Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/130319

Unleashing the Power of Goal Programming: A Comprehensive Literature Review on Optimal Financial Portfolio Selection

Mohammed Hassanien [1,], Hussein Sayed [2] and Ramadan Hamed [2]*

[1] Macro Economic Policies Center, Institute of National Planning (INP), Cairo, Egypt
[2] Department of Statistics, Faculty of Economics and Political Science, Cairo University, Giza, Egypt

Received: 17 Jul. 2023, Revised: 11 Nov. 2023, Accepted: 14 Nov. 2023.
Published online: 1 May 2024

Abstract: This paper aims to provide a comprehensive presentation and evaluation of various goal programming techniques applied to the optimization of financial portfolio selection. There has been a remarkable surge of interest in investment strategies for stock market analysis. Active and passive portfolio management strategies have emerged as the primary approaches for effectively managing investment portfolios. The fundamental objective of every investor and portfolio manager is to construct a portfolio that maximizes returns while minimizing the potential risks associated with market conditions. To tackle the complexities of the portfolio selection process, goal programming has gained widespread recognition as a practical tool for addressing multiple, conflicting, and incomparable objectives simultaneously. Additionally, we conduct a bibliometric analysis to shed light on the research and publications related to the application of goal programming models in the context of optimal financial portfolio selection.

Keywords: Fund Management Strategies; Goal Programming, Index Tracking Portfolio; Optimal Portfolio; Tracking Error.

1. Introduction

The importance of stock markets for a country's economy cannot be overstated. The liquidity in stock markets helps economic growth and the enhancement of production processes and provides a general assessment of the strength and stability of the economy. There are several financial institutions in a country that help to manage its financial performance. These financial institutions include banks, stock markets, and other mortgages firms. A stock market is simply defined as a place where the shares of publicly held corporation are traded (Almomen, 2016) [1].

Financial investments, especially in stock exchange markets, require investors or hired fund managers to control a large amount of money to be invested in large number of assets. Fund managers usually provide investment management services including financial statement analysis, assets/stocks selection, and directing management and investments with all associated monitoring and evaluation processes. The main aim of fund managers is to achieve and maintain high capital growth and income over both short and long terms (Edirisinghe, 2013) [2].

One of the ways to manage and mitigate investment risk is to form a stock portfolio and diversify all types of assets. Assets can be categorized into many classes such as equity securities, debt securities, real estate, merchandise, etc. Different stocks have different levels of risk, and the presence of risk complicates the selection of stocks. Therefore, one of the most critical concerns for investors in the financial markets is how to choose a portfolio that is optimal in terms of both profitability and risk. Investors' preferences are reflected in the allocation process of invested money among different stocks that compose the portfolio (Bogdan et al., 2010) [3].

Investors are faced with numerous risks, including tax risks, market risks, credit risks, political risks, liquidity risks, and exchange rate risks. Risk could be defined as the lack of predictability of outcomes affecting the set of financial transactions. To successfully manage risk, investors must be able to measure it and to determine the level of risk that could be acceptable according to their preferences (Bansal., 2007) [4].

Many approaches have been applied to find the optimal financial portfolio such as goal programming, mixed integer programming, quadratic linear programming, stochastic linear programming, and hybrid genetic algorithms. This paper reviews the existing literature on the use of goal programming models to optimal financial portfolio selection.

Azmi and Tamiz (2010) [5] presented a brief review on the use of Goal Programming for Portfolio Selection problem.

*Corresponding author e-mail: Mohamed.hassanien@inp.edu.eg

They discussed the use of Multi-Criteria Decision Analysis in Portfolio Selection and the importance of Goal Programming. Moreover, they highlighted the theoretical and practical developments in the use of goal programming for optimal selection of the tracking portfolio. Different variants of goal programming, including weighted, lexicographic, MiniMax, and fuzzy goal programming models were identified for the selection problem.

Aouni et al. (2014) [6] presented a review on different variants of goal programming models that have been applied to financial portfolio management. They aimed in their review to update the literature for the benefit of both researchers and practitioners who are interested in this subject. Their review and analysis were limited only to the use of the goal programming approach in this area and were not subject to other multiple criteria decision-making models. The main conclusion they have stated is that there is still a lack in the development of computerized decision support systems.

Masmoudi and Abdelaziz (2018) [7] presented a review on the application of multiple objective deterministic and stochastic programming models for the portfolio selection problem. They highlighted the different concepts related to portfolio selection theory, including pricing models and portfolio risk measures. They discussed the models proposed to solve the problem and compared their relevant different assumptions and solving techniques. They concluded that there is a need for more in-depth search about developing models that takes into account the risk in terms of ambiguous environment and volatile market conditions all over the world.

This paper can be viewed as an extension and update for the previous reviews on the use of goal programming in financial portfolio management. We highlight the main variants of goal programming in the process of selecting the optimal portfolio and the approaches to prioritize among different desired goals and targets. The paper starts with a presentation of the basic and general formulation and principles of the optimal financial portfolio selection process. The formulation of the goal programming models and their application to the problem is then discussed. Finally, we perform a bibliometric analysis of the applications of goal programming to optimal portfolio selection.

2. General Formulation of the Optimal Portfolio Selection Problem

2.1. Portfolio Construction

The financial decision maker (FDM) in stock market and investment analysis could have more than one objective to achieve (Sharpe, 1985) [8]. Generally, there are three main objectives the FDM should be interested in; (i) stock analysis, which includes estimates of future outcomes for individual stocks and their relation to other stocks, (ii) portfolio analysis to determine the set of efficient portfolios, where an efficient portfolio usually offers a higher overall expected return than any other portfolio with comparable risk, and (iii) portfolio selection which includes the selection of a portfolio from the efficient set, which depends on the decision maker's preferences toward risk and expected return.

The origin of the modern portfolio theory dates back to (Markowitz, 1952) [9] who introduced the mean-variance model for selecting the most efficient portfolios that maximize the expected rate of return of the investment money within a minimum level of potential risk. According to Markowitz, investors prefer the efficient portfolio with higher return and lower risk, which gives a maximum return for a given risk, or a minimum risk for a given return. Based on his model, investors should weigh risk versus return, and allocate funds among investment options based on the trade-off between them.

Fig. 1: Efficient Portfolio Construction

Source*: Williams, R. T. (2011). An introduction to trading in the financial markets: trading, markets, instruments, and processes. Academic Press.*

Figure (1) summarizes Markowitz's theory where an efficient portfolio is defined as the group of securities with the highest possible return for a given amount of risk. In this context, the constructed optimum sets of portfolios are those for which it is not possible to get a higher return on the portfolio without having more risk. Conversely, if a portfolio is efficient, it is not possible to reduce risk without reducing the return.

The model can be viewed as a bi-objective model, which is formulated as a quadratic optimization problem involving the minimization of risk, subject to a constraint on the minimum desired level of return as follows:

$$\boldsymbol{Max.} \sum_{j=1}^{n} r_j w_j \quad (1)$$

$$\boldsymbol{Min.} \sum_{j=1}^{n} \sum_{k=1}^{n} w_j \sigma_{jk} w_k \quad (2)$$

subject to:

$$\sum_{j=1}^{n} w_j = 1 \,, \quad (3)$$

$$w \in F \,, \quad (4)$$

Objective 1 presents the expected return of the constructed portfolio, while objective 2 is the variance of the constructed portfolio to be minimized.

w_j : the amount to be invested in stock *j*.

r_j : the expected return of stock *j*.

σ_{jk} : the covariance of returns for stocks *j* and *k*.

F: the set of feasible solutions.

The above model has the following set of assumptions:

- Investors are risk-averse and have increasing expected utility, and the final utility curve of their wealth is decreasing.
- Investors choose their stock portfolio based on the average and variance of expected returns.
- Investors prefer higher returns at a certain level of risk, and vice versa. In other words, for a certain level of return, they seek the lowest risk rate (anti-recession assumption).

Investment strategies could be broadly classified into two types of management: active and passive management. In active fund management, managers deal directly by buying or selling stocks to achieve returns. Moreover, fund managers have more confidence in their ability to estimate cash flows, growth rates, and discount rates. The objective of active fund management is to identify stocks that do considerably better than the market in terms of higher expected returns over a certain period (Boubakari, 2010) [10].

Fund managers in active fund management aim at maximizing return by selling and buying decisions based on their experience, knowledge, and expectations. Usually, this type of management is associated with risk-taking people who wish to maximize their return on investments and have a high degree of flexibility and tend to invest in stocks whose values are going to outperform other stocks over time (Mutunge and Haugland, 2018) [11].

