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A Reliability-Based Risk Analysis of a Hollow Buttress Gravity Concrete Dam

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Abstract: In this paper we investigate the suitability of using first-order reliability method (FORM) analysis against a more vigorous Monte Carlo Simulation (MCS) for reliability-based analysis of the existing buttress dam in compliance with ICOLD (2005) requirements. Extreme rainfall events due to climate change critically impact the reliability and safety of the existing dams, making it an important consideration in dam risk analysis to prevent dam failure. The sliding mode is the most dominant mode of failure, and the overturning failure mode has the least significant effect. A strong linear correlation between the probability of failure and the safety factor for sliding has been established. An excellent linear correlation between the FORM-Taylor Series and Monte Carlo analysis for sliding has been obtained. The Taylor Series method is based on a simplified first-order probabilistic analysis, although somewhat conservative but adequate to be used at the preliminary design stage. The friction angle for sliding has the highest sensitivity of 94.8%, followed by the density of concrete of 4.0% and the least is the cohesion of 1.2%. The unconditional and conditional probability of failure were then evaluated against the ICOLD (2005) requirements. This study's significance indicates the FORM is too conservative to be used for the risk-based safety evaluation of the old existing hollow buttress dam at the operation stage. The Monte Carlo analysis provides a more precise form of risk analysis suitable for both the existing old dam and the new dam at the design, construction and operation stages.

Keywords: climate change, buttress dam, reliability analysis, first-order reliability method, Monte Carlo analysis.

1 Introduction

Buttress dams are hydraulic concrete engineering structures composed of inclined panels or arches or buttress heads that are firmly supported by buttress web. The Ambursen type buttress dam constructed in the early 1960's has a unique configuration where the upstream inclined flat slabs are situated and are supported by the corbels of the buttresses with a distinct structural arrangement. Hollow buttress dams pose less concern about uplift by foundation water pressures due to its smaller base than the gravity dams. For lateral stability, the stiffeners are incorporated as flanges with braced struts spanning between the web buttresses. The paper seeks to address the challenge of utilizing FORM (First Order Reliability Method) and MCS (Monte Carlo Simulation) for the reliability risk assessment of an aging concrete buttress dam in accordance with USBR-USACE (2019) and ICOLD (2005) guidelines.

Climate change with extreme rainfall events today leading to higher Probable Maximum Flood level plays a critical impact on the Reliability and safety of the dams, making it an important consideration in dam risk analysis (Fluixá-Sanmartín et al., 2018; Loza and Fidélis 2021; Chen and Lin, 2018; Kunkel et al., 2013; Salas et al., 2020; Sibuea et al., 2021) to meet the current USBR-USACE (2019) and ICOLD (2005) guidelines. Conversely, drought forecasting effect the operation of the hydroelectric power dam. The impacts and indices analysis of drought forecasting in Asia Continent was conducted by Khan et al. (2018).

The traditional design method for concrete gravity dams still uses the deterministic approach with a factor of safety, whereby the uncertainties associated with input variables are not directly accounted in the structural safety analysis (Pires et al., 2019). Probabilistic analyses offer a more comprehensive assessment compared to deterministic analyses with just a factor of safety owing to the incorporation of input variables with its range of standard Deviation depicting the actual set of measurements (Muench, 2010). The utilization of probabilistic techniques to assess the safety and integrity of a structural system has also been deliberated by Garcia et al. (2012). The use of safety factors for safety quantification of structures should be used cautions as the structure with the same safety factors but different coefficients of variation can have different probabilities of failure in the order 10^{-4} (ICOLD, 1993). Christian et al. (1994), Tang et al. (1999), provide a clear underlying theories and examples on the use of Reliability in geotechnical engineering. Reliability analysis, on the other

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hand, furnish a robust mechanism for assessing the cumulative impacts of uncertainties and offer a valuable insight of discerning situations of notably significant or insignificant uncertainties (Duncan, 2000). The design and safety check of concrete gravity dams using reliability analysis can effectively overcome the shortcomings of the safety factor method (Pei et al, 2011, Sharafati et al., 2020).

According to ICOLD (2005), risk analysis provides a pivotal tool in the risk management process. In the context of dam safety, the reliability risk analysis requires the identification of the dam potential failure modes and the quantification of probabilities of the structural responses to various loading conditions. In the context of dam block stability analysis, the First Order Reliability Method (FORM) can be used with relative ease, without additional dataset beyond the conventional analyses using the factor of safety approach. The analytical value can be significantly increases with only a modest additional computational effort (Yang and Ching, 2020). Hariri-Ardebili (2018) provides a comprehensive state-of-the-art review towards an effective risk-based probabilistic approach for dam safety. Pires et al. (2019) demonstrated the use of probabilistic risk-based analysis of a concrete gravity dam with its failure modes. The use of structural reliability methods in concrete dams is not widespread in its use in practice (Pires et al., 2019).

This research uses the FORM and MCS reliability method for the risk assessment of an aging old concrete Buttress Dam. Other techniques such as Probabilistic Density Evolution Method (PDEM) and Direct Probability Integral Method (DPIM) which is more suitable for stochastic structure system under random dynamic loads has been used for probabilistic reliability analysis. Pang et al. (2023) and Das and Tesfamariam (2023) have shown a good result when PDEM is compared with MCS. Lu, et al. (2024) has shown a precision result when DPIM is used to solve the dynamic response problems of nonlinear materials of a high concrete face rock dam. A non-intrusive random seepage method using 10,000 deterministic calculations for the high earth-rockfill dam requires only 1/11 of the computation time at the 10^{-3} failure level compared with the MCS and the error between the two methods is less than 1 % (Xu et al., 2023).

This case study aims to evaluate the probability of failure modes of the aging concrete buttress dam using the simplified FORM-Taylor series analysis against a more precise Monte Carlo Simulation (MCS) and assess its suitability to be used for risk-based safety evaluation against the USBR-USACE (2019) and ICOLD (2005) guidelines.

2 Reliability-Based Design Methods

The reliability-based analysis of a typical cross-section of the RCC dam is analyzed by First Order Reliability Method (FORM) using first-order Taylor series approximation and compared with a more complex Monte Carlo Simulation (MCS) approach. FORM-Taylor Series approximation is mathematically simpler, though somewhat less precise, that can be performed using an excel spreadsheet that is used by the design practitioners. MCS which is coded in MATLAB provides a more rigorous and precise analysis which is suitable for construction and operation stage assessment and a research-based environment.

2.1 First-Order Reliability Method (FORM)

First-Order Reliability Method (FORM)" originates from the approximation of the performance function g(X) by the linearization of first-order Taylor expansion.

A performance function or limit state function, g(x) is defined as the failure state (g(x) < 0) and safety state (g(x) > 0) where x = (x_1, x_2... $_n$) is a random variable vector.

The performance function (Phoon, 2019) is widely adopted:

$$g\ (\mathrm{x}) = g\ (x_1,\ x_2\dots{}_n) = F_s\ (x_1, x_2, \dots, x_n) - 1.0 \tag{1}$$

where F_s is the factor of safety and the prescribed acceptable safety factor is 1.0 (Liang et al., 1999).

The probability of failure can be defined as

$$Pf_{\mathrm{FORM}} = P\ (g\ (\mathrm{x}) < 0) = \int g(\mathrm{x}) \leq 0\ f(\mathrm{x})\ \mathrm{dx} \tag{2}$$

where f(x) is the joint probability density function of x.

