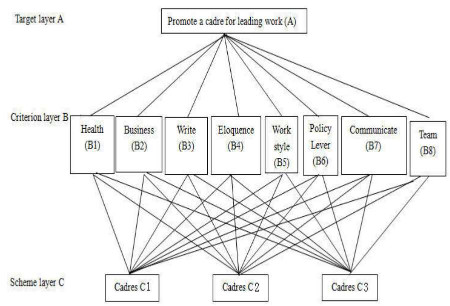

Figure 1. Hierarchy Structure Figure

$$A_2 = \begin{vmatrix} 1 & 1/5 & 4 & 1/4 & 1 & 2 & 3 & 8 \\ 5 & 1 & 9 & 1 & 5 & 7 & 8 & 13 \\ 1/4 & 1/9 & 1 & 1/8 & 1/4 & 1/2 & 1 & 4 \\ 4 & 1 & 8 & 1 & 4 & 6 & 7 & 12 \\ 1 & 1/5 & 4 & 1/4 & 1 & 2 & 3 & 8 \\ 1/2 & 1/7 & 2 & 1/6 & 1/2 & 1 & 1 & 6 \\ 1/3 & 1/8 & 1 & 1/7 & 1/3 & 1 & 1 & 5 \\ 1/8 & 1/13 & 1/4 & 1/12 & 1/8 & 1/6 & 1/5 & 1 \end{vmatrix}$$

Revised matrix with the method of optimal matrix transmitting in [5] is

$$A_2 = \begin{vmatrix} 1 & 0.296 & 2.937 & 0.332 & 1 & 1.848 & 2.360 & 7.480 \\ 3.376 & 1 & 9.914 & 1.122 & 3.376 & 6.24 & 7.97 & 25.78 \\ 0.340 & 0.101 & 1 & 0.113 & 0.340 & 0.629 & 0.804 & 2.60 \\ 3.01 & 0.99 & 8.849 & 1 & 3.01 & 5.56 & 7.10 & 23.65 \\ 1 & 0.296 & 2.940 & 0.333 & 1 & 1.85 & 2.36 & 7.63 \\ 0.541 & 0.160 & 1.605 & 0.180 & 0.541 & 1 & 1.28 & 4.13 \\ 0.423 & 0.125 & 1.24 & 0.142 & 0.424 & 0.780 & 1 & 3.23 \\ 0.134 & 0.039 & 0.385 & 0.042 & 0.131 & 0.242 & 0.310 & 1 \end{vmatrix}$$

Sort weight vector is (0.15, 0.21, 0.08, 0.19, 0.15, 0.11, 0.09, 0.02), consistency index is 0.8051, so it not passes the consistency test of judgment matrix.

Revised matrix by the method in this paper is

$$A_2' = \begin{vmatrix} 1 & 0.2000 & 3.9986 & 0.2598 & 1.0026 & 2.0140 & 3.0000 & 8.003 \\ 5.000 & 1 & 9.0007 & 1.0098 & 4.9986 & 7.0179 & 8.0039 & 12.9961 \\ 0.2501 & 0.1111 & 1 & 0.1251 & 0.2501 & 0.5039 & 0.9907 & 3.9975 \\ 3.8491 & 0.9902 & 7.9836 & 1 & 3.9932 & 5.9867 & 6.9869 & 11.9978 \\ 0.9974 & 0.2001 & 3.9984 & 0.2504 & 1 & 1.9889 & 3.0006 & 7.9929 \\ 0.4974 & 0.1424 & 1.9845 & 0.1670 & 0.5028 & 1 & 0.9757 & 5.9947 \\ 0.3333 & 0.1249 & 1.0094 & 0.1431 & 0.3333 & 1.0249 & 1 & 4.8389 \\ 0.1251 & 0.0769 & 0.2502 & 0.0833 & 0.1251 & 0.1668 & 0.2067 & 1 \end{vmatrix}$$

Sort weight vector is (0.1112,0.3252,0.0346, 0.3065, 0.1117, 0.0512, 0.0426,0.0170), consistency index is 0.0345, which passes the consistency test of judgment matrix obviously. The deviation with the method in [5] between judgment matrix and the original judgment matrix is too big, however, the deviation with genetic ants searching algorithm is lesser. The sorting weight vectors from two methods are consistent.

In order to find out the score, we must obtain the biggest feature vector, which is from scheme layer to target layer.

$$(W_1^1, W_2^1, W_3^1) = (0.1025, 0.1225, 0.0305),$$
$$(W_1^2, W_2^2, W_3^2) = (0.5000, 0.5000, 0.8750),$$
$$(W_1^3, W_2^3, W_3^3) = (0.8333, 0.1667, 0.7500),$$
$$(W_1^4, W_2^4, W_3^4) = (0.1667, 0.1520, 0.2753),$$
$$(W_1^5, W_2^5, W_3^5) = (0.1071, 0.3214, 0.6592),$$
$$(W_1^6, W_2^6, W_3^6) = (0.7014, 0.4286, 0.0348),$$
$$(W_1^7, W_2^7, W_3^7) = (0.4917, 0.9624, 0.0914),$$
$$(W_1^8, W_2^8, W_3^8) = (0.1071, 0.5627, 0.0108).$$

$$\text{Score of C1} = \sum_{i=1}^{8} (W_i \times W_i^1) \times 100 = 32.45.$$

$$\text{Score of C2} = \sum_{i=1}^{8} (W_i \times W_i^2) \times 100 = 32.68.$$

$$\text{Score of C3} = \sum_{i=1}^{8} (W_i \times W_i^3) \times 100 = 47.79.$$

Cadre C3 would be promoted to a higher leader result of his score is the highest.

5 Summary

The paper established a nonlinear programming model on judgment matrix consistency modified and had proved the existence of its global optimal solution, trying to design corresponding genetic ant algorithm to solve the nonlinear programming. Compared the method with existing methods at last, it was proved to complement the shortage of theory in research judgment matrix consistency problem effectively; it has been applied to realistic enterprise decision-making, which makes this method have practical significance.

Acknowledgements

This research was partially supported by the educational part research project of Hebei province (Grant No. 2009159)

References

[1] LIU Xingtang, WU Xiaoyan. Modern System Modeling and Emulation Technology [M]. Xi'an: Northwest Polytechnic University Press, 2001.
[2] CHEN Lai'an, LU Junling. System Engineering Principle and Application [M]. Beijing: science journal press, 1988.
[3] ZHU Jianjun, LIU Shixin. A new method of improving inconformity judgment matrix [J]. System Engineering Theory and Practice, 2003, 6(11): 95-110.
[4] LI Lingjuan, DOU Kun. Consistency Search of Judgment

Matrix in Analytic Hierarchy Process [J]. Computer Technology and Development, 2009, 19(10): 131-133.

[5] MA Yundong, HU Mingdong. Improved AHP method and its application in multi-objective decision-making [J]. System Engineering theory and practice, 1997, 17(6): 40-44.

[6] GUO Zhumei. Improved New Method to Solve Consistency of Judgment Matrix in AHP [J]. Qiqihar university journal, 2010, 26(6): 84-86.

[7] DING Jianli, CHEN Zengqiang, YUAN Zhuzhi. Blend of genetic algorithm and ant algorithm [J]. Computer Research and Development, 2003, 40(9): 1352-1357.

[8] LIU Yong, KANG Lishan. Numerical Juxtaposition Algorithm—genetic algorithm [M]. Beijing: science press, 1998.

[9] WANG Lianfen. The Introduction and The guide of Analytic Hierarchy Process [M]. Beijing: people university press in China, 1989.