On the other hand, in passive fund management, this process is done through the so-called stocks portfolio. The goal of investing in an index-tracking portfolio is to invest in a diversified portfolio that gives the same return as the market while minimizing transaction costs and possible risk. Passive fund managers have to follow a predefined set of constraints and criteria that guarantee a certain level of return given a certain level of risk. This type is more associated with risk-averse people who aim to reach a return close as much as possible to a reference market index (Torrubiano and Suarez, 2009) [12].

Two ways could be followed in passive portfolio management to track an index: full replication or partial replication. Full replication is considered the simplest way to track an index, whereas all the stocks that make up the index are purchased in the same proportions as in the index. This approach can achieve a perfect match. However, it has many disadvantages. As certain stocks may only contribute a tiny proportion to the whole index, reconstruction/rebalancing of the index may require manipulating all stocks that incur high transaction costs. Furthermore, stocks of small companies may be illiquid which also entails relatively high transaction and administrative costs (Wang et al., 2018) [13].

On the other hand, in partial replication, fund managers usually select a subset of the stocks in the index. This approach is usually more favored in index tracking to perform in a more efficient way than full replication of all stocks. The problem in this context is the selection of the portfolio that best matches the performance of the market index, i.e., to identify the best/optimal subset of stocks that constructs the portfolio and specify its corresponding weights reflected in the amount of money to be invested in each stock. Another important aspect to be considered is that the evolution of the value of the tracking portfolio over time depends on the assets that make up the portfolio. There are two ways to describe the positions of these assets within the portfolio. The first way is to use the number of units of each asset. The second is to use the weight of each asset (Canakgoz and Beasley, 2009) [14].

Index tracking is a popular form of passive fund/portfolio management where the concern is to track/reproduce the performance of a stock market index. Passive fund management has, in general, lower fixed and transaction costs compared to active fund management, although once the market falls, the invested fund would be highly affected. In passive fund management, the expected return to the portfolio is lower than that to an active fund management, whereas it involves lower risk. Historical evidence, however, reveals that active fund managers averagely underperform their corresponding benchmarks (Mutunge and Haugland, 2018). Researchers also found that some of the best active fund managers did perform reasonably well in some periods, but most of them failed to carry their success over a long-term period. Therefore, greater attention has been directed toward passive fund management (Garcia et al., 2013) [15].

Two main questions arise when designing a portfolio. The first relates to the optimal set of stocks that should compose the investment portfolio, while the second is how to allocate the amount to be invested among those different stocks. Many approaches and techniques were developed in this context. Usually, the ultimate goal of a management strategy is to select the portfolio that achieves maximum return with low risk levels. The selection process of stocks to construct the portfolio and the determination of allocation weights associated with each stock are conditioned on many factors, resulting in a multi-objective process (Kolm et al., 2014) [16].

Typically, the problem of index tracking portfolio is formulated as a mathematical programming problem (quadratic programming, mixed integer programming, goal programming, etc.). When a cardinality constraint is included into the problem to determine a fixed number of stocks in the portfolio, the computational complexity of the problem substantially increases (Torrubiano and Suarez, 2009; Mutunge and Haugland, 2018) [11,12]. Heuristics methods are then required to reach near-optimal solutions when the cardinality constraint is considered in the problem.

2.2. Tracking Performance of the Constructed Portfolio

In passive fund management, the tracking performance of a portfolio is measured by the tracking error. Several measures of this error had been proposed in the literature. Most of these measures either depend on the correlation between the tracking portfolio and the market index or depend on different estimates of the variance of the difference between the mean returns of the market index and the portfolio. Rudolf (1999) [17] defined the tracking error as the absolute value of historical differences between the returns from the tracked index and the tracking portfolio and formulate the tracking error in time t as $|R_P - R_I|$; where R_P denotes the return of the tracking portfolio and R_I is the return of the tracked index.

Since tracking portfolio cannot replicate the index perfectly, it is necessary to measure the performance of the constructed portfolio, i.e., how good the tracking error is. In addition, a performance measure helps to conduct and compare different portfolios. It is worth mentioning that even though a low tracking error in the past does not guarantee a high tracking accuracy in the future, tracking portfolios with a lower tracking error tend to have a higher tracking accuracy in the future. Most measures of tracking quality are based on the concept of tracking error. There are the group of quadratic measures, such as mean squared error (MSE) and tracking error variance (TEV); and the group of linear measures, including mean absolute deviation (MAD) and maximal absolute deviation (MAXD) (Karlow, 2013) [18]. The TE measures the difference between the returns of the tracking portfolio and the returns of the index.

It can be formulated as:

$$TE = R_{I,t} - R_{P,t} = R_{I,t} - \sum_{i=1}^{N} w_i \, r_{i,t} \tag{5}$$

where w_I is the weight of asset i, and N is the number of assets, and the sum of the weights in the tracking portfolio has to be one i.e. $\sum_{i=1}^{N} w_i = 1$.

Konno and Yamazaki (1991) [19] presented the Mean-Absolute Deviation (MAD) model as a substitute for the Markowitz model. The MAD model reduces a risk indicator, where the indicator is the mean absolute deviation. Furthermore, as an alternative to the well-known asset allocation plan, first, the fund is divided among indexes representing various asset classes, and then it is divided among specific assets using models that are appropriate for each asset class. The tracking error formula in this model measures the average of the absolute deviations between the returns of the index and the returns of portfolio during a given time period t.

$$MAD = \frac{1}{T}\sum_{t=1}^{T}\left|R_{I,t} - R_{P,t}\right| = \frac{1}{T}\sum_{t=1}^{T}|TE_t| \tag{6}$$

where $t = 1, 2, 3, \ldots, T$ are the time periods. Focardi and Fabozzi (2004) [20] proposed an index tracking portfolio optimization model with a new measure for the variance between index return and the return on the portfolio. They compared their results to the classical Markowitz mean-variance optimization model. The goal of the study was to find a portfolio that minimizes the variance of the difference between return of the portfolio R_p, and the index return R_I i.e., $var(R_p - R_I)$.

Another measure for the tracking error measure is MSE, which is the mean of the squared differences between the returns of the index and the portfolio during a given time, period:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(R_{I,t} - R_{P,t})^2 = \frac{1}{T}\sum_{t=1}^{T}(TE_t)^2 \quad (7)$$

Where $t = 1, 2, 3, \ldots, T$ are the periods. In addition, the root mean square error $RMSE$ is used to measure the tracking error with the advantage of preserving the same scale of MSE $where, RMSE = \sqrt{MSE}$. Another well-known used measure of the tracking error is the variant of MSE the so called tracking error variance (TEV). It is defined as the variance of the differences between the return of the index and the returns of the constructed portfolio;

$$TEV = \frac{1}{T}\sum_{t=1}^{T} R_{I,t} - R_{P,t} - (\bar{R}_I - \bar{R}_P)^2 \quad (8)$$

Where, $\bar{R}_I$ is the mean of the return s of the portfolio, $\bar{R}_P$ is the mean of the returns of the index. It is worth mentioning that no preference for any of the different formulas of tracking error was stated clearly in the literature however, the majority of authors use the quadratic form in defining the tracking error function in the formulation of their problems (Chi et al., 2019) [21].

Dose and Cincotti (2005) [22] proposed a clustering technique of financial time series with application to index tracking problem. Their goal was to minimize a generalized objective function which contains a tradeoff between tracking error and excess return. Their hierarchical clustering approach used was based on two dissimilar measures. The first measure is the correlation distance in return; assuming two-time series on return of two stocks, (*X*, *Y*) then,

$$d(x, y) = \sqrt{2(1 - c_{xy})} \quad (9)$$

where $X = (x_1, x_2, \ldots, x_T)$ and $Y = (y_1, y_2, \ldots, y_T)$and c_{xy} is the correlation coefficient between X and Y. Another measure of similarity was used based on the percentage difference of price of stocks in the form, $d(x, y)$= $\min\{d_1, d_2\}$, where:

$$d_1(x, y) = \sum_{a\in R}^{min}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{x_t - ay_t}{x_t}\right)^2\right\};\ d_2(x, y) = \sum_{a\in R}^{min}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{x_t - ay_t}{ay_t}\right)^2\right\} \quad (10)$$

The results of the comparison of the clustering-based techniques with random selection demonstrated the advantage of clustering in noise reduction and obtaining robust forecasts.

Chen and Kwon (2012) [23] developed a portfolio selection model for index tracking problem using binary integer programming to maximize the similarity between the selected assets and the target index in a robust way.

Jimbo et al. (2017) [24] defined the reliability of a portfolio as; $R = \left|\frac{\sigma_{pred} - \sigma_{real}}{\sigma_{real}}\right|$, and a portfolio is more reliable when R is small. An optimal Stock portfolio investment strategy should show the investors how much to invest in each asset in a given portfolio. They investigated the optimization of the portfolio under cardinality constraints, aiming at answering how much that K should be where $K \leq N; N$ is the total number of stocks.

