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A New Extended Exponentiated Lomax (EEtLx) Distribution for Life Data

*Braimah Joseph Odunayo** *and Fabio Mathias Correa*

Department of Mathematical Statistics and Actuarial Science, University of the Free State, South Africa

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Abstract: This paper proposes a new life distribution with both unbounded and bounded support. The proposed Extended Exponentiated Lomax (EEtLx) distribution is derived from the New Extended Exponentiated-G (NEET-G) family of Elgarhy, Haq, Gozel, and Nasir (2017). The maximum likelihood estimation method is used to estimate parameters of the proposed distribution and derive the relevant properties. The model is applied to patient relief times (minutes) after receiving a particular analgesic using different selection criteria (Log likelihood, Akaike Information Criteria (AIC), and Bayesian Information Criteria (BIC)). Resulting distributions are compared to well-established lifetime distributions in literature. The total time on test (TTT) plot is also used to determine whether the hazard rate function for this data increases or decreases, which subsequently reveals a decreasing trend. The proposed distribution remains the best fit compared to competing distributions. It is recommended that this distribution can be applied, not only in medical science, but also in reliability science, engineering, and economics fields.

Keywords: Life distribution, Reliability, Exponential

1 Introduction

Generalization of conventional distributions is an attractive component of the distribution theory where lifetime data models are developed by combining relevant lifetime distributions. Researchers have modeled lifetime data for a variety of distributions with unlimited support and exponential distribution generalizations and modifications. Some notable distribution families include the Topp Leone Kumaraswamy-G [1], the Kumaraswamy-G [2], Topp Leone-G [3], Odd Lindley-G family [4], Gompertz-G family [5], and Odd Frechet G family by [6]. A generalization of the exponential distribution based on family of distributions in this context was proposed by Elgarhy, Haq, Gözel, and Nasir (2017). The exponential (Ex) distribution has been generalized by many authors. For example, [7] proposed the exponentiated Ex (EEx) distribution, [8] proposed the beta Ex (BEx) distribution, [9] developed the Kumaraswamy Ex (KEx) distribution as a special case of the Kumaraswamy Weibull distribution and proposed the Kumaraswamy Weibull distribution.

The generalized odd Burr III (GOBIII) distribution was developed by [10] and is a continuous distribution with two extra shape parameters. The basic properties, such as generating functions, quantiles, entropy measurements, ordinary moments, and order statistics, were determined. In addition to deriving properties of truncated moments and hazard functions for the GOBIII-G distribution, three unique models from the family were also provided. To estimate the model parameters, they applied the maximum likelihood method.

Weibull-G class family of distributions with Truncated Cauchy Power were developed by [11]. The expansion of the density function, moments, incomplete moments (IMOs), residual life and reversed residual life functions, and entropy were among the statistical features of the family that were investigated. To show the applicability and adaptability of their recommended approach, they contrast the performances of their proposed estimators using actual data sets.

The Burr X distribution was developed using the "truncated-composed" scheme by [12] in order to inspire the creation of a new family of univariate continuous-type distributions known as the truncated Burr X generated family. They asserted that it offers greater modeling flexibility for any parental distribution and is straightforward mathematically.

* Corresponding author e-mail: braimahjosephodunayo@aauekpoma.edu.ng

The ”generalized inverted Kumaraswamy-G” family of distributions is introduced by [13] along with a unique generator function based on the inverted Kumaraswamy distribution. Among its mathematical characteristics that they describe are the quantile and generating functions, order statistics, ordinary and incomplete moments.

A class called the Marshall-Olkin odd Burr III family is proposed by [14] to generate continuous distributions and extract some of its statistical features. The percentile approach, least squares, and maximum likelihood were used to estimate the parameters. They use three engineering-related applications to demonstrate the model's utility.

A family proposed by [15] incorporates additional shape and dispersion parameters, along with the skewness and weight of the tails, based on a reduced form of the generalized Fréchet distribution. The primary functions of the new family, the stress-strength parameter, various functional series expansions, incomplete moments, and entropy measures were also examined, along with the estimation of theoretical and practical parameters, bivariate extensions via copulas, and model parameter estimation.

