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Characteristics of stochastic solutions to the long–short-wave interaction model through Brownian process

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Abstract: This article aims to offer stochastic solutions for the nonlinear long-short wave interaction system (NLSWIS) via Itô sense. Using He's semi-inverse approach, some innovative travelling wave solutions are produced. These solutions are obtained using the Ritz technique. We produce the potential model that corresponds to the NLSWIS model's energy equation. We also explain how multiplicative noise affects the solutions. A few graphs are also displayed using the Matlab packet software. The developed solutions can be applied in space plasma, ocean waves, regenerating infectious for people crowds, and economic viability as a consequence of infectious these issues being controlled. In fact, the He's semi-inverse approach exhibits promise for resolving a range of nonlinear systems that arise in the applied sciences.

Keywords: Long–short-wave interaction system, Brownian process, solitary wave solution.

1 Introduction

In the applied sciences, nonlinear partial differential equations (NPDEs) are utilised to describe a variety of complex processes, such as materials science, optical fiber communications, superfluid, quantum mechanics, plasma physics, chemical engineering, kinematics and many other [1–4]. The fundamental approach one may take to forecast, manage, and quantify the underlying characteristics of a system under investigation is to model the system in terms of some mathematical equations, which are typically nonlinear, and then use a suitable technique to find exact analytical solutions of such equations.

In the present period of science and technology, several investigators have been engaged to develop various analytical methods for obtaining precise solutions for non-partial differential equations NPDEs [5–9]. The phenomena mimicked by these NPDEs can be better understood when exact solutions are provided.

A stochastic process explains the temporal development of a random phenomenon. Mathematical foundations for the science of stochastic processes were laid around 1950. Since that time, stochastic processes have spread among mathematicians, physicists, and engineers as a standard tool. Stock price modeling, rational option pricing theory and NPDEs all make use of this theory [10]. A common stochastic process that combines characteristics of a Markov process with a martingale is the Brownian motion process [10]. Brownian motion process is a widely used stochastic process in dispersive environments [11, 12]. Moreover, the this process enters in various real life problems, such as molecules of water, crystalline interface, crystalline structures, semiconductors, solid state physics and so forth. Recent improvements in stochastic calculus have been made possible by stochastic partial differential

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equations (SPDEs) are expected to create the framework for thoroughly modelling real-world models [11]. More than anyone else, mathematicians are most comfortable using SPDEs and stochastic processes to natural models.

In the ongoing study, we consider the nonlinear long-short wave interaction system (NLSWIS). This system was initially developed by Benney (1977) for considering a generic framework for interactions between short and long waves [13]. Sakkaravarthi et al. [14] examined the nonlinear resonance interaction of numerous short waves with a long wave in two spatial dimensions using Hirota's bilinearization approach.Uncovering basic physical interactions leads to further inquiry and analysis of various nonlinear interactions underlying the overall solution structure, such as analytical, dark, and approximate solutions, is very stimulating [15]. In addition to being rich, sophisticated models for strong nonlinearities, NLSWISs serve as the foundational models for a variety of fascinating interaction phenomena that arise from applications in nonlinear optics, plasma and biophysics, gravity and water waves, and other fields [16]. The long-short-wave interaction system is written as

$$\begin{aligned} &\psi_t + \psi_x + (|\,\phi\,|^2)_{xx} = 0\,, \\ &i\phi_t + \phi_{xx} - \psi\,\phi = 0, \end{aligned} \tag{1}$$

$\psi(x,t)$ denotes a real function describing characterizes the longitudinal wave and $\phi(x,t)$ is a complex function describing the slowly varying envelope of the short transverse wave. Most authors investigate this model in deterministic case, using various analytical methods [16–19] and references therein. Wang et al. [17] introduced periodic wave solutions for the NLSWISs, using F-expansion approach. Khater et al. [18] applied the extended F-expansion method to constructs some new solutions for this. By using the simplest equation technique, Triki et al. [19] were able to study the system (1) and get soliton solutions as well as additional solutions like plane waves and singular periodic solutions. For the first time based on our knowledge, we examine the NLSWISs in the Itô sense that are compelled by multiplicative noise as follows

$$\begin{aligned} &\psi_t + \psi_x + (|\,\phi\,|^2)_{xx} = 0\,, \\ &i\phi_t + \phi_{xx} - \psi\,\phi - i\sigma\,\phi\,\Xi_t = 0, \end{aligned} \tag{2}$$

σ represents the noise strength, whereas noise Ξ_t represents the time derivative of the Brownian motion $\Xi(t)$ [10].

