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The Proposed RSSE Method to Estimate Type One Censoring Data with S-function for COVID-19

Khaldoun Mohammad Ayyalsalman[1,], Iden Hassan Alkanani[2], Adeeb Ahmed AL Rahamneh[3], Muhammad Turki Alshurideh[4], Khaleel Al-Daoud[5] and Sulieman Ibraheem Shelash Al-Hawary[6]*

[1]Department of Financial and Banking Sciences, Amman University College, Al-Balqa Applied University, Amman, Jordan
[2]Department of Mathematics, College of Science for Women, University of Baghdad, Baghdad, Iraq
[3]Department of Economics, Faculty of Business, Al-Balqa Applied University, Salt, Jordan
[4]Department of Marketing, School of Business, The University of Jordan, Amman, Jordan
[5]Department of Accounting, Business School Faculty, Al-Ahliyya Amman University, Amman, Jordan
[6]Department of Business Administration, School of Business, Al al-Bayt University, Mafraq, Jordan

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Abstract: To get the estimated values for parameters, this study use the recommended Ranking set sampling estimator method (RSSE), which is based on the Newton Raphson approach. For type one censored data, the parameters of the Exponential-Rayleigh distribution of order statistic are estimated. subsequently utilising real COVID-19 data from Health and Environment Ministry in Iraqi and Al Karkh General Hospital. Fuzzy numbers are obtained by estimating their values using the nonlinear membership function (S-function). Finally, the ranking algorithm is applied to turn the fuzzy numbers into crisp numbers.

Keywords: Exponential-Rayleigh distribution (ERD), Order Statistic, Type One Censored Data, Nonlinear Membership Function, COVID-19, Ranking function

1 Introduction

A new distribution was proposed and introduced by [1], which they called Exponential-Rayleigh distribution (ERD), it was mixed between the cumulative function of Exponential distribution with one parameter, which is a scale, and the cumulative function of Rayleigh distribution with one parameter also of scale type.

Experimenters and researchers may not receive all of the information regarding the failure times of experimental units in many life testing and reliability studies; this is known as censoring data. According to [2], there are three categories for censored samples: interval-censored, left-censored, and right-censored. Additionally, there are three branches within the right-censored sample: the singly type 1 sample of censoring, the singly type 2 sample of censoring, and the sample of progressively censoring.

Zadeh discovered fuzzy set theory in 1965 [3]. Bradshaw employed fuzzy sets for the first time in 1983 to explain how each product unit's standards relate to the measurement of conformance [4]. In many respects, fuzzy set theory has been a major area of study. There are several applications of fuzzy sets in fields including computer science, operation research, logic, statistics, and medicine [5].

When the lifespan observations were given as fuzzy numbers, researchers employed the Bayesian estimation approach to estimate the values of the parameters of the exponential distribution according to the type 2 censoring scheme [2]. Chaturvedi et al. [6] published parameter estimation techniques utilising Type-II progressively hybrid censored fuzzy lifespan data. Morover, authors like [7] estimated Three novel mixture distribution parameters under type one censored sample, using (MLE) based on the Newton Raphson procedure, by employing the real data for lung cancer and stomach cancer.

Hussein et al [1] proposed a new distribution, they were called Exponential-Rayleigh distribution, they were finding the estimation values for survival function and hazard rate function by using computation of greatest likelihood method. Utilising the ranking function to

* Corresponding author e-mail: khaldoun.alsoudy@bau.edu.jo

transform fuzzy values into crisp ones, this study proposes the Ranking Set Sampling Estimator Method (RSSE), which is based on the nonlinear Membership Function S-function., to estimate the parameters of (ERD) under an order statistic for type one censored sample. Subsequently, apply the mean square error method to determine which is optimal for both conventional and fuzzy numbers.

This research contains, section two includes some properties of Exponential-Rayleigh distribution, section three deriving the proposed (RSSE) method, section four comprises some definitions of fuzzy sets, section five contain ranking function, section six about descriptive data, section seven consisting of numerical results. Finally, section eight about the conclusions.