Due to the inherent complexity of the multidimensional integral in equation (2), the reliability index β is typically computed in engineering practice and the failure probability is subsequently estimated by

$$Pf_{\mathrm{FORM}} \approx \Phi\ (-\beta) = 1\text{-}\ \Phi\ (\beta) \tag{3}$$

where Φ (·) is the standard normal cumulative distribution function.

Since FORM only gives a linear approximation of the limit-state function at the design point, the reliability index may be over- or underestimated for the functions with considerable curvature as such Monte Carlo Simulation provide a more accurate solution for a multi-number of the variable of a large complex model with non-linear limit state functions.

2.2 Monte Carlo Simulation

Monte Carlo simulation (MCS) provides a versatile approach capable of solving large complex models with multi-variables, whether linear or nonlinear of single or multiple limit state functions. This method involves sample trials of random variables, taken from the joint density function f(x) in Equation 2. The probability of failure is then estimated as in Equation 4.

$$P_{fMCS} = \frac{1}{N}\sum_{i=1}^{N} I[X_i] = \frac{N_f}{N} \tag{4}$$

where P_{fMCS} is the estimated probability of failure. I[ ] denotes the indicator function, Xi represents the sample vector i, Nf signifies the points within the failure domains, and N stands for the total number of trials. The number of trials must be sufficiently large to attain a precise estimation of the probability of failure with minimum statistical errors.

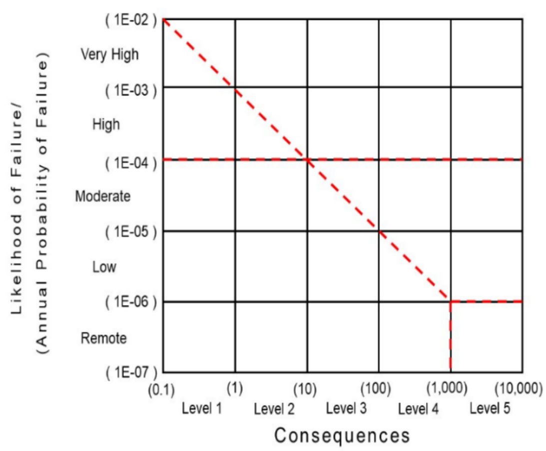
$$I(X_i) = \begin{cases} 1 \text{ if } g(x) \leq 0 \\ 0 \text{ if } g(x) > 0 \end{cases}$$

Finally, the MCS-based reliability index is expressed as:

$$\beta_{MCS} \approx -\Phi^{-1}(Pf_{MCS}) = \Phi^{-1}(1 - Pf_{MCS}) \quad (5)$$

3 Tolerable Risk Guidelines

A risk matrix provides a valuable tool for the likelihood of failure and the consequences arising from identified risk drivers associated with significant potential failure modes. In Figure 1, a dam risk matrix is depicted, employing general categories of failure likelihood and severity of the consequence.



Consequences Category – (Life Loss) and Level of Economic Loss

Fig. 1: Dam Risk Matrix (USBR-USACE, 2019)

The vertical axis of the matrix delineates the likelihood of failure and the annual probability of failure (APF), while the horizontal axis delineates the corresponding consequences, including loss of life and economic impacts categorized as follows: Level 1 <\$10 million, Level 2 \$10-\$100 million, Level 3 \$100-\$1 billion, Level 4 \$1-\$10 billion, and Level 5 >\$10 billion (USBR-USACE, 2019). However, further studies need to be carried out on the life and economic loss as a consequence of the probability of failure associated with the dams. ICOLD (2005) uses the horizontal dashed line value for the probability of failure of 10^{-4} for the high-risk dams.

For existing dams, the APF for an individual risk to the identifiable person or group, defined by a location should be limited to the value of less than 10^{-4} per year, except in exceptional circumstances (ANCOLD, 2003). The USACE (2014) policy for the estimated APF of greater 10^{-4} per year is unacceptable except in exceptional circumstances with the justification to implement risk reduction actions. If the APF is less than 10^{-4} per year, the other tolerable risk guidelines are met, and the implementation of the risk reduction actions diminishes.

4 Buttress Gravity Concrete Dam – A Case Study

The hollow buttress dam's typical cross-section of the buttress hollow spillway and its forces are shown in Figure 2.

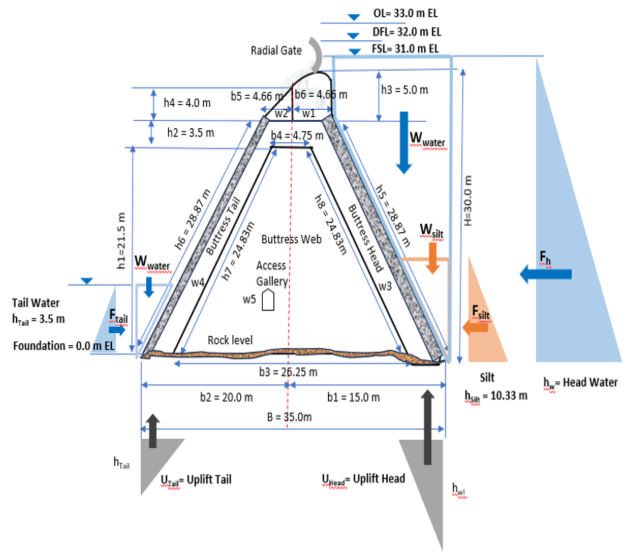


Fig. 2: A typical cross-section of the buttress hollow spillway and its forces

In the analysis, the assumption is that the exterior load on the slab is directly transferred to the buttresses and cohesion forces are subsequently transferred from the slab to the bedrock trench. The uplift pressure is exerted beneath the whole front plate and also beneath the 'thick' buttress web. If the thickness of web "t" is bigger than 2 m, the uplift reduction must be considered over a distance of "t/2" else if web "t" is less than 2 m, uplift is applied only for a whole front plate with no uplift under the buttress web.

4.1 Design Data of Hollow Buttress Dam

The design data of the aging hollow buttress dam which was constructed in the early 1960's is given in Table 1.

Table 1: Design Data of the Dam

General	SI units
Full supply level (FSL)	31.00 m EL
Design flood level (DFL)	32.00m EL
Overtopping level (OL) – PMF level	33.00m EL
Silt level - assumed 1/3 of FSL level	10.33m EL
Foundation level	00.00m EL
Width of base	35.0 m
Thickness of the buttress web	2.0 m
Spacing of the buttress web - centre to centre	7.5 m

4.2 Variability of Design Parameters

Variable material parameters involved in the reliability risk analysis of gravity dam include the density of concrete, shearing friction coefficient and cohesion of dam concrete-rock interface. Volume weight of concrete, friction angle and cohesion were determined by site-specific laboratory test samples. The sample size must meet the statistical requirements and be treated as a random variable (Xin and Chongshi, 2016).

Economic and safety reasons made it desirable to use the actual site-specific values of shear strength and uplift based on the actual monitoring system rather than generic values as specified in the guidelines for concrete dam stability analyses. The use of instrumented measured uplift and shear strength with thorough knowledge of site geology will reduce the uncertainty in stability evaluations of the risk analysis and economic cost. Uplift pressures over the maximum design have been reported in some dams (Spross, J. et al., 2014). Degradation phenomena in dam-foundation contacts have been also reported in some cases that are needed for remedial actions (Barpi, F. and Valente, S., 2008).