The primary goal of this work is to examine the solutions for the NLSWISs induced by multiplicative noise in Itô sense. Namely, we introduce the exact solution for this system. We present the corresponding energy equation with potential for the NLSWIS. Indeed, we provide a very vital hyperbolic travelling wave solution for this system in the presence of Itô sense, using He's variations technique [20–22]. We also study the influence of this of a noise parameter on the propagation of the solution.

This paper's layout is organised as follows. Sec. 2 shows a quick discussion of the Brownian motion process. Sec. 3 introduce the mathematical analysis for the NLSWIS. Sec. 4 introduces a very vital hyperbolic travelling wave solution for this system in the presence of Itô sense. The explantation for the acquired stochastic solutions is given in Sec. 5. Conclusions are provided in Sec. 6.

2 Brownian motion process

The Brownian motion process is a very widely-used random process. It has been used to the physical sciences, engineering, and finance. It is a stochastic process $\{\Xi(t)\}_{t\geq 0}$ satisfies:

(i) $\Xi(t), t \geq 0$ are continuous functions of time t, $\Xi(t) \sim N(0,t)$.
(ii) For $s < t < u < c$, $\Xi(s) - \Xi(t)$, $\Xi(c) - \Xi(u)$ are independent.
(iii) $\Xi(t) - \Xi(s)$ follows a normal distribution with zero mean and variance $t - s$, i.e. $\Xi(t) - \Xi(s) \sim \sqrt{t-s}N(0,1)$, $N(0,1)$ represents a standard normal distribution.

3 Mathematical investigation

Using wave transformation

$$\phi(x,t) = U(\eta)e^{i\theta+\sigma\Xi(t)-\sigma^2 t}; \psi(x,t) = \chi(\eta) \tag{3}$$

$$\eta = \omega\,x + \rho\,t,\ \theta = Kx + \lambda t,$$

ρ, k, λ, ω denote constants, whereas σ is the noise strength, produces

$$-K^2U(\eta) + \omega^2 U''(\eta) - \lambda U(\eta) - \chi(\eta)U(\eta) = 0 \tag{4}$$

$$2\omega U(\eta)U'(\eta) + (\rho+\omega)\chi'(\eta)e^{2\sigma(\sigma t-\beta(t))} = 0 \tag{5}$$

Taking expectation for Eq. (5), yields

$$2\omega U(\eta)U'(\eta) + (\rho+\omega)\chi'(\eta)e^{2\sigma^2 t}\,E(e^{-2\sigma\Xi(t)}) = 0, \tag{6}$$

since $E(e^{-2\sigma\Xi(t)}) = e^{-2\sigma^2 t}$, Eq. (6) becomes

$$2\omega U(\eta)U'(\eta) + (\rho+\omega)\chi'(\eta) = 0.$$

Solving the last equation gives

$$\chi(\eta) = -(\frac{\omega}{\rho+\omega})U^2(\eta). \tag{7}$$

Eq. (4) becomes

$$U''(\eta)+\frac{1}{\omega(\rho+\omega)}U^3(\eta)-\frac{(\lambda+K^2)}{\omega^2}U(\eta)=0. \quad (8)$$

On the other hand we have

$$2k\omega U'(\eta)+\rho U'(\eta)-\sigma^2 U(\eta)=0, \quad (9)$$

with constraint equation

$$e^{\frac{2\eta\sigma^2}{2k\omega+\rho}}\left(\frac{\sigma^4}{(2k\omega+\rho)^2}-\frac{\lambda+K^2}{\omega^2}\right)+\frac{1}{\rho\omega+\omega^2}=0. \quad (10)$$

Eq. (12) illustrates an energy equation with potential

$$V=-\frac{K^2U(\eta)^2}{2\omega^2}-\frac{\lambda U(\eta)^2}{2\omega^2}+\frac{U(\eta)^4}{4\omega(\rho+\omega)}. \quad (11)$$

4 He's semi-inverse technique

From equation (8) and the constraint condition (10), we have

$$\omega^2 U''(\eta)+\frac{1}{(1-2k)}U^3(\eta)-\left(\lambda+K^2\right)U(\eta)=0. \quad (12)$$

In line with He's semi-inverse method mentioned in [20–22], the variational formulation from Eq. (12) is as follows:

$$J(U)=\int_0^\infty\left\{\frac{\omega^2}{2}(U')^2-\frac{1}{4(1-2k)}U^4+\frac{1}{2}\left(\lambda+K^2\right)U^2\right\}d\eta. \quad (13)$$