2 The Exponential Rayleigh distribution:

The ERD's CDF is:

$$F(t;\theta,\beta) = 1 - e^{-(\theta t + \frac{\beta}{2}t^2)} \quad (1)$$

Using ERD, the probability density function is:

$$f(t;\theta,\beta) = \begin{cases} (\theta+\beta t)e^{-(\theta t+\frac{\beta}{2}t^2)} & ,t>0 \\ 0 & ,Otherwise \end{cases} \quad (2)$$

ERD has two scale parameters which are denoted by θ,β, the parameter space is as follows:

$$\Omega = \{(\theta,\beta);\theta>0,\beta>0\}$$

Given is the Survival function, which is:

$$S(t;\theta,\beta) = e^{-(\theta t+\frac{\beta}{2}t^2)} \quad (3)$$

The Hazard rate function is obtained by:

$$h(t;\theta,\beta) = \theta+\beta t \quad (4)$$

3 Parameters Estimation

This section estimates the unknown parameters of the ERD and illustrates the derivative using the (RSSE) approach for statistic type 1 censoring sample.

3.1 Type-One Censoring Data

One of the most widely used right-censored kinds, it has an observed, random number of failures time denoted by m and a specified experiment period indicated by t. Conversely, (n-m) denotes the subjects or units that did not fail or survive, and n stands for the subjects or objects that are included in the study.

3.2 Order Statistic

Let $X_1,X_2,X_3,...,X_n$ be a random sample from continuous probability density function f(x) with provided $a<x<b$, let Y_1 be the smallest of these x_i,Y_2 be the next in order to magnitude and Y_n be the largest of these x_i. That means

$Y_1<Y_2<...<Y_n$ denotes $X_1,X_2,X_3,...,X_n$ when the latter are arranged in a sending order of magnitude. Then $Y_1,Y_2,...,Y_n$ are denoted of the order statistic of random sample $X_1,X_2,X_3,...,X_n$. Now easy to formulate the probability density function of any order statistic, called Y_i in term of f(x) and F(x) as follows:

$$g(y_i) = \begin{cases} \frac{m!}{(i-1)!(m-i)!}\left[F(y_i)\right]^{i-1}\left[1-F(y_i)\right]^{m-i} & a<y_i<b \\ 0 & Otherwise \end{cases} \quad (5)$$

3.3 Ranking Set Sampling Estimator Method (RSSE)

This estimator method's concept is to apply the order statistic to the (ERD) in equation 5. Lastly, identify the parameters of this distribution by maximising these parameters using the greatest likelihood estimator. First apply the order statistic as follows:

$$g(t_{(i)}) = \frac{m!}{(i-1)!(m-i)!}\left[F(t_{(i)})\right]^{i-1} \left[1-F(t_{(i)})\right]^{m-i} f(t_{(i)}) \quad (6)$$

After that finding the likelihood function for equation 6 by utilizing the progressively censoring formula as follows:

$$g(t_{(i)}) = \frac{m!}{(i-1)!(m-i)!}\left[1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{i-1} \left[e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{m-i+1}(\theta+\beta t_{(i)}) \quad (7)$$

$$L = \frac{n!}{(n-m)!}\left[s(t_{(m)};\theta,\beta)\right]^{n-m}\prod_{i=1}^{m} g\left[t_{(i)}\right] \quad (8)$$

$$L = \frac{n1}{(n-m)!}\left[e^{-(\theta t_{(m)}+\frac{\beta}{2}t_{(m)}^2)}\right]^{n-m} \left[\frac{m!}{(i-1)!(m-i)!}\right]^m \prod_{i=1}^{m}\left[1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{i-1} \prod_{i=1}^{m}\left[e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{m-i+1}\prod_{i=1}^{m}\left[\theta+\beta t_{(i)}\right] \quad (9)$$

$$let \frac{n!}{(n-m)!} = a, \frac{m!}{(i-1)!(m-i)!} = b$$

Then we get:

$$\begin{aligned} L = a\left[e^{-(n-m)(\theta t_{(m)}+\frac{\beta}{2}t_{(m)}^2)}\right] \\ b^m \prod_{i=1}^{m}\left[1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{i-1} \\ \prod_{i=1}^{m}\left[e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}\right]^{m-i+1} \prod_{i=1}^{m}[\theta+\beta t_{(i)}] \end{aligned} \tag{10}$$

