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Nonlinear Control Technique for Hyperchaotic Complex Nonlinear Systems

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Abstract: This work presents hyperchaos synchronization between two identical hyperchaotic systems by using nonlinear control technique. This technique is applied to achieve chaos synchronization for two identical hyperchaotic complex Chen systems. The idea of hyperchaos synchronization is to use the output of the drive system to control the response system so that the output of the response system follows the output of the drive system asymptotically. Lyapunov functions are derived to prove that the error systems are asymptotically stable. Expressions are derived for the control functions which are used to achieve chaos synchronization. Numerical simulations show the validity of these expressions.

Keywords: Hyperchaos, Synchronization, Nonlinear control, Error system, Lyapunov functions, Complex

1 Introduction

Research in the area of the synchronization of dynamical systems dates back over 300 years. Huygens, most famous for his studies in optics and the construction of telescopes and clocks, was probably the first scientist who observed and described the synchronization phenomenon as early as in the 17th century. The pioneering paper on the concept of chaos synchronization was not presented until 1990. Pecora and Carroll introduced a method [1] to synchronize two identical chaotic systems with different initial conditions. Because of their works, chaos synchronization has been intensively studied in the last few years. It has been widely explored in a variety of fields including physical, chemical and ecological [2] systems, secure communications [3–5].

Hyperchaos synchronization is a very important nonlinear phenomenon, which has been studied to date on dynamical systems described by real variables. There also exist, however, interesting cases of dynamical systems, where the main variables participating in the dynamics are complex, as for example when amplitudes of electromagnetic fields are involved. Another example is when chaos synchronization is used for communications, where doubling the number of variables may be used to increase the content and security of the transmitted information. A similar generalization of the real Lorenz system to the corresponding one with complex ODEs has been introduced to describe and simulate the physics of laser and thermal convection of liquid flows [6–10]. The electric field amplitude and the atomic polarization amplitude are both complex, for details see, e.g. [11–14] and references therein.

Recently, we have introduced the hyperchaotic complex Chen system [15]. This system is hyperchaotic and exhibit chaotic and hyperchaotic attractors. The fixed points and their stability are studied of these complex systems. The main goal of this paper is to investigate and study the chaos synchronization of two identical hyperchaotic complex Chen systems by using nonlinear control technique [16–20]. The hyperchaotic complex Chen system expressed by:

$$\begin{aligned}
\dot{x} &= \alpha\,(y-x)\,,\\
\dot{y} &= (\gamma-\alpha)\,x - xz + \gamma y + w,\\
\dot{z} &= 1/2\,(\overline{x}y + x\overline{y}) - \beta z + w,\\
\dot{w} &= 1/2\,(\overline{x}y + x\overline{y}) - dw,
\end{aligned} \tag{1}$$

where α, β, γ are positive parameters and d is control parameter, $x = u_1 + iu_2$, $y = u_3 + iu_4$ are complex

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function, $i=\sqrt{-1}$ and z,w are real function. Dots represent derivatives with respect to time and $\overline{(\dots)}$ the complex conjugate function. For the case $\alpha=32$, $\beta=4$, $\gamma=25$ and $d=5$ with the initial conditions $t_0=0$, $u_1(0)=1$, $u_2(0)=2$, $u_3(0)=3, u_4(0)=4$, $u_5(0)=5$ and $u_6(0)=6$ we calculate the Lyapunov exponents as:
$\lambda_1=2.23$, $\lambda_2=1.13$, $\lambda_3=0$, $\lambda_4=-8.41$, $\lambda_5=-11.07$, $\lambda_6=-17.06$ [21].

This paper is organized as follows: In Section 2, we study the synchronization of two identical hrperchaotic complex Chen systems with parameter perturbation via nonlinear control technique. Expressions are derived for the control functions which are used to achieve hyperchaos synchronization. Numerical simulations show the validity of these expressions. Some figures are presented to show our results for chaos synchronization and their errors. Finally in Section 3 we summarize the main conclusions of our investigations.

