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Fuzzy System to Control a Liquid Mixing Tank

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Abstract: This paper aims to develop a fuzzy system to control the level and temperature of a fluid in a mixing tank by dynamically adjusting the flow rates of cold and hot liquids. The system involves a temperature and level loop, with the control variables being the rates of cold water and hot water flow, while also considering the outlet flow from the reservoir as a disruptive factor. The main goal of this control strategy is to maintain the liquid level and temperature within specified reference values. The system is a multivariable process that uses a fuzzy control system to develop a technique that, based on the state variables, can adapt the controller to achieve optimal control performance. In the development of this application, some linear controllers were tested, each implemented around specific regions within the phase plane. The designed controllers are then employed in conjunction with the fuzzy system to evaluate the system's state variables. Therefore, an interpolation of these linear controllers is conducted, allowing for their application in a nonlinear system, with the aim of enhancing control performance near the boundaries between regions. The results show that the fuzzy approach is promising and provides convergence to the reference values of the liquid level and temperature in the mixing tank.

Keywords: mixing tank, control fuzzy, non-linear systems, interpolation

1 Introduction

A system is defined as a junction of particles that act together in order to achieve a specified objective [1]. A linear system is defined by any system that meets the properties of homogeneity and additivity, which can be understood as the superposition property [2]. As a consequence, any system that does not meet this criteria for being classified as nonlinear system.

Despite of the non-linearity present in most industrial processes, it is common to approximate them by linear dynamics, due to a wide range of tools to analyze these systems and the ease of designing the controller that will be applied. One of the techniques that can be used is the process linearization system, which allows a linear analysis around a certain operating region. If this region is compatible with the equilibrium point and small signals are identified, there is the possibility of linearizing the system around such region [1].

Using the premise of this technique, switching control can be performed, dividing the phase plane of a system into sections and for each of these, design a controller different that only operates in the given region. However, when analyzing borders, this switching will happen abruptly, which may be impossible in practical situations or even lead to unstable behavior. As a way to solve this problem, [3] used fuzzy inference in conjunction with the linearization technique to propose a another solution to control non-linear systems.

One of the first works to use fuzzy logic as a tool for identification of systems using a fuzzy model was developed by Takagi and Sugeno [4]. That method uses input and output data, whose inputs present uncertainties and are in a fuzzy subspace, resulting in a linear input-output description. In this sense, [5] proposes a fuzzy application as a supervisory controller to assist in determining the gains K_p and K_i of a Proportional Integral (PI) controller, so that the membership functions define the proportion with which each controller operates in different regions.

Considering this scenario, the objective of this paper is to control, by varying the rates of cold and hot liquid flow, the level and the temperature of a fluid in a mixing tank. Such equipment consists of a temperature and level

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loop, where the regulating variables are the cold water flow and the hot water flow, also containing the reservoir outlet flow as a disturbing variable. The purpose of the control is to ensure that the level and temperature of the liquid in the reservoir remain stabilized at given reference values. The system is a multivariable process whose objective is to use a fuzzy control system for development of a technique that allows, according to the state variables, to modify the controller to achieve optimized control performance. In order to develop this application, different linear controllers were designed for different regions of the phase plane. With the designed controllers, fuzzy inference is used to analyze the system state variables. Then, it is possible to perform an interpolation of linear controllers to be applied in a non-linear system, in order to smooth the control close to the limits between regions.

This paper is organized as follows. Section 2 presents the considered non-linear model, along with the details of the local linear controllers. The fuzzy control system, proposed in this paper, responsible for combining the linear controllers in order to stabilize the non-linear dynamics is detailed in Section 3. Section 4 presents the simulation results that illustrate the validity of the proposed method, and Section 5 concludes the paper.

2 Modeling the Dynamic Liquid Mixing System

The system under study consists of three tanks: a hot water tank in the temperature of 50^oC, a cold water tank at a temperature of 24^oC and the main tank, where the mixture of the liquids occur. The objective of the control system is to guarantee, using the flow of the cold and hot liquids as the control input, that both the level h $[mm]$ and the temperature T $[^oC]$ of the liquid in the mixture tank follow the independent reference signals $r_h(t)$ and $r_T(t)$, respectively.