Calculating both sides of the equation's natural logarithm:

$$\begin{aligned} lnL = lna - (n-m)(\theta t_{(m)} + \frac{\beta}{2}t_{(m)}^2) + m\, lnb \\ + \sum_{i=1}^{m}(i-1)ln(1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}) \\ - \sum_{i=1}^{m}(m-i+1)(\theta t_{(i)} + \frac{\beta}{2}t_{(i)}^2) \\ + \sum_{i=1}^{m} ln(\theta+\beta t_{(i)}) \end{aligned} \tag{11}$$

Equation 11 is partially derived with regard to θ and β, respectively, and its value is set to zero.

$$\begin{aligned} \frac{\partial lnl}{\partial\theta} = -(n-m)t_{(m)} + \sum_{i=1}^{m}\frac{(i-1)t_{(i)}e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}} \\ - \sum_{i=1}^{m}(m-i+1)t_{(i)} + \sum_{i=1}^{m}\frac{1}{(\theta+\beta t_{(i)})} = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\partial lnl}{\partial\beta} = -\frac{1}{2}(n-m)t_{(m)}^2 \\ + \frac{1}{2}\sum_{i=1}^{m}\frac{(i-1)t_{(i)}^2 e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}} \\ - \frac{1}{2}\sum_{i=1}^{m}(m-i+1)t_{(i)}^2 + \sum_{i=1}^{m}\frac{t_{(i)}}{\theta+\beta t_{(i)}} = 0 \end{aligned} \tag{13}$$

Now, we can put $\frac{\partial lnl}{\partial\theta}$ as a function f(θ) and put $\frac{\partial lnl}{\partial\beta}$ as a function f(β)

$$\begin{aligned} f(\theta_k) = -(n-m)t_{(m)} + \sum_{i=1}^{m}\frac{(i-1)t_{(i)}e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}} \\ - \sum_{i=1}^{m}(m-i+1)t_{(i)} + \sum_{i=1}^{m}\frac{1}{(\theta+\beta t_{(i)})} \end{aligned} \tag{14}$$

$$\begin{aligned} f(\beta_k) = -\frac{1}{2}(n-m)t_{(m)}^2 \\ + \frac{1}{2}\sum_{i=1}^{m}\frac{(i-1)t_{(i)}^2 e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}} \\ - \frac{1}{2}\sum_{i=1}^{m}(m-i+1)t_{(i)}^2 + \sum_{i=1}^{m}\frac{t_{(i)}}{(\theta+\beta t_{(i)})} \end{aligned} \tag{15}$$

Observing that equations 14 and 15 are unable to resolve them, use an iterative technique like the following is the Newton-Raphson method to calculate the values of $\hat{\theta}$ and $\hat{\beta}$:

$$\theta_{k+1} = \theta_k - \frac{f(\theta_k)}{\acute{f}(\theta_k)}$$

Where

$$\begin{aligned} \acute{f}(\theta_k) = -\sum_{i=1}^{m}\frac{(i-1)t_{(i)}^2 e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{(1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)})^2} \\ - \sum_{i=1}^{m}\frac{1}{(\theta+\beta t_{(i)})^2} \end{aligned} \tag{16}$$

And

$$\beta_{k+1} = \beta_k - \frac{f(\beta_k)}{\acute{f}(\beta_k)}$$

Where

$$\begin{aligned} \acute{f}(\beta_k) = -\frac{1}{4}\sum_{i=1}^{m}\frac{(i-1)t_{(i)}^4 e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)}}{(1-e^{-(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2)})^2} \\ - \sum_{i=1}^{m}\frac{t_{(i)}^2}{(\theta+\beta t_{(i)})^2} \end{aligned} \tag{17}$$

