Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/180319

The Analysis of Black-Scholes Model of Option Pricing with Time-Varying Parameters on Share Prices for Capital Market

Innocent U. Amadi[1], *Nmerukini Akani*[2], *Chisara P. Ogbogbo*[3,*], *Simon I. Aboko*[1] *and Adaobi M. Udoye*[4]

[1]Department of Mathematics/ Statistics, School of Science and Technology, Captain Elechi Amadi Polytechnic, Rumuola Portharcourt, Nigeria
[2]Department of Mathematics/Statistics, Faculty of Natural Sciences, Ignatius Ajuru University of Education, Rumuolumeni, Portharcourt, Nigeria
[3]Department of Mathematics, College of Basic and Applied Sciences, University of Ghana. Legon. Accra. Ghana
[4]Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti, Nigeria

Received: 21 Dec. 2023, Revised: 21 Feb. 2024, Accepted: 23 Mar. 2024
Published online: 1 May 2024

Abstract: This paper studied the perception of European option which is geared towards valuation of financial assets at a prescribed time in the future. In particular, the analytic formula of Black-Scholes model was considered for share prices of Fidelity and Access banks which gave closed form prices of call options. The explicit price on the variations of maturity days is found accordingly. The closed form prices of both banks were compared. The simulation results show that: the share price determines the value of call option prices. An increase on the maturity days dominantly increases the value of call option for both Fidelity Bank, Access Bank and their future merger Bank. Merging of the two banks improved the value of call option prices. Fidelity bank has a good maximum value of call option prices during the period of investments, Kolmogorov ?Smirnov (KS) test shows that the two call option prices (Fidelity and Access) do not come from a common distribution, the normality test of both banks are not significant. The results presents to the Banks management, a basis for taking vital decisions, depending on the levels of their investments.

Keywords: Black-Scholes Model, Financial Asset, Mathematical finance, Share Prices, Kolmogorov-Smirnov test

1.1 Introduction

The modeling of financial quantities or variables has recorded increased admiration, due to its copious submissions in the new emergent field of science and technology. Such applications include the following: valuing of financial derivatives which strengthens the financial power of every investment and enhances efficiency for option traders, the stock volatility which helps investors to measure rise and fall of stock prices, insurance of risky investments, etc. The case of option pricing, requires some estimation in day-to-day activities in the financial markets, with respect to time of maturity. Scholars who have dealt comprehensively in financial markets who obtained interesting results, include [1]-[4]. Understanding financial variables, its dynamic relationships and how it affects investors or traders is of great importance. It has therefore become imperative to understand the dynamic nature of the physical problem to be solved, and also know why such method is good. Physical problems of this nature need analytic approach, which can give exact solutions. This is necessary for proper mathematical predictions. On the general note, it is essential to study financial market related problems, well formulated and accurate analytical solutions in order to stimulate realistic results; therefore, the analytical method is adopted based on Black-Scholes model for European call options.

There is enormous interest in financiers, mathematicians and statisticians over the Black-Scholes (BS) model discovered by [5] to analyze the European option on a stock market (which does not pay a dividend during the option's life) as well as solving PDEs in Sobolev spaces.

* Corresponding author e-mail: cpnogbogbo@ug.edu.gh

For instance, [6] studied implied volatility and implied risk- free rate of return in solving systems of Black-Scholes equations. In their research they established that option prices provide important information for market participants for future expectations and market policies. In the same vein [7] analyzed Black-Scholes formula for the valuation of European options; Hermite polynomials were applied. They concluded that BS formula can easily be achieved devoid of the use of partial differential equation. In another study of BS [8] considered the BS terminal value problem and observed that their proposed method is better, and simpler than the previous methods. In the work of [9], time varying factor was incorporated in the explicit formula for different aspect of options with the aim of providing exact solution for dividend paying equity of option. Details of more financial models can be seen in the works of [10]-[15].

This study is targeted at solving BS model for estimation of call option prices which can be used for investment decisions for Fidelity and Access banks. To establish its effects in financial market; therefore, different variations of maturity days and other comparisons were made. Some statistical tests were done to adequately understand the dynamics of financial market as it affects Fidelity and Access banks. This is a novel contribution in this dynamics area of mathematical finance.

The paper is arranged in the following ways: Section 2.1 Mathematical framework of Black-Scholes model, results and discussion is seen in 3.1. This paper is concluded in Section 4.1.