A new family of distributions type II exponentiated half logistic was designed by [16]. Explicit expressions for the quantile function, moments, probability weighted moments, mean deviation, order statistics, and Rényi entropy are among the mathematical properties of this new family that were derived. By using the maximum likelihood, the distribution's parameter estimate was investigated.

A family of distributions was presented by [17] and was based on the Lehmann type II distribution, the inverse exponential distribution, the T-X transformation, and the odds function. They look into its broad mathematical characteristics, such as entropies, order statistics, quantile function, moments, and moment generating function. Both the greatest likelihood and the least squares methods were used to estimate the parameters.

While there are many distributions with unbounded support in survival and reliability studies in literature, there are only a few with finite support. Furthermore, in the literature on dependability, there does not appear to be probability distribution with both unbounded and bounded support. The purpose of this research is to introduce a new life distribution that may be employed in a variety of situations with unbounded and bounded support.

1.1 Motivation of the Lomax Distribution

The Lomax distribution is motivated by the fact that it is a heavy-tail probability distribution that finds applications in medicine, agriculture, engineering, business, economics, actuarial science, reliability, queuing theory, and Internet traffic modeling [18]. A Pareto Type I distribution that has had its shape parameter moved to zero is known as the Lomax distribution. One particular instance of the generalized Pareto distribution is the Lomax distribution.

A particular instance of the beta prime distribution is the Lomax distribution, where the scale parameter $\lambda = 1$. The F-distribution becomes a Lomax distribution with shape parameter $\alpha = 1$ and scale parameter $\lambda = 1$. The Lomax distribution is extended by the exponential to support on a bounded interval. A Lomax variable (shape = 1.0, scale = λ) logarithm is distributed logistically, with shape $log\lambda$ and scale 1.0. This suggests that a log-logistic distribution is equivalent to a Lomax(shape = 1.0 , scale = λ)-distribution. The lomax distribution also has the ability to assume a variety of real-life data types, including steady, declining, rising, and upside-down data. Its density function may be symmetrical or right-skewed. Additionally, the hazard rate function may be declining, inverted, or constant. The cumulative distribution function of a Lomax distribution is presented in a closed form and also its hazard rate function [19]. Type II Topp Leone-power Lomax (TIITL-PL) distribution as developed by [20] has the ability to fit several kinds of real life data sets. The outcome demonstrates the flexibility of the Lomax model as an alternative to various real life data fit and analysis. [21] demonstrated the ability of a Lomax distribution to model real life data using percentile and bootstraping. The Marshall-Olkin Power Lomax distribution as examined by [22] also resolved that Lomax distribution can fit several real life data set.

All these unique characteristics of Lomax distribution have motivated us to extend the family of Exponentiated distribution to Lomax distribution.

1.2 The Advantage of the New Model

In literature-based survival and reliability research, there are many distributions with unbounded support, but very few with finite support. On the other hand, different data in practical applications have semi-bounded or bounded support. Moreover, there doesn't seem to be a probability distribution with both bounded and unbounded support in the reliability literature. As a result, we may model the distribution of this kind of data by using this new distribution. Choosing an appropriate distribution improves modeling effectiveness. The goal of this research is to present a novel life distribution with both bounded and unbounded support that can be used in a range of scenarios. Furthermore, improved modeling can also enhance the statistical model's practical performance.

The organization of the remaining part of this paper is structured as follows. Section 2 contains the methodology. Section 3 has the derivation of the probability denity funcion (pdf) and the cumulative distribution function (cdf) of the proposed Extended Exponentiated Lomax (EEtLx) distribution. Section 4 contains the distribution's derived mathematical properties. Section 5 discussed application to real-life data sets, while Section 6 concluded followed by acknowledgements and references.

