Appl. Math. Inf. Sci. 6 No. 1S pp. 25S-29S (2012)

Applied Mathematics & Information Sciences
An International Journal

Automatic and Precise Image Registration Based on Phase-Correlation Combined with Surface Fitting

Wang Weihong[1], Xia Liegang[1,2], Luo Jiancheng[2] and Hu Xiaodong[2]
[1]College of Computer Science and Technology, Zhejiang University of Technology, China
[2]Institute of Remote Sensing Application, Chinese Academy of Science, Beijing, China
Email Address: wwh@zjut.edu.cn

Received: Received Dec. 22, 2010; Revised Feb. 21, 2011; Accepted Apr. 23, 2011
Published online: 1 January 2012

Abstract: The original method for image registration based on phase-correlation is only valid, if the shift vector is of integer values. So the error could be up to half of a pixel in theory. Since the estimation of the parameters on rotation and scaling is based on the estimation of these shift values, their accuracy is also restricted. In response to this limitation, we improved the method of search the maximum point of the impulse function. Using surface fitting methods like Parabolic Interpolation or Gaussian Function Fitting Method we could get the location of extreme point in sub-pixel level. So the error would be significantly reduced. The result of some registration experiments showed that the algorithm was valid for dealing with shifted, rotated and scaled images. Through the analysis we can see the improved method greatly increased the accuracy of registration without increasing the computational complexity.

Keywords: Automatic Image Registration; Phase Correlation; Accuracy of Registration; Surface Fitting

1 Introduction

Image registration can be formally defined as the transformation of one image with respect to another so that the properties of any resolution element of the object being imaged is addressable by the same coordinate pair in either one of the images. By setting up the geometry parallelism among images to transform, compare and analyze images according to the same reference frame image [1]. In the process of multiple data-source images analysis, such as remote sensing, image fusion, computer graphics, computer vision, environment supervision [2], map drawing and medical imaging, the registration is one of the most important steps.

For the problem of automatic image registration (globally shift, rotation and scaling) based on frequent domain framework, many researchers have proposed two kinds of methods [3, 4] relies on the Fourier shift theorem and cross-correlation of separation variables in log-polar coordinate. The algorithms have been tested on the registration of remotely sensed images by IDL ENVI implementation [5]. Despite insensitive to linear change of image brightness and having a certain degree of anti-noise performance, the methods can only estimate shift values in pixel level rather than in sub-pixel level [6-8], thus to generate errors which will be cumulated when estimating rotation and scaling parameters after log-polar coordinate conversion. Consequently, we should estimate the more accurate values of shift to improve the accuracy of all the estimated parameters.

In this paper, the errors of original algorithm, generating in the method of calculating the most extreme point position in impulse function directly, are put forward and given the in-depth analysis. And the improvements are proposed accordingly, that is to combine with the method of surface fitting to calculate a local extreme point position (sub-pixel shift value). It reduced the error of original algorithm to register the image with non-integer pixels' shift and improved the estimated accuracy of rotation and scaling parameters. The result of experiment proved our assumption.

2 Improvement of Phase-Correlation Algorithm

Phase-Correlation based Registration Algorithm. It is known that if two images differ only by a shift, then they should satisfy the formula that

$$f_2(x,y)=f_1(x-x_0,y-y_0), \tag{2.1}$$

where x_0 and y_0 are shift values. To get these values, we make the Fourier inverse transform for cross-power spectrum, that is

$$F^{-1}\left(\frac{F_1F_2^*}{\left|F_1F_2^*\right|}\right)=F^{-1}(e^{-j2\pi(ux_0+vy_0)}), \tag{2.2}$$

where F_2^* is complex conjugate of $F_2(u,v)$ and the right-hand side is a 2-D impulse function. According to the nature of impulse function we can get the values of x_0 and y_0, which are shift parameters, through seeking the peak point (x_0, y_0) of $\delta(x-x_0, y-y_0)$.