2.1 Mathematical Framework of Black-Scholes Model

The Black-Scholes model is made up of seven assumptions: the asset price has characteristics of a Brownian motion with and as constants, the transaction costs or taxes are not allowed, the entire securities are absolutely divisible, dividend is not permitted during the period, riskless arbitrage opportunities are not acceptable, the security trading is constant, the option is exercised at the time of expiry for both call and put options.

Consider a market where the underlying asset price v, $0 \leq t \leq T$ on a complete probability space $(\Omega, f, \wp)$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dW(t). 0 < v < \infty \tag{1}$$

Theorem 1.1 (Ito's formula) Let $(\Omega, \beta, \alpha, F(\beta))$ be a filtered probability space. $X = \{X_t, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, F(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \mathscr{R}$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \Box))$

$$du(t, X(t)) = \left(\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right)dt + f\frac{\partial u}{\partial x}dW(t)$$

Using theorem 1.1 and equation (1) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \exp\left(\sigma dW(t) + (\alpha - \frac{1}{2}\sigma^2)t\right), \forall\, t \in [0,1].$$

In mathematical finance, an arbitrage arguments show that any derivative $V(S,t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence we have the following:

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} + rS\frac{\partial V(S,t)}{\partial S} - rV(S,t) = 0 \tag{2}$$

where r represents interest rate, σ represents volatility of the underlying assets and t represents time of maturity.

With boundary conditions:

$$V(S,t) \to \infty \text{ as } S \to \infty \text{ on } [0,T).. \tag{3}$$

$$V(S,t) \to 0 \text{ as } S \to 0 \text{ on } [0,T).. \tag{4}$$

and final time condition given by :

$$V(S_T, T) = (S_T - k)^+ = f(S_T) \text{ on } [0,\infty) \tag{5}$$

Equation (4) is the value of asset and is worthless when the stock price is zero [19]. The details of the above option model can expressly be found in the following books: [17]-[19] etc.

To eliminate the price process in (2) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\begin{aligned} C &= SN(d_1) - Ke^{-rt}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned} \tag{6}$$

where C is price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate, T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and is the cumulative normal distribution.

However, this study will effectively and adequately analyze Fidelity, Access banks and their future merging according to [16] in order to realistically assess the value of share prices.

3.1 Results and Discussion

This Section presents the simulated results for the problem stated in Section 2.1. The Table results are implemented using Matlab programming software

Table 1 describes the variations of maturity days as it affects the value of Fidelity bank share prices. An increase in the maturity days increases the value of the call option prices. That is to say, that share prices is time dependent in time varying investments. This situation informs the management of Fidelity bank, PLC on the proper ways of taking decisions in order to manage their investments.

Table 2 portrays call option disparities of maturity days in the value of access bank share prices. It is clear that increase in the number of maturity days increases the value of the option prices. This remark is encouraging in every investments because it is profit maximizing which will guide the management of Access bank, PLC, the ways of taking decisions based on the levels of their investments.

Table 3 shows the value of Fidelity and Access banks share prices as they merge in future. It can be seen from the variations of maturity days that increase in the maturity days increases the value of call option prices. Careful looking at the call option prices as they merge; one will understand that it is more profitable for the two banks to merge because the value of their assets will increase tremendously as seen above.

3.1.1 Goodness of Fit test

H_0 : *The call option prices of Fidelity* and *Access banks* comes from a common distribution
H_1 : They are all not from a common distribution

In Table 4, the alternative hypothesis of KS was obviously accepted; hence, the p-values are non-significant. The result shows that at α=0.01, the values of two call option prices does not come from the same distribution which has financial inferences in respect to asset returns; with this result traders or investors will be well informed on how to take some vital decisions based on the levels of their investments.

In Table 5 are the initial share prices of Fidelity and Access banks with their respective call option prices. This means the value of each asset in both banks in their corresponding years of trading. Within the period of trading, Fidelity bank has good maximum number of call option prices than Access bank; which implies that Fidelity bank is profit indexed in terms of investments plans, see column 6.