We employ the Ritz method to search for a solitary wave solution in the form

$$U(\eta)=A\,sech(B\eta), \quad (14)$$

A,B are an unknown constant. Substituting Eq. (14) into Eq. (13), yields

$$J=\int_0^\infty\left[\frac{\omega^2}{2}A^2B^2 sech^2(\eta)\tanh^2(\eta)-\frac{1}{4(1-2k)}A^4 sech^4(\zeta)+\frac{1}{2}\left(\lambda+K^2\right)A^2 sech^2(\eta)\right]d\eta$$
$$=\frac{\omega^2A^2B}{6}-\frac{A^4}{6B(1-2k)}+\frac{1}{2B}\left(\lambda+K^2\right)A^2.$$

Differentiating J with respect to A,B and setting $\frac{\partial J}{\partial A}=0$ and $\frac{\partial J}{\partial B}=0$ produces

$$\frac{\partial J}{\partial A}=\frac{\omega^2AB}{3}-\frac{2A^3}{3B(1-2k)}+\frac{1}{B}\left(\lambda+K^2\right)A.$$

$$\frac{\partial J}{\partial B}=\frac{\omega^2A^2}{6}+\frac{A^4}{6B^2(1-2k)}-\frac{1}{2B^2}\left(\lambda+K^2\right)A^2.$$

Solving these equations yields:

$$A=\pm\sqrt{2(1-2K)(\lambda+K^2)},\qquad B=\pm\frac{1}{\omega}\sqrt{(\lambda+K^2)}. \quad (15)$$

Using (3) and (7), we get

$$U(x,t)=\pm\sqrt{2(1-2K)(\lambda+K^2)}\times sech(\pm\tfrac{1}{\omega}\sqrt{(\lambda+K^2)}(\omega x-2k\omega t)), \quad (16)$$

$$\chi(x,t)=2(\lambda+K^2)sech^2(\pm\frac{1}{\omega}\sqrt{(\lambda+K^2)}(\omega x-2k\omega t)). \quad (17)$$

Thus the stochastic solution of (2) is

$$\phi(x,t)=\pm\sqrt{2(1-2K)(\lambda+K^2)}\,e^{i(Kx+\lambda t)+\sigma\Xi(t)-\sigma^2t}\times sech(\pm\tfrac{1}{\omega}\sqrt{(\lambda+K^2)}(\omega x-2k\omega t)). \quad (18)$$

5 Results and Discussions

It has been claimed that the explicit stochastic solutions to the NLSWIS via Brownian motion process, specifically hyperbolic function solutions, were obtained. Brownian motion is a fundamental building element of stochastic calculus and the key for describing stochastic models. This process is a highly effective approach for coping with a wide range of random events in real life. The $\Xi(t)$ function is used to convert the stochastic NLSWIS model to nonlinear ordinary differential equations. We investigate the NLSWIS model specifically using the Brownian motion technique.

The majority of standard publications investigated the suggested NLSWIS model in deterministic scenarios. In contrast to other methods, we analyse this model in a stochastic scenario, that is, when it is created by multiplicative noise via the Brownian motion process. As a successful technique for generating NLSWIS solution sets, we constructed closed-form wave formations using the energy equation with potential V. Equation (11) provides the potential for the energy equation and the matching precise solution of Eq. (2). To achieve critical hyperbolic secant stochastic solutions, we have been using He's semi-inverse technique to the NLSWIS model with multiplicative noise in the Itô sense. This type of solution secant solution occurs in the profile of a laminar jet [23]. The He's semi-inverse method was utilised to provide innovative and succinct random solutions for the NLSWIS model with multiplicative random parameters. The key advantages of this technique over others are that it can solve a broader range of physical models and removes costly and time-consuming computations.

Because of its crucial uses, the impact of a noise parameter on the propagation of soliton solutions has

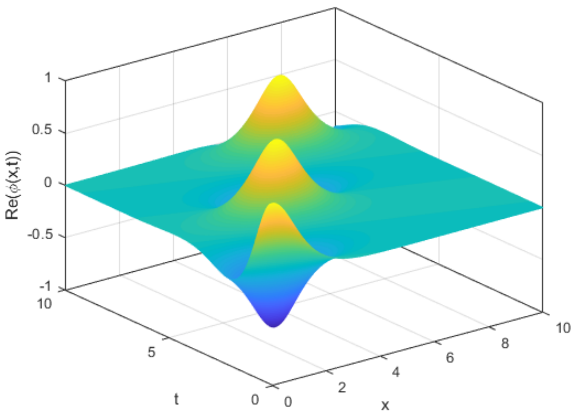

Fig. 1: 3D envelope wave of solution (18) for $\sigma = 0$.