4.2.1 Density of Concrete Material

CIB (1991) gives a mean value of 23.5 kN/m^3 with a standard deviation, σ of 0.940 kN/m^3 or the coefficient of variation (COV) of 0.04 for concrete of compressive strength 20 MPa, and 24.5 kN/m^3 with a standard deviation, σ of 0.735 kN/m^3 or COV = 0.03 for concrete of compressive strength greater than 40 MPa. Assume the compressive strength for the buttress dam is 40MPa.

4.2.2 Friction Angle Parameters of Concrete to Rock and Concrete to Concrete Interface

Category II rock mass of medium sound was adopted with the friction angle, ϕ with the value of 50.0º and standard Deviation, σ of 13.26º and cohesion, C of 1.2 MPa and σ of 0.44 MPa. These pro-rated values are based on the China Electric Council (2010) recommendation for the rock-concrete interface at the foundation level as given in Table 2.

4.3 Load Cases

The load case events adopted in the analyses are as follows;

1. Usual Load Case: Full supply level (FSL) with AEP of 1
2. Unusual Load Case: Design Flood Level (DFL) with AEP of 1:1,000
3. Extreme Load Case: Overturning Level (OL) at PMF level with AEP of 1:10,000

The unusual load case at the DFL of an old existing dam constructed in the early 1960's was designed with a yearly probability of 1:1,000. The extreme load case due to PMF for OL need to be checked with an AEP of 1:10,000 for safety requirements as required by ICOLD (2005).

The assumed silt and tail-water level is 10.33 m and 3.5 m respectively for all load cases under FSL, DFL and OL conditions.

The reliability-based using FORM-Taylor Series Approximation and Monte Carlo Simulation (MCS) are carried out based on the above conditions to determine the annual probability of failure for the sliding and overturning failure modes that are applied to the RCC dam main spillway section.

5 Methodology

Two methods of reliability risk analysis - the simplified FORM with Taylor Series approximation and Monte Carlo analysis - have been used in this paper. The first Taylor Series method is a probabilistic simplified analysis, though somewhat less precise, and was used by USACE (1997 and 1998) and Duncan (2000). The second Monte Carlo probabilistic analysis was carried out using a 10 million sample population provides a more rigorous and precise form of analysis than the FORM-Taylor Series method.

5.1 Reliability-based analysis using FORM-Taylor Series Approximation.

A simplified reliability analysis using the FORM-Taylor series approximation as proposed by Duncan (2000) is carried out for the RCC concrete dam for the stability checks against sliding and overturning, mathematically simpler, though somewhat less precise but adequate for design practice, that can be performed using excel spreadsheet. The terms involved in computing the sliding factor of safety FOS [C, $W_{concrete}$, tan ϕ] and overturning factor of safety FOS [$W_{concrete}$] all involve some degree of uncertainty. Therefore, the computed value of the sliding and overturning factor of safety also involves some uncertainty.

Table 2: Friction Angle and Cohesion Parameters of Rock Interface

Rocks properties of dam foundation	Friction angle ϕ º		Cohesion C, MPa	
	Mean	Standard Deviation, s	Mean	Standard Deviation, s
Category I: dense and sound, the distance between cracks > 1 m	56.31 52.43	16.70 14.57	1.5 1.3	0.54 0.47
Category II: sound, weakly weathered massive rock with crack spaces distance between 0.5-1m	52.43 47.73	14.57 11.86	1.3 1.1	0.47 0.40
Category III: Rock mass of medium sound with crack spaces distance between 0.3-0.5m	47.73 41.99	11.86 11.31	1.1 0.7	0.40 0.28

It is useful to be able to assess the Reliability of sliding and overturning factors of safety, as well as the best estimate of its value. Therefore, the computed value of the sliding and overturning factor of safety also involves some uncertainty. It is useful to be able to assess the Reliability of sliding and overturning factors of safety, as well as the best estimate of its value.

The calculation steps using the reliability-based FORM-Taylor Series approximation are as follows:

Step 1. Determine the most likely values of the parameters involved and compute the factor of safety by the normal (deterministic) method for sliding and overturning. This is sliding F_{MLV} or overturning F_{MLV}.

$$F_{MLV} = \frac{\{CA + (\sum W_{conc} + \sum W_{water} + \sum W_{silt} - \sum U_{uplift}) \tan \emptyset\}}{F_h + F_{silt} - F_{tail}} \quad (5)$$

$$F_{MLV} = \frac{\{\sum W_{conc} \cdot x_{conc} \cdot + \sum W_{water} \cdot x_{water} + \sum W_{silt} \cdot x_{silt} + F_{tail} \cdot \frac{h_{tail}}{3}\}}{F_h \cdot \frac{h_w}{3} + F_{silt} \cdot \frac{h_{silt}}{3} + \sum U_{uplift} x_u} \quad (6)$$

The above deterministic analysis using the above factor of safety can be easily extended into the first-order reliability analysis using first-order Taylor Series approximation.

Step 2. Estimate the mean and standard deviations of the parameters that involve uncertainty. i.e., angle of friction, ϕ and Density of concrete, γ_{conc} are considered as random variables with normal distributions.

1 CIB (1991) gives the concrete Density, γ_{conc} of a mean value of 24.5 kN/m^3 with a standard deviation, σ of 0.735 kN/m^3 for concrete of compressive strength of 40 MPa.

2 China Electric Council (2010) Category III rock mass of medium sound with the friction angle value, ϕ of 50.0^o and standard deviation, σ of 13.26^o and cohesion value, C value of 1.19 MPa and standard Deviation, σ of 0.24 MPa.

Step 3. Use the Taylor series technique (Wolff, 1994; USACE 1997, 1998 and Duncan, 2000) to estimate the standard Deviation and the coefficient of variation of the factor of safety for cohesion, weight of concrete and friction angle using these formulas:

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \left(\frac{\Delta F_3}{2}\right)^2} \quad (7)$$

$$V_F = \frac{\sigma_F}{F_{MLV}} \quad (8)$$

Compute the factor of safety with each parameter increased by one standard Deviation and then decreased by one standard Deviation from its most likely value, with the values of the other parameters equal to their most likely values. These calculations result in N values of F^+ and N values of F^-. Using these values of F^+ and F^-, compute the values of ΔF for each parameter and compute the standard Deviation of the factor of safety (σ_F) using (7) and the coefficient of variation of the factor of safety (V_F) using (8). To calculate β, the First Order Reliability Method (FORM) method uses a Taylor series expansion as above, simplified by using only the first term (hence, "First Order").

Step 4. Use an Excel spreadsheet to determine the value of FMLV from the first step and the value of VF from the third step to determine the value of Pf. The key to computing more precise values of P_f is to compute the value of the lognormal reliability index, β_{LN}, using the following formula (Scott et al. 2001):

$$\beta_{LN} = \frac{\ln\left(F_{MLV}/\sqrt{1+V^2}\right)}{\sqrt{\ln(1+V^2)}} \quad (9)$$

where β_{LN} = lognormal reliability index; V = coefficient of variation of a factor of safety; and F_{MLV} = most likely value of factor of safety.

Step 5 When β_{LN} has been computed using (9), the value of P_f can be determined accurately using the built-in function NORMSDIST in Excel. The argument of this function is the reliability index, β_{LN}. In Excel, under "Insert Function," "Statistical," choose "NORMSDIST," and type the value of β_{LN}.

The excel spreadsheet output of the Reliability-based analysis using FORM-Taylor Series approximation for a design flood level case is given in Appendix 1.