2 Synchronization of two identical hyperchaotic complex Chen systems

2.1 Theoretical results

This subsection is devoted to study the hyperchaos synchronization of the complex Chen system using the idea of nonlinear control technique as follows :
We assume that we have two hyperchaotic complex Chen systems and the drive system with the subscript ”d” is to control the response system with subscript ”r”. The derive and response systems defined respectively as:

$$\begin{aligned}
\dot{x_d} &= \alpha\,(y_d - x_d),\\
\dot{y_d} &= (\gamma-\alpha)\,x_d - x_d z_d + \gamma y_d + w_d,\\
\dot{z_d} &= 1/2\,(\overline{x_d} y_d + x_d \overline{y_d}) - \beta z_d + w_d,\\
\dot{w} &= 1/2\,(\overline{x_d} y_d + x_d \overline{y_d}) - d w_d
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\dot{x_r} &= \alpha\,(y_r - x_r) + (v_1 + i v_2),\\
\dot{y_r} &= (\gamma-\alpha)\,x_r - x_r z_r + \gamma y_r + w_r + (v_3 + i v_4),\\
\dot{z_r} &= 1/2\,(\overline{x_r} y_r + x_r \overline{y_r}) - \beta z_r + w_r + v_5,\\
\dot{w_r} &= 1/2\,(\overline{x_r} y_r + x_r \overline{y_r}) - d w + v_6,
\end{aligned} \tag{3}$$

where $x_d=u_{1d}+iu_{2d}$, $y_d=u_{3d}+iu_{4d}$ are complex state variables, $z_d=u_{5d}$, $w_d=u_{6d}$ are real state variable, $x_r=u_{1r}+iu_{2r}$, $y_r=u_{3r}+iu_{4r}$ and $z_r=u_{5r}$, $w_r=u_{6r}$, $\overline{(\dots)}$ denotes the complex conjugate variable and v_1+iv_2, v_3+iv_4 and v_5, v_6 are complex and real control functions respectively, which are to be determined.
The complex system (2) can be rewritten as a five real first order ODEs of the form :

$$\begin{aligned}
\dot{u}_{1d} &= \alpha\,(u_{3d} - u_{1d}),\\
\dot{u}_{2d} &= \alpha\,(u_{4d} - u_{2d}),\\
\dot{u}_{3d} &= (\gamma-\alpha)\,u_{1d} - u_{1d}u_{5d} + \gamma u_{3d} + u_{6d},\\
\dot{u}_{4d} &= (\gamma-\alpha)\,u_{2d} - u_{2d}u_{5d} + \gamma u_{4d},\\
\dot{u}_{5d} &= u_{1d}u_{3d} + u_{2d}u_{4d} - \beta u_{5d} + u_{6d},\\
\dot{u}_{6d} &= u_{1d}u_{3d} + u_{2d}u_{4d} - d u_{6d},
\end{aligned} \tag{4}$$

and the response system (3) can be rewritten as a five real first order differential equations of the form:

$$\begin{aligned}
\dot{u}_{1r} &= \alpha\,(u_{3r} - u_{1r}) + v_1,\\
\dot{u}_{2r} &= \alpha\,(u_{4r} - u_{2r}) + v_2,\\
\dot{u}_{3r} &= (\gamma-\alpha)\,u_{1r} - u_{1r}u_{5r} + \gamma u_{3r} + u_{6r} + v_3,\\
\dot{u}_{4r} &= (\gamma-\alpha)\,u_{2r} - u_{2r}u_{5r} + \gamma u_{4r} + v_4,\\
\dot{u}_{5r} &= u_{1r}u_{3r} + u_{2r}u_{4r} - \beta u_{5r} + u_{6r} + v_5.\\
\dot{u}_{6r} &= u_{1r}u_{3r} + u_{2r}u_{4r} - d u_{6r},
\end{aligned} \tag{5}$$

In order to obtain the active control signals, we define as the error states between the response system that is be controlled and the controlling derive system as :

$$\begin{aligned}
e_{u_1} + i e_{u_2} &= x_r - x_d = (u_{1r} - u_{1d}) + i(u_{2r} - u_{2d}),\\
e_{u_3} + i e_{u_4} &= y_r - y_d = (u_{3r} - u_{3d}) + i(u_{4r} - u_{4d}),\\
e_{u_5} &= z_r - z_d = u_{5r} - u_{5d}.\\
e_{u_6} &= w_r - w_d = u_{6r} - u_{6d}.
\end{aligned} \tag{6}$$