Lam et al. (2017) [25] proposed a two-stage mixed integer programming model to improve the existing single stage model for tracking FBMKLSI index in Malaysia. In their proposed model, besides the aim of minimizing the tracking error they considered generating higher portfolio mean return than the benchmark index return. In the first stage of the model the aim is to minimize the tracking error, while in the second stage the aim is to maximize the portfolio mean return at minimum tracking error.

To sum up, index tracking problem can be viewed as a dual or multi-objective optimization problem, where a trade-off between maximizing expected performance and minimizing tracking error. Unlike liner programming which tries to maximize or minimize the objective directly, goal programming seeks to minimize the unwanted deviations from any single goal. Hence, it is very useful and practical in handling such a problem with competing goals in construing portfolios with multiple objectives (Liang et al, 2007) [26]. Different goal programming models variants that have been applied to the optimal financial portfolio selection will be discussed in the following section.

3. Goal Programming as a Multiple Criteria Decision Making (MCDM) for the Optimal Portfolio Selection

Multiple criteria decision-making (MCDM) is a term used to describe a subfield in operations research and management science. Zionts (1981) [27] defined MCDM as a means to solve decision problems that involve multiple and sometimes conflicting objectives. MCDM refers to making decisions in the face of multiple, frequently contradictory, and incommensurable factors.

Zopounidis & Doumpos (2013) [28] discussed the wide application of multi criteria decision systems for financial decisions, in particular portfolio selection and corporate performance evaluation. They also highlighted the need to enhance exiting techniques of operations research to be applied in finance, especially in the era of uncertain and ambiguous financial decisions and global investment environment.

Jing et al. (2023) [29] introduced a comprehensive modeling for the optimal selection of stock portfolios using multi criteria decision-making methods. They were interested in maximizing stock portfolio returns and minimizing risk. They applied their model to a sample of 79 companies listed on the Tehran Stock Exchange. They were able to construct an optimal portfolio in terms of high return and low risk measures.

Goal Programming (GP) was first introduced by Charnes, Cooper, and Fergusin (1955) [30]. Goal Programming is a linear programming extension that can deal with multiple objectives and is used to solve a multi-objective optimization problem that tackles a trade-off among conflicting objectives. In general, three main types of analysis could be considered in goal programming:

- Determining the resources required to achieve a desired set of goals.
- Determining the extent to which goals can be met given the resources available.
- Providing the most satisfactory solution given different resource levels and goal priorities.

Goal programming is a well-known MCDM technique based on the distance function concept, in which the Decision-Maker (DM) seeks the solution that minimizes the absolute deviation between the objective's achievement level and its aspiration level. It is considered a powerful tool that draws upon the highly developed and tested technique of linear programming which provides a simultaneous solution to a complex system of competing objectives. GP is a powerful and adaptable technique that has been applied to a wide range of decision-making problems involving conflicting multiple objectives, in economics, accounting, engineering, agriculture, marketing, transportation, finance, and other types of application areas. The main two advantages of goal programming models are their flexibility in terms of the constraint functions and their aim of reaching practical and attainable real-world solutions to decision problems rather than seeking idealistic or optimal solutions satisfying the mathematical conditions (Charnes et al., 1957) [31].

When applied to portfolio selection, a GP model enables the financial decision maker to aggregate several financial aspects and preferences to select the best compromise portfolio. The financial decision maker, whether an investor, a portfolio manager, or a financial analyst, can choose the appropriate goal-programming variant to deal with a specific portfolio selection based on the availability of information and data about market conditions (Lee, 1973) [32].

The main idea behind goal programming is to set goals and try to minimize the deviations between the goals and their desired targets or outcomes. If all deviations can reach zero, then all goals can be achieved. In general, however, to minimize either under or over achievement of a certain goal, a deviational variable d is assigned to each goal (Tamiz et al, 1995) [33]. The main aim of GP then is to minimize the deviation between the goals, say $Z_i(x)$, X= $(x_1, x_2, \dots, x_n)$, and these acceptable aspiration levels, g_i ; $i = 1,2, \dots, k$.

Minimize $\sum_{i=1}^{k} |Z_i(x) - g_i|$ (11)

Subject to:

$x \in X = \{x \in \mathbb{R}^n; A_x \leq b; x \geq 0\}$ (12)

$Z_i(x) - g_i = d_i^+ - d_i^-; d_i^+, d_i^- \geq 0$ (13)

$d_i^- \times d_i^+ = 0$ (14)

$d_i^-, d_i^+ \geq 0$ (15)

where Z_i ; the linear function of the i^{th} goal and g_i is the aspiration level of i^{th} goal, K is the total number of goals, b is the right-hand side of the constraint coefficient, $Z_i(x)$ is the k^{th} objective and g_i is the aspiration level of the k^{th} goal. Equation (11) states that z is the function of unwanted positive and negative deviations for all decision goals. d_i^-

represents the extent of underachievement of goal i relevant to its specified aspiration level. d_i^+ represents the extent of over achievement of goal i relevant to its specified aspiration level.

The achievement function is considered one of the main key elements in goal programming models for portfolio selection. It measures the extent of achievement for the minimization of unwanted deviations in the model. The goal programming model is a distance function where the unwanted negative and positive deviations between the achievement and aspiration levels are to be minimized (Aouni, 2015) [34].

The problem of optimal portfolio selection in the context of goal programming varies according to the targets and goals of the investor or decision maker as will be illustrated. In general, to customize the general goal programming formulation to either index or enhanced index tracking problem, some constrained goals are stated. Two objectives must be achieved: maximizing returns of the portfolio and minimizing tracking error. In applications, it is customary to set the target values, or acceptable levels, at 3% for the tracking error and at 7% for the rate of return. This specification allows for direct tradeoffs between all unwanted deviational variables by placing them in a weighted, normalized single achievement function to be minimized (Beasley, 2003) [35].

The major difference between goal programming and other linear programming models is the explicit consideration of goals and the various priorities associated with the different goals (Aouni et al. 2014) [6]. Goal programming models can be categorized in terms of the importance comparability of the goals. When the goals are of roughly comparable importance, goal programming is known as non-preemptive (Bravo et al. 2010) [36]. The following subsections discuss the main Goal Programming variants and their application to the portfolio selection process.

3.1. Weighted Goal Programming

In this type of goal programming model, the user orders the unwanted deviational variables according to their relative importance. The model attaches weights associated with each goal according to its relative importance in the models. The decision makers, or the experts, usually identify importance based on previous experience. The goal in such a case is to minimize the summation of the unwanted weighted deviations. More specifically, in the case of portfolio selection, the model aims to minimize the measure of the risk and maximize the return of the constructed portfolio, taking the prioritization of goals into consideration (Ballestero et al. 2015) [37]. The mathematical formulation of the weighted goal programming applied to the optimal portfolio selection problem could be expressed as follows:

$$Min. Z = \sum_{i=1}^{p} we_i^+ \delta_i^+ + we_i^- \delta_i^- \quad (16)$$

Subject to:

$$f_i(w) + \delta_i^- - \delta_i^+ = g_i \quad ; (i = 1,2, \dots, p), \quad (17)$$

$$\sum_{j=1}^{n} w_j = 1 , \quad (18)$$

$$w \in F , \quad (19)$$

$$\delta_i^- , \delta_i^+ \geq 0 \quad ; (i = 1,2, \dots, p). \quad (20)$$

A weighted goal programming (WGP) model for investment-planning with profit and risk or safety goals was first introduced by Callahan in 1973[38]. The selection problem using weighted goal programming was initially addressed by Sharma et al. (1995) [39]. Tamiz et al. (1996) [40] provided a WGP formulation for the Portfolio Selection Problem in two stages: (i) prediction of the shares' sensitivity to particular economic indicators, and (ii) choice of the best portfolio based on the financial decision maker's (FDM) preferences.

The maximization of the utility or satisfaction of the FDM is the goal of all rational investment decisions, according to Tamiz (1996) [40]. Bravo& Garcia (2010) [36] investigated the issue of portfolio selection using more than one criterion. They considered weights for goals based on investor's preferences and the application of Arrow's absolute risk aversion coefficients.

Cooper et al. (1997) [41] presented a weighted goal programming for the assessment of security portfolio and regression relationships using the Warsaw Stock Exchange, Dominiak (1997) [42] demonstrated the use of interactive multiple objective programming for optimal portfolio selection. Another weighted goal programming model was applied to the selection and composition of mutual funds in the Greek market (Pendaraki et al., 2004) [43].