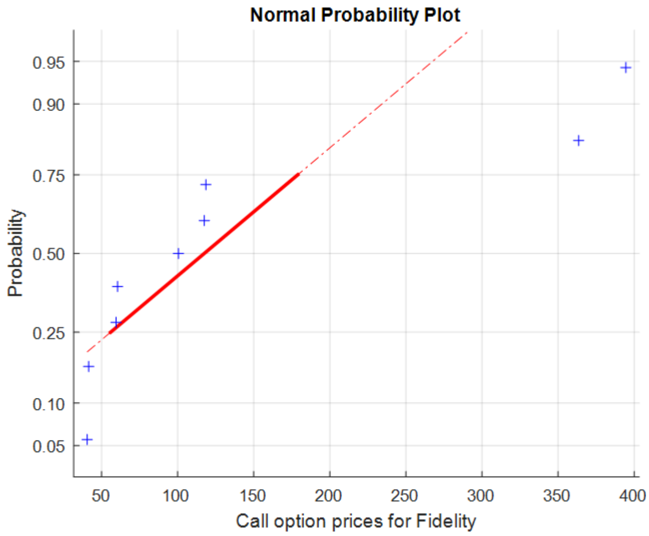

Fig. 1: The normality text of Fidelity bank, PLC call option prices

Figure 1 is the normality test of on the values of call options prices for Fidelity bank; it shows that call option prices do not come from a population with a common distribution. This also have the same interpretations in Figure 2.

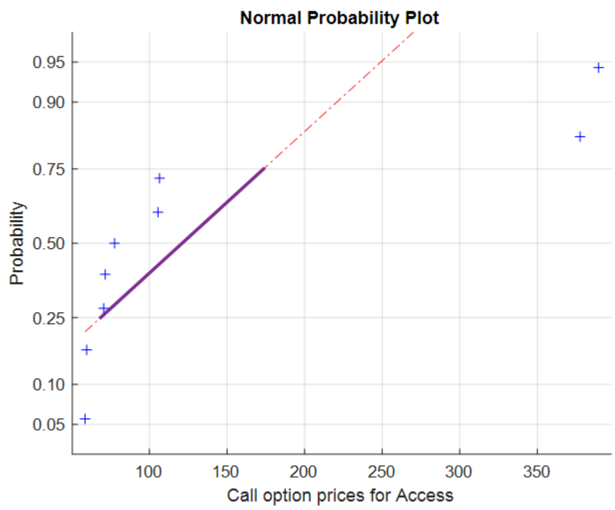

Fig. 2: The normality test of Access bank, PLC call option price

Table 1: The value of Share price of Fidelity Bank, PLC and variations of maturity days with the following parameter values: $r = 0.03, \sigma = 0.25, k = 450$ **and** $t = 1$

Initial Share Price (S_0)	Call Option Price when time $t = 4$	Call Option Price when time $t = 6$	Call Option Price when time $t = 12$
415	79.6936	100.3379	140.2049
62	6.9477e-004	0.0156	0.445
138	0.4181	1.6534	8.0033
61	5.9530e-004	0.0139	0.4165
121	0.1713	0.8520	5.2318
81	0.0076	0.0877	1.2679
139	0.4384	1.7127	8.1878
80	0.0069	0.0812	1.2099
384	61.8659	81.0953	118.9472

Table 2: The value of Share price of Access Bank, PLC with variations of maturity days with the following parameter values: $r = 0.03, \sigma = 0.25, k = 450$ **and** $t = 1$

Initial Share Price (S_0)	Call Option Price when time $t = 4$	Call Option Price when time $t = 6$	Call Option Price when time $t = 12$
410	76.6809	97.1219	136.6974
80	0.0069	0.0812	1.2099
126	0.2269	1.0494	5.9757
79	0.0062	0.0752	1.1537
98	0.0360	0.2706	2.5420
92	0.0219	0.1880	2.0277
127	0.2396	1.0925	6.1316
91	0.0200	0.1764	1.9490
378	58.6580	77.5702	114.9730

Table 3: The value of Share price of Fidelity-Access Merged with variations of maturity days with the following parameter values: $r = 0.03, \sigma = 0.25, k = 450$ **and** $t = 1$

Initial Share Price (S_0)	Call Option Price when time $t = 4$	Call Option Price when time $t = 6$	Call Option Price when time $t = 12$
825	428.2075	449.4113	490.8146
142	0.5036	1.8997	8.7561
264	14.4793	24.9310	49.8843
140	0.4594	1.7735	8.3748
219	6.0058	12.5047	30.7737
173	1.6856	4.7180	15.9111
266	14.9692	25.5976	50.8306
171	1.5756	4.4827	15.3788
762	367.5961	389.3919	431.7473