The level and temperature dynamics are obtained from [6] and are represented by equations (1) and (2), respectively. The variables $u_c(t)$ and $u_h(t)$ are the cold and hot water flows, respectively, the internal area of the tank is $33250mm^2$ and t is the time in seconds.

$$\frac{dh}{dt} = \frac{1}{33250}\left(u_c(t) + u_h(t) - \frac{h(t)}{0.0032}\right) \quad (1)$$

$$\frac{dT}{dt} = \frac{1}{33250h(t)}\left(u_c(t)(24 - T(t)) + u_h(t)(50 - T(t))\right) \quad (2)$$

Note that the dynamics (1)-(2) are nonlinear which, along with the multivariable nature of the system, difficults the synthesis of appropriate controllers. In order to apply the design technique proposed in this paper, a linearization procedure is applied [8]. The resulting linear system is denoted as

$$\dot{x}(t) = A(h,T)x(t) + B(h,T)u(t), \quad (3)$$

where the states $x(t)$ and the inputs $u(t)$ are defined as

$$x(t) = \begin{bmatrix} h(t) \\ T(t) \end{bmatrix}, u(t) = \begin{bmatrix} u_c(t) \\ u_h(t) \end{bmatrix}$$

The linearized matrices $A(h,T)$ and $B(h,T)$ are obtained from the Jacobian matrices presented in (4), considering $\frac{dh}{dt} = \dot{h} = f_h$ and $\frac{dT}{dt} = \dot{T} = f_T$.

$$A(h,T) = \begin{bmatrix} \frac{\partial f_h}{\partial h} & \frac{\partial f_h}{\partial T} \\ \frac{\partial f_T}{\partial h} & \frac{\partial f_T}{\partial T} \end{bmatrix}, \quad B(h,T) = \begin{bmatrix} \frac{\partial f_h}{\partial u_c} & \frac{\partial f_h}{\partial u_h} \\ \frac{\partial f_T}{\partial u_c} & \frac{\partial f_T}{\partial u_h} \end{bmatrix}. \quad (4)$$

The proposed control approach consists in determining a series of local state-feedback controllers, constructed from the Jacobian matrices in (4) for predefined fixed reference values of h_r and T_r, and then joining the gains using a dedicated Fuzzy system. In order to assure the desired reference tracking properties, two integrators are coupled to the control loop, resulting in two extra states $q_r(t)$ and $q_T(t)$, whose dynamics are given by

$$\dot{q}_r(t) = r_h(t) - h(t), \quad \dot{q}_T(t) = r_T(t) - T(t).$$

The addition of both states yields the augmented system

$$\dot{\hat{x}}(t) = \begin{bmatrix} A(h,T) & 0 \\ -I & 0 \end{bmatrix}\hat{x}(t) + \begin{bmatrix} B(h,T) \\ 0 \end{bmatrix}u(t),$$
$$\hat{x}(t) = \begin{bmatrix} x(t) \\ \begin{bmatrix} q_r(t) \\ q_T(t) \end{bmatrix} \end{bmatrix}. \quad (5)$$

For a fixed pair (h_r, T_r), the local controllers are given by

$$u(t) = K(h_r, T_r)\hat{x}(t), \quad (6)$$

being the state-feedback gain $K(h_r, T_r)$ resultant from the application of the LQR synthesis method [9] on system (5). Briefly stated, such method consists of computing the gain $K(h_r, T_r)$ solution of the optimization problem

$$\min_{K(h_r,T_r)} \int_0^\infty \hat{x}(\tau)^T Q\hat{x}(\tau) + u(\tau)^T Ru(\tau)d\tau. \quad (7)$$

For this paper, each gain results from setting the parameters $Q = 10I$ and $R = I$, being I the identity matrix of appropriate dimensions.

3 Fuzzy Control System

The so-called "gains" of controllers, denoted by K, are adjusted to regulate the behavior of a control system and ensure that it meets desired specifications. Therefore, controllers use gains to adjust the system's response to changes in controlled conditions or variables. In this work, the proposal is that the controller gains are provided by the fuzzy system, to ensure that the control system meets the desired performance requirements, interpolating the controllers local linear lines in order to smooth out the control close to the boundaries between regions. Proper adjustment of these gains is essential to optimize control system performance.