5.2 Reliability-based analysis using Monte Carlo Simulation

A practical alternative is to develop probability distributions for the various parameters and apply a more rigorous Monte Carlo Simulation (MCS) with a higher degree of accuracy to determine the probability that the safety factor is below some threshold value associated with instability or other types of bad performance.

The basic procedure using the Monte Carlo analysis coded in Matlab for the buttress concrete dam is listed below:

Step 1 Build a probabilistic model of limit state analysis for a safety factor for sliding and overturning moment as given in Equations 5 and 6, respectively.

Step 2 Assign the mean and probability distributions to the model inputs for uncertainty in material properties, i.e., angle of friction ϕ and Density of concrete, γ_{conc} are considered random variables with normal distributions with no correlation.

1 CIB (1991) gives a concrete density of a mean value of 24.5 kN/m^3 with a standard deviation of 0.735 kN/m^3 for concrete of compressive strength of 40 MPa.

2 China Electric Council (2010) Category II rock mass of medium sound with a friction angle value of 50.0^o with standard Deviation, σ the value of 13.26^{o,} and cohesion, C of 1.2 MPa with σ a value of 0.44 MPa were used for the concrete dam foundation concrete-rock interface.

Step 3 Sample the model inputs based on their normal distributions and constraints using the 3-sigma rule.

Step 4 Input all the constant or determinate values.

Step 5 Run the model for the safety factor for sliding and overturning.

Step 6 Record the model output factor of safety.

Step 7 Repeat for the specified samples of the model inputs. Ten million input samples are used.

Step 8 Compute the number of samples with the factor of safety < 1.0; however, the safety factor is a constraint to be greater than zero.

Step 9 Evaluate the probability distribution for the model outputs with $N=10^6$.

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I[X_i] = \frac{N_f}{N} \tag{10}$$

Probability of Failure = No of samples with the factor of safety < 1.0 / Total No of Samples.

```
for i=1:MCS_Sample
W_Concrete(i) =
Area_of_Concrete*Density_of_Concrete_distribu
tion(i);
Sliding_FOS(i) =
((Cohesion_distribution(i)*Area_of_Contact+((
W_Concrete(i)+W_Water+W_Silt-
Total_Uplift_Force)
*tand(Friction_Angle_distribution(i)))))
/((Horizontal_Hydraulic_Force_Upstream+Horizo
ntal_Force_Silt-
Horizontal_Hydraulic_Force_Downstream));
end
FOS_Below_One = nnz(Sliding_FOS<1);
Probability_of_Failure =
FOS_Below_One/MCS_Sample;
```

Matlab Code for Sliding Mode of Failure

Matlab Code for Overturning Mode of Failure

```
for i=1:MCS_Sample
    W_Concrete(i) =
Volume_of_Concrete*Density_of_Concrete_distr
ibution(i);
    Overturning_FOS(i) =
(Cohesion_distribution(i)*Area_of_Contact+(W
_Concrete(i)*Lever_arm_Xconcrete)+(W_Water*L
ever_arm_Xwater)+(W_Silt*Lever_arm_Xsilt))/(
(Horizontal_Hydrostatic_Force_Upstream*Lever
_arm_Ywater_Upstream)+(Horizontal_Force_Silt
*Lever_arm_Ysilt)+Total_Uplift_Moment-
(Horizontal_Hydrostatic_Force_Downstream*Lev
er_arm_Ywater_Downstream));
end
FOS_Below_One = nnz(Overturning_FOS<1);
Probability_of_Failure =
FOS_Below_One/MCS_Sample;
```

Step 10 Calculate the Reliability Index. β. Set β > 8 if the number of samples with a safety factor < 1.0 is zero, i.e., probability of failure = 0.

Matlab Code for Reliability Index

```
% Calculate the reliability index (Beta) using
the inverse CDF of the standard normal
distribution
Beta = -norminv(Probability_of_Failure);
```

Step 11 Display the output and plot the number of samples against the safety factor graph.

The output file from the MCS analysis for the sliding and overturning failure modes for the design flood level case is given in Appendix 2.

6 Results and Discussion

The concrete gravity dam's two predominant probabilities of failures – sliding and overturning modes have been analyzed and discussed in this section.

6.1 Factor of Safety, Probability of Failure, and Reliability Index Concrete-Foundation Level

The hollow section between the buttresses is open to the outside air and, as such, does not experience any extreme uplift condition. Only the buttress head and tail at the slab wall have an uplift pressure.

The summary of the hollow spillway section results for sliding, overturning, and FORM-Taylor Series and Monte Carlo simulation probability of failure is shown in Table 3.

The above sliding factor of safety for full supply level (FSL), design flood level (DFL), and overtopping level (OL) are compared with the minimum sliding factor of safety for usual, unusual, and extreme flood is 2.0, 1.5, and 1.3 respectively for a well-defined friction and cohesion present as given Table C.8 of MyDAMS (2017). The sliding safety factor of 2.24 at FSL is higher than 2.0 for well-defined conditions with 80% confidence on test data where friction and cohesion are present. The sliding safety factor for DFL (2.11) and OL (2.00) is greater than 1.5 and 1.3, respectively.

The above sliding factor of safety for full supply level (FSL), design flood level (DFL), and overtopping level (OL) are compared with the minimum sliding factor of safety for usual, unusual, and extreme flood is 2.0, 1.5, and 1.3 respectively for a well-defined friction and cohesion present as given Table C.8 of MyDAMS (2017). The sliding safety factor of 2.24 at FSL is higher than 2.0 for well-defined conditions with 80% confidence on test data where friction and cohesion are present. The sliding safety factor for DFL (2.11) and OL (2.00) is greater than 1.5 and 1.3, respectively.

The FSL has the highest sliding reliability index (β_{TS} = 1.406, β_{MCS} =1.658), and OL has the lowest (β_{TS} = 1.173, β_{MCS} = 1.441). In the Monte Carlo Simulation (MCS), the number of samples run is 10 million, and zero failure is recorded when β> 7.87375, i.e., P_f = 00.000E+00.

The FSL has the highest overturning reliability index (β_{TS} = 54.150, β_{MCS} >8), and OL has the lowest (β_{TS} = 48.6042, β_{MCS} >8). In the MATLAB coding for MCS, the operational limit for no likelihood of failure is set at β > 8.

The no likelihood of the probability of overturning failure in the FORM-TS is similar to MCS values. There is no probability of overturning failure (P_{fTS} = 00.000E+00, P_{fMCS} = 00.000E+00 for all the events under FSL, DFL, and OL.

Figure 3 indicates the sliding factor of safety and reliability index β for the FORM and Monte Carlo.

The safety factor for sliding follows the same trend line as the reliability index, β_{TS} of FORM-Taylor Series, and β_{MCS} of MCS. β_{MCS} values of MCS are higher than β_{TS} values of the FORM-Taylor Series.

However, no direct comparison or trend between the reliability index, β TS for FORM-Taylor Series, and β MCS for MCS can be established. In the MATLAB coding for MCS, the operational limit where no probability of failure occurs is set as $\beta > 8$, i.e., $P_f = 00.000E+00$.