and using

$$\begin{aligned}
u_{1d}u_{5d} - u_{1r}u_{5r} &= -u_{1r}u_{5r} + u_{1r}u_{5d} - u_{1r}u_{5d} + u_{1d}u_{5d}\\
&= -(u_{5r} - u_{5d})u_{1r} - u_{5d}(u_{1r} - u_{1d})\\
&= -e_{u_5}u_{1r} - u_{5d}e_{u_1},\\
u_{2d}u_{5d} - u_{2r}u_{5r} &= -u_{2r}u_{5r} + u_{2r}u_{5d} - u_{2r}u_{5d} + u_{2d}u_{5d}\\
&= -(u_{5r} - u_{5d})u_{2r} - u_{5d}(u_{2r} - u_{2d})\\
&= -e_{u_5}u_{2r} - u_{5d}e_{u_2},\\
u_{3r}u_{1r} - u_{1d}u_{3d} &= u_{3r}u_{1r} - u_{3r}u_{1d} + u_{3r}u_{1d} - u_{1d}u_{3d}\\
&= u_{3r}(u_{1r} - u_{1d}) + u_{1d}(u_{3r} - u_{3d})\\
&= u_{3r}e_{u_1} + u_{1d}e_{u_3},\\
u_{4r}u_{2r} - u_{2d}u_{4d} &= u_{4r}u_{2r} - u_{4r}u_{2d} + u_{4r}u_{2d} - u_{2d}u_{4d}\\
&= u_{4r}(u_{2r} - u_{2d}) + u_{2d}(u_{4r} - u_{4d})\\
&= u_{4r}e_{u_2} + u_{1d}e_{u4}.
\end{aligned} \tag{7}$$

Subtracting (2) from (3) using (6) and(7) to get :

$$\begin{aligned}
\dot{e}_{u_1} + i\dot{e}_{u_2} &= \alpha\left[(e_{u_3} - e_{u_1}) + i\,(e_{u_4} - e_{u_2})\right]\\
&\quad + (v_1 + i v_2),\\
\dot{e}_{u_3} + i\dot{e}_{u_4} &= -\alpha\,(e_{u_1} + i e_{u_2}) + \gamma[(e_{u_1} + e_{u_3})\\
&\quad + i(e_{u_2} + e_{u_4})] - e_{u_5}\,(u_{1r} + i u_{2r})\\
&\quad - u_{5d}\,(e_{u_1} + i e_{u_2}) + e_{u_6} + (v_3 + i v_4),\\
\dot{e}_{u_5} &= -\beta e_{u_5} + u_{1d}e_{u_3} + u_{3r}e_{u_1} + u_{2d}e_{u_4}\\
&\quad + u_{4r}e_{u_2} + e_{u_6} + v_5,\\
\dot{e}_{u_6} &= -d e_{u_6} + u_{1d}e_{u_3} + u_{3r}e_{u_1} + u_{2d}e_{u_4}\\
&\quad + u_{4r}e_{u_2} + v_5.
\end{aligned} \tag{8}$$

Equation (8) describes a dynamical system via which the “errors” evolve in time and finally the ODEs of this system in real form become:

$$\begin{aligned}
\dot{e}_{u_1} &= \alpha\,(e_{u_3} - e_{u_1}) + v_1,\\
\dot{e}_{u_2} &= \alpha\,(e_{u_4} - e_{u_2}) + v_2,\\
\dot{e}_{u_3} &= -\alpha e_{u_1} + \gamma(e_{u_1} + e_{u_3}) - e_{u_5}u_{1r}\\
&\quad - u_{5d}e_{u_1} + e_{u_6} + v_3,\\
\dot{e}_{u_4} &= -\alpha e_{u_2} + \gamma(e_{u_2} + e_{u_4}) - e_{u_5}u_{2r}\\
&\quad - u_{5d}e_{u_2} + e_{u_6} + v_4,\\
\dot{e}_{u_5} &= -\beta e_{u_5} + u_{1d}e_{u_3} + u_{3r}e_{u_1} + u_{2d}e_{u_4}\\
&\quad + u_{4r}e_{u_2} + e_{u_6} + v_5,\\
\dot{e}_{u_6} &= -d e_{u_6} + u_{1d}e_{u_3} + u_{3r}e_{u_1}\\
&\quad + u_{2d}e_{u_4} + u_{4r}e_{u_2} + v_6.
\end{aligned} \tag{9}$$