To develop this application, the first step is to define different linear controllers for different regions of the phase plane. Then, the fuzzy system is used to combine the use of these controllers in the different regions of the phase plane. Therefore, 4 local linear controllers are defined, considering the 4 extreme points of the phase plane, where a given point X has as references the values of the level and temperature of the tank ($X(h,T)$):

–Point 1: low level tank, $h = 10$, and cold temperature $T = 24$. The matrices are: $A(10,24)$ and $B(10,24)$. Then, the controller is $K_1 = K(10,24)$.
–Point 2: high level tank, $h = 150$, and cold temperature $T = 24$. The matrices are: $A(150,24)$ and $B(150,24)$. Then, the controller is $K_2 = K(150,24)$.
–Point 3: low level tank, $h = 10$, and hot temperature $T = 50$. The matrices are: $A(10,50)$ and $B(10,50)$. Then, the controller is $K_3 = K(10,50)$.
–Point 4: high level tank, $h = 150$, and hot temperature $T = 50$. The matrices are: $A(150,50)$ and $B(150,50)$. Then, the controller is $K_4 = K(150,50)$.

For the other points of the phase plane, the fuzzy controller will indicate the combination of these local controllers to achieve the desired level and temperature.

When applying this fuzzy control approach, depending on the region where the system trajectory passes, there is a change on the controllers' gain, creating the need to interpolate the controllers in the transition regions, so that there is no abrupt change or switching between controllers, which could lead to performance issues or even instability.

The input variables, which are the level h and temperature T of the liquid in the tank, are modeled by Gaussian membership functions, considering the discretization intervals shown in Table 1.

Table 1: Discretization of inputs

Input	linguistic value	numeric range
h	low	[1 50]
	middle	[40 100]
	high	[80 150]
T	cold	[24 35]
	warm	[30 40]
	hot	[35 50]

From the definition of the linguistic intervals, the Gaussian membership functions of the inputs are determined according to Fig. 1 for the liquid level input h and according to Fig. 2 for the temperature T.

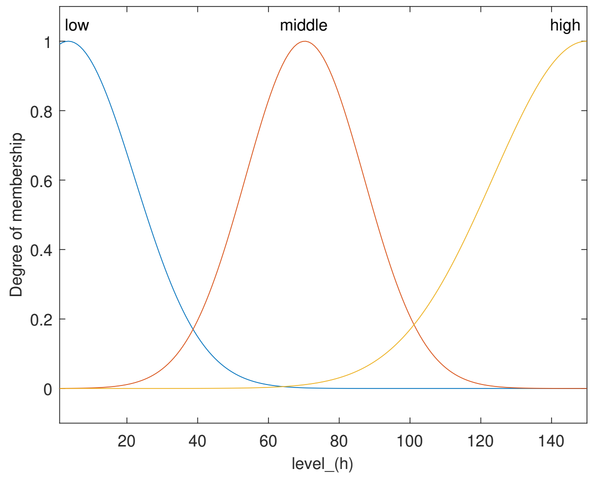

Fig. 1: Gaussian membership function for h input.

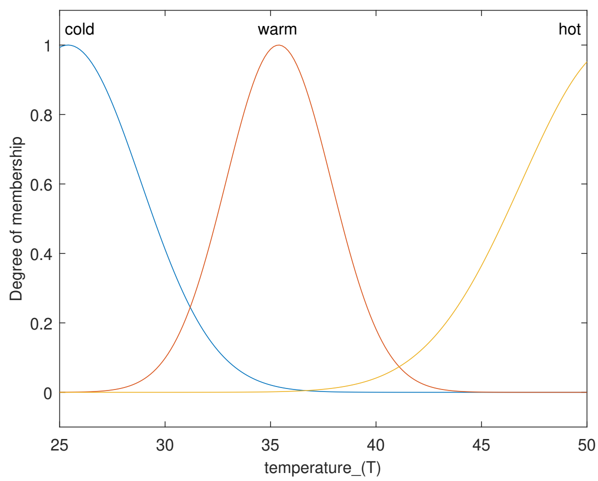

Fig. 2: Gaussian membership function for T input.

Similarly, the output of the fuzzy system, which consists of the values of the controllers' gains K, is discretized into 4 linguistic intervals, according to the K_1, K_2, K_3, K_4, previously defined, as can be seen in Table 2.