4.1 Conclusion

This paper investigated the effectiveness of Black-Scholes analytical solution on the share price of Fidelity and Access banks from 2016-2022 retrievable from [16]. From the study the following results were obtained: (i) the higher the share price determines the value of call option prices (ii) an increase on the maturity days dominantly increases the value of call option for both Fidelity, Access and their future merging (iii) merging of the two banks improved on the value of call option prices (iv) Fidelity bank has a good maximum value of call option prices during the period of investments (v) KS test shows that the two call option prices (Fidelity and Access) do not come from a common distribution (vi) the normality test of both banks are not significance. To this end, studying the delay in the valuation of call option is an interesting area to explore.

Acknowledgment

The authors are grateful to the anonymous referee for a thorough checking of the details and for helpful comments that improved this paper.

Table 4: Goodness of fit test for assessment of Fidelity and Access banks

Fidelity -call option	Access-call option	Mean	STD	KS Stat	P-value	Decision
394.5317	389.5317	392.0317	3.5355	1.0000	0.2890	Accept
41.5305	59.5317	392.0317	3.5355	1.0000	0.2890	Accept
117.5317	105.5317	111.5317	8.4853	1.0000	0.2890	Accept
40.5302	58.5317	49.5310	12.7290	1.0000	0.2890	Accept
100.5317	77.5317	89.0317	16.4853	1.0000	0.2890	Accept
60.5317	71.5317	66.0317	7.7782	1.0000	0.2890	Accept
118.5317	106.5317	112.5317	8.4853	1.0000	0.2890	Accept
59.5317	70.5317	65.0317	7.7782	1.0000	0.2890	Accept
363.5317	377.5317	370.5317	9.8995	1.0000	0.2890	Accept

Table 5: The value of Share price of Fidelity-Access banks with yearly comparisons of maximum call option prices $r = 0.2, \sigma = 0.25, k = 25$ **and** $t = 1$

Initial Share Price	Fidelity-call option prices	Initial Share Price	Access-call option prices	Maximum
415	394.5317	410	389.5317	394.5317 *F
62	41.5305	80	59.5317	59.5317 ** A
138	117.5317	126	105.5317	117.5317 * F
61	40.5302	79	58.5317	58.5317 ** A
121	100.5317	98	77.5317	100.5317 * F
81	60.5317	92	71.5317	71.5317 ** A
139	118.5317	127	106.5317	118.5317 * F
80	59.5317	91	70.5317	70.5317 ** A
384	383.5317	378	377.5317	383.5317 * F

References

[1] M. S. Umar and B. M. Tijjani, *Stock prices and firm earning per share in Nigeria*. Journal of Research in National Development **11**(2), 21-33 (2013).

[2] F. N. Nwobi, *Application of symmetry analysis of partial differential equations arising from mathematics of Finance. Doctor of Philosophy in school of Mathematical Sciences University of KwaZulu–Natal Durban South Africa*, 2011.

[3] B. O. Osu, *A stochastic model of the variation of the capital market price*. International Journal of Trade, Economics and Finance **1**(2), 297 (2010), IACSIT Press.

[4] Davies, Iyai, Uchenna Amadi Innocent, Roseline Ndu, and others. *Stability analysis of stochastic model for stock market prices*. International Journal of Mathematical and Computational Methods **4** (2019), International Association of Research and Science.

[5] F. Black and M. Scholes, *The pricing of options and corporate liabilities*. Journal of political economy **81**(3), 637-654 (1973). The University of Chicago Press.

[6] R. Cont, *Model uncertainty and its impact on the pricing of derivative instruments*. Mathematical finance **16**(3), 519-547 (2006), Wiley Online Library.

[7] E. Lindström, J. Ströjby, M. Brodén, M. Wiktorsson and J. Holst, *Sequential calibration of options*. Computational Statistics & Data Analysis **52**(6), 2877-2891 (2008), Elsevier.

[8] F. Lorella, F. Mariani, M. C. Recchioni, F. Zirilli, and others, *Calibration of a multiscale stochastic volatility model using European option prices*, Mathematical methods in Economics and Finance **3**(1), 49-61 (2008).

[9] B. Marcelo, M. Scott and S. Marco, *Implied Volatility and the Risk-free Rate of Return in option Markets [preliminary draft]. Comments welcome* (2014).