Halim et al. (2015) [44] proposed a weighted goal programming model to find the optimal solution for six goals related to banking performance, namely asset accumulation, liability reduction, shareholders' wealth, earning, profitability and optimum management of all items in the financial statement.

Wang et al. (2018) [13] proposed a weighted mixed integer-programming model for the index-tracking problem with stratified sampling and optimal allocation. They were the first to incorporate the optimal allocation in stratified sampling into the indexation problem. Besides the aim of minimizing the tracking error, they considered the estimation accuracy derived from the use of stratified sampling. By adding the stratification criteria into their model, they aimed at enhancing the efficiency of the assets selection and produced more accurate forecasting estimates.

3.2. Lexicographic Goal Programming

In this type of model, the objective function is made up of an ordered vector with a dimension equal to q, the number of priority levels associated with the model. Each component in the vector represents the unwanted deviations of goals for each priority level. Usually in this model, goals are ranked according to priority levels representing their importance to the decision maker. The most important objectives are accorded the highest level of priority, while the less important are assigned lower priority levels. The obtained values of the deviations of a higher level of priority are then introduced as constraints within the mathematical programs related to the objectives placed at lower levels of priority. In this way, objectives assigned lower priority levels will play marginal roles in the decision-making process (Babaei et al., 2009) [45].

Lexicographic Goal Programming (LGP) could deal with many priority levels in portfolio selection problems. Goal constraints are included according to their importance of achievement in the model structure as follows:

Maximizing the portfolio's expected return, while minimizing some measurement of portfolio's risk.

Minimizing other portfolios risks (e.g., the systematic risk as measured by Beta Coefficient).

Minimizing the cost of rebalancing the portfolio. In addition to any other priority levels.

The mathematical formulation of the lexicographic goal programming (LGP) model for the optimal portfolio selection is as follows:

$$\boldsymbol{Lex\ Min}\ L = \left[\, l_1(\delta^-, \delta^+), l_2(\delta^-, \delta^+), \ldots, l_q(\delta^-, \delta^+)\right] \tag{21}$$

subject to:

$$f_i(w) + \delta_i^- - \delta_i^+ = g_i \qquad ; (i = 1,2, \ldots, p) ; \tag{22}$$

$$\textstyle\sum_{j=1}^{n} w_j = 1 , \tag{23}$$

$$w \in F , \tag{24}$$

$$\delta_i^- , \delta_i^+ \geq 0 \qquad ; (i = 1,2, \ldots, p). \tag{25}$$

where L represents the ordered vector of the unwanted deviations and q indicates different priority levels associated with the model according to their relative importance for the financial decision maker.

Lee (1973) [32] proposed the initial formulation of the LGP model for the portfolio selection problem. He considered the following three goals: (i) minimal dividends, (ii) earnings growth, and (iii) a 50% dividend payout ratio. Both Lee & Lerro (1973) [32] and Sharma et al. (2006) [46] used the LGP to deal with mutual funds in stock markets. They emphasized that their models, through taking into account the trade-offs between financial risk and inflation risk, produced better results than the original classical model developed by Markowitz (1952) [6].

Kumar et al. (1978) [47] presented a LGP for dual-purpose funds where investment companies issue two different types of securities: (a) income shares and (b) capital shares. They offered empirical examples to demonstrate the potential for dual-purpose fund managers to increase in the future. They applied an LGP model for managing a dual- purpose fund. They determined their goals as cash allocation, income return, capital return, individual security allocation.

Babaei et al. (2009) [45] applied a lexicographic goal programming model for portfolio optimization problem. They considered five different criteria for the selection of the portfolio. They used ranks for the objectives of the model according to weights determined through the decision maker's preferences.

Pardalos et al. (2012) [48] applied a successive lexicographical goal programing model for the optimal portfolio selection. The main idea of the successive models is that sequence of lexicographic goal programming problems is solved with different reference points resulting in different solutions. They also aimed at generating solutions with approximately uniform distribution in a Pareto set.

Garcia et al. (2013) [15] proposed a multi objective model for passive portfolio management with an application on the S&P 100 Index. They utilized the model to satisfy different investment profiles with the same set of stocks but with different weights. In other words, all the optimal portfolios that would be acquired contained the same set of selected stocks but with different shares of money to be invested in each. Their model considered the minimization of the tracking error, i.e., minimizing the variance in returns between the tracking portfolio and the market index, with the possibility of achieving excess return. They imposed no constraints on the cardinality of the portfolio number of stocks to be selected to compose the portfolio.

Siew et al. (2014) [49] proposed a goal programming approach for portfolio optimization in enhanced index tracking. They formulated the problem of enhanced index tracking as a dual-objective problem in shaping the trade-off between maximizing the mean return and minimizing the risk of the tracking error. They applied their proposed model on FTSE Bursa Malaysia Kuala Lumpur Composite index (FBMKLCI) which is the leading indicator of the performance of the Malaysia stock market and economy for the largest 30 companies listed in the main market.

In another Malaysian application, Siew and Hoe (2016) [50] proposed the use of goal programming as a strategic tool for the fund managers to track the benchmark technology index. They used data on weekly return of the companies of the technology sector in Malaysia stock market. Their dual objective formulation of the goal-programming problem aimed at minimizing the portfolio tracking, as a first goal, while the other goal is to maximize the portfolio mean return. They reached an optimal portfolio that consists of 10 selected stocks out of the 17 stocks in the main index and was able to generate an excess mean weekly return of 0.3% at minimum tracking error 2.1%.

Mohiti et al. (2019) [51] proposed a two-stage robust goal programming model for portfolio selection. In the first stage of their model, the desired percentage of investment in each industrial group is determined based on return and risk measures. In the second stage, after considering systematic and non-systematic risk, the amount of investment in each stock is determined. Based on actual data they demonstrated that their model could outperform other existing models under the conditions of high uncertainty in terms of higher returns.

3.3. Stochastic Goal Programming

A stochastic linear program is a mathematical program where the objective function and/or the constraints parameters are treated as random variables and deal with problems where its probability distribution is known (Aouni et al. 2012) [52]. The financial decision maker in some cases takes decisions and utilizes investment preferences under uncertainty. In specific, the objective function and the corresponding goals and constraints are random variables with specific distribution. The mathematical formulation of the stochastic goal programming models can be represented as follow:

$$\boldsymbol{Maximize}\,.Z = \sum_{i=1}^{p}(\tilde{\delta}_i^{+} + \tilde{\delta}_i^{-}) \tag{26}$$

$subject\ to$:

$$\sum_{j=1}^{n} a_{ij}\, w_j + \tilde{\delta}_i^{-} - \tilde{\delta}_i^{+} = \tilde{g}_i \ ; \forall\, i \in I, \tag{27}$$

$$\lambda + \delta_i^{-} - \delta_i^{+} \leq 1 \ ; \ \forall\, i \in I, \tag{28}$$

$$w \in F\,; \tag{29}$$

$$\tilde{\delta}_i^{-}, \tilde{\delta}_i^{+} \geq 0\,; \forall\, i \in I, \tag{30}$$

where $\breve{g}_i \in N(\mu_i, \sigma_i^2)$.

Aouni et al. (2005) [53] applied a stochastic goal programming model for portfolio selection using data from the Tunisian Stock Market with imperfect information about the returns of stocks and preferences of the investor. To guarantee the diversity of the constructed portfolio, they make the condition of investing less than 10% in any stock and less than 30% in the bank shares, leasing, insurance and investment corporation sectors.

Ji et al. (2005) [54] proposed a stochastic linear goal programming approach to multistage portfolio management. Their model took into account both the investment goal and the risk control at each stage. Through matching the moments and fitting the descriptive features of the asset returns, a linear programming model is used to generate the single-stage scenarios. They provided a practical case as an illustrative example of the application of the model and scenario generation approach.

Abdelaziz et al. (2009) [55] proposed a stochastic goal programming model to construct a satisfying portfolio for the United Arab Emirates equity market. Using monthly equity data in UAE from 2002 to 2005, their constructed model performed better when compared to the traditional Markowitz model assuming the non-normality of equity returns.

Recently, Wu et al. (2022) [56] proposed a stochastic dominance approach for stock selection in modeling index-tracking portfolio. They developed a three-step approach to determine the number and identify candidate stocks in the constructed portfolio. In terms of the standard deviation, they empirically illustrated that their suggested model could be used to efficiently construct a partial tracking portfolio and replicate the return of the main index. They applied the suggested model to data on the financial times stock exchange 100 index FTSE 100, which is the major stock index in the United Kingdom.

