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On the modulation of exact structural solitons in interacting waves model

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Abstract: The nonlinear model for short and long interacting waves is considered one of the most important mathematical systems that studies the interaction between complex types of waves in different media and environments. The exact structure solution are produced. The dynamical energy equation is presented for studying stable and unstable solution conditions. Some stable and unstable solutions are obtained in the form of periodic envelopes, explosive and stationary soliton solutions. The obtained solutions can be used in space plasma, ocean waves, regenerative infectious for people crowds and economic viability resulting from controlling infectious these problems.

Keywords: Long–short-wave interaction system, solitary solutions, periodic envelopes.

1 Introduction

Nonlinear evolution equations (NLEEs) are commonly employed to describe complicated physical phenomena in the world around us since the environment is fundamentally nonlinear [1–5]. These equations regulate a large number of mathematical physics problems. In nonlinear science, finding analytical solutions is essential to understanding and characterising such physical processes. Dispersion, dissipation, reaction, diffusion and convection are nonlinear wave processes that are crucial to understanding nonlinear wave equations [6]. The pursuit of solutions for NLEEs has emerged as a very rigorous and dynamic field of study [7–9].

Benney [10] introduced broad theory of interactions between short and long waves is taken into consideration in the nonlinear long-short wave interaction systems (NLSWIS). These processes represent the nonlinear dynamical interaction of low-frequency long waves and high-frequency short waves. [11]. Finding fundamental physical relationships that spur more research into the different nonlinear interactions behind the general solution structure such as analytical, dark, and approximate solutions is incredibly inspiring. Subsequently, two waves interacting in the following domains: lower hybrid oscillation, lightwave, acoustic wave, electrostatic ion cyclotron wave, and plasma oscillation, etc. This mechanism was also described mathematically using (NLSWIS) [12].

The following nonlinear long-short wave interaction system is the subject of this investigation [13–15]:

$$\begin{aligned} &\psi_t + (|\,\phi\,|^2)_{xx} + \psi_x = 0\,, \\ &i\phi_t - \psi\,\phi + \phi_{xx} = 0, \end{aligned} \tag{1}$$

where $\psi(x,t)$ is a real function, which represents the long longitudinal wave. Whereas ϕ is a complex function, which denotes the slowly varying envelope of the short transverse wave. We introduced the exact structural solutions for model (1). We present the corresponding the dynamical energy equation which used for studying stable and unstable solution conditions. We also provide some

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stable and unstable solutions in the form of periodic envelopes, explosive and stationary soliton solutions. The solutions that are achieved have applications in the ocean waves, space plasma, regenerative infectious disease management for crowded areas, and economic viability that arises from managing infectious diseases.

The article is arranged as follows: Sec. 2 the mathematical analysis corresponding to nonlinear long-short wave interaction system. We present the exact structure solution. We also introduce the dynamical energy equation for studying stable and unstable solution conditions. Sec. 3 illustrates the explantation for the provided solutions. Sec. 4 provides the study's conclusion.

2 Mathematical analysis

Using wave transformation

$$\phi(x,t) = M(\Gamma)e^{i\Theta}; \psi(x,t) = \chi(\Gamma) \tag{2}$$

$$\Gamma = \omega\, x + \delta\, t,\ \Theta = Kx + \Omega t,$$

K, δ, Ω and ω are constants, yields

$$K^2(-M(\Gamma)) + \omega^2 M''(\Gamma) - \chi(\Gamma)M(\Gamma) - \Omega M(\Gamma) = 0 \tag{3}$$

$$2\omega(\omega - 2K\omega)M(\Gamma)\chi'(\Gamma)M'(\Gamma) = 0. \tag{4}$$

Solving the last equation provides

$$\chi(\Gamma) = -(\frac{\omega}{\delta + \omega})M(\Gamma)^2.$$

Eq. (3) becomes

$$-K^2 M(\Gamma) + \frac{\omega M(\Gamma)^3}{\omega - 2K\omega} + \omega^2 M''(\Gamma) - \Omega M(\Gamma) = 0. \tag{5}$$

On the other hand we have

$$2k\omega M'(\Gamma) + \delta M'(\Gamma) = 0, \tag{6}$$

with constraint equation

$$\Omega = \frac{\omega^2}{\omega^2 - 2K\omega^2} - K^2.$$

Eq. (5) illustrates an energy equation

$$-\frac{M(\Gamma)^2}{2\left(\omega^2 - 2K\omega^2\right)} + \frac{M(\Gamma)^4}{4\omega(\omega - 2K\omega)} + \frac{1}{2}M'(\Gamma)^2 = 0. \tag{7}$$

