Applied Mathematics & Information Sciences
An International Journal

Soft Lattice Implication Subalgebras

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Received: 31 Oct. 2012, Revised: 30 Dec. 2012, Accepted: 14 Jan. 2013
Published online: 1 May 2013

Abstract: In this paper, we introduce the concepts of soft lattice implication subalgebras, endow a parameter set as a lattice implication algebra, and further discuss its equivalent characterization. Then, new operations of soft lattice implication subalgebras are introduced, under which two soft lattice implication subalgebras is also a soft lattice implication subalgebra. Finally, the concepts of image and preimage of a soft lattice implication subalgebra and their properties are presented.

Keywords: soft set, lattice implication algebra, operation, cartesian product, soft image.

1. Introduction

There still exist some complicated problems in engineering, economics, sociology, medical science and many other fields [1], which cannot be successfully dealt with by classical methods. The reason is that various uncertainties are typical for these problems. Thus, in order to solve these problems, many scholars proposed a great deal of theories gradually, such as probability theory, fuzzy set theory [2,3], rough set theory [4,5,6], vague set theory [7] and interval mathematics [8]. Though they are all feasible methods to describe uncertainties, each of these theories has its inherent difficulties, as mentioned by Molodtsov [9]. Therefore, Molodtsov proposed soft set theory, which can model vagueness and uncertainty. With the introduction of soft sets by Molodtsov, plenty of scholars did researches on its properties and application. Maji et al.[10]defined several operations on soft sets and showed the applications of soft set theory in decisions making problem. Chen et al.[11]proposed a definition for soft set parameterization reduction and investigated an application to another decision making problem. Aktas et al. [12] compared soft sets to the related concepts of fuzzy sets and rough sets. Meanwhile, they defined soft groups and obtained some related properties. Sezgin et al.[13]introduced the concepts of normalistic soft group and normalistic soft group homomorphism. Kong et al.[14]further studied the problem of the reduction of soft sets and fuzzy soft sets by introducing a definition of normal parameter reduction.

In 2008 , Yuan Xuehai's graduated student presented the new definitions of soft subgroups and normal soft subgroups and obtained some primary results. These work greatly enriched the algebraic structure of soft set theory. Yi Liu and Yang Xu introduced and studied soft lattice implication algebras[15]. So far, there did not exist so many profound results on the study of soft algebra. Lidong Wang and Xiaodong Liu, Cesim Temel [16,17] presented homomorphisms on some special algebras. We see that variable threshold concept lattice was proposed by Ma et al., which provide a new parameterized way to obtain formal concepts from data with fuzzy attributes [18]. The way that we endow a parameter set as a lattice implication algebra can make more achievements.

The purpose of the present paper is a further attempt to broaden the theoretical aspects of soft lattice implication algebras. In this paper, we endow a parameter set as a lattice implication algebra. On the above basis, new operations of soft lattice implication subalgebras are introduced, under which two soft lattice implication subalgebras is also a soft lattice implication subalgebra.

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2. Preliminaries

In this section, we present the concepts of lattice implication algebras and soft sets. Then we list some preliminary theorems that are needed in this paper.

2.1 The Related Concepts of Lattice Implication Algebra

Definition 2.1[19] (Lattice implication algebra) Let (L, $\vee$, $\wedge$,O,I) be a bounded lattice with an order-reversing involution " $'$ ", I and O the greatest and the smallest element of L respectively, and $\to$:$L \times L \to L$ be a mapping. (L, $\vee$, $\wedge$,$'$, $\to$,O,I) is called a lattice implication algebra if the following conditions hold for any x,y,$z \in L$:

(I_1) $x \to (y \to z) = y \to (x \to z)$;
(I_2) $x \to x = I$;
(I_3) $x \to y = y' \to x'$;
(I_4) $x \to y = y \to x = I$ implies $x = y$;
(I_5) $(x \to y) \to y = (y \to x) \to x$;
(I_6) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$;
(I_7) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$.