Finally, using the following formula to stop the Newton-Raphson method:

$$\begin{bmatrix}\theta_{k+1} - \theta_k \\ \beta_{k+1} - \beta_k\end{bmatrix} \leq \begin{bmatrix}\in_\theta \\ \in_\beta\end{bmatrix}$$

4 Fuzzy set theory

This section will be talking about definitions of fuzzy sets, using the nonlinear S-function, and integrating the ranking function.

4.1 Fuzzy Set

Z will be a universal set, $\tilde{A}$ in Z is a fuzzy set which is defined by $\mu_{(\tilde{A})}(z) \to [0,1]$, where $\mu_{(\tilde{A})}(z)$ is a membership function,$\forall z \in Z$, and [0,1] is a membership set [2].

4.2 α-Cut

It is a crisp set defined as: $A_\alpha = \{z \in Z; A(z) \geq \alpha, \alpha \in [0,1]\}$ [8].

4.3 Fuzzy Number

It is a fuzzy subset in R, (real number), which is both normal and convex, where $sup(\tilde{A}) = \{x \in R, \mu_{\tilde{A}} > 0\}$ [8].

4.4 S-Function

The membership definition for S-function, S: R $\to$ [0, 1] will be:

$$S(x;a,b,c) = \begin{cases} 0 & x \leq a \\ 2(\frac{x-a}{c-a})^2 & a < x \leq b \\ 1-2(\frac{x-a}{c-a})^2 & b < x \leq c \\ 1 & x > c \end{cases}$$

With

$$b = \frac{a+c}{2}$$

4.5 Fuzzy Numbers Finding [a,b,c]

Suppose that $\Delta = 0.001$, Replace it in cases 1 and 2 to determine the approximate values.

Case 1:

Consider $a_1 = \hat{\theta} - \Delta$, $b_1 = \hat{\theta}$, $c_1 = \hat{\theta} + \Delta$ then $a_1 = \hat{\theta} - 0.001$, $b_1 = \hat{\theta}$, $c_1 = \hat{\theta} + 0.001$

Case 2:

Consider $a_2 = \hat{\beta} - \Delta$, $b_2 = \hat{\beta}$, $c_2 = \hat{\beta} + \Delta$ then $a_2 = \hat{\beta} - 0.001$, $b_2 = \hat{\beta}$, $c_2 = \hat{\beta} + 0.001$

5 Ranking Function

The ranking function was established by Jain in 1976, and Yeager proposed four indices in 1981. These might be used to sort fuzzy quantities in [0,1] [5]. Mapping from fuzzy numbers to real numbers is called a fuzzy number ranking function.

Ranking Function Algorithm

A nonlinear ranking function is used to transform the fuzzy number into a crisp number by (s-function), resulting in the following ranking function.

Let

$$\alpha = 2(\frac{x-a}{c-a})^2$$

Taking root for both sides, we get:

$$\sqrt{\frac{\alpha}{2}} = \frac{x-a}{c-a}$$

$$x = a + (c-a)\sqrt{\frac{\alpha}{2}}$$

$$\tilde{A}^1_{(a)} = a + (c-a)\sqrt{\frac{\alpha}{2}} \tag{18}$$

Let

$$\alpha = 1 - 2(\frac{x-a}{c-a})^2$$

Taking root for both sides, we get:

$$\sqrt{\frac{1-\alpha}{2}} = \frac{x-a}{c-a}$$

$$x = a + (c-a)\sqrt{\frac{1-\alpha}{2}}$$

$$\tilde{A}^\mu_{(a)} = a + (c-a)\sqrt{\frac{1-\alpha}{2}} \tag{19}$$

Where

$\tilde{A}^1_{(a)}$: is bounded left continuous maximizing function over $[0,\lambda]$.