To model the fuzzy system output, triangular membership functions are defined, since this variable

Table 2: Discretization of output

Output	linguistic value	numeric range
K	K_1	[00.33]
	K_2	[00.66]
	K_3	[0.331]
	K_4	[0.661]

presents peak values. Fig. 3 illustrates the triangular membership functions for the gains K.

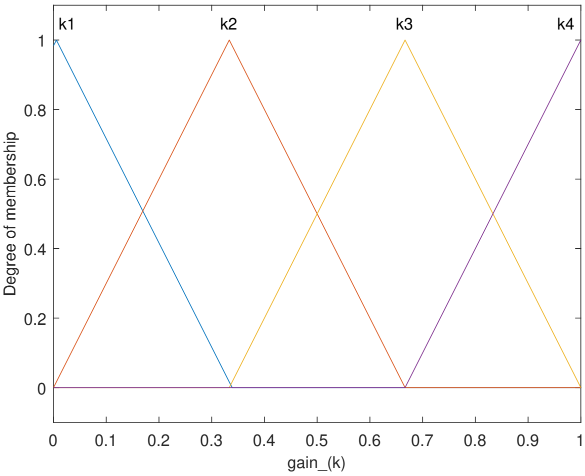

Fig. 3: Triangular membership function for output K.

The performance of a fuzzy system is described by its linguistic rule base, which indicates the corresponding outputs. To build the rule base and sets used in a fuzzy system, it is commonly used operator experience of the analyzed system, machine training and a combination of the previous ones [7]. Table 3 shows the possible linguistic rules that combine the use of the controllers in different regions of the phase plan.

Table 3: Fuzzy rule base

	temperature		
level	cold	warm	hot
low	K_1	K_2	K_2
middle	K_3	K_3	K_2
high	K_3	K_3	K_4

In order to perform the combination of input variables in the antecedent of the database, linguistic rules, the "and" operator was applied. This definition is responsible for executing the logical operations of conjunction, also known as T-norm [7]. As a step prior to the execution of the T-norm, the "product" operation was defined. Thus, the product of the entries is performed before executing the T-norm.

4 Results

Experiments were conducted using the software Matlab, from *mathworks.com*. The tests developed aim to analyze the performance of the proposed control combination technique, compare it with the results obtained when apply mismatched controllers and identify possible improvements. All tests were executed computationally through the "ode45" command, from Matlab, which computes the solutions for the different environments and situations tested, mainly providing information about the behavior of state variables.

In order to illustrate the advantages of the proposed technique, three different control systems are compared:

– Controller 1: Usual static state-feedback gain, without using the described Fuzzy structure. This gain is synthesized from system (5) considering $h_r = 75$ and $T_r = 37$, which is an average point within the given operation intervals;
– Controller 2: Stems from the proposed Fuzzy control system as described in Section 3, being each gain K_1 to K_4 obtained using the LQR synthesis method with $Q = 10I$ and $R = I$;
– Controller 3: Similar to Controller 2, but setting different values for the parameter LQR matrix Q for each point described in Section 3. Describing $Q(h,T)$ the parameter matrix depending on the reference level and temperature as

$$Q(h,T) = \text{diag}(q_1,q_2,q_3,q_4) = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}, \quad (8)$$

then the matrices used in the experiments are

$$Q(10,24) = \text{diag}(100,100,100,100), \quad (9)$$
$$Q(150,24) = \text{diag}(100,1,100,1), \quad (10)$$
$$Q(10,50) = \text{diag}(100,10,100,10), \quad (11)$$
$$Q(150,50) = \text{diag}(1,10,1,10). \quad (12)$$

For all cases, it is set $R = I$.

It is important to highlight that the defined values of Q and R for the three controllers resulted from a series of previous simulations.