Theorem 2.1.1[19]Let L be a lattice implication algebra, then for any x,y,$z \in L$,

(1) $If\, I \to x = I, then\, x = I$;
(2) $I \to x = x \; and \; x \to O = x'$;
(3) $O \to x = I \; and \; x \to I = I$;
(4) $(x \to y) \to ((y \to z) \to (x \to z)) = I$;
(5) $(x \to y) \to x' = (y \to x) \to y'$;
(6) $x \wedge y = ((x \to y) \to x')'$;
(7) $x \wedge y = ((x \to y) \to x')'$.

Definition 2.1.2[19] (Lattice implication subalgebra) Let L be a lattice implication algebra. $S \subseteq L$ is called a lattice implication subalgebra of L , if the following conditions hold :

(1) $(S, \vee, \wedge, ')$ is bounded sublattice of $(L, \vee, \wedge)$ with an order-reversing involution $'$;
(2) If x,$y \in S$, then $x \to y \in S$.

It is clear that lattice implication subalgebra S is a lattice implication algebra.

Theorem 2.1.2[19] Let L be a lattice implication algebra, $S \subseteq L$, if

(1) $O \in S$;
(2) For any x,$y \in S$ implies $x \to y \in S$.

Then S is a lattice implication subalgebra of L .

Definition 2.1.3[19] (Homomorphism) Let L_1 and L_2 be lattice implication algebras, f:$L_1 \to L_2$ a mapping from L_1 to L_2. Then f is called a lattice implication homomorphism from L_1 to L_2 , if the following conditions hold for any $x, y \in L$:

(1) $f(x \to y) = f(x) \to f(y)$;
(2) $f(x \vee y) = f(x) \vee f(y)$;
(3) $f(x \wedge y) = f(x) \wedge f(y)$;
(4) $f(x') = (f(x))'$.

2.2 The Related Concepts of Soft Set

Definition 2.2.1(Cartesian product) Let L_1,L_2 be non-empty sets. $L_1 \times L_2 = \{(x,y)|x \in L_1, y \in L_2\}$ is cartesian product of L_1,L_2.

Definition 2.2.2[20] (Soft set) Let U be an initial universe set and E a set of parameter. Let $P(U)$ denote the power set of U and $A \subset E$. A pair (F,A) is called a soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

Definition 2.2.3[21] Let (H,E) be a soft set over X , a pair (A_H,X) is called the duality of (H,E) , where A_H is a mapping given by:

$A_H : \quad X \to P(E), x \mapsto A_H(x) = \{\varepsilon \in E | x \in H(\varepsilon)\}$
for all $x \in X$.

Definition 2.2.4[20] Intersection of two soft sets $(F$,$A)$ and $(G$,$B)$ over a common universe U is the soft set $(H$,$C)$, where C=$A \cap B$, and for any $x \in C$,$H(x)$=$F(x) \cap G(x)$.We write $(F$,$A) \cap (G$,$B)$= $(H$,$C)$.

Definition 2.2.5[20] (AND operation on two soft sets) If $(F$,$A)$ and $(G$,$B)$ are two soft sets, then " $(F$,$A)$ AND$(G$, $B)$ " denoted by $(F$,$A) \wedge (G$,$B)$ is defined as $(F$,$A) \wedge (G$, $B)$= $(H$,$A \times B)$, where $H(\alpha$,$\beta)$=$F(\alpha) \cap G(\beta)$, for any $(\alpha$,$\beta) \in A \times B$.

3. Soft Lattice Implication Subalgebra

In this section, we introduce the concepts of soft lattice implication subalgebras. We endow a parameter set as a lattice implication algebra, and discuss its equivalent characterization.

Definition 3.1 Let L be a lattice implication algebra, U an initial universe set, and H a soft set. If the following conditions hold for any $x, y \in L$:

(1) $H(O) \supseteq H(x)$;
(2) $H(x \vee y) \supseteq H(x) \cap H(y)$;
(3) $H(x \wedge y) \supseteq H(x) \cap H(y)$;
(4) $H(x') \supseteq H(x)$;
(5) $H(x \to y) \supseteq H(x) \cap H(y)$.

Then soft set H is called a soft lattice implication subalgebra of L.

Theorem 3.1 H is a soft lattice implication subalgebra of L if and only if the following conditions hold for any $x, y \in L$:

(1) $H(O) \supseteq H(x)$;
(2) $H(x \to y) \supseteq H(x) \cap H(y)$.