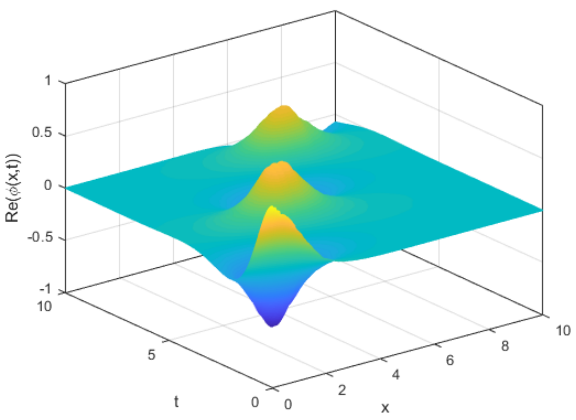

Fig. 2: 3D envelope wave of solution (18) for $\sigma = 0.3$.

received increased attention in recent decades. The significance of the stochastic solutions is in their ability to shed light on the propagation of NLSWIS waves in various physical viewpoint areas when they arise. We have created some related profile images to demonstrate the dynamical nature of these solutions. Figs. 1-4 illustrate the envelope waves for the presented solution. It was observed that the waves with varied frequencies had less amplitude as σ is raised. As seen in Figs. 2-4, increasing σ values cause the wave's crest to gradually shift until it vanishes.

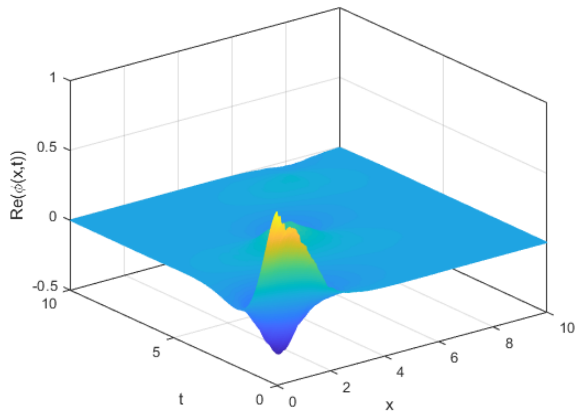

Fig. 3: 3D envelope wave of solution (18) for $\sigma = 0.6$.

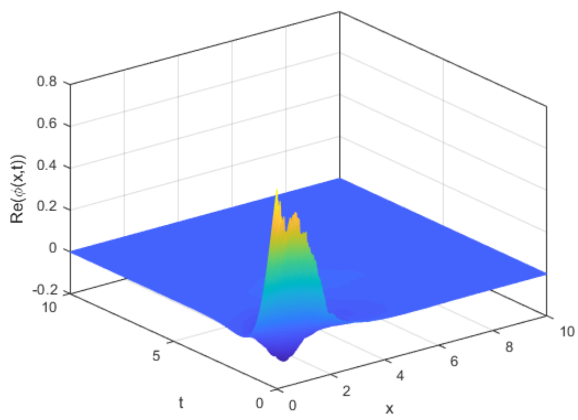

Fig. 4: 3D envelope wave of solution (18) for $\sigma = 0.9$.

6 Conclusions

Using He's semi-inverse technique, we explored the NLSWIS caused by multiplicative noise through Itô sense. We use this method to produce some innovative travelling wave solutions. This method's primary benefits over others are that it can handle a wider range of scientific issues and eliminates time-consuming and expensive computations. The energy equation's potential model was put out. We show how multiplicative noise affects the behaviour of the reported solutions. The solutions obtained have implications for the expanding field of mathematical medical genomics, the control of infectious diseases that lead to economic viability, auroral plasma, deep ocean interacting waves, microbial pathogens in human bodies, and shaped genetic variation in modern populations.

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Conflicts of Interest

The author declares no conflict of interests.

References

[1] M. Younis, N. Cheemaa, S. Mahmood and S. T. R. Rizvi, On optical solitons: the chiral nonlinear schrödinger equation with perturbation and bohm potential, *Opt. Quant. Electron.* **48** (2016) p. 542.