[10] O. L. Babasola, I. Irakoze and A. A. Onoja, *Valuation of European options within the Black-Scholes framework using the Hermite Polynomial*. J Sci Eng Res **5**(2), 200–13 (2008).

[11] B. Shin and K. Hwajoon, *The solution of Black-Scholes terminal value problem by means of Laplace transform*. Glob J Pure Appl Math **12**, 4153–8 (2016).

[12] M. R. Rodrigo and R. S Mamon, *An alternative approach to solving the Black–Scholes equation with time-varying parameters*. Applied Mathematics Letters **19**(4), 398-402 (2006), Elsevier.

[13] B. O. Osu and C. Olunkwa, *Weak solution of Black-Scholes equation option pricing with transaction costs*. Int J Appl Math **1**(1), 43-55 (2014).

[14] F. N. Nwobi, M. N. Annorzie and I. U. Amadi, *Crank-Nicolson finite difference method in valuation of options*. Commun Math Fin, **8**(1), 93-122 (2019).

[15] B. O. Osu, S. C. Emenyonu, C. P. Ogbogbo and C. Olunkwa, *Markov Models on Share Price Movements in Nigeria Stock Market Capitalization*. Applied Mathematics and Information Sciences an International Journal N **2** (2019).

[16] S. E. Fadugba, O. E. Helen and others, *A new approach for solving boundary value problem in partial differential equation arising in financial market*, Applied Mathematics **7**(09), 840, Scientific Research Publishing (2016).

[17] D. J. Higham, *An introduction to financial option valuation: mathematics, stochastics and computation*. Cambridge University Press **13** (2004).

[18] p. Wilmott, S. Howison and J. Dewynne, *The mathematics of financial derivatives: a student introduction*. Cambridge university press (1995).
[19] J. C. Hull *Other Derivatives (Eight Edition)*. New Jersey: PrenticeHall (2012).

Amadi Innocent Uchenna obtained a BSc degree in Mathematics (Pure and Applied), MSc in Applied Mathematics, MSc Statistics from Rivers State University, Nigeria, and PhD Statistics (specializing in Mathematical finance) from Imo State University, Owerri, Nigeria. He is a lecturer in the Dept of Mathematics/Statistics at Captain Elechi Amadi, Polytechnic, Rumuola Portharcourt, Nigeria. He is also a visiting scholar at Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt. He has supervised so many post graduate students. An editor of international Research journal of innovations in Engineering and Technology, he has published so many articles in reputable journals.His research interests are as follows: Stochastic Approximation, Mathematics of finance, Statistics, Stochastic Differential Equation and Partial differential equation. His love for Mathematics is second to none.

Akani Nmerukini Akani Nmerukini holds a BSc degree in Mathematics and Statistics from University of Port Harcourt and currently pursuing her MSc Statistics at Ignatius Ajuru University of Education, Rumuolumeni Port Harcourt, Nigeria.

Chisara P. Ogbogbo holds a PhD in Mathematics from the University of Ibadan. Nigeria. (With specialization in Financial Mathematics). She is currently the Head of Department, for the Department of Mathematics, University of Ghana. Legon. Her research interests are Financial Mathematics, computational Finance, Mathematical modeling, especially stochastic modeling involving the financial market. Chisara has written a number of research papers published in competitive peer reviewed, international journals of Mathematics and Science. She has a significant number of collaborations globally. She has reviewed a number of journal articles for national and international journals. She has supervised many students, and she is a mentor and advocate for Girls in STEM.

Aboko Igboye Simon is a Senior Lecturer in the Department of Mathematics and Statistics of the Captain Elechi Amadi Polytechnic, Rumuola, Port Harcourt. He has a B. Sc in Mathematics (Pure and Applied Option), M.Sc in Applied Statistics and a Doctor of Philosophy (PhD)in Applied Statistics all from the River State University, Port Harcourt. He co-authored the books FUNDAMENTAL OF STATISTICS and also INFERENTIAL STATISTICS.He is a member of Nigeria Statistical Association (NSA) and also Member Nigeria Mathematical Association (MAN). He has published extensively in National and International Journals.

Adaobi Mmachukwu Udoye obtained a BSc., MSc. and PhD degree in Mathematics from the University of Ibadan, Ibadan, Nigeria. She is a lecturer in the department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti, Nigeria. Her research interest is in the area of Applied Mathematics.