Proof. Suppose that H is a soft lattice implication subalgebra of L , by Definition3.1, $H(O) \supseteq H(x)$, $H(x \to y) \supseteq H(x) \cap H(y)$, holds for any $x, y \in L$. Conversely, assume (1) and (2) holds, it follows that

(i) $H(x \vee y) = H((x \to y) \to y) \supseteq H(x \to y) \cap H(y) \supseteq (H(x) \cap H(y)) \cap H(y) = H(x) \cap H(y)$;

(ii) $H(x') = H(x \to O) \supseteq H(x) \cap H(O) = H(x)$;

(iii) $H(x \wedge y) = H((x' \vee y')') \supseteq H(x' \vee y') \supseteq H(x') \cap H(y') \supseteq H(x) \cap H(y)$.

Therefore, H is a soft lattice implication subalgebra of L.

Theorem 3.2 Let L be a lattice implication algebra and H a soft lattice implication subalgebra of L , where H is a mapping given by $H : L \to P(U)$. Then the following statements hold:

(1) H is a soft lattice implication subalgebra of L if and only if $A_H(u)$ is a lattice implication subalgebra of L, where the condition $A_H(u) \triangleq \{x | u \in H(x)\}$ holds for any $u \in U$;

(2) If $A : U \to P(L)$ is a soft set , then $A(u)$ is a lattice implication subalgebra of L for any $u \in U$ if and only if H_A is a soft lattice implication subalgebra of L, where the condition $H_A(x) \triangleq \{u | x \in A(u)\}$ holds.

Proof.(1) "⇒" For any $x, y \in A_H(u)$, then $u \in H(x)$ and $u \in H(y)$, it follows that $u \in H(x) \cap H(y) \subseteq H(x \to y)$, thus $(x \to y) \in A_H(u)$. And $u \in H(x) \subseteq H(O)$, so $O \in A_H(u)$. Then $A_H(u)$ is a lattice implication subalgebra of L by Theorem 2.1.2.

"⇐" For any $u \in H(x)$, since $A_H(u)$ is a lattice implication subalgebra, we have $O \in A(u)$. Then $u \in H(O)$. Thus $H(O) \supseteq H(x)$. And for any $u \in H(x) \cap H(y)$, obviously, $u \in H(x)$ and $u \in H(y)$. Then $x \in A_H(u)$ and $y \in A_H(u)$. Since $A_H(u)$ is a lattice implication subalgebra of L, we have $(x \to y) \in A_H(u)$. Then $u \in H(x \to y)$. That is to say, $H(x \to y) \supseteq H(x) \cap H(y)$, H is a soft lattice implication subalgebra of L by Theorem 3.1.

(2) "⇒" (i) For any $x, y \in L$, $u \in H_A(x) \cap H_A(y)$, then $u \in H_A(x)$, $u \in H_A(y)$. Hence $x \in A(u)$, $y \in A(u)$. Since $A(u)$ is a soft lattice implication subalgebra of L , and $(x \to y) \in A(u)$, we have $u \in H_A(x \to y)$. Hence $H(x \to y) \supseteq H(x) \cap H(y)$.

(ii) For any $u \in H_A(x)$, since $A(u)$ is a lattice implication subalgebra of L , there exists $O \in A(u)$. Then $u \in H_A(O)$, hence $H_A(O) \supseteq H_A(x)$.

Consequently, we have H_A is a soft lattice implication subalgebra of L.

"⇐" For any $x \in A(u)$, there exists $u \in H_A(x) \subseteq H_A(O)$. Then $O \in A(u)$. But also for any $x, y \in A(u)$, hence $u \in H_A(x)$ and $u \in H_A(y)$. Thus $u \in H_A(x) \cap H_A(y) \subseteq H_A(x \to y)$. It follows that $(x \to y) \in A(u)$ holds. Thus $A(u)$ is a soft lattice implication subalgebra of L by Theorem 3.1.

Theorem 3.3 Let L_1, L_2 be lattice implication subalgebras of L , then $L_1 \cap L_2$ is also lattice implication subalgebra of L .