2 Methodology

New Extended Exponentiated-G (NEET-G) family was proposed by [23], which encompasses a broader range of distributions. The NEET-G family's cumulative distribution function (cdf) and probability density function (pdf) is given by:

$$F_{EtE-G}(x;\theta,\alpha,\lambda,\gamma)=\left\{1-\left[1-[G(x;\gamma)]\right]^{\alpha\lambda}\right\}^{\theta} \tag{1}$$

and

$$f_{EtE-G}(x;\theta,\alpha,\lambda,\gamma)=\theta\alpha\lambda g(x;\gamma)\left[1-\left[G(x;\gamma)\right]\right]^{\alpha\lambda-1}\left\{1-\left[1-G(x;\gamma)\right]^{\alpha\lambda}\right\}^{\theta} \tag{2}$$

where $\theta,\alpha,\gamma\geq 0$ are the shape parameters, respectively.
The cdf and pdf of Lomax distribution are given respectively as:

$$G(x;\beta,\sigma)=1-(1+\beta x)^{-\sigma} \tag{3}$$

and

$$g(x;\beta,\sigma)=\sigma\beta(1+\beta x)^{-\sigma-1} \tag{4}$$

3 The proposed Extended Exponentiated Lomax (EEtLx) distribution

In this section, the Extended Exponentiated Lomax (EEtLx) distribution is derived. Some of its properties such as: moments, quantile, moment generating function, hazard function, survival function, odd function, distribution of order statistic, parameters estimate by means of maximum likelihood method are also derived. To obtain the new life distribution, (3) is inserted into (1) and is given as:

$$F_{EEtLx}(x;\theta,\alpha,\lambda,\beta,\sigma)=[1-(1+\beta x)^{-\sigma\alpha\lambda}]^{\theta} \tag{5}$$

On differentiating (5), we obtained the pdf of the new model as

$$f_{EEtLx}(x;\theta,\alpha,\lambda,\beta,\sigma)=\theta\alpha\lambda\sigma\beta(1+\beta x)^{-\sigma\alpha\lambda-1}[1-(1+\beta x)^{-\sigma\alpha\lambda}]^{\theta-1} \tag{6}$$

where $x\geq 0$ and $\theta,\alpha,\lambda,\beta,\sigma>0$.
The pdf and cdf plot for the newly proposed distribution are shown in Figure 1.

3.1 Important representation

Using binomial expansion on the last term in (6), we can represent the model as:

$$\left[1-(1+\beta x)^{-\sigma\alpha\lambda}\right]^{\theta-1}=\sum_{i=0}^{\infty}\frac{(-1)^{i}\Gamma(\theta)}{i!\Gamma(\theta-i)}(1+\beta x)^{-\sigma\alpha\lambda i} \tag{7}$$

and

$$f_{EEtLx}(x;\theta,\alpha,\lambda,\beta,\sigma)=\theta\alpha\lambda\beta\sigma\sum_{i=0}^{\infty}\frac{(-1)^{i}\Gamma(\theta)}{i!\Gamma(\theta-i)}(1+\beta x)^{-\sigma\alpha\lambda(i+1)-1} \tag{8}$$

Using (8), we can derive the properties for the EEtLx distribution.

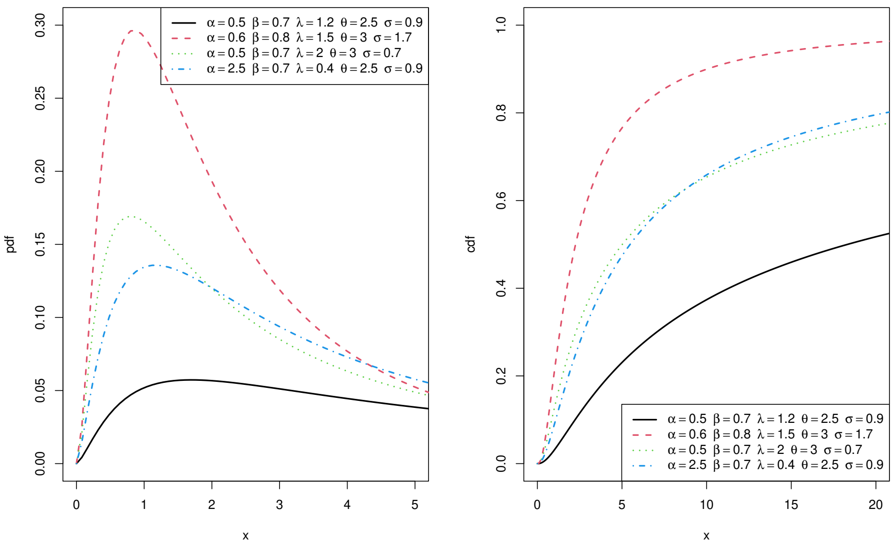

Fig. 1: Probability density function and cumulative density function of the EEtLx distribution