The corresponding exact solutions of model is

$$M(\Gamma) = \frac{2\sqrt{2}\exp\left(\frac{i\sqrt{-\omega^2(\omega-2K\omega)}\sqrt{\frac{\omega^2}{\omega^2-2K\omega^2}}(x\omega-2Kt\omega)}{\omega^2\sqrt{\omega-2K\omega}}\right)}{1+\exp(\frac{2i\sqrt{-\omega^2(\omega-2K\omega)}\sqrt{\Lambda}(x\omega-2Kt\omega)}{\omega^2\sqrt{\omega-2K\omega}})}. \tag{8}$$

$$\phi(x,t) = \frac{2\sqrt{2}}{1+\exp\left(\frac{2i\sqrt{-\omega^2(\omega-2K\omega)}\sqrt{\frac{\omega^2}{\omega^2-2K\omega^2}}(x\omega-2Kt\omega)}{\omega^2\sqrt{\omega-2K\omega}}\right)} \times \exp\left(i\left(t\left(\frac{1}{1-2K}-K^2\right)+Kx\right)-\sqrt{\frac{1}{1-2K}}(x-2Kt)\right). \tag{9}$$

$$\psi(x,t) = -\frac{8\exp\left(\frac{2i\sqrt{-\omega^2(\omega-2K\omega)}\sqrt{\frac{\omega^2}{1-2K}}(x\omega-2Kt\omega)}{\omega^2\sqrt{\omega-2K\omega}}\right)}{\left(1+\exp\left(\frac{2i\sqrt{-\omega^2(\omega-2K\omega)}\sqrt{\frac{\omega^2}{\omega^2-2K\omega^2}}(x\omega-2Kt\omega)}{\omega^2\sqrt{\omega-2K\omega}}\right)\right)^2}. \tag{10}$$

3 Results and Discussions

The interacting nonlinear model (1) with its effects of both short- and long-wave interactions is treated mathematically. A dynamical differential energy equation (7) with a restricted dispersion relation is obtained from equation (1). The dynamical kinetic energy, potential, and phase portrait behaviors for the examined equation (7) were depicted in the Figs. 1(a,b,c). As seen from potential and energy in figures 1(a,b,c), it was introduced that there are two unstable and one stable point. It was reported that, some solutions were expected in the form of periodic solitary, explosive and dissipative solitons.

Also, figures 2(a,b) show the effect of K and ω on the potential V. It was noted that K and ω affects both the solitary amplitude and width. To discuss the obtained solutions of equation (7) some graphs were plotted to describe the different wave behaviors that propagates in this system model. The breather periodic solution for equation (9) is obtained in Fig. 3(a). The rational explosive soliton with rapid increasing amplitude for equation (10) is shown in Fig. 3(b). Also, the three dimensional stationary soliton and contour form of equation (9) and (10) are depicted in Figs. 4(a,b).

On the other hand, the parametric effects on the obtained solutions are shown in Figs. 5(a,b). In figure Figs. 5(a) the compressive stationary soliton for $\phi(x,t)$ is obtained for $K = 0.1$, by increasing K the soliton reverse its direction and converts to rarefactive soliton with increasing amplitude and decreasing width. Finally, the compressive stationary soliton for $\psi(x,t)$ is obtained with reasing amplitude and reduced width as in shown in Fig. 5(b). In summary, the structural solitary nonlinear solutions for short and long interacting waves model modulated by the parametric model effects.