Proof. Because L_1, L_2 are lattice implication subalgebras of L , then $O \in L_1$ and $O \in L_2$, we have $O \in L_1 \cap L_2$. For any $x, y \in L_1 \cap L_2$, thus $x, y \in L_i (i = 1, 2)$. Since $L_i (i = 1, 2)$ are lattice implication subalgebras of L, hence $(x \to y) \in L_i (i = 1, 2)$ and so $(x \to y) \in L_1 \cap L_2$. By Theorem 2.1.2, $L_1 \cap L_2$ is a lattice implication subalgebra of L .

Theorem 3.4 Let L_1, L_2 be lattice implication subalgebras of L. Suppose H_1, H_2 be soft lattice implication subalgebras of L_1, L_2 respectively, and $(H, L_1 \cap L_2) = (H_1, L_1) \cap (H_2, L_2)$, then H is a soft lattice implication subalgebra of $L_1 \cap L_2$.

Proof. By Theorem 3.3, we have $L_1 \cap L_2$ is a lattice implication subalgebra of L . For any $x, y \in L_1 \cap L_2$,we have $H(x) = H_1(x) \cap H_2(x) \subseteq H_1(O) \cap H_2(O) = H(O)$, and $H(x \to y) = H_1(x \to y) \cap H_2(x \to y) \supseteq (H_1(x) \cap H_1(y)) \cap (H_2(x) \cap H_2(y)) = (H_1(x) \cap H_2(x)) \cap (H_1(y) \cap H_2(y)) = H(x) \cap H(y)$, thus H is a soft lattice implication subalgebra of $L_1 \cap L_2$.

Definition 3.2 Let L_1, L_2 be lattice implication algebras, $L = L_1 \times L_2$, for any $x, y \in L$, denote

$$x \to y \triangleq (x_1 \to y_1, x_2 \to y_2) ,$$

$$x \vee y \triangleq (x_1 \vee y_1, x_2 \vee y_2),$$

$$x \wedge y \triangleq (x_1 \wedge y_1, x_2 \wedge y_2) ,$$

$$x' \triangleq (x_1', x_2') ,$$

where $x = (x_1, x_2)$, $y = (y_1, y_2)$, $x_i, y_i \in L_i (i = 1, 2)$.

Definition 3.3 (Lattice implication algebra operation of cartesian product) Let L_1, L_2 be two lattice implication algebras. Define operations on $L_1 \times L_2$ by for any $(x_1, x_2), (y_1, y_2) \in L_1 \times L_2$

(1) $(x_1, x_2) \vee (y_1, y_2) = (x_1 \vee y_1, x_2 \vee y_2)$;

(2) $(x_1, x_2) \wedge (y_1, y_2) = (x_1 \wedge y_1, x_2 \wedge y_2)$;

(3) $(x_1, x_2)' = (x_1', x_2')$;

(4) $(x_1, x_2) \to (y_1, y_2) = (x_1 \to y_1, x_2 \to y_2)$.

Theorem 3.5 Let L_1, L_2 be two lattice implication algebras. Then $L_1 \times L_2$ under the above operations constructs is a lattice implication algebra.

Proof. For any $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in L_1 \times L_2$,then