4 Mathematical properties

The major mathematical properties of the EEtEx family are derived and presented in this section.

4.1 Moments

$$E(X^r) = \int_0^\infty x^r f(x)dx \tag{9}$$

$$E(X^r) = \theta\alpha\lambda\beta\sigma \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(\theta)}{i!\Gamma(\theta-i)} \int_0^\infty x^r (1+\beta x)^{-\sigma\alpha\lambda(i+1)-1} dx \tag{10}$$

Suppose Y is a random variable with Lomax distribution and parameters β and σ, the r^{th} moment of Y is given by:

$$E(Y') = \left(\frac{\sigma}{B'}\right) B(r+1, \sigma - r) \tag{11}$$

Suppose X is a random variable that assumes the EEtLx distribution. From equation (10), the r^{th} moment of X is therefore obtained as:

$$E(X') = \alpha\theta\lambda\sigma\beta K \left[\frac{\sigma\alpha\lambda(i+1)}{\beta'}\right] B(r+1, \sigma\alpha\lambda(i+1) - r) \tag{12}$$

where $K = \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(\theta)}{i!\Gamma(\theta-i)}$

4.2 Quantile function

The EEtLx distribution is simply approximated by inverting (5) as follows: If u is a random uniform distribution $U(0,1)$, then the equation's solution is:

$$[1-(1+\beta x)^{-\sigma\alpha\lambda}]^{\theta}=u$$
$$u^{\frac{1}{\theta}}=1-(1+\beta x)^{-\sigma\alpha\lambda}$$
$$1-u^{\frac{1}{\theta}}=1-(1+\beta x)^{-\sigma\alpha\lambda}$$
$$(1-u^{\frac{1}{\theta}})^{\frac{-1}{\sigma\alpha\lambda}}=1+\beta x$$
$$(1-u^{\frac{1}{\theta}})^{\frac{-1}{\sigma\alpha\lambda}}-1=\beta$$

Then, the equation (13) is the quantile function of the EEtLx distribution.

$$X=Q(u)=\frac{1}{\beta}\left[(1-u^{\frac{1}{\theta}})^{\frac{-1}{\sigma\alpha\lambda}}-1\right] \tag{13}$$

And, the equation (14) is the median of the EEtLx distribution is:

$$Q(0.5)=\frac{1}{\beta}\left[\left(1-0.5^{\frac{1}{\theta}}\right)^{\frac{-1}{\sigma\alpha\lambda}}-1\right] \tag{14}$$

4.3 Hazard function

As little time Δt approaches zero, the hazard function is defined as the instantaneous failure rate of an item or component of a product and is stated as:

$$h(x;\alpha,\beta,\lambda,\theta,\sigma)=\frac{\theta\alpha\lambda\sigma\beta(1+\beta x)^{-\sigma\alpha\lambda-1}[1-(1+\beta x)^{-\sigma\alpha\lambda}]^{\theta-1}}{1-[1-(1+\beta x)^{-\sigma\alpha\lambda}]^{\theta}} \tag{15}$$

4.4 Survival function

The survival function is defined as the probability that an item will not fail before a certain time (t). It is represented as $S(x;\alpha,\beta,\lambda,\theta,\sigma)=1-F(x;\alpha,\beta,\lambda,\theta,\sigma)$, then:

$$S(x;\alpha,\beta,\lambda,\theta,\sigma)=1-[1-(1+\beta X)^{-\sigma\alpha\lambda}]^{\theta} \tag{16}$$