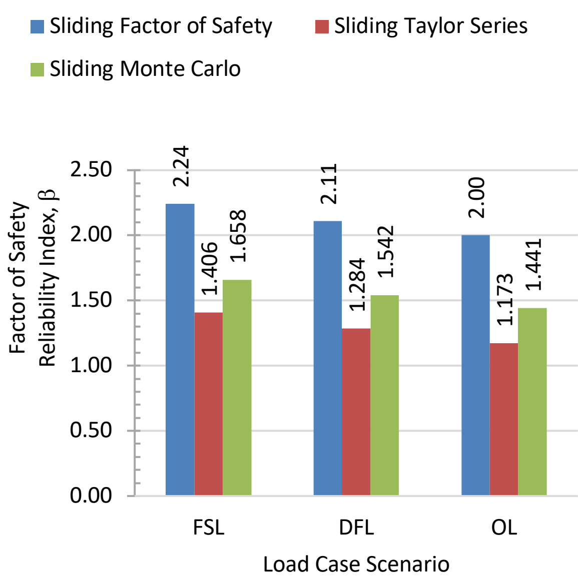


Fig. 3: Sliding Factor of Safety and Reliability Index β

Figure 4 shows the relationship between the probability of failure P_f and sliding factor of safety, F for FORM-Taylor Series and Monte Carlo Simulation.

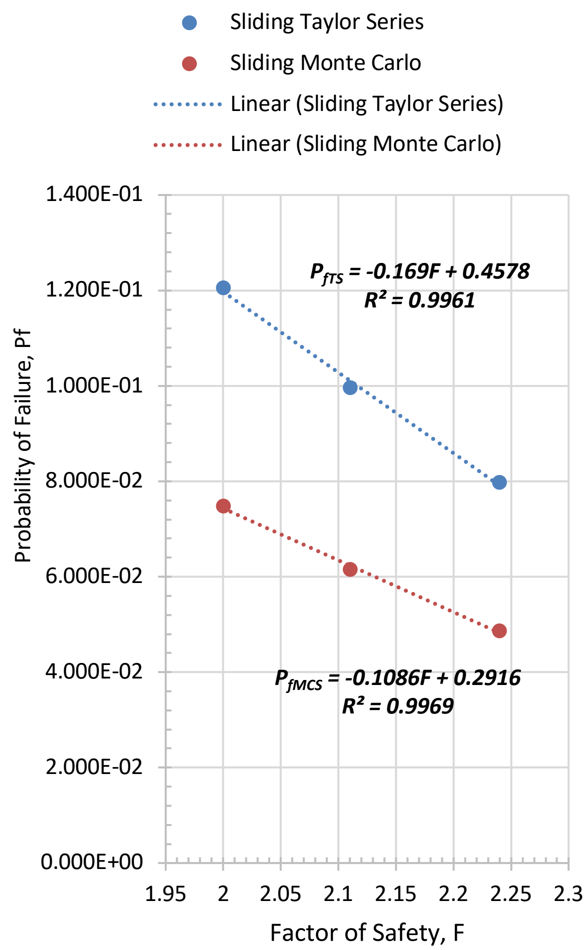


Fig. 4: Relationship between the Sliding Probability of Failure P_f and Factor of Safety, F

Table 3: Sliding and Overturning Factor of Safety and Probability of Failure of Buttress Dam

			Sliding			**Overturning**		
Load Case Scenario	h_w (m)	h_{tail} (m)	Sliding F_{MLV}	Taylor Series P_f (b_{TS})	Monte Carlo P_f (b_{MCS})	Overturning F_{MLV}	Taylor Series P_f (b_{TS})	Monte Carlo P_f (b_{MCS})
Full Supply Level Usual Event	31.0	3.5	2.24	7.981E-02 (1.406)	4.863E-02 (1.658)	2.82	0.00E+00 (54.150)	0.00E+00 (>8)
Design Flood Level Unusual Event	32.0	3.5	2.11	9.965E-02 (1.284)	6.152E-02 (1.542)	2.65	0.00E+00 (51.364)	0.00E+00 (>8)
Overtopping Level Extreme Event	33.0	3.5	2.00	1.205E-01 (1.173)	7.477E-02 (1.441)	2.48	0.00E+00 (48.604)	0.00E+00 (>8)

FSL is full supply level, DFL is design flood level, OL is overtopping level, h_w is upstream head of water, and *P_f Probability of Failure* and *b Reliability Index are given in bracket.*

The results indicate a very strong linear correlation between the probability of failure P_f and sliding factor of safety, F with $R^2 > 0.96$ for both FORM-Taylor Series and Monte Carlo Simulation with the following relationship;

- FORM-Taylor Series

$$P_{fTS} = -0.169F + 0.9961 \quad \text{with } R^2 = 0.9961 \tag{11}$$

- Monte Carlo Simulation

$$P_{fMCS} = -0.1086F_S + 0.2916 \quad \text{with } R^2 = 0.9969 \tag{12}$$

The probability of failure P_f decreases linearly with the increase in the safety factor, F, for both the FORM-Taylor Series and Monte Carlo analysis. The FORM-Taylor Series, P_{fTS} is higher than the Monte Carlo probability of failure P_{fMCS} for the given sliding factor of safety.

Figure 5 shows the sliding probability of failure P_f for FORM-Taylor Series and Monte Carlo Simulation and their normalized values.

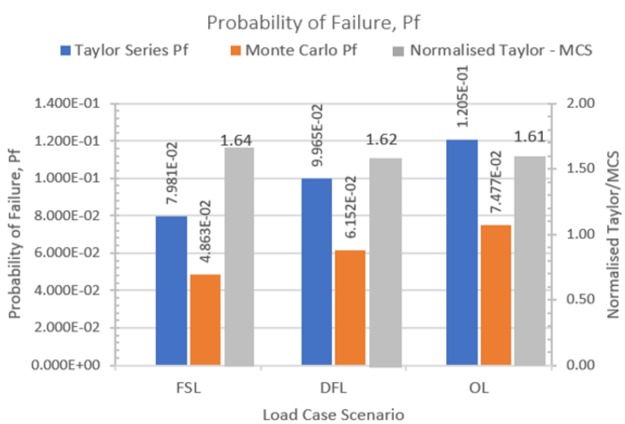


Fig. 5: Sliding Probability of Failure, P_f and

Normalized Taylor Series-Monte Carlo Values

It is interesting to note that the risk probability of failure for a DFL event is lower than for FSL and OL events due to the stabilizing effect of the tail-water force against the destabilizing headwater effect on the horizontal forces. The normalized FORM-Taylor to MCS probability of failures P_f values ranges from 1.61 to 1.64 with an average value of 1.625.

Figure 6 illustrates the correlation between the FORM-Taylor Series and Monte Carlo analysis for the sliding probability of failure.

The results indicate an excellent linear correlation between the FORM-Taylor Series, P_{fTS}, and Monte Carlo analysis P_{fMCS} for sliding with $R^2 = 1$ with the following relationship.

$$P_{fTS} = 1.6202\ P_{fMCS} \tag{13}$$

FORM-Taylor Series, P_{fTS} values are 1.62 times more conservative than Monte Carlo analysis P_{fMCS} for sliding as given in the above correlation equation. Thus, FORM-Taylor probability values are too conservative in evaluating the risk on the probability of failure than the more precise MCS reliability analysis of the existing old concrete buttress dam.

No direct comparison can be made between the reliability indexes, β_{TS} for FORM-Taylor Series, and β_{MCS} MCS. In the MATLAB coding for MCS, the operational limit where no probability of failure occurs is set at $\beta > 8$, i.e., P_f = 00.000E+00.