(I_1) $x \to (y \to z)$
$= (x_1,x_2) \to (y_1 \to z_1, y_2 \to z_2)$
$= (x_1 \to (y_1 \to z_1), x_2 \to (y_2 \to z_2))$
$= (y_1 \to (x_1 \to z_1), y_2 \to (x_2 \to z_2))$
$= (y_1,y_2) \to (x_1 \to z_1, x_2 \to z_2)$
$= y \to (x \to z)$.
(I_2) $x \to x$
$= (x_1,x_2) \to (x_1,x_2)$
$= (x_1 \to x_1, x_2 \to x_2)$
$= (I_1,I_2) = I$.
(I_3) $x \to y$
$= (x_1 \to y_1, x_2 \to y_2)$
$= (y_1{}' \to x_1{}', y_2{}' \to x_2{}')$
$= (y_1{}', y_2{}') \to (x_1{}', x_2{}')$
$= y' \to x'$.
(I_4) If $(x_1 \to y_1, x_2 \to y_2) = x \to y = I = y \to x = (y_1 \to x_1, y_2 \to x_2)$,
then $x_1 \to y_1 = y_1 \to x_1 = I_1$, and $x_2 \to y_2 = y_2 \to x_2 = I_2$.
Hence $x_1 = y_1$, $x_2 = y_2$.
Thus $x= (x_1,x_2) = (y_1,y_2) =y$.
(I_5) $(x \to y) \to y$
$=((x_1 \to y_1) \to y_1, (x_2 \to y_2) \to y_2)$
$=((y_1 \to x_1) \to x_1, (y_2 \to x_2) \to x_2)$
$=(y \to x) \to x$.
(I_6) $(x \vee y) \to z$
$= (x_1 \vee y_1, x_2 \vee y_2) \to (z_1,z_2)$
$= ((x_1 \vee y_1) \to z_1, (x_2 \vee y_2) \to z_2)$
$= ((x_1 \to z_1) \wedge (y_1 \to z_1), (x_2 \to z_2) \wedge (y_2 \to z_2))$
$= (x_1 \to z_1, x_2 \to z_2) \wedge (y_1 \to z_1, y_2 \to z_2)$
$= (x \to z) \wedge (y \to z)$.
(I_7) $(x \wedge y) \to z$
$= (x_1 \wedge y_1, x_2 \wedge y_2) \to (z_1,z_2)$
$= ((x_1 \wedge y_1) \to z_1, (x_2 \wedge y_2) \to z_2)$
$= ((x_1 \to z_1) \vee (y_1 \to z_1), (x_2 \to z_2) \vee (y_2 \to z_2))$.
By Definition 2.1.1, we have $L_1 \times L_2$ is a lattice implication algebra.

Theorem 3.6 Let L_1,L_2 be lattice implication algebras, H_1,H_2 be soft lattice implication subalgebras of L_1, L_2 respectively, and $L = L_1 \times L_2$. Then $(H,L) = (H_1,L_1)$
$\wedge (H_2,L_2)$ is a soft lattice implication subalgebra of L.

Proof. Since L_1,L_2 are lattice implication algebras, by Theorem 3.5, we know that $L = L_1 \times L_2$ is a lattice implication algebra.

(i) For any $x \in L$, $x = (x_1,x_2)$, $x_i \in L_i (i = 1,2)$, we have
$H(x) = H(x_1,x_2) = H_1(x) \cap H_2(x) \subseteq H_1(O) \cap H_2(O)$
$= H(O)$.

(ii) For any $x,y \in L$, $x = (x_1,x_2)$, $y = (y_1,y_2)$, $x_i,y_i \in L_i (i = 1,2)$, we have

$$\begin{aligned}H(x \to y) &= H(x_1 \to y_1, x_2 \to y_2)\\ &= H_1(x_1 \to y_1) \cap H_2(x_2 \to y_2) \supseteq\\ &\quad (H_1(x_1) \cap H_1(y_1)) \cap (H_2(x_2) \cap H_2(y_2))\\ &= (H_1(x_1) \cap H_2(x_2)) \cap (H_1(y_1) \cap H_2(y_2))\\ &= H(x_1,x_2) \cap H(y_1,y_2) = H(x) \cap H(y).\end{aligned}$$

Hence H is a soft lattice implication subalgebra of L.

4. Image and Preimage of a Soft Lattice Implication Subalgebra

In this section, we introduce the image and preimage of a soft lattice implication subalgebra. Then their properties are discussed.

Definition 4.1 Let L_1,L_2 be lattice implication algebras, U an initial universe set, $f : L_1 \to L_2$ a mapping, and $H_1 : L_1 \to P(U)$,$H_2 : L_2 \to P(U)$ soft sets. Define

$$f(H_1)(x_2) = \begin{cases} \bigcup\limits_{f(x_1)=x_2} H_1(x_1) & f^{-1}(x_2) \neq \phi \\ \phi & f^{-1}(x_2) = \phi \end{cases}$$

and $f^{-1}(H_2)(x_1) = H_2(f(x_1))$.