With coordinate conversion, the rotation and scaling of original image can be converted into shift parameters in log-polar coordinates. Suppose $f_1(x,y)$ and $f_2(x,y)$ only differ by rotation and scaling, which means that

$$f_{2\rho l}(r,\theta)=f_{1\rho l}(r-r_0,\theta-\theta_0), \tag{2.3}$$

where r is the coordinate value of logarithmic axis and θ is coordinate value of angle axis so as to get the shift parameters of r_0 and θ_0 in log-polar coordinate system. With the (2.3), we can get the value of r_0 and θ_0, and obtain r and θ on the reverse as well. The equation proposed by Chen can be adopted which is

$$\begin{cases} x(m,k)=\dfrac{N/2-1}{M-1}(M-1)^{m/(M-1)}\cos(\pi k/K)+N/2 \\ y(m,k)=\dfrac{N/2-1}{M-1}(M-1)^{m/(M-1)}\sin(\pi k/K)+N/2 \end{cases}, \tag{2.4}$$

where m is coordinate value of logarithmic axis and k is coordinate value of angle axis in log-polar coordinate system, while x and y are coordinate value in Cartesian coordinate before conversion. In log-polar coordinate system, having obtained the offset (m,k), σ and θ_0 can be acquired by the equations below:

$$\begin{cases} \sigma=(M-1)^{m/(M-1)}\cdots\cdots\cdots 0\le m<M/2 \\ \sigma^{-1}=(M-1)^{(M-m)/(M-1)}\cdots M/2<m<M \end{cases}$$
$$\text{and}\begin{cases} \theta_0=\pi k/K\cdots\cdots\cdots\cdots 0\le k<K/2 \\ \theta_0=-\pi(K-k)/K\cdots K/2<k<K \end{cases} \tag{2.5}$$

Indubitably, the error will be cumulated due to the facts that parameters m and k generate the estimates of σ and θ_0. The step size of rotation angle in (2.4) is π/K, which cause the maximum error is $\pi/2K$ while the error should be double when $\theta\in[0,2\pi)$. And for scaling parameter, as $b=(M-1)^{1/(M-1)}$ in (2.4) and get the result that $\sigma=b^m$. However, the error positively depends on m-error, maximum up to $b^m(\sqrt{b}-1)$.

Algorithm Improvement. From the above analysis, we can find that in order to improve the accuracy of rotation and scaling parameters, a simple method is to increase m and k in (2.4) and reduce the step size (the quantization error of shift) as well. However, the biggest problem brought about is the multiple cost of computation and moreover the high requirement of the interpolation accuracy during the coordinate conversion. And the most important is that the method is ineffective when analyze the source of errors. If the error comes from the noise and other external factors, such improvements in accuracy will also be in vain.

In order to improve the accuracy of shift in sub-pixel level, some scholars have put forward algorithms combined with genetic algorithm or normalized cross correlation algorithm to implement the registration by two phases. The original phase-correlation algorithm used as the coarse registration and then the other support methods combined as the fine registration. With the secondary registration algorithm, it can improve the accuracy (generally, only effective for the accuracy of shift) but increase the computational cost inevitably.

Through digging the information of impulse function $\delta(x,y)$, we use two methods to fit local surface near the extreme point location to get the value of sub-pixel displacement $(\Delta x, \Delta y)$ (suppose the peak point (x_0, y_0) have been found by the original method).

Parabolic Interpolation. Take peak point (x_0, y_0) and the field points around as the points satisfying the below-parabolic equation and put them into the following equations.

$$\delta(x,y)=d_2(x-x_0)^2+e_2(y-y_0)^2 \\ +d_1(x-x_0)+e_1(y-y_0)+f \tag{2.6}$$

By the nature of extreme point, we can get the results:

$$\begin{cases} d_1=(\delta(x_0+1,y_0)-\delta(x_0-1,y_0))/2, d_2 \\ \quad =(\delta(x_0-1,y_0)+\delta(x_0+1,y_0)-2\delta(x_0,y_0))/2 \\ e_1=(\delta(x_0,y_0+1)-\delta(x_0,y_0-1))/2, e_2 \\ \quad =(\delta(x_0,y_0-1)+\delta(x_0,y_0+1)-2\delta(x_0,y_0))/2 \\ \Delta x=-d_1/2d_2, \Delta y=-e_1/e_2 \end{cases} . \tag{2.7}$$