3.4. Fuzzy and Otherwise Imprecise Goal Programming

The Fuzzy Goal Programming (FGP) model was developed to deal with some decisional situations where the decision-maker can only give vague and imprecise goal values (Tsaur et al. 2021) [57]. The FGP is based on the fuzzy set's theory developed by Zadeh (1965) [58] and by Bellman and Zadeh (1970) [59]. Fuzzy goal programming is considered a relatively recent approach to optimal portfolio selection. Fuzzy goal programming is used to handle the uncertainty found in the settings of the optimization problem. In the fuzzy goal programing model for portfolio selection, the decision maker has to identify the aspiration level for each objective in the model. In other words, FGP is a goal programming extension that is used to handle decision issues involving multiple objectives in an uncertain environment where aspiration levels are not precisely known. The mathematical formulation of the fuzzy goal programming model for the optimal portfolio selection is as follows:

$$\textbf{Maximize}\ . Z = \lambda \tag{31}$$

subject to:

$$\frac{f_i(w)}{\Delta_i} + \delta_i^- - \delta_i^+ = \frac{g_i}{\Delta_i} \quad ; \forall\, i \in I, \tag{32}$$

$$\lambda + \delta_i^- - \delta_i^+ \leq 1 \quad ; \quad \forall\, i \in I, \tag{33}$$

$$w \in F\,; \tag{34}$$

$$\lambda, \delta_i^-, \delta_i^+ \geq 0\,;\ \text{for all } i \in I, \tag{35}$$

where Δ_i is the constant of deviations of the aspiration levels g_i.

Parra et al. (2001) [60] applied a fuzzy goal programming model for the portfolio selection problem. They considered return, risk and liquidity as fuzzy terms and applied their model to 132 Spanish mutual funds. Chen and Tsai (2001) [61] developed a preemptive GP model in a fuzzy framework. Bilbao et al. (2006) [62] integrated the knowledge of the expert and the preferences of the FDM into their developed model. They made an extension of Sharpe model where the data are fuzzy, and the betas are estimated using historical data. They illustrated their model through a case study of 26 Spanish mutual funds covering the period (1996–2000).

Abdelaziz et al. (2007) [63] proposed a new deterministic formulation for the multi objective problem for portfolio selection. They combined compromise programming and chance constraint programming. Application of their model to the Tunisian stock market revealed that results of the optimal solution when the expected returns are normally distributed are close to results when rate of returns are known with certainty.

Bilbao et al. (2007) [64] designed flexible decision-making models for portfolio selection including expert's knowledge and imprecise preferences and included them in a GP decision-making model for portfolio selection.

Mansour et al. (2007) [65] also developed an imprecise GP model for portfolio selection based on the satisfaction functions. The FDM's intuition, experience and judgment were explicitly expressed through the satisfaction functions. Their model was applied to select a financial portfolio within the Tunisian Stock Exchange market listed during the period from January 1999 to December 2002). Three objectives were considered: the rate of return, the liquidity and risk.

Hasuike et al. (2009) [66] considered the optimal portfolio selection with uncertain expected return rates. They proposed single and bi-criteria optimization models for random fuzzy portfolio selection problems. Applying their suggested model to real data, the results indicated that their model outperforms other models as it demonstrated a higher flexibility in dealing with ambiguous conditions.

Pakdel et al. (2012) [67] developed an imprecise goal programming model for portfolio selection problem characterized by imperfection of information by the decision maker. They applied their model to a set of 15 companies listed in Tehran stock exchange market. The basic advantage of their applied model is that it was able to choose the efficient stock baskets as a goal with first priority, then to satisfy other stated goals according to the preferences of the decision maker.

Ghahtarani and Najafi (2013) [68] proposed a robust optimization model for the portfolio selection problem using fuzzy goal programming approach. Their model aimed to deal with some uncertain coefficients that exist in both single and multi-objective models which affect the feasibility and optimality of solutions.

Kazemi et al. (2017) [69] introduced a fuzzy goal programming model for efficient portfolio selection. They added the objective of divided yield to the constraints of the presented model aimed at covering more financial indices. They applied the proposed model to data from the New York Stock Exchange to investigate its efficiency and flexibility.

Shapcott (1992) [70] solved the index tracking problem using a genetic algorithm and a quadratic programming technique. The genetic algorithm was used to generate and select the subset composing the portfolio, while the weights of how much money should be invested in each stock were found using the quadratic programming technique. He applied his model on the FTSE-100 Index and ended with 20 assets composing the optimal portfolio.

Recently, Tsaur et al. (2021) [57] stated that that the spreading of COVID-19 has greatly affected the global economy in many aspects, including significant reeducation in income and production, rise in unemployment, recession in travel and tourism sectors in addition to many other forms of instability in the global economy. Such features of the economy negatively affect the investment environment and financial decisions. No longer are investors able to predict the behavior of stock markets' performance based on past historical data and practices. Therefore, they proposed a fuzzy portfolio model to reach the optimal and efficient portfolio within a vague economic environment. The main advantage of their proposed model is that whether investors feel optimistic, neutral, or pessimistic, they can select their imprecise goals of expected return and associated risk.

Pahade & Jha (2021) [71] proposed a trapezoidal fuzzy model for the optimal portfolio selection. In their mode, they incorporated the return to stocks as a fuzzy variable to capture the uncertainty nature of stock markets. They considered the proposed model as extension of the mean–variance model under a fuzzy environment They applied their model on data of stocks form the Bombay Stock Exchange which is the premier market for financial stocks in India. A polynomial goal programming approach was followed to solve their multi-objective model.

Fiala & Borovička (2022) [72] proposed a complex user-friendly model as a supporting tool for investment decision making in the optimal portfolio selection problem. Their suggested model is based on interactive goal programming approach that takes into account many factors such as, multiple criteria, investor's preferences, specific investment input data, … etc. According to this approach, decision space is searched by changes of aspiration levels.

4. Bibliometric Analysis of Goal Programming Applications in Optimal Financial Portfolio Selection

After introducing the main types of goal programming models applied to the portfolio selection problem, this part presents a bibliometric analysis of the utilization of goal programming in addressing the optimal financial portfolio selection problem. The aim of this part is to shed light on the existing literature discussing the application of goal programming in the financial portfolio management. Many aspects can be studied in this context such the area of research, scope of the journal, and country from which the publications had been done.

Bibliometric analysis is a quantitative method to analyze scientific publications and citations, through quantitative representation of existing work in a specific topic considering the broad scope, multidisciplinary, and diversity in the topic under investigation. Bibliometric analysis aims to explore, map, and analyze the evolution of research fields/topics (Zhou et al. 2020) [73]. The popularity of bibliometric analysis in social research is due to its utility for handling large and broad volumes of scientific data in addition to, the accessibility of scientific databases, such as Scopus, Web of Science (WoS), and Google Scholar, which easily offer rich, updated and large volumes of publications (Donthu et al., 2021) [74].

Colapinto et al. (2019) presented a bibliometric analysis based on the descriptors "Goal Programming", "Financial Portfolio", and "Portfolio Management". Within the context of their analysis, they concluded that there is a steady increase in the number of related papers over the past two decades where they reached out to 91 papers tackling this topic. They also noted that no longer is portfolio selection research limited to people specialized in finance as researchers in the field of management science and operations research become very active and professional in applying the goal programming models to portfolio selection. Our results, presented below, confirm such increased openness of optimal portfolio selection studies.

Milhomem and Dantas (2020) [75] presented a systematic literature review analysis of new approaches used in portfolio optimization. They aimed to perform a comprehensive review on the exact and heuristic methods, software/programming languages, and constraints that had been used to solve the portfolio optimization problem. They reviewed 41 published articles in scientific journals that addressed topics related to portfolio optimization in general.

They concluded that techniques to make estimates of future returns were becoming trendy and very common in studies on portfolio optimization with more factors that should be considered in the selection process.

Zhou et al. (2022) [76] presented a bibliometric analysis for the evolution of portfolio optimization. They aimed to comprehensively investigate the development of portfolio optimization research and to introduce some deep insights into this area of research. Their analysis explored the status quo and emerging trends of portfolio optimization research on various aspects such as authors, countries and journals. They found out that the number of publications on portfolio optimization research had been rapidly growing between 2005 to 2020, reaching 651 articles in 2020. Such a result indicates an increasing number of scholars and researchers who are paying attention to this topic.

Recently, Ghanbari et al. (2023) [77] presented a bibliometric analysis for risk measures used in portfolio optimization. They provided a comprehensive review and analysis for the different published articles during the period 2000 and 2022. Their results indicated that the number of articles on risk measures for portfolio optimization has increased since 2001. Moreover, they found out that China and the Unites States were the two largest contributing countries in this field.

In line with the previous literature, the aim of our analysis is to explore and investigate the published work on the use of goal programming for optimal financial portfolio selection. To achieve this, we used the database of Web of Science where the descriptors used in inquiries were “goal programming” and “optimal portfolio selection”, in order to limit results to the process of the optimal portfolio selection and exclude other aspects of portfolio management such as, liquidity of assets, distribution of revenues, risk decomposition …etc.