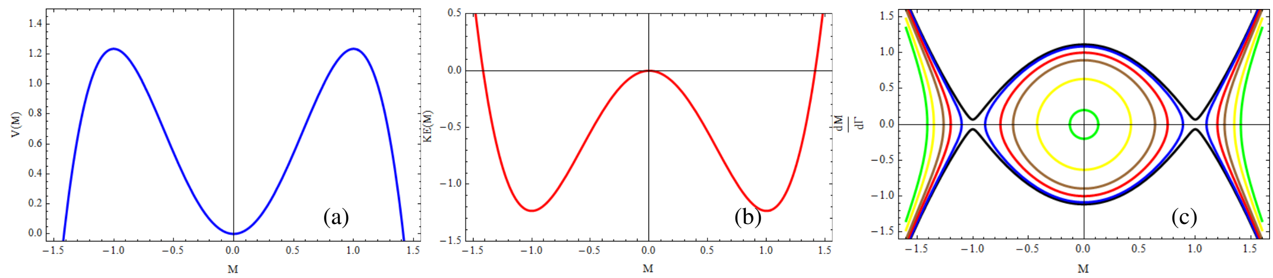

Fig. 1: (a) Variation of **V** with M (b) Variation of **KE** with M (c) Plot of **V** and $\frac{dM}{d\Gamma}$ with M; for the model parameters are $\omega = 1.4$ and $K = 0.3$.

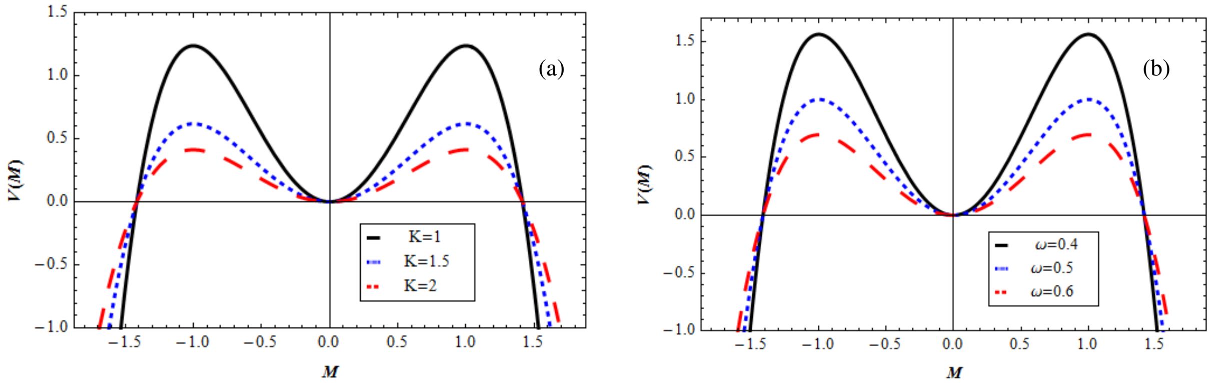

Fig. 2: (a) Variation of **V** with M for $\omega = 1.4$. (b) Variation of **V** with ω for $K = 0.3$.

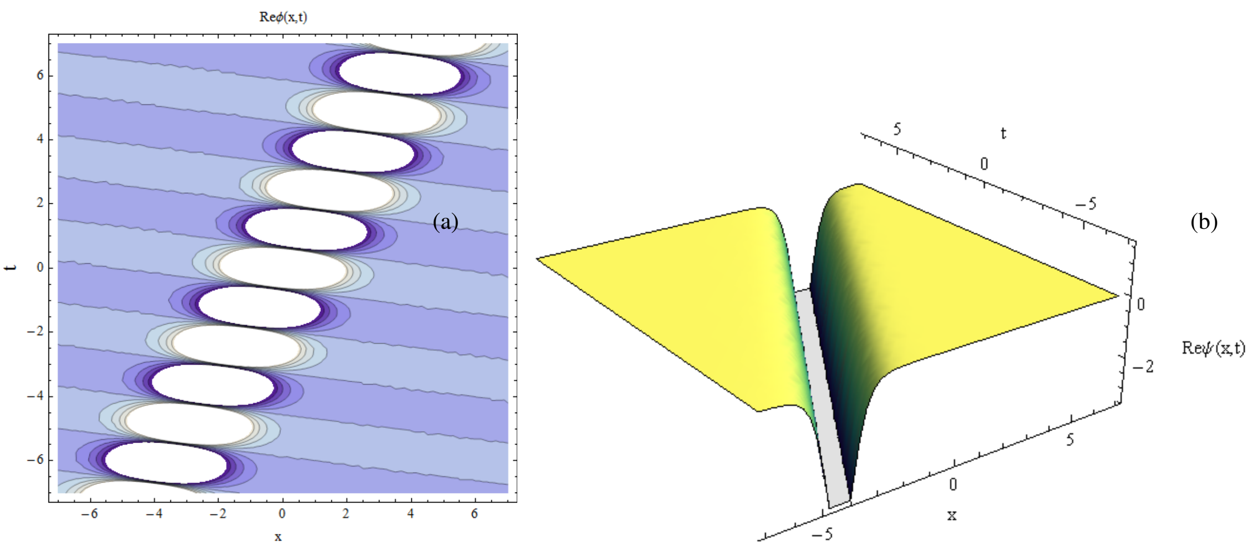

Fig. 3: (a) Contour plot of Variation of **Re**$\phi(x,t)$ with x,t and (b) Variation of **Re**$\psi(x,t)$ with x,t; where the model parameters are $\omega = 1.4$ and $K = 0.3$.