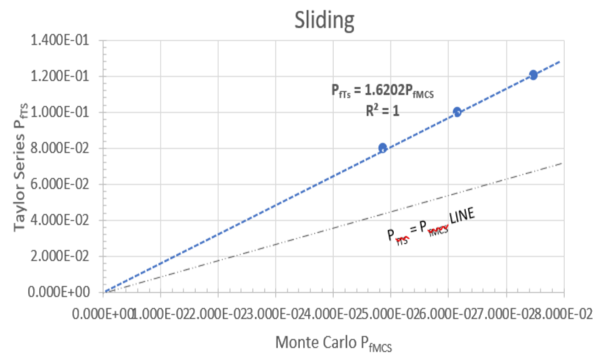


Fig. 6: Sliding Probability of Failure – Correlation between FORM-TS and Monte Carlo Analysis

From the reliability analysis, the sliding failure mode was the dominant mode over the overturning mode of failure. The most probably been friction angle is the most influential random variable in this failure mode. The overturning had a significantly lower probability of occurrence than sliding. The overturning modes had a very low probability of occurrence, the reason being that the stabilizing weight of concrete with only a negligible destabilizing uplift force for the buttress dam.

The calculation results indicate that FORM is too conservative than MCS to be used for the probabilistic risk-based analysis as a reference for the design of the buttress concrete dam for sliding failure mode. At no likelihood of overturning failure, FORM and MCS have the same probability value. FORM can only be adequately used at the preliminary design stage for the buttress dam reliability risk analysis. A more precise Monte Carlo Simulation is recommended to be used as a reference for the concrete buttress dam's design, construction and operation stage.

6.2 Sensitivity Analysis

Sensitivity analysis measures how the impact of one or more input variables can lead to uncertainties in the output variables. Table 4 and Figure 7 indicate the sensitivity analysis on the independent input variable friction angle, cohesion and concrete Density for sliding. No sensitivity analysis is carried out for overturning as it has only one input variable, i.e., density of concrete.

ΔF values measure the swing on the safety factor for the friction angle, cohesion, and the density of concrete taken from the FORM-Taylor Series analysis for sliding, as shown in Appendix 1.

The friction angle swing ΔF ranges from 2.07 to 2.32, cohesion swing ΔF ranges from 0.0257 to 0.0288, and concrete swing F's Density ranges from 0.0865 to 0.0978. The sensitivity for the friction angle is 94.8%, the density of concrete is 4.0%, and cohesion is 1.2%. The sliding mode of failure is very sensitive to friction angle and less sensitive to the density of concrete and cohesion. In this sense, the Taylor Series Method can be viewed as a structured sensitivity analysis or parametric study.

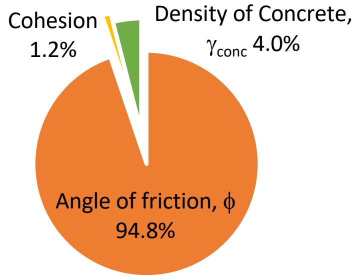


Fig. 7: Sensitivity Analysis for Sliding

6.3 Unconditional Probability of Failure

The annual exceedance probability (AEP) for full supply level (FSL) under usual operating conditions and design flood level (DFL) and overtopping level (OL) is assumed to be 1, $10^{-3,}$ and 10^{-4}.year^{-1} respectively.

$$Unconditional\ probability\ of\ failure, Pf_{unconditional} = \sum Pf_{conditional} \times \sum Pf_{water\ level} \quad (14)$$

The conditional probability of failure provides insights into the susceptibility of a dam to specific triggers, while the unconditional probability of failure offers a broader assessment of the overall failure likelihood. These probabilities assist in prioritizing risk management strategies, determining maintenance needs, and making informed decisions regarding dam safety measures. Table 5 shows the results of the sliding unconditional probability of failure for the FORM-Taylor Series and Monte Carlo simulation.

All the sliding combined or unconditional probability of failure under DFL and OL cases is less than 1.0 x 10^{-4} (ICOLD, 2005) except for FSL. The highest is under FSL (P_{fTS}= 7.981-02, P_{fMCS}= 4.863E-02) and lowest is under OL (P_{fTS}= 1.205E-05, P_{fMCS}= 17.477E-06). This uncertainty can be further reduced by accurate and extensive in situ testing, and accounting for the buried part of the slab in the foundation, the calculated reliability index can be further increased.

The unconditional and conditional probability of failure for overturning is 00.00E+00. As such, there is no likelihood of the overturning probability of failure for the hollow buttress dam for all the load cases.

Table 4: Sensitivity Analysis for Sliding

Input	Full Supply Level (FSL)		Design Flood Level (DFL)		Overtopping Level (OL)		Average
Variable	Swing DF	Sensitivity %	Swing DF	Sensitivity %	Swing DF	Sensitivity %	Sensitivity %
Angle of friction, ϕ°	2.32	94.8	2.19	94.8	2.07	94.8	94.8
Cohesion, C	0.0288	1.2	0.0272	1.2	0.0257	1.2	1.2
Density of concrete, g_{conc}	0.0978	4.0	0.0919	4.0	0.0865	4.0	4.0

Table 5: Sliding Unconditional Probability of Failure: FORM -TS and Monte Carlo Simulation

Load Cases	Water Level AEP Event	Taylor Series *Conditional P_f*	Taylor Series *Unconditional, P_f*	*Monte Carlo Simulation Conditional P_f*	*Monte Carlo Simulation Unconditional , P_f*
Full Supply Level (FSL) Usual	1	7.981E-02	7.981E-02	4.863E-02	4.863E-02
Design Flood Level (DFL) Unusual	10^{-3}	9.965E-02	9.965E-05	6.152E-02	6.152E-05
Overtopping (OL) Extreme	10^{-4}	1.205E-01	1.205E-05	7.477E-02	7.477E-06

7 Conclusions

1. The results indicate that FORM is conservative for the probabilistic risk-based analysis as a reference for the buttress concrete dam design for the sliding failure mode. At no likelihood of overturning failure, FORM and MCS have the same probability value. MCS is thus recommended to be used as the design reference for the concrete buttress dam.
2. Sliding is the most dominant failure mode for the buttress concrete dam. The overturning mode has no probability of occurrence, with the most probable reason being that the concrete density and coefficient of hydraulic inefficiency presented balanced contributions.
3. A strong linear correlation with $R^2 > 0.996$ between the sliding probability of failure P_f and the sliding factor of safety has been established.
4. The FORM-Taylor Series- Monte Carlo analysis for sliding has an excellent linear correlation with $R^2 = 1$. FORM is 62.0% more conservative than the Monte Carlo analysis for the sliding mode of failure.
5. The conditional and unconditional sliding probability of failure for the full supply level (FSL) case is higher than 1.0×10^{-4} as per ICOLD (2005) requirement. Unconditional sliding probability of failure for design flood and overtopping level load cases are lower than 1.0×10^{-4}.
6. No likelihood of unconditional or conditional overturning probability of failure for all load case (FSL, DFL, and OL) events.
7. The major sensitivity of 94.8% for sliding failure is the friction angle, whereas the Density and cohesion have a very low sensitivity of 4.0% and 1.2%, respectively.