1-D Fitting Gaussian Function. According to the symmetry of the impulse function on both sides of the extreme point, we use 1-D fitting Gaussian Function in x direction and y direction respectively to get the point's location. Taking x direction as example, three points (x_0-1,y_0), (x_0,y_0), (x_0+1,y_0) are substituted into following function:

$$\delta(x,y_0)=e^{-\pi(x-x_0-\Delta x)^2/\tau^2}/|\tau|. \tag{2.8}$$

Then we got

$$\begin{cases} \Delta x=\dfrac{\ln\delta(x_0,y_0)-\ln\delta(x_0-1,y_0)}{2\ln\delta(x_0,y_0)-\ln\delta(x_0-1,y_0)-\ln\delta(x_0+1,y_0)}-1/2 \\ \Delta y=\dfrac{\ln\delta(x_0,y_0)-\ln\delta(x_0,y_0-1)}{2\ln\delta(x_0,y_0)-\ln\delta(x_0,y_0-1)-\ln\delta(x_0,y_0+1)}-1/2 \end{cases} . \tag{2.9}$$

Implementation for Improved Algorithm. The final algorithm flow for determining rotation, scaling, and shift is:

1) For reference image I_1 and input image I_2, convert polar coordinates in the logarithmic coordinate system to get $I_{1\rho l}$ and $I_{2\rho l}$.

2) Make Fourier Transform for $I_{1\rho l}$ and $I_{2\rho l}$ respectively, seek cross-power spectrum and make a Fourier inverse transform according to (2.2).

3) Find peak point location (m,k) and get the sub-pixel level displacement to amend the peak point, getting result of $(x_0+\Delta x, y_0+\Delta y)$. Then we can get the values of σ and θ_0 by (2.5).

4) Make inverse transform of rotation and scaling for I_2 and obtain the value (x_0,y_0) by I_1 to amend $(x_0+\Delta x, y_0+\Delta y)$.

3 Experiment and Analysis

Through our remotely sensed image processing platform, we implement the traditional image registration algorithm and the two above-mentioned improved methods. In order to test and verify their efficiency and accuracy adequately, we designed the experiment.

First we conduct a TM image to a certain degree of rotation and scaling （$\sigma=1.39,\theta_0=55°$） and cut an image with size of 512*512 from original image. Then we got the reference image and image to be registered for experiment in Fig 3.1.

We registered the images mentioned above by Gaussian algorithm and the original algorithm respectively and the estimated parameters selected in the process are shown in Table 3.1. In order to better compare the effect of two algorithms, we select Link function of ENVI to compare the reference image and the images after registration, the result shown in Fig 3.2. On the whole, we see that phase-correlation algorithm has reached the requirements of registration if ignore the errors of interpolation during the process of image scaling. Comparing all the image results, the registration image by Gaussian algorithm is closer to the reference image, so it is obviously that the improved algorithm had reduced the registration errors from original algorithm efficiently.

Table 3.1 estimated parameters (rotation and scaling) comparison after registration by two kinds of algorithm

	m	k	Scale	angle
Actual Value	27.188	156.4444	1.3927	55
Original algorithm	28	156	1.4066	54.8438
Gaussian function method	27.5288	156.4578	1.3985	55.0047