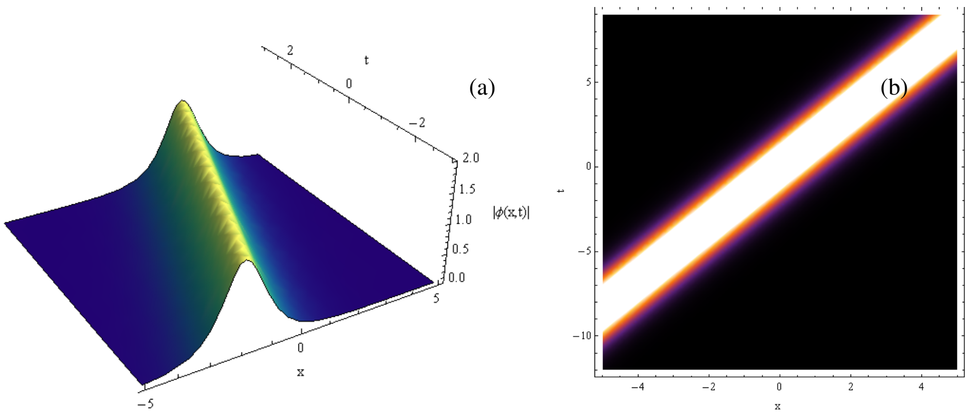

Fig. 4: (a) 3D plot of Variation of $|\phi(x,t)|$ with x,t and (b) Contour plot of variation $|\psi(x,t)|$ with x,t; where the model parameters are $\omega = 1.4$ and $K = 0.3$.

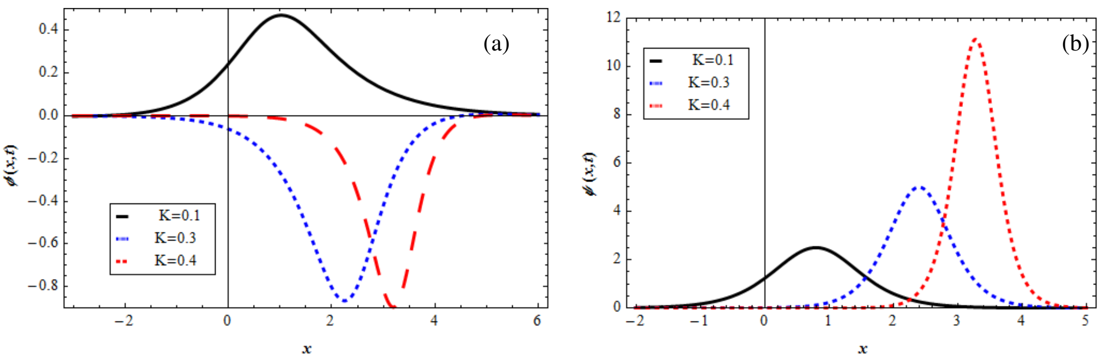

Fig. 5: (a) Variation of $|\phi(x,t)|$ with x,K for $t = 4, \omega = 1.4$ and (b) Variation of $|\psi(x,t)|$ with x,K for $t = 4, \omega = 1.4$.

4 Conclusion

The equations for the model of interacting waves have been reduced to a nonlinear form, which can be solved by an accurate dynamical energy equation. Phase plane analysis has been used to assess the stability of equation dynamics in order to determine the suitable solutions for this model. Periodic envelopes, explosive and stationary soliton solutions, and other stable and unstable solutions are produced. The solutions that are achieved have applications in auroral plasma, deep ocean interacting waves, microbial pathogens in humans body and shaped genetic variation in modern populations, and this has significant ramifications for both the management of infectious illnesses that promote economic viability and the expanding area of mathematical medical genetics.

Conflicts of Interest

The author declares no conflict of interests.

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