4.5 Odds function

The odds function is derived using the relation:

$$O(x;\alpha,\beta,\lambda,\theta,\sigma)=\frac{F(x;\alpha,\beta,\lambda,\theta,\sigma)}{S(x;\alpha,\beta,\lambda,\theta,\sigma)}$$

$$O(x;\alpha,\beta,\lambda,\theta,\sigma)=\frac{[1-(1+\beta X)^{-\sigma\alpha\lambda}]^{\theta}}{1-[1-(1+\beta X)^{-\sigma\alpha\lambda}]^{\theta}} \tag{17}$$

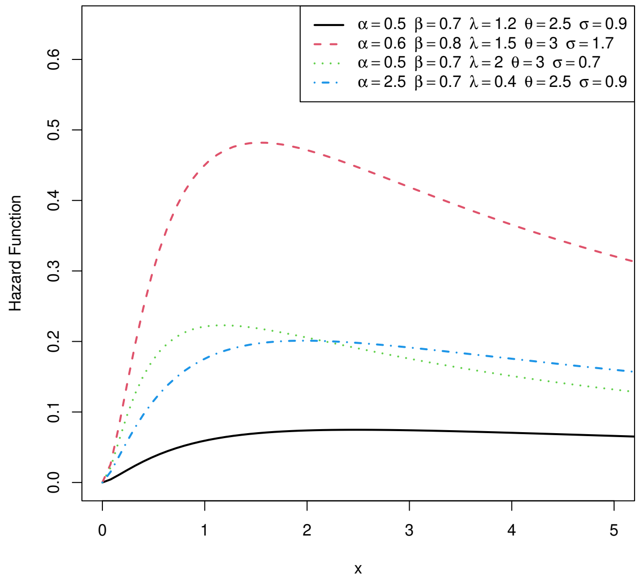

Fig. 2: Harzard function of the EEtLx distribution

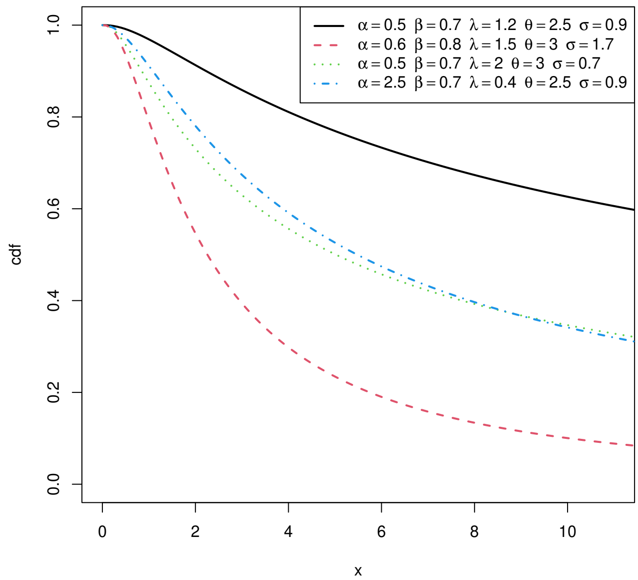

Fig. 3: Plots of the cdf of the EEtLx distribution

4.6 Distribution of Order Statistics

Let $X_1, X_2, \cdots, X_n$ be n independent random variable from the extended exponentiated Lomax distribution. Also, let $X_1 \leq X_2 \leq \cdots \leq X_n$ be the corresponding order statistic. If $F_{r:n}(x)$ and $f_{r:n}(x)$, such that $r = 1, 2, 3, \cdots, n$ signify the cdf and pdf of the r^{th} order statistics of $X_{r:n}$. The pdf of the r^{th} order statistics of $X_{r:n}$ is given by:

$$f_{r:n}(x;\alpha,\beta,\lambda,\theta,\sigma) = \frac{1}{B(r,n-r+1)} \sum_{i=0}^{n-r} \frac{(-1)^i \Gamma(n-r+1)}{i!\Gamma(n-r+1-i)} [F(x)]^{r+i-1} f(x) \tag{18}$$