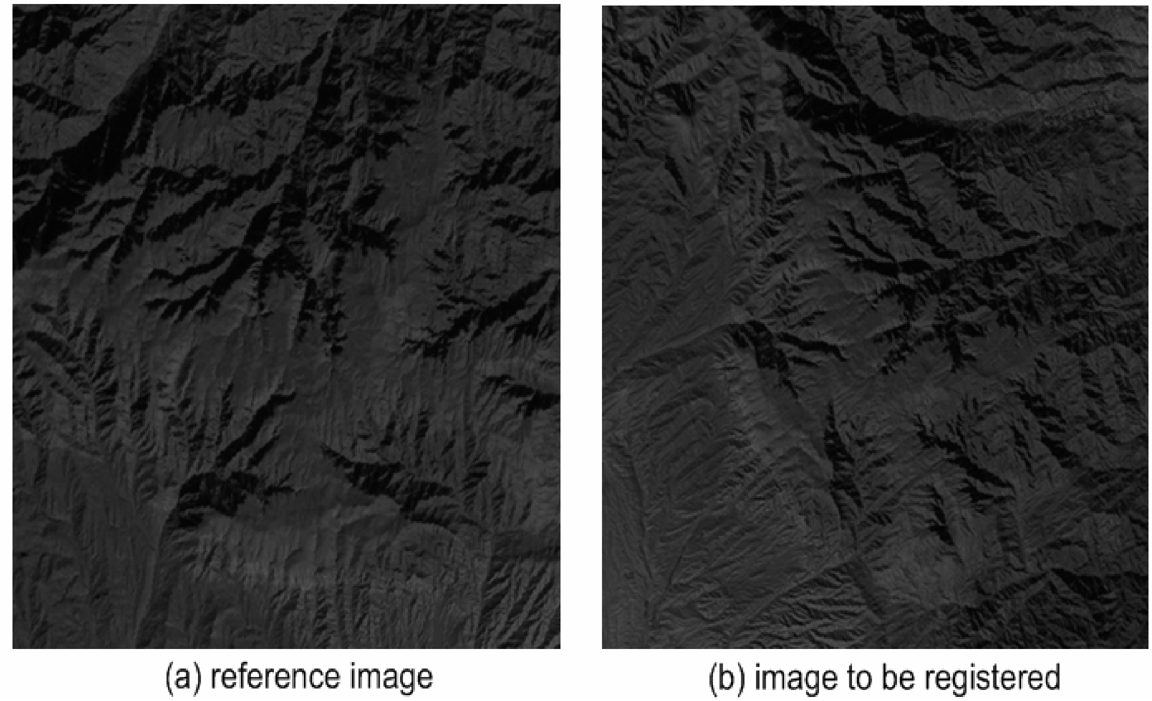

Fig3.1 image results after registration by two kinds of algorithm

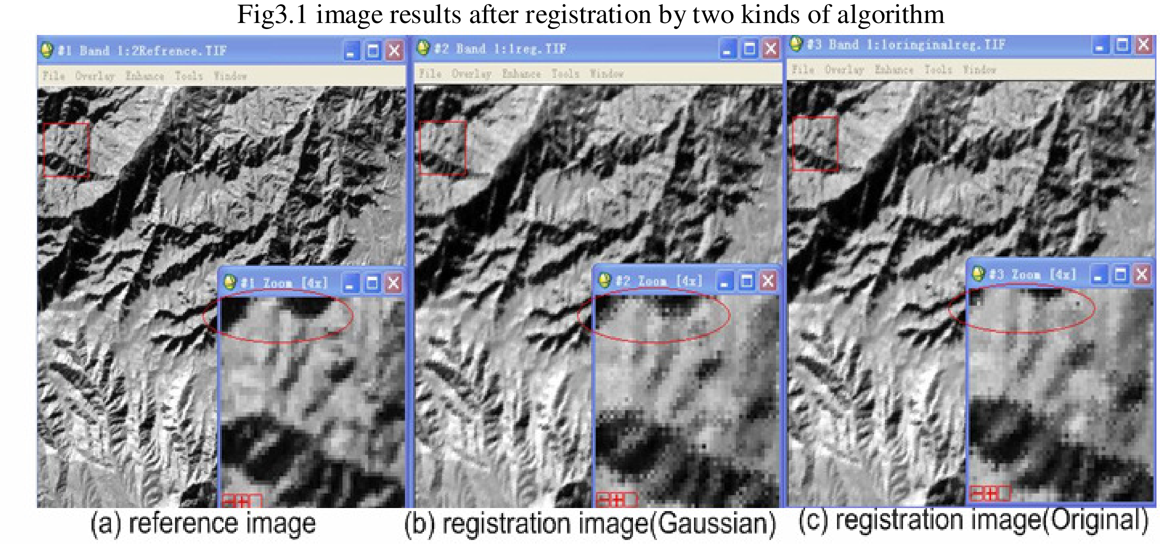

Fig3.2 comparison between image results after registration by two kinds of algorithm and reference image (show with Link in ENVI, the area of bottom right corner of the window is 4 times lager than the area of related upper left corner)