The query results have revealed that 67 manuscripts published within the period from 1997 to 2023 have tackled this topic. the majority of these manuscripts are articles in scientific journals (85%), while 9% are conference papers, and only about 4% are book chapters. Figure (2) shows the distribution of the published works on the use of goal programming models for optimal portfolio selection problem according to the scientific discipline.

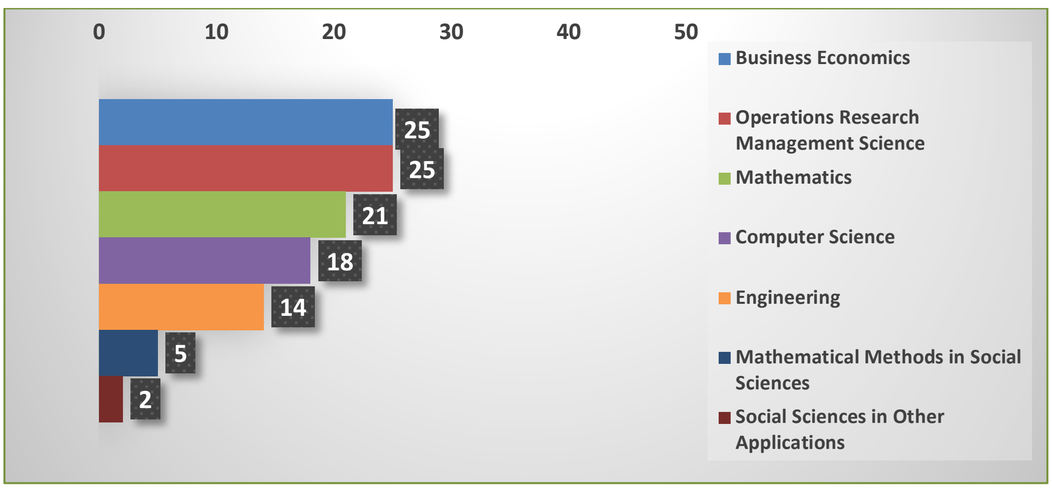

Fig. 2: Number of Publications on Goal Programming for Optimal Index Tracking Portfolio Selection by Research Area of Web of Science, 1997-2023

Figure (3) further classifies the identified publications according to the area of research category in Web of Science. The two areas with the most publications (25, representing 37% of the publications) are in business economics and operations research management science. These are followed by mathematics (21 publications/31%) and computer science (18 publications/27%) followed by engineering (14 publication /21%). The areas with the lowest number of published papers are mathematical methods in social science and social sciences in other topics. It is worth mentioning that a single paper can be classified according to more than one area of research.

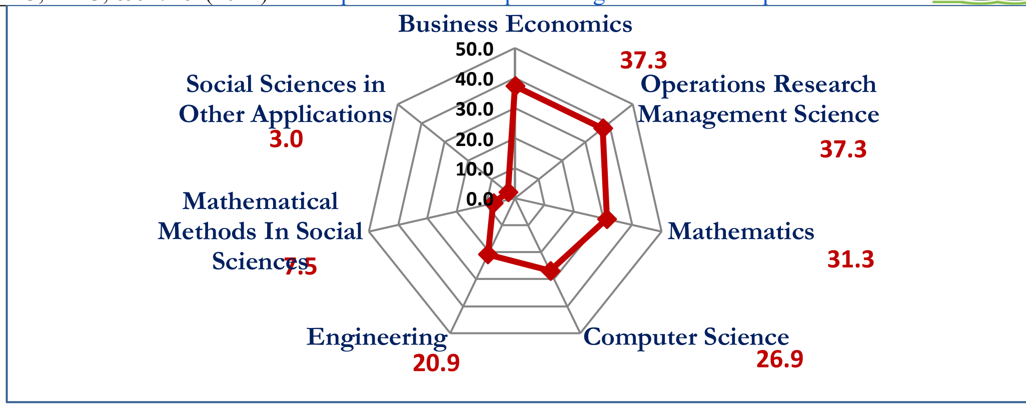

Fig. 3: Distribution of Publications on goal programming for Optimal Index tracking portfolio selection optimal by Research Area of Web of Science, 1997-2023

Another important aspect to consider is the country where the paper is published. According to Figure (4), the majority of publications were published from China (16 papers), followed by the United States of America (14 paper) and Iran (10 papers). While the rest of the countries as shown vary according to the number of published works from five to a single publication.

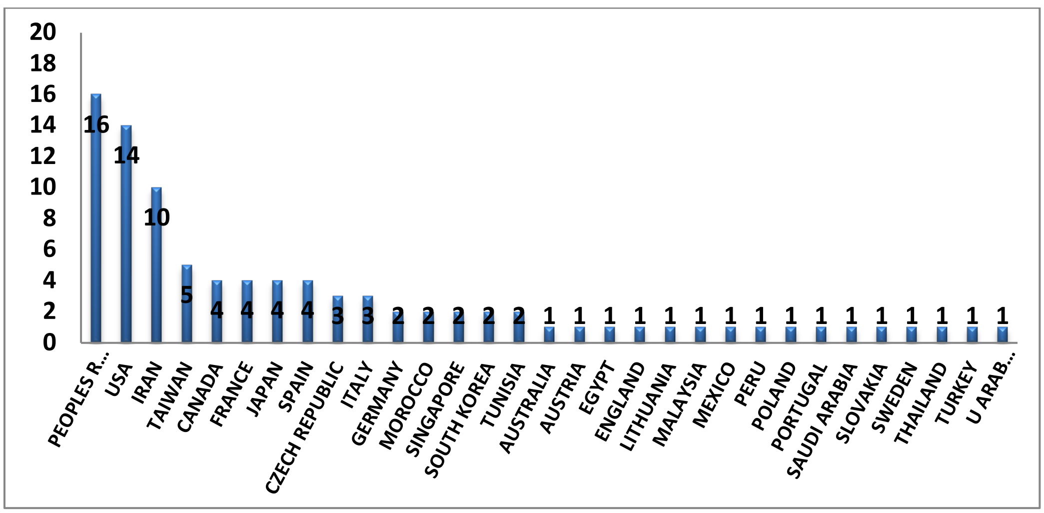

Fig. 4: Number of Published Work by Country of Publication using Web of Science Data, 1997-2023

These results are similar to the results of Zhou et al. (2022) [76], where China is found to be the most productive contributor followed by the United States of America in the second place. It should be noted in this context that researchers could change universities and countries throughout their academic life. Moreover, some researchers might have multiple affiliation, which affects the results presented here. Colapinto et al. (2019) [78] have also noted that it is impossible to reach and provide an accurate measure of the contribution of single author / Country.

Figure (5) depicts the distribution of published works by year of publication. In general, there is an increasing trend in the number of published works on the use of goal programming in the selection problem of the optimal financial portfolio.

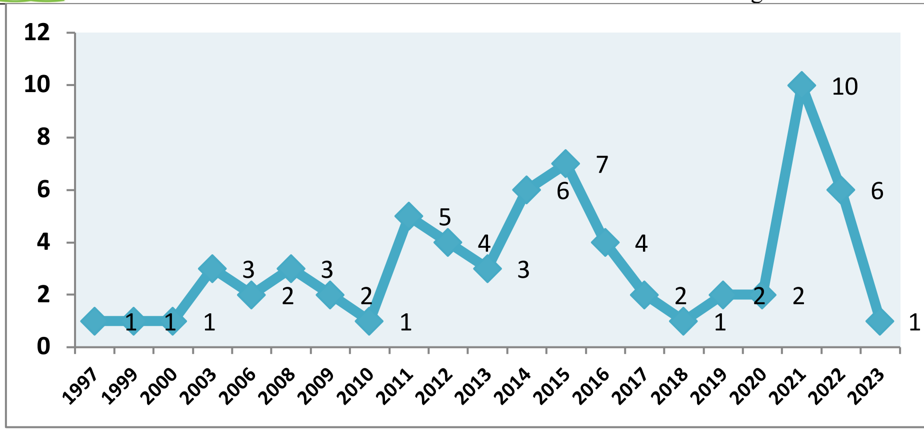

Fig. 5: Distribution of Published Work by Years

This result is in agreement with that of Colpinto et al. (2019) [78], who concluded that portfolio managers are increasingly focusing on quantitative investing in order to take advantage of the benefits of diversity and other aspects that might affect the decisions of portfolio selection. According to Colpinto et al. (2019) [78], the rapid increase of goal programming variants for optimal portfolio selection could be attributed to the fact that it is an easy and practical tool for application and that there are available many commercial optimizations and solving software for this purpose.