Using the cdf and pdf of extended exponentiated Lomax distribution in (5) and (6), we have

$$f_{r:n}(x;\alpha,\beta,\lambda,\theta,\sigma) = \frac{\alpha\beta\lambda\theta\sigma}{B(r,n-r+1)} \sum_{i=0}^{n-r} \frac{(-1)^i \Gamma(n-r+1)}{i!\Gamma(n-r+1-i)} (1+\beta x)^{-\sigma\alpha\lambda-1} [1-(1+\beta X)^{-\sigma\alpha\lambda}]^{\theta(i+1)-1} \tag{19}$$

Equation (19) is the r^{th} order statistic of the extended exponential Lomax distribution. The minimum order statistics is then obtained by setting r = 1 in equation (18) as:

$$f_{1:n}(x;\alpha,\beta,\lambda,\theta,\sigma) = \eta\alpha\beta\lambda\theta\sigma \sum_{i=0}^{n-1} \sum_{j=0}^{\infty} \frac{(-1)^{i+j} \Gamma[\theta(i+1)]\Gamma(n)}{j!\Gamma[\theta(i+1)-j]\Gamma(n-1)} (1+\beta x)^{-\sigma\alpha\lambda(j+1)-1} \tag{20}$$

The maximum order statistics is also obtained by setting $r = n$ in (18) as

$$f_{n:n}(x;\alpha,\beta,\lambda,\theta,\sigma) = \eta\alpha\beta\lambda\theta\sigma \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(\theta)}{j!\Gamma(\theta-j)} (1+\beta x)^{-\sigma\alpha\lambda(j+1)-1} \tag{21}$$

4.7 Parameter estimation

In this section, the parameters of the EEtLx distribution are estimated using the maximum likelihood estimation (MLE) method. For a random sample X, with $X_1, X_2, \cdots, X_n$ of size n from the EEtLx $(\alpha, \beta, \theta, \lambda, \sigma)$, the log-likelihood function $L(\alpha, \beta, \theta, \lambda, \sigma)$ of (6) is given by:

$$log(L) = nlog(\theta) + nlog(\alpha) + nlog(\lambda) + nlog(\beta) + nlog(\sigma) - (\sigma\alpha\lambda+1) \sum_{i=1}^{n} log(1+\beta X_i) + (\theta-1) \sum_{i=1}^{n} log[1-(1+\beta X_i)^{-\sigma\alpha\lambda}] \tag{22}$$

Differentiating equation (22) with respect to each parameter and equating to zero, we have

$$\frac{\partial log(L)}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n} log[1-(1+\beta X_i)^{-\sigma\alpha\lambda}] \tag{23}$$

$$\frac{\partial log(L)}{\partial\alpha} = \frac{n}{\alpha} - \sigma\lambda \sum_{i=1}^{n} log(1+\beta X_i) - (\theta-1) \sum_{i=1}^{n} \left[\frac{(1+\beta X_i)^{-\alpha} log(1+\beta X_i)}{1-(1+\beta X_i)^{-\sigma\alpha\lambda}} \right] \tag{24}$$

$$\frac{\partial log(L)}{\partial\lambda} = \frac{n}{\lambda} - \sigma\alpha \sum_{i=1}^{n} log(1+\beta X_i) - (\theta-1) \sum_{i=1}^{n} \left[\frac{(1+\beta X_i)^{-\lambda} log(1+\beta X_i)}{1-(1+\beta X_i)^{-\sigma\alpha\lambda}} \right] \tag{25}$$

$$\frac{\partial log(L)}{\partial\sigma} = \frac{n}{\sigma} - \alpha\lambda \sum_{i=1}^{n} log(1+\beta X_i) - (\theta-1) \sum_{i=1}^{n} \left[\frac{(1+\beta X_i)^{-\sigma} log(1+\beta X_i)}{1-(1+\beta X_i)^{-\sigma\alpha\lambda}} \right] \tag{26}$$

$$\frac{\partial log(L)}{\partial\beta} = \frac{n}{\beta} - (\sigma\alpha\lambda+1) \sum_{i=1}^{n} \frac{X_i}{1+\beta X_i} + \sigma\alpha\lambda(\theta-1) \sum_{i=1}^{n} \left[\frac{X_i(1+\beta X_i)^{-\sigma\alpha\lambda-1}}{1-(1+\beta X_i)^{-\sigma\alpha\lambda}} \right] \tag{27}$$

Equations (23), (24), (25), (26), and (27) do not have their simple form and therefore, are intractable, thus, iterative procedures must be used to obtain parameter estimates.

5 Application to real-life data set

The efficiency of the proposed life distribution is demonstrated in this section using real-life data sets. The data set contains information about the relief times (in minutes) of patients who received an analgesic over the course of their lives (Ibrahim, Hammed and Olalekan 2022). The data set consists of twenty (20) observations as presented in Table (1):

Table 1: Data set of relief times (in minutes)

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0

To start, we must determine whether the Extended Exponentiated Lomax (EEtLx) distribution fits the data. The empirical and theoretical density functions (pdf), theoretical distribution function (cdf), quantile (Q-Q), and probability (P-P) plots are all utilized to determine whether the data fits the theoretical distribution. The findings of the tests of goodness of fit were acceptable, indicating that the underlined distribution was well-fit (Figure 4), where $\hat{\alpha} > 0$, $\beta > 0$, $\hat{\theta} > 0$, $\hat{\sigma} > 0$ and $\lambda > 0$.

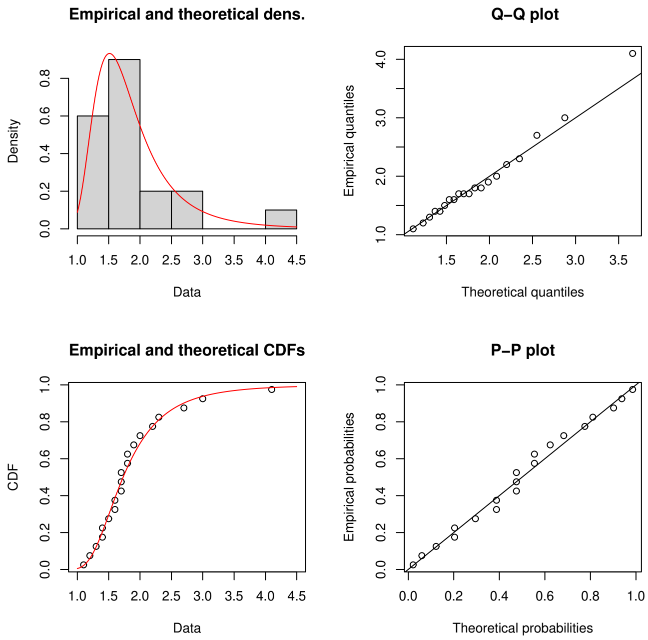

Fig. 4: Fitted cdf, pdf, Q-Q and P-P plots of the data set.

The Kumaraswamy Exponentiated Lomax (KEtLx) distribution with five parameters, Topp Leone Kumaraswamy Lomax (TLKLx) distribution with five parameters, Exponentiated Lomax (ExLx) distribution with four parameters and Lomax (Lx) distribution with two parameters are used as competing models in this study. The pdf of the competing distributions are listed below.

Kumaraswamy Exponentiated Lomax (KEtLx) distribution (El-Batal and Kareem 2014)

$$f(x) = \alpha\theta\beta\lambda(1+\beta X)^{-(\sigma+1)}[1-(1+\beta X)^{-\sigma}]^{\alpha\lambda-1}\{1-[1-(1+\beta X)^{-\sigma}]^{\alpha\lambda}\}^{\theta-1}$$

The Topp Leone Kumaraswamy Lomax (TLKLx) distribution (Sule et al. 2021)

$$f(x) = \frac{2\alpha\theta\lambda\sigma\beta}{(1+\beta X)^(\sigma+1)}[1-(1+\beta X)^{-\sigma}]^{\alpha-1}\{1-[1-(1+\beta X)^{-\sigma}]^{\alpha}\}^{2\lambda-1}\{1-[1-(1-(+\beta X)^{-\sigma})^{\alpha}]^{2\lambda}\}^{\theta-1}$$

Exponentiated Lomax (ExLx) distribution (Salem 2014)

$$f(x) = \theta\sigma\beta(1+\beta X)^{-(\sigma+1)}[1-(1+\beta X)^{-\sigma}]^{\theta-1}$$

Lomax (Lx) distribution (Parmil K, Kirandeep K and Jaspreet K 2018)

$$f(x) = \sigma\beta(1+\beta X)^{-(\sigma+1)}$$

Table 2: The MLEs and Information Criteria of the models based on the data set.

Models	Parameters								
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$	LL	AIC	BIC	p-Value
EEtLx	34.2138	0.0739	45.1243	49.2290	0.0201	-16.0408	42.0816	47.0602	0.0024*
TLKLx	4.9977	0.6116	3.2129	1.4974	5.5974	-16.5950	43.1901	48.1688	0.4475
KExLx	6.1755	0.0801	3.8225	12.4937	1.4593	-17.6547	45.3094	50.2880	0.4671
ExLx	-	0.2222	14.0376	8.9812	-	-18.5090	43.0180	46.0052	0.0494*
Lx	-	0.0586	-	8.4837	-	-33.8818	71.7636	73.7551	0.5183

The estimated parameters, log-likelihood, AIC, and BIC values are provided in Table (2). The goodness of fit values for the EEtLx distribution are clearly the lowest, indicating that the proposed life distribution provides a better fit. Figure (5) shows how the data is correctly skewed and can explain both the proposed and competing life distributions. Figure (4) also shows that the EEtLx distribution fits the data well, as shown by the QQ-plot and PP-plot.

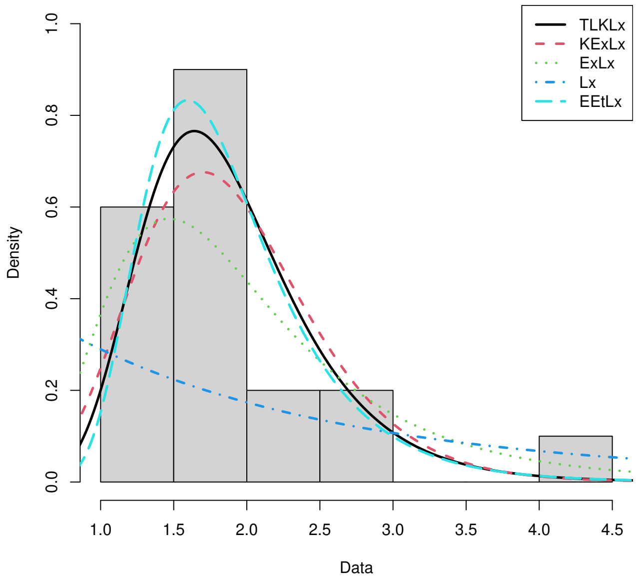

Fig. 5: Fitted pdfs of the TLKLx, KExLx, ExLx, Lx and EEtLx models for the data set.

6 Conclusion

We propose the Extended Exponentiated Lomax (EEtLx) distribution, which is derived from Elgarhy, Haq, Gözel, and Nasir's (2017) New Extended Exponentiated-G (NEET-G) family of distributions. Monte-Carlo simulations estimate the parameters of the proposed distribution using the maximum likelihood method. Moments, mean, quantile function, order statistics, hazard rate function, odd function, and parameter estimation are among the properties derived in the study. The proposed distribution is compared to existing distributions after being applied to a real-life data set. The proposed life distribution is right-skewed, and different shapes are obtained by changing parameter values. The EEtLx is more efficient than the competing distribution pdfs for different shape parameter values from a real-life data set, as shown in the Tables and Graphs. We conclude that EEtLx provides a better fit than the competing distributions based on the values of model selection criteria (Log-likelihood, Akaike Information Criteria, and Bayesian Information Criteria). Therefore, the proposed lifetime distribution compares well with other well-established lifetime distributions. This proposed distribution is recommended to be applied, not only in medical science, but also in reliability science, engineering, and economics fields.

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