[2] O. Gaxiola and A. Biswas, Akhmediev breathers, peregrine solitons and kuznetsov-ma solitons in optical fibers and pcf by laplace-adomian decomposition method, *Optik* **172** (2018) p. 930–939.

[3] A. R. Alharbi, M. A. E. Abdelrahman and M. B. Almatrafi, Analytical and numerical investigation for the dmbbm equation, *CMES-Computer Modeling in Engineering & Sciences* **122**(2) (2020) 743–756.

[4] M. A. E. Abdelrahman, M. B. Almatrafi and A. R. Alharbi, Fundamental solutions for the coupled kdv system and its stability, *Symmetry* **12** (2020) p. 429.

[5] M. Mirzazadeh, M. Eslami and A. Biswas, 1-soliton solution of kdv equation, *Nonlinear Dyn.* **80**(1(2)) (2015) 387–396.

[6] X. F. Yang, Z. C. Deng and Y. Wei, A riccati-bernoulli sub-ode method for nonlinear partial differential equations and its application, *Adv. Diff. Equa.* **1**()) (2015) 117–133.

[7] Y. F. Alharbi, M. A. Sohaly and M. A. E. Abdelrahman, Fundamental solutions to the stochastic perturbed nonlinear schrödinger's equation via gamma distribution, *Results in Physics* **25** (2021) p. 104249.

[8] M. B. Almatrafi, A. R. Alharbi and C. Tunç, Constructions of the soliton solutions to the good boussinesq equation, *Adv. Diff. Equ.* **2020** (2020) p. 629.

[9] M. A. E. Abdelrahman and G. Alshreef, Closed-form solutions to the new coupled konno–oono equation and the kaup-newell model equation in magnetic field with novel statistic application, *Eur. Phys. J. Plus* **136**(4) (2021) 1–10.

[10] I. Karatzas and S. E. Shreve, Brownian motion and stochastic calculus, 2nd ed., *Springer-Verlag, Berlin* (1991).

[11] H. Pishro-Nik, Introduction to probability, statistics and random processes, *Kappa Research, LLC* (2014).

[12] Y. F. Alharbi, E. K. El-Shewy and M. A. E. Abdelrahman, New and effective solitary applications in schrödinger equation via brownian motion process with physical coefficients of fiber optics, *AIMS Math.* **8**(2) (2023) 4126–4140.

[13] D. J. Benney, A general theory for interactions between short and long waves, *J. Stud. Appl. Math.* **56**(2) (1977) 81–94.

[14] K. Sakkaravarthi, T. Kanna, M. Vijayajayanthi and M. Lakshmanan, Multicomponent long-wave-short-wave resonance interaction system: bright solitons, energy-sharing collisions, and resonant solitons, *Phys. Rev. E* **90** (2104) p. 052912.

[15] A. Chowdhury and J. A. Tataronis, Long wave–short wave resonance in nonlinear negative refractive index media, *Physical Review Letters* **100** (2008) p. 153905.

[16] A. A. Zabolotskii, Inverse scattering transform for the yajima-oikawa equations with nonvanishing boundary conditions, *Physical Review A* **80** (2009) p. 063616.

[17] M. L. Wang, Y. M. Wang and J. L. Zhang, The periodic wave solutions for two systems of nonlinear wave equations, *Chin. Phys.* **12** (2003) 1341–1348.

[18] A. H. Khater, M. M. Hassan and D. K. Callebaut, Travelling wave solutions to some important equations of mathematical physics, *Rep. Math. Phys.* **66**(1) (2010) 1–19.

[19] A. Triki, M. Mirzazadeh, A. H. Bhrawy, P. Razborova and A. Biswas, Solitons and other solutions to longwave short-wave interaction equation, *Rom. J. Phys.* **60** (2015) 72–86.

[20] J. H. He, Semi-inverse method of establishing generalized variational principles for fluid mechanics with emphasis on turbomachinery aerodynamics, *Int. J. Turbo Jet-Engines* **14**(1) (1997) 23–28.

[21] J. H. He, Variational principles for some nonlinear partial dikerential equations with variable coencients, *Chaos Solitons Fractals* **19**(4) (2004) 847–851.

[22] J. H. He, Some asymptotic methods for strongly nonlinear equations, *Internat, J. Modern Phys. B* **20**(10) (2006) 1141–1199.

[23] E. W. Weisstein, Concise encyclopedia of mathematics, 2nd edn, *CRC Press, New York* (2002).