$\tilde{A}^\mu_{(a)}$:is bounded left continuous minimizing function over $[0,\lambda]$.

Ordered pair of functions can be expressed to represent arbitrary fuzzy numbers as an $[\tilde{A}^1_{(a)}, \tilde{A}^\mu_{(a)}]$.

Where $\tilde{A}^1_{(a)} \leq \tilde{A}^\mu_{(a)}$, let $w = \frac{1}{2}$ be the weight for $\tilde{A}^1_{(a)}$ and $(1-w) = \frac{1}{2}$ be the weight for $\tilde{A}^\mu_{(a)}$.

$$R(\tilde{A}) = \frac{1}{2}\int_0^\lambda (\tilde{A}^1_{(a)} + \tilde{A}^\mu_{(a)})da \tag{20}$$

where $\lambda \in [0,1]$

$$R(\tilde{A}) = \frac{1}{2}\int_0^{\lambda}(2a+(c-a)\sqrt{\frac{\alpha}{2}}+(c-a)\sqrt{\frac{1-\alpha}{2}})da$$

$$R(\tilde{A}) = \frac{1}{2}\int_0^{\lambda} a\,d\alpha + \frac{c-a}{2\sqrt{2}}\int_0^{\lambda}\sqrt{a}\,d\alpha + \frac{c-a}{2\sqrt{2}}\int_0^{\lambda}\sqrt{1-a}\,d\alpha$$

$$R(\tilde{A}) = a\lambda + \frac{c-a}{3\sqrt{2}}\lambda^{\frac{3}{2}} - \frac{c-a}{3\sqrt{2}}(1-\lambda)^{\frac{3}{2}} + \frac{c-a}{3\sqrt{2}}$$

$$R(\tilde{A}) = a\lambda + \frac{c-a}{3\sqrt{2}}(1-\lambda^{\frac{3}{2}}) - \frac{c-a}{3\sqrt{2}}(1-\lambda)^{\frac{3}{2}} \quad (21)$$

6 Data Description

This research is based on actual data and a sample that were obtained from AL Karkh General Hospital, the Ministry of Health and Environment in Iraq. Concerning COVID-19. The study took place over the course of four months, or 120 days. The study comprised 1058 patients in total; six cases were excluded, comprising 26 prisoners, 48 individuals with negative swabs, 29 patients whose exit status was unknown, 2 patients who managed to escape the hospital, 35 patients who were transferred to other hospitals, and 133 patients who were discharged at their own responsibility. After then, there were 785 patients, and 88 of them passed away throughout the course of the investigation.

7 Numerical Results

Matlab programming (version 2021) was used to calculate the parameter values: $\hat{\theta} = 0.01804, \hat{\beta} = 0.00137$ with initial values $\theta_0 = 0.001, \beta_0 = 0.0002$.

There were fifteen patients who passed away in a single day, seven patients who passed away two days later, five patients who passed away three days later, nine patients who passed away four days later, four patients who passed away five days later, seventeen patients who passed away six days later, three patients who passed away nine days later, two patients who passed away ten days later, six patients who passed away twelve days later, three patients who passed away fifteen days later, and four patients who passed away eighteen days later. We wrote the data down briefly while organizing the data, taking note of the fact that number of patients passed away in the same day and avoiding repetition.

Next, assess of outcomes for the hazard rate function, survival function, and probability death density function. The fuzzy numbers are arranged after the table with crisp values.