5. Conclusion

In this paper, we presented a review and a bibliometric analysis of the use of goal programming in the optimal financial portfolio selection problem. To conclude, based on the aforementioned review and analysis, goal programming could be used for incorporating multi period, extended factors, and different risk measures for portfolio selection. Additionally, investors could assign target values not only for the goals but also for the relevant achievement function. Further development could be found in the applications of goal programming for mutual funds and considering other extended factors that affect investment decisions.

However, goal programming models are widely applied in deterministic context for optimal portfolio selection models. More applications concerned with fuzzy and stochastic decision-making environments are needed. In general, extra comprehensive research is required to address other forms of uncertainty while considering the dynamic and stochastic nature of the investment process. Such uncertainties could include the risk of not achieving a desired or planned portfolio return, which is the most challenging issue in portfolio selection.

Additionally, there is a crucial need to develop models that reassess the risk in a way that takes into account the sensitivity of the environment and the inherent ignorance of market trends. New trends in portfolio selection models should consider many other factors in the investment decisions such as, uncertain probability distribution; uncertain risk measures; socially responsible investment; strategic behaviors, group strategic behaviors and dynamic aspects.

It is also of note that many studies have indicated that the behavior of financial portfolio management is affected by several extended factors that should be controlled or monitored during the modeling process of the portfolio. These include factors such as the international economy, global political stability, natural phenomena and disasters, in addition to the phycology and subjectivity of the financial decision maker. And as sustainable development, social responsibility, environmental impact, and other aspects beyond monetary profits accrue higher salience, they should figure more in investment decisions and strategies including in portfolio selection. Given the number of options and different choices available in capital markets for the investor, more complex and detailed models are required to fully capture the real environment of investment. Future developments and applications of goal programming models in portfolio selection problems would be of a great interest from the researchers in the future. In addition, reaching more concrete and reliable solving models could greatly benefit from active collaborations between academic researchers and field practitioners.

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

References

[1] Almomen, H. (2016). "Role of stock market in the economy of a country," *International Journal of Management and Commerce Innovations*,3 (2):609-611.

[2] Edirisinghe, N. C. P. (2013). "Index tracking optimal portfolio selection," *Quantitative Finance Letters*, 1(1):16-20.

[3] Bogdan,S., Baresa, S., and Ivanovic, S. (2010). "Portfolio analysis based on the example of Zagreb Stock Exchange," *UTMS Journal of Economics*, 1(1): 39-52.

[4] Bansal,R. (2007). "Long-Run Risks and Financial Markets", *National Bureau of Economic Research*, NBER Working Paper Series, 13196:283-300.

[5] Azmi, R., & Tamiz, M. (2010). "A review of goal programming for portfolio selection". New developments in multiple objective and goal programming, 15-33.

[6] Aouni, B., Colapinto, C., & La Torre, D. (2014). "Financial portfolio management through the goal programming model: Current state-of-the-art". European Journal of Operational Research, 234(2), 536-545.

[7] Masmoudi, M., & Abdelaziz, F. B. (2018). "Portfolio selection problem: A review of deterministic and stochastic multiple objective programming models". Annals of Operations Research, 267, 335-352.

[8] Sharpe, W., (1985). Investments (3rd edn). Prentice-Hall, Englewood Cliffs, NJ.

[9] Markowitz, H. (1952). "Portfolio selection," *The Journal of Finance*, 7(1): 77-91.

[10] Boubakari, A. (2010). "The role of stock market development in economic growth: evidence from some Euronext countries," International Journal of Financial Research.1 (1):14-20.

[11] Mutunge, P. and Haugland, D. (2018). "Minimizing the tracking error of cardinality constrained portfolios," Computers and Operations Research. 90: 33-41,

[12] Torrubaino, R. and Suarez, A. (2009). "A hybrid optimization approach to index tracking," Annals of Operations Research, 166(1): 57-61.

[13] Wang, M., Xu, F. and Dai, Y. H. (2018). "An index tracking model with stratified sampling and optimal allocation," Applied Stochastic Models in Business and Industry, 34:144-157.

[14] Canakgoz, N. and Beasley, J. (2009). "Mixed integer programming approach for index tracking and enhanced indexation," European Journal of Operational Research, 196(1):384-399.

[15] Garcia, F., Guijarro, F., and Moya, I. (2013). "A multi objective model for passive portfolio management: an application on the S and P 100 index," Journal of Business Economics and Management, 14(4):758-775.

[16] Kolm, P. N., Tütüncü, R., & Fabozzi, F. J. (2014). "60 Years of portfolio optimization: Practical challenges and current trends. European Journal of Operational Research", 234(2), 356-371.

[17] Rudolf, M. (1999). "A linear model for tracking error minimization," Journal of Banks and Finance, 23(1): 85-103.

[18] Karlow, D. (2013). Comparison and development of methods for index tracking (Doctoral dissertation, Frankfurt School of Finance & Management (Frankfurt/Main)).

[19] Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. Management science, 37(5), 519-531.

[20] Focardi, S. and Fabozzi, F. (2004). "A methodology for index tracking based on time-series clustering," Quantitative Finance. 4(4): 417-425.

[21] Chi, G., Ding, S., & Peng, X. (2019). "Data-driven robust credit portfolio optimization for investment decisions in P2P lending". Mathematical Problems in Engineering.

[22] Dose, C. and Cincotti, S. (2005). "Clustering of financial time series with application to index and enhanced index tracking portfolio," Physica A: Statistical Mechanics and its Applications, 355(1):145-151.

[23] Chen, C. and Kwon, H. (2012). "Robust portfolio selection for index tracking," Computers and Operations Research, 39(2): 829-837.doi.org/10.3390/math11020415.

[24] Jimbo, H., Nogongo, I., Andjiga, N., Suzuki, T. and Onana, C. (2017). "Portfolio optimization under cardinality

constraints: a comparative study," Open Journal of Statistics, 7(2):731-742.

[25] Lam,W. Jaaman,H. and Lam,W. (2017). "Emhamced Index Tracking in Portfolio Optimization with Two Stage Mixed Integar Programming Model". Journal of Fundamental and Applied Sciences,9 (5S):1-12.

[26] Liang-Chuan, W., Chou, S. C., Chau-Chen, Y., & Ong, C. S. (2007). "Enhanced index investing based on goal programming". Journal of Portfolio Management, 33(3), 49.

[27] Zionts, S. (1981). "A multiple criteria method for choosing among discrete alternatives". European Journal of Operational Research, 7(2), 143-147.

[28] Zopounidis, C., & Doumpos, M. (2013). Multicriteria decision systems for financial problems. Top, 21, 241-261.

[29] Jing, D.; Imeni, M.; Edalatpanah, S.A.; Alburaikan, A.; Khalifa, H.A.E.-W.(2023). "Optimal Selection of Stock Portfolios Using Multi-Criteria Decision-Making Methods". Mathematics, 11, 415.

[30] Charnes, A., Cooper, W. W., & Ferguson, R. O. (1955). "Optimal estimation of executive compensation by linear programming". Management science, 1(2), 138-151.

[31] Charnes, A., & Cooper, W. W. (1957). "Management models and industrial applications of linear programming". Management science, 4(1), 38-91.

[32] Lee, S. M., & Lerro, A. J. (1973). "Optimizing the portfolio selection for mutual funds". The Journal of Finance, 28(5), 1087-1101.

[33] Tamiz, M., Jones, D. F., & El-Darzi, E. (1995). "A review of goal programming and its applications". Annals of operations Research, 58, 39-53.

[34] Aouni, B., Colapinto, C., La Torre, D., Liuzzi, D., & Marsiglio, S. (2015). "On dynamic multiple criteria decision making models: A goal programming approach". Multiple criteria decision making in finance, insurance and investment, 31-48.

[35] Beasley, J. E., Meade, N. and Chang, T. (2003). "An evolutionary heuristic for the index tracking problem," European Journal of Operational Research, 148(3) 621-643.

[36] Bravo, M., Pla-Santamaria, D., & Garcia-Bernabeu, A. (2010). "Portfolio selection from multiple benchmarks: A goal programming approach to an actual case". Journal of Multi-Criteria Decision Analysis, 17(5-6), 155-166.

[37] Ballestero, E., Garcia-Bernabeu, A., & Hilario, A. (2015). Portfolio Selection by Goal Programming Techniques. Socially Responsible Investment: A Multi-Criteria Decision Making Approach, 111-129.

[38] Callahan, J. (1973). "Introduction to Financial Planning Through Goal Programming". Cost and Management, 47(1), 7-12.