Then $f(H_1)$, $f^{-1}(H_2)$ are soft sets of L_1 and L_2 respectively. $f(H_1)$ is called the image of H_1 under f and $f^{-1}(H_2)$ is called the preimage (or inverse image) of H_2 under f.

Theorem 4.1 Let L_1,L_2 be lattice implication algebras, U an initial universe set, $f : L_1 \to L_2$ a lattice implication homomorphism from L_1 to L_2, and $H_1 : L_1 \to P(U)$ is a soft set, then

(1) If H_1 is a soft lattice implication subalgebra of L_1, then $f(H_1)$ is a soft lattice implication subalgebra of L_2.

(2) If H_2 is a soft lattice implication subalgebra of L_2, then $f^{-1}(H_2)$ is a soft lattice implication subalgebra of L_1.

Proof. (1) (i) For any $x_2 \in L_2$, , $u \in f(H_1)(x_2)$, then

$u \in \bigcup\limits_{f(x_1)=x_2} H_1(x_1) \subseteq H_1(O) \subseteq \bigcup\limits_{f(x)=O} H_1(x) = f(H_1)(O)$.

Hence $f(H_1)(O) \supseteq f(H_1)(x_2)$.

(ii) For any $x_2,y_2 \in L_2$, $u \in f(H_1)(x_2) \cap f(H_1)(y_2)$,

we have $u \in f(H_1)(x_2) \bigcup\limits_{f(x_1)=x_2} H_1(x_1)$ and $u \in f(H_1)(y_2) = \bigcup\limits_{f(y_1)=y_2} H_1(y_1)$. Hence exist $x_1 \in L_1$, $y_1 \in L_1$, thus $x_2 = f(x_1), u \in H_1(x_1)$, $y_2 = f(y_1)$, $u \in H_1(y_1)$. Since H_1 is a soft lattice implication subalgebra of L_1, we have $u \in H_1(x_1) \cap H_1(y_1) \subseteq H_1(x_1 \to y_1)$, and f is a lattice implication homomorphism, it follows that $f(x_1 \to y_1) = f(x_1) \to f(y_1) = x_2 \to y_2$,
$f(H_1)(x_2 \to y_2) = \bigcup\limits_{f(x)=x_2 \to y_2} H_1(x) \supseteq H_1(x_1 \to y_1)$.
That is $u \in f(H_1)(x_2 \to y_2)$, therefore $f(H_1)(x_2 \to y_2) \supseteq f(H_1)(x_2) \cap f(H_1)(y_2)$.

Consequently, we have $f(H_1)$ is a soft lattice implication subalgebra of L_2 by Theorem 3.1.

(2) (i) For any $x_1 \in L_1$, we have $f^{-1}(H_2)(x_1) = H_2(f(x_1)) \subseteq H_2(O) = H_2(f(O)) = f^{-1}(H_2)(O)$.

(ii) For any $x_1, y_1 \in L_1$, we have $f^{-1}(H_2)(x_1) \cap f^{-1}(H_2)(y_1) = H_2(f(x_1)) \cap H_2(f(y_1)) \subseteq H_2(f(x_1) \to f(y_1)) = H_2(f(x_1 \to y_1)) = f^{-1}(H_2)(x_1 \to y_1)$. By Theorem 3.1, thus $f^{-1}(H_2)$ is a lattice implication subalgebra of L_1.

5. Conclusion

First, we introduced the concepts of soft lattice implication subalgebras, endowed a parameter set as a lattice implication algebra, and further discussed its equivalent characterization. Then, new operations of soft lattice implication subalgebras were introduced, under which two soft lattice implication subalgebras was also a soft lattice implication subalgebra. Finally, the concepts of image and preimage of a soft lattice implication subalgebra and their properties were presented. To extend this work, one could study on it to get further profound results.

Acknowledgement

This work is supported by National Undergraduate Training Programs for Innovation and Entrepreneurship of China (No.2050205) ; National Natural Science Foundation of China (No.61170121) .

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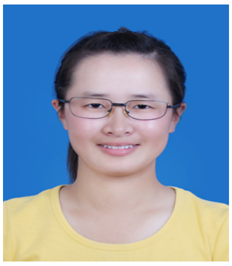

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