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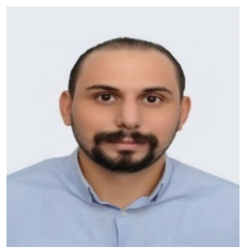

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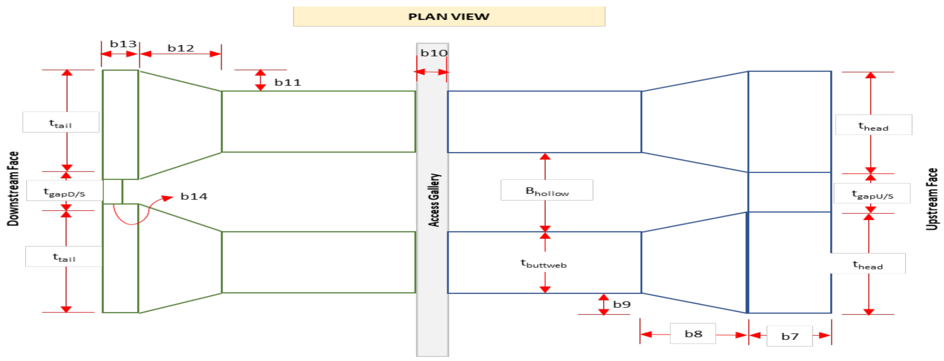
Appendix 1: Reliability Risk Analysis Form - Taylor Series Approximation of a Concrete Buttress Dam

Basic Data:

GENERAL		UNIT
Date of construction	1960	
Dam Crest Level	32.00	m asl
Full Supply Level (FSL) - AEP 1:1	31.00	m asl
Overtopping level (PMF Level) - AEP 1:10,000)	33.00	m asl
Design Flood level (DFL) - AEP 1:1,000	32.00	m asl
Minimum operating level	22.50	m asl
Sedimentation Level = 1/3 of FSL	10.33	m asl



RANDOM VARIABLE	UNIT	Mean, μ	Std. Dev, σ
Friction angle, Ø	Degree	50	13.26
Internal friction angle, tan Ø	-	1.19	0.24
Cohesion coeff., C'	kN/m^2	1.2	0.44
Unit weight of concrete, γ_{conc}	kN/m^3	24.5	0.735
DETERMINISTIC	**UNIT**	**Mean**	
Unit weight of water, γ_w	kN/m^3	10.0	
Unit weight of silt, γ_{silt}	kN/m^3	9.0	

Calculation of Forces

1. Self-Weight of the dam

Section No.	Particulars	Vertical Force (kN)	Lever arm (m)		Moment (kNm)
w1	b6*h3*B*γ_{conc} =	4283.21	b2 + b6/2 =	22.33	95648.42
w2	1/2*b5*h4*B*γ_{conc} =	1713.29	b2-(1/3*b5) =	18.45	31603.26
U/S	1/2*(b6 + b1) * (h1+h2)* B* γ_{conc} =	45161.16	L/2 =	17.50	1781617.03
D/S	1/2*(b5 + b2) * (h1+h2)* B* γ_{conc} =	56645.53			
Hollow Buttress Web	1/2*(b3 + b4) * h1* B_{hollow}* γ_{conc} =	-44905.44	b12+b13+ (b3/2) =	16.07	-721675.29
Hollow Buttress Head and Tail	[(0.5*($t_{gapD/S}$+B_{hollow})*b12 *h7) +(0.5*($t_{gapU/S}$+B_{hollow}) *b8*h8)]* γ_{conc} =	-12257.19	L/2 =	17.50	-214500.82
	Total	**50640.56**			**972692.60**

2. Hydrostatic Load U/S

b) Reservoir at Design Flood Level (DFL)

Height of water, h_{wups} = 32.00 m

Section No.	Particulars	Force (kN)	Lever arm (m)		Moment (kNm)
Horizontal	1/2*h_{wups}^2*B* γ_w =	38400.00	h_{wups} /3 =	10.67	409600.00
Vertical	1/2 *b1* h_{wups} *B*γ_w =	18000.00	b2+b6+(2/3* (b1-b6)) =	31.55	567972.00

3. Hydrostatic Load D/S

b) Reservoir at Design Flood Level (DFL)

Height of Tail Water, $h_{TWL\,s}$ = 3.50 m

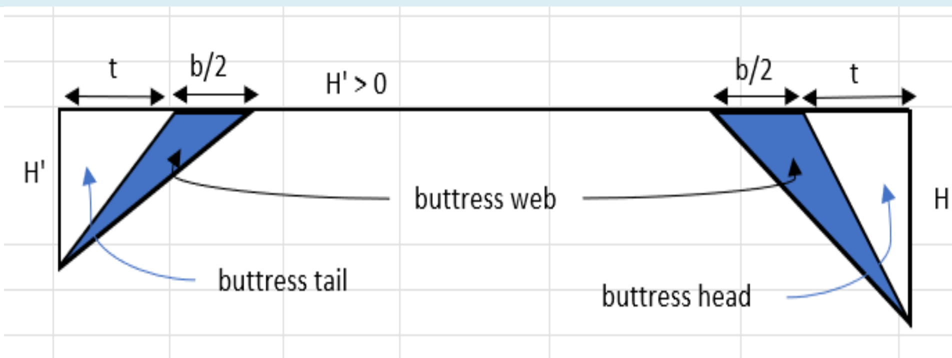
Section No.	Particulars	Force (kN)	Lever arm (m)		Moment (kNm)
Horizontal	1/2*h_{TWL}^2*B* γ_w =	459.38	h_{TWL} /3 =	1.17	535.94
Vertical	1/2*h_{TWL}*(b2-b5)*B* γw =	4932.13	1/3*(b2-b5) =	5.11	25216.31

4. Silt Load

Depth of Silt, h_{silt} =	10.33	m		A third of FSL
Sediment, φ' =	25	°		
Coefficient of active pressure, Ka =	0.41			

Section No.	Particulars	Force (kN)	Lever arm (m)		Moment (kNm)
Horizonta	$1/2*h_{silt}{}^2*B*\gamma_{silt}*Ka=$	1462.61	$h_{silt}/3=$	3.44	5037.89
Vertical	$1/2*h_{silt}*(b1-b6)*B*\gamma_{silt}*Ka=$	1463.27	b2+b6+(2/3*(b1-b6))=	31.55	46172.12

5. Uplift Pressure



b) Reservoir at Design Flood Level (DFL)

Height of water, H =	32	m
Height of water, H' =	3.50	m

	Section No.	Particulars	Uplift Vertical Force (kN)	Lever arm (m)		Moment (kNm)
U/S	Buttress Head	$1/2*\gamma_w*H*[[0.5*(t_{head}-t_{buttweb})*b8]+[(t_{head}-t_{buttweb})*b7]+(t_{gapU/S}*b7)]=$	2924.64	L-1/3*(b7+b8)=	33.07	96712.00
U/S	Buttress Head + b/2*web	$1/2*\gamma_w*H*[t_{buttweb}*(b7+b8+(t_{buttweb}/2))]=$	2174.72	$L-1/3*(b7+b8+t_{buttweb}/2)=$	32.73	71188.73
D/S	Buttress Tail	$1/2*\gamma_w*H'*[[0.5*(t_{tail}-t_{buttweb})*b12]+[(t_{tail}-t_{buttweb})*b13]+(t_{gapD/S}*b14)]=$	138.11	1/3*(b12+b13)=	0.98	135.62
D/S	Buttress Tail + b/2*web	$1/2*\gamma_w*H'*[t_{buttweb}*(b12+b13+(t_{buttweb}/2))]=$	138.11	$1/3*[b12+b13+(t_{buttweb}/2)]=$	1.32	181.66
		Total	**5375.58**			**168218.01**

6.0 Load Combinations

c) Reservoir at Design Flood Level (DFL)