[39] Sharma, J. K., Sharma, D. K., & Adeyeye, J. O. (1995). "Optimal portfolio selection: A goal programming approach". Indian Journal of Finance and Research, 7(2), 67-76.

[40] Tamiz, M., Hasham, R., Jones, D. F., Hesni, B., & Fargher, E. K. (1996). "A two staged goal programming model for portfolio selection". Multi-objective programming and goal programming: Theories and applications, 286-299.

[41] Cooper, W. W., Lelas, V., & Sueyoshi, T. (1997). "Goal programming models and their duality relations for use in evaluating security portfolio and regression relations". European Journal of Operational Research, 98(2), 431-443.

[42] Dominiak, C. (1997). "An application of interactive multiple goal programming on the Warsaw stock exchange". In Advances in Multiple Objective and Goal Programming: Proceedings of the Second International Conference on Multi-Objective Programming and Goal Programming, Torremolinos, Spain, May 16–18, 1996 (pp. 66-74). Springer Berlin Heidelberg.

[43] Pendaraki, K., Doumpos, M., & Zopounidis, C. (2004). "Towards a goal programming methodology for constructing equity mutual fund portfolios". Journal of Asset Management, 4, 415-428.

[44] Halim, B. A., Abd Karim, H., Fahami, N. A., Mahad, N. F., Nordin, S. K. S., & Hassan, N. (2015). Bank financial statement management using a goal programming model. Procedia-social and behavioral sciences, 211, 498-504.

[45] Babaei, H., Tootooni, M., Shahanaghi, K., & Bakhsha, A. (2009). "Lexicographic goal programming approach for portfolio optimization". Journal of Industrial Engineering International, 5(9), 63-75.

[46] Sharma, H. & Sharma, D. (2006). A multi-objective decision-making approach for mutual fund portfolio. Journal of Business & Economics Research (JBER), 4(6).

[47] Kumar, P. C., Philippatos, G. C., & Ezzell, J. R. (1978). “Goal programming and the selection of portfolios by dual-purpose funds”. The Journal of Finance, 33(1), 303-310.

[48] Pardalos, P. M., Steponavičė, I., & Žilinskas, A. (2012). “Pareto set approximation by the method of adjustable weights and successive lexicographic goal programming”. Optimization Letters, 6, 665-678.

[49] Siew.L, Hafizah, S. and Hamizun, I. (2014). “Portfolio optimization in enhanced index tracking with goal programming approach,” The American Institute of Physics Conference Proceedings, 1614(1), 968: https://doi.org/10.1063/1.4895332.

[50] Siew,L. and Hoe.L. (2016). “Strategic decision making in portfolio management with goal programming model”. American Journal of Operations Management and Information Systems, 1 (1): 34-38: https://www.sciencepublishinggroup.com/j/ajomis.

[51] Mohiti, M., Monsef, H., Anvari-Moghaddam, A., & Lesani, H. (2019). “Two-stage robust optimization for resilient operation of microgrids considering hierarchical frequency control structure”. IEEE Transactions on Industrial Electronics, 67(11), 9439-9449.

[52] Aouni, B., Ben Abdelaziz, F., & La Torre, D. (2012). The stochastic goal programming model: theory and applications. Journal of Multi-Criteria Decision Analysis, 19(5-6), 185-200.

[53] Aouni, B., Abdelaziz, F. B., & Martel, J. M. (2005). “Decision-maker's preferences modeling in the stochastic goal programming”. European Journal of Operational Research, 162(3), 610-618.

[54] Ji, X., Zhu, S., Wang, S., & Zhang, S. (2005). A stochastic linear goal programming approach to multistage portfolio management based on scenario generation via linear programming. IIE Transactions, 37(10), 957-969. DOI: 10.1080/07408170591008082

[55] Abdelaziz, F. B., El Fayedh, R., & Rao, A. (2009). A discrete stochastic goal program for portfolio selection: The case of United Arab Emirates equity market. INFOR: Information Systems and Operational Research, 47(1), 5-13.

[56] Wu, L., Wang, Y., & Wu, L. H. (2022). “Modeling index tracking portfolio based on stochastic dominance for stock selection”. The Engineering Economist, 67(3), 172-194. https://doi.org/10.1080/0013791X.2022.2047851 .

[57] Tsaur, R. C., Chiu, C. L., & Huang, Y. Y. (2021). “Fuzzy portfolio selection in COVID-19 spreading period using fuzzy goal programming model”. Mathematics, 9(8), 835. https:// doi.org/10.3390/math9080835

[58] Zadeh, L. A. (1965). “Fuzzy sets”. Information and control, 8(3), 338-353.

[59] Bellman, R. E., & Zadeh, L. A. (1970). “Decision-making in a fuzzy environment”. Management science, 17(4), 141-164.

[60] Parra, M. A., Terol, A. B., & Urıa, M. R. (2001). “A fuzzy goal programming approach to portfolio selection”. European Journal of Operational Research, 133(2), 287-297.

[61] Chen, L. H., & Tsai, F. C. (2001). “Fuzzy goal programming with different importance and priorities”. European journal of operational research, 133(3), 548-556.

[62] Bilbao,Terol, A., Pérez,Gladish, B., Arenas,Parra, M., & Rodríguez,Uría, M. V. (2006). “Fuzzy compromise programming for portfolio selection”. Applied Mathematics and computation, 173(1), 251-264..

[63] Abdelaziz, F. B., Aouni, B., & El Fayedh, R. (2007). “Multi-objective stochastic programming for portfolio selection”. European Journal of Operational Research, 177(3), 1811-1823.

[64] Bilbao, A., Arenas, M., Rodríguez, M. V., & Antomil, J. (2007). “On constructing expert betas for single-index model”. European journal of operational research, 183(2), 827-847.

[65] Mansour, N., Rebai, A., & Aouni, B. (2007). “Portfolio selection through imprecise Goal Programming model: Integration of the managers’ preferences”. Journal of Industrial Engineering International, 3(5), 1-8.

[66] Hasuike, T., Katagiri, H., & Ishii, H. (2009). “Portfolio selection problems with random fuzzy variable returns”, Fuzzy Sets and systems, 160(18), 2579-2596.

[67] Pakdel,A., Noroozzadeh,R., and Sadeghi,P. (2012). “Select Efficient Portfolio through Goal Programming Model”. Australian Journal of Basic and Applied Sciences (7): 189-194.

[68] Ghahtarani, A., & Najafi, A. A. (2013). “Robust goal programming for multi-objective portfolio selection problem”. Economic Modelling, 33, 588-592.

[69] Kazemi, A., Shakourloo, A., & Alinezhad, A. (2017). “A Fuzzy Goal Programming Model for Efficient Portfolio Selection”. Journal of Optimization in Industrial Engineering, 10(22), 61-71.

[70] Shapcott, J. (1992). “Index Tracking: Genetic Algorithms for Investment Portfolio Selection”. Edinburgh Parallel Computing Center, Technical Repor.t EPCC-SS92-24.

[71] Pahade, J. K., & Jha, M. (2021). “Credibilistic variance and skewness of trapezoidal fuzzy variable and mean–variance–skewness model for portfolio selection”. Results in Applied Mathematics, 11, 100159.

[72] Fiala, P., & Borovička, A. (2022). “Stock portfolio selection using aspiration level-oriented procedure: real case on the RM-SYSTEM Czech stock exchange”. Central European Journal of Operations Research, 30(2), 781-805.

[73] Zhou, W., Gu, Q., & Yu, D. (2020). Knowledge Framework and Evolution of Fuzzy Portfolio Research: A Bibliometric Analysis. Mathematical Problems in Engineering, 2020, 1-18.

[74] Donthu, N., Kumar, S., Mukherjee, D., Pandey, N., & Lim, W. M. (2021). How to conduct a bibliometric analysis: An overview and guidelines. Journal of Business Research, 133, 285-296.

[75] Zhou, W., Gu, Q., & Yu, D. (2020). Knowledge Framework and Evolution of Fuzzy Portfolio Research: A Bibliometric Analysis. Mathematical Problems in Engineering, 2020, 1-18.

[76] Zhou, W., Zhu, W., Chen, Y., & Chen, J. (2022). “Dynamic Changes and Multi-dimensional Evolution of Portfolio Optimization”. Economic Research-Ekonomska Istraživanja, 35(1), 1431-1456 .DOI: 10.1080/1331677X.2021.1968308

[77] Ghanbari, H., Safari, M., Ghousi, R., Mohammadi, E., & Nakharutai, N. (2023). “Bibliometric analysis of risk measures for portfolio optimization”. Accounting, 9 (2), 95-108.

[78] Colapinto, C., La Torre, D., & Aouni, B. (2019). “Goal programming for financial portfolio management: a state-of-the-art review”. Operational Research, 19(3), 717-736.