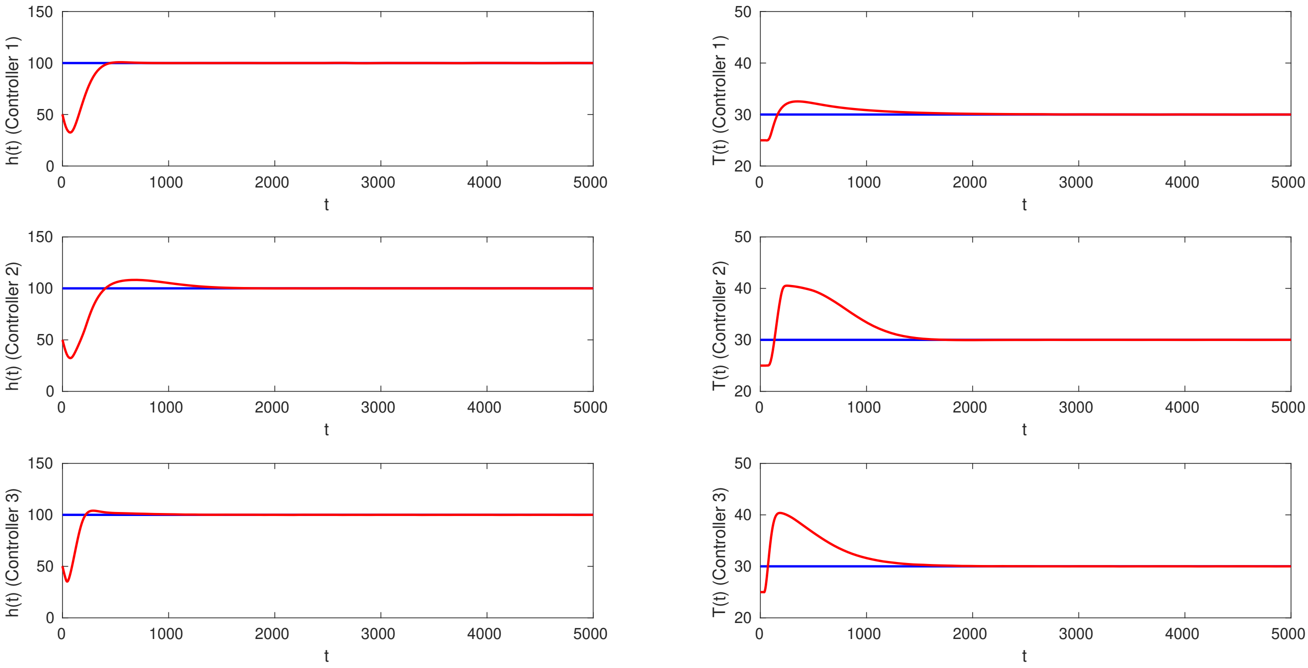

Fig. 4: Simulation results obtained by setting the level and temperature references, respectively, to $r_h = 100$ and $r_T = 30$.