No.	Particulars	Horizontal Force (kN)	Overturn Moment (kNm)	Vertical Force (kN)	Resist Moment (kNm)
1	Self-Weight of the Concrete			50640.56	972692.60
2	Horizontal Hydrostatic Pressure U/S	38400.00	409600.00		
3	Vertical Hydrostatic Pressure U/S			18000.00	567972.00
4	Horizontal Silt Pressure	1462.61	5037.89		
5	Vertical Silt Pressure			1463.27	46172.12
6	Horizontal Hydrostatic pressure D/S	-459.38	-535.94		
7	Vertical Hydrostatic pressure D/S			4932.13	
8	Uplift Pressure		168218.01	-5375.58	
	Total	**39403.24**	**582319.96**	**69660.38**	**1540664.60**

6.0 Factor of Safety (F_{MLV})

$$Sliding\ F_{MLV} = \frac{\{C_u A + (\sum W_{conc} + \sum W_{water} + \sum W_{silt} - \sum U_{uplift}) \tan \emptyset\}}{F_h + F_{silt} - F_{tail}}$$

$$Overturning\ F_{MLV} = \frac{\{\sum W_{conc} . x_{conc} . + \sum W_{water} . x_{water} + \sum W_{silt} . x_{silt} + F_{tail} . \frac{h_{tail}}{3}\}}{F_h . \frac{h_w}{3} + F_{silt} . \frac{h_{silt}}{3} + \sum U_{uplift} x_u}$$

Load Combination	F_{MLV} Sliding	F_{MLV} Overturning
Full Supply Level (FSL)	2.24	2.82
Overtopping Level (OL)	2.00	2.48
Design Flood Level (DFL)	2.11	2.65

SUMMARY RESULTS

Case	U/S Head, h_w (m)	D/S Head, h_{tail} (m)	Sliding F_{MLV}	Reliability Index, β_{LN}	Failure Probability, P_f	Overturn F_{MLV}	Reliability Index, β_{LN}	Failure Probability, P_f
Full Supply Level	31.00	3.50	2.24	1.406	7.981E-02	2.82	54.150	0.000E+00
Design Flood Level	32.00	3.50	2.11	1.284	9.965E-02	2.65	51.364	0.000E+00
Overtopping Level	33.00	3.50	2.00	1.173	1.205E-01	2.48	48.604	0.000E+00

7.0 FORM – Taylor Series Reliability Analysis

Sliding at DFL				
Variables	**Values**	**Factor of Safety**		**ΔF**
Concrete unit weight, γ_c				
Most Likely Value (MLV) + σ	25.235 kN/m^3	F+	2.15E+00	9.19E-02
Most Likely Value (MLV) - σ	23.765 kN/m^3	F-	2.06E+00	
Friction Angle, Ø				
Most Likely Value (MLV) + σ	63.26 °	F+	3.51E+00	2.19E+00
Most Likely Value (MLV) - σ	36.74 °	F-	1.32E+00	
Cohesion, C				
Most Likely Value (MLV) + σ	1.64 °	F+	5.15E-02	2.72E-02
Most Likely Value (MLV) - σ	0.76 °	F-	2.44E-02	

	F_{MLV} Sliding	=	2.11
$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2}$	Standard Deviation, σ_F	=	1.096
$V_F = \frac{\sigma_F}{F_{MLV}}$	Coefficient of Variant , V_F	=	0.519
$\beta_{LN} = \frac{ln(F_{MLV}/\sqrt{1+V^2})}{\sqrt{\ln(1+V^2)}}$	Realibility Index, β_{LN}	=	1.284
	Probability of Safety, P(S)	=	0.900
	Probability of Failure, P(f)	=	9.965E-02

Overturning at DFL				
Variables	**Values**	**Factor of Safety**		**ΔF**
Concrete unit weight, γ_c				
Most Likely Value (MLV) + σ	25.235 kN/m^3	F+	2.78E+00	1.00E-01
Most Likely Value (MLV) - σ	23.765 kN/m^3	F-	2.67E+00	

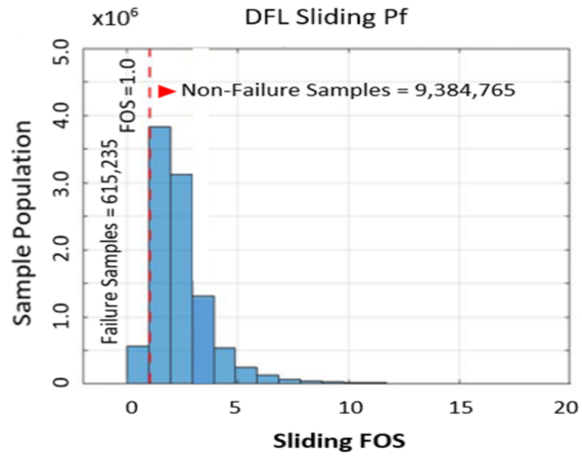
	F_{MLV} Overturning	=	2.65
$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2}$	Standard Deviation, σ_F	=	0.050
$V_F = \frac{\sigma_F}{F_{MLV}}$	Coefficient of Variant , V_F	=	0.019
$\beta_{LN} = \frac{ln(F_{MLV}/\sqrt{1+V^2})}{\sqrt{\ln(1+V^2)}}$	Realibility Index, β_{LN}	=	51.364
	Probability of Safety, P(S)	=	1.000
	Probability of Failure, P(f)	=	0.00E+00

Appendix 2: Reliability Risk Analysis - Monte Carlo Simulation

Output File

Title: Hollow Buttress Section - Sliding Probability of Failure at Design Flood Level (DFL)

Volume of Concrete in m^3 is 2.0670e+03
Volume of Water in m^3 is 2.2932e+03
Weight of Water in kN is 2.2496e+04
Submerged Density of silt in kg/m^3 is 10
Volume of Silt in m^3 is 162.5900
Weight of Silt in kN is 1.6259e+03
Total Uplift Force in kN is 5.3756e+03
Total Horizontal Hydraulic Force upstream in kN is 38400
Total Horizontal Silt Force upstream in kN is 1.4626e+03
Total Horizontal Tail Hydraulic Force downstream in kN is 459.3800
Area of Contact m2 is 96.1500
Total Number of Monte Carlo Sample is 10000000
Total Number of Failures out of Total Samples are 615235
Probability of Failure is 6.15235e-02
Reliability Index (Beta) is 1.542



Title: Hollow Buttress Section - Overturning Probability of Failure at Design Flood Level (DFL)

Volume of Concrete in m^3 is 2.0670e+03
Lever arm, X Concrete in m is 19.2100
Volume of Water in m^3 is 1800
Weight of Water in kN is 17658
Lever arm, X Water in m is 31.5500
Submerged Density of silt in kg/m^3 is 9.0
Volume of Silt in m^3 is 162.5900
Weight of Silt in kN is 1.4633e+03
Lever arm, X silt in m is 31.5500
Total Horizontal Hydrostatic Force upstream in kN is 5.9214e+04
Lever arm, Y Water upstream in m is 10.6700
Total Horizontal Silt Force upstream in kN is 1.4626e+03
Lever arm, Y silt in m is 3.4400
Total Uplift Moment in kNm is 1.6822e+05
Total Horizontal Hydrostatic Force downstream in kN is 59.3800
Lever arm, Y Water downstream in m is 1.1700
Area of Contact m2 is 96.1500
Total Number of Monte Carlo Sample is 10000000
Total Number of Failures out of Total Samples are 0
Probability of Failure is 0
Reliability Index (Beta) > 8.0