4. Conclusion

The paper analyzed the accuracy of phase-correlation based image registration algorithm, and on this basis we proposed an improved method for image registration. For getting the most accuracy point location in sub-pixel level, we designed two algorithms combined with the surface fitting technology. By this way we improved the performance of registration on images with shift, which means the performance of scaling and rotation would be improved correspondingly. From the experimental results we found both of our algorithms can reduce the estimated errors of original algorithm. Specially, one-dimensional Gaussian function was better in the universal cases. Compared with the other similar algorithms, method in this paper improved the accuracy without adding computational complexity and also suitable for all the registration parameters.

However, there are still many problems need to be considered in the future research, such as multi-peak point problem in impulse function will limit the applicability of phase-correlation method, which means maybe we need to find an efficient pretreatment for some special images; Another problem is how to seek a more appropriate method of surface fitting which is so important to our method.

Acknowledgements
This research was supported by grant 40971228, 40871203 and 60873033 from the Natural Science Foundation of China and grant R1090569 from Zhejiang Province Natural Science Foundation for Distinguished Young Research Team. The authors thank the reviewers for their comments.

References

[1] Barbara Zitova, Jan Flusser. Image registration methods: a survey. *Image and Vision Computing.* Vol.22, No.11, (2003), 977-1000.
[2] Shah, C.A., Y.W. Sheng and L.C. Smith, Automated Image Registration Based on Pseudo invariant Metrics of Dynamic Land-Surface Features. *IEEE Transactions on Geoscience and Remote Sensint.* Vol.46, No.11,(2008). 3908-3916.
[3] Chen Q, Dfrise M, Deconinck F. Symmetric phase only matched filtering of Fourier-mellin transform for image registration and recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence.* Vol.16, No.12, (1994), 1156-1168.
[4] B. Srinivasa Reddy and B.N. Chatterji. An FFT Based Technique for Translation, Rotation, and Scale-Invariant Image Registration. *IEEE Transactions on Image Processing.* Vol.5, No.8,(1996), 1266-1271.
[5] Hongjie Xie, Nigel Hicks, et.al. An IDL/ENVI implementation of the FFT-based algorithm for automatic image registration. *Computers & Geosciences* 29(2003), 1045-1055.
[6] Yang, J., T.K. Sarkar and P. Antonik, Applying the Fourier-modified mellin transform (FMMT) to Doppler-distorted waveforms. *Digital Signal Proceeding.* Vol.17, No.6(2007),1030-1039.
[7] Balci M, Foroosh H. Subpixel estimation of shifts directly in the Fourier domain[J]. *IEEE Transactions on Image Processing.* Vol.15, No.7(2006),1965-1972.
[8] Li L, Zeng Q S, Meng F F. Translation, rotation, and scale invariant image registration technique using angular and radial difference functions. *Chinese Optics Letters.* Vol.6, No.11(2008), 827-829.

Wang Weihong received the MS degree in computer science from Zhejiang University, He is currently a Professor with the Department of Computer Science and Technology at Zhejiang University of Technology, China. He has published over 30 papers in international peer reviewed journals. His research interests are in the areas of image processing, computational intelligence and knowledge discovery.

Luo Jiancheng received the PhD degree from Chinese Academy of Sciences, major in remote sensing. He is currently a Professor in Institute of Remote Sensing Applications, CAS. He has published over 100 papers and three monographs in cooperation. His research interests are in the areas of information extraction from remotely sensed imagery and geo-computation. Email: luojc@irsa.ac.cn.

Xia Liegang received the MS degree in computer S&T from Zhejiang University of Technology, He is currently an Student in Institute of Remote Sensing Applications, CAS. His research interests are in the areas of remotely sensed imagery processing, image understanding, computational intelligence and knowledge discovery.

Hu Xiaodong received the PhD degree from Chinese Academy of Sciences. His research interests are in the areas of image processing and computational theory. Email: mydonne@163.com