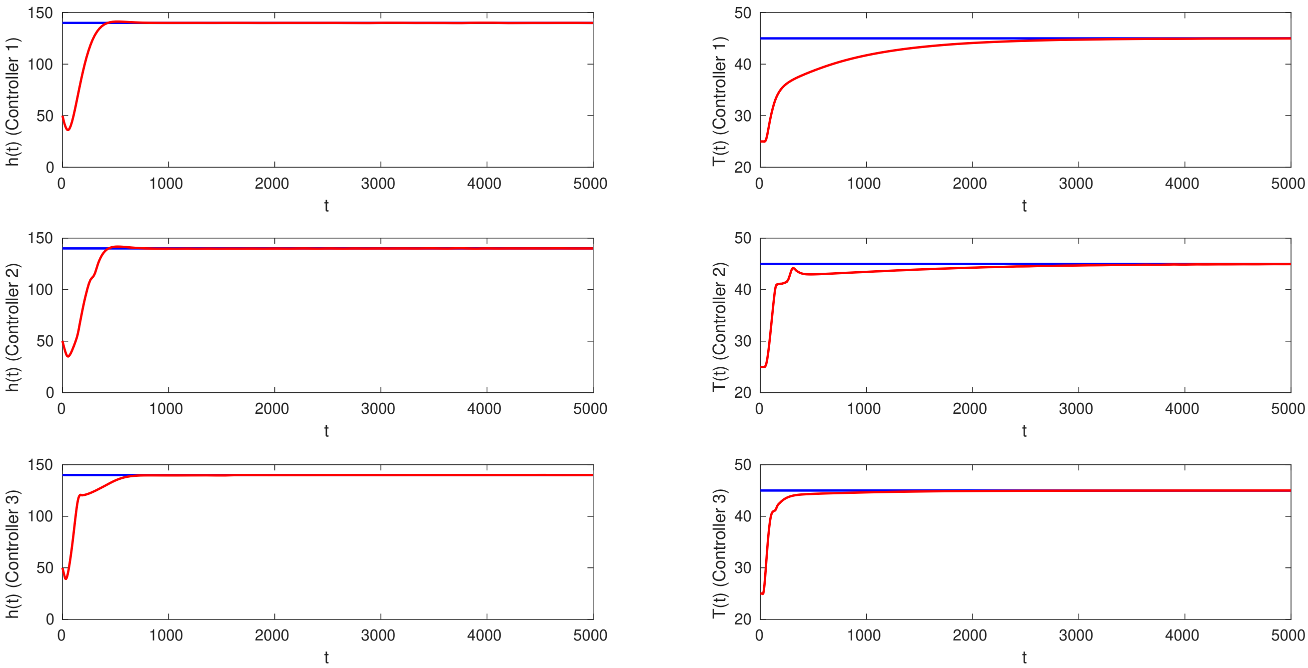

Fig. 5: Simulation results obtained by setting the level and temperature references, respectively, to $r_h = 140$ and $r_T = 45$.

Fig. 4 presents the level and temperature states resulting from the three controllers for references $r_h = 100$ and $r_T = 30$. Since such values are close to the average point $h = 75$ and $T = 37$, Controller 1 is expected to have a satisfactory behavior. In fact, such controller presented a better behavior than Controller 2, which is reasonable since the latter in synthesized around extreme situations. Nevertheless, the fuzzy system is capable of joining the extreme gains into a controller capable of stabilizing the plant around an average point. On the other hand, Controller 3 presented a better performance than Controller 2, showing that synthesizing the gains using different optimization parameters for each situation indeed improves the performance of the controllers.

The outputs of the system considering $r_h = 140$ and $r_T = 45$, which are near to the extreme situations, are presented in Fig. 5. It can be seen that the convergence time for the levels from the three controllers are somewhat similar. Analyzing the temperature, Controller 2 presents a faster convergence than 1 during the first $500s$, with a slower behavior from then on. The temperature from Controller 3, on the other hand, has an uniform and faster convergence.

Both experiments show the advantages of the proposed Fuzzy control system over a static one. The results also indicate that the behaviors could be improved if regions were considered for the Fuzzy system; However, setting a considerable higher quantity of points would be ill advisable, since the computational burden would be heavier, probably without a considerable performance improvement. Another possibility on refining the proposed control system is on the synthesis of the state-feedback gains, which can take into account the specificities of each region and could possibly include other criteria, such as robustness indexes.

5 Conclusion

This study aimed to propose a control technique for non-linear systems based on a combination of local controllers, through a fuzzy approach that generates the controllers' gains in order to smooth such combination. This way, control action in transition regions can be done more smoothly to achieve the desired stabilization properties.

The proposed fuzzy control technique was applied to control the level and temperature of the liquid in a mixing tank that uses cold and hot water to reach the expected level and temperature. Such system is both non-linear and multivariable, thus requiring an appropriate control system.

The specifications of the hardware used in computational experiments are composed of an Intel Core I7 processor, RAM memory 16GB and operational system Microsoft Windows 10. The runtime of the used technique is not a parameter of comparison of its performances, but it is mentioned here since it may be expected by readers. Using this hardware configuration and the Matlab software, a fraction of a second is needed to run the system, about 0.31s.

The achieved results show that the fuzzy control system has better performance in relation to the non-fuzzy system, since the fuzzy approach provides convergence and is promising. A technical limitation of the presented results that can be cited is that the stability is achieved through numerical experiment, at points other than those used for generation of control. This stabilization region may vary depending on the synthesis technique used at each point. A theoretical proof is yet to be developed in a future work, as well as using other control structures to be combined with the fuzzy system.

In conclusion, it is shown that the proposed control technique is capable of smooth the transition between controllers and build a combined controller that can be implemented in a non-linear system, achieving the control objective.

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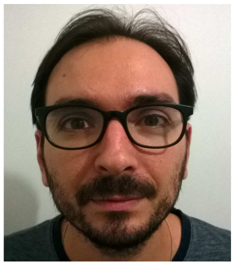

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