Table 1: Represent $\hat{f}(t)$, $\hat{S}(t)$, $\hat{F}(t)$ and $\hat{h}(t)$

t	$\hat{f}(t)$	$\hat{S}(t)$	$\hat{F}(t)$	$\hat{h}(t)$
1	.019049929	.981449	.018551	.01941
2	.019988777	.961924	.038076	.02078
3	.02085414	.941496	.058504	.02215
4	.021644062	.920241	.079759	.02352
5	.022357037	.898234	.101766	.02489
6	.022992012	.875553	.124447	.02626
7	.023548383	.852276	.147724	.02763
8	.024025982	.828482	.171518	.029
9	.024425075	.80425	.19575	.03037
10	.024746343	.779658	.220342	.03174
11	.024990865	.754783	.245217	.03311
12	.025160101	.729701	.270299	.03448
13	.025255869	.704487	.295513	.03585
15	.025235915	.65395	.34605	.03859
16	.025125392	.628764	.371236	.03996
17	.024951742	.60372	.39628	.04133
18	.024718178	.57888	.42112	.0427

$\hat{f}(t)$ values rises till t=13, subsequent to which the $f(x)$ falls between $15 \le t \le 18$. $\hat{F}(t)$ values rises as failure times rises, while the values of $\hat{S}(t)$ decrease and the $\hat{h}(t)$ values rises with the rises in failure times. Survival function has a mean square error of 0.373006.

Now, we can find the fuzzy numbers as:

Case 1:

$a_1 = 0.01704,\ b_1 = 0.01804,\ c_1 = 0.01904$

Case 2:

$a_2 = 0.00037,\ b_2 = 0.00137,\ c_2 = 0.00237$

From case1, case2 and by equation 21 we substitute

$\lambda = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ to determine which is optimal to use (MSE).

Table 2: Represent $\hat{f}(t)$, $\hat{S}(t)$, $\hat{F}(t)$ and $\hat{h}(t)$ when $\lambda = 0.8$

t	$\hat{f}(t)$	$\hat{S}(t)$	$\hat{F}(t)$	$\hat{h}(t)$
1	.018455	.983555	.016445	.018764
2	.022277	.963167	.036833	.023129
3	.025819	.939094	.060906	.027494
4	.029044	.911635	.088365	.031859
5	.031918	.881123	.118877	.036224
6	.034416	.847924	.152076	.040589
7	.036522	.812422	.187578	.044954
8	.038223	.775016	.224984	.049319
9	.039517	.736112	.263888	.053684
10	.040409	.696115	.303885	.058049
11	.040908	.655425	.344575	.062414
12	.041031	.614425	.385575	.066779
13	.0408	.573482	.426518	.071144
15	.039386	.493098	.506902	.079874
16	.038266	.454252	.545748	.084239
17	.036916	.416643	.583357	.088604
18	.035373	.380484	.619516	.092969

Table 3: Represent the values of $\hat{f}(t)$, $\hat{S}(t)$, $\hat{F}(t)$ and $\hat{h}(t)$ when $\lambda = 0.9$

t	$\hat{f}(t)$	$\hat{S}(t)$	$\hat{F}(t)$	$\hat{h}(t)$
1	.017099	.983348	.016652	.017389
2	.017947	.96582	.03418	.018582
3	.018736	.947473	.052527	.019775
4	.019466	.928367	.071633	.020968
5	.020135	.908562	.091438	.022161
6	.020741	.888119	.111881	.023354
7	.021285	.8671	.1329	.024547
8	.021765	.84557	.15443	.02574
9	.022182	.823592	.176408	.026933
10	.022535	.801228	.198772	.028126
11	.022826	.778542	.221458	.029319
12	.023055	.755596	.244404	.030512
13	.023222	.732453	.267547	.031705

Noting that values of $\hat{f}(t)$ are increasing till t=12, the $f(x)$ is decreasing when $13 \leq t \leq 18$. $\hat{F}(t)$ values are increasing with the rises of failure times, $\hat{S}(t)$ values are decreasing with the rises of failure times, and $\hat{h}(t)$ values are increasing with the failure times increases. Survival function (MSE) is 0.027478.

8 Conclusions

Using mean square error, Tables 1, 2, and 3 demonstrate that fuzzy numbers outperform crisp numbers; that is, the (MSE) of fuzzy numbers is lower than the (MSE) of traditional numbers. When $\lambda \in [0.8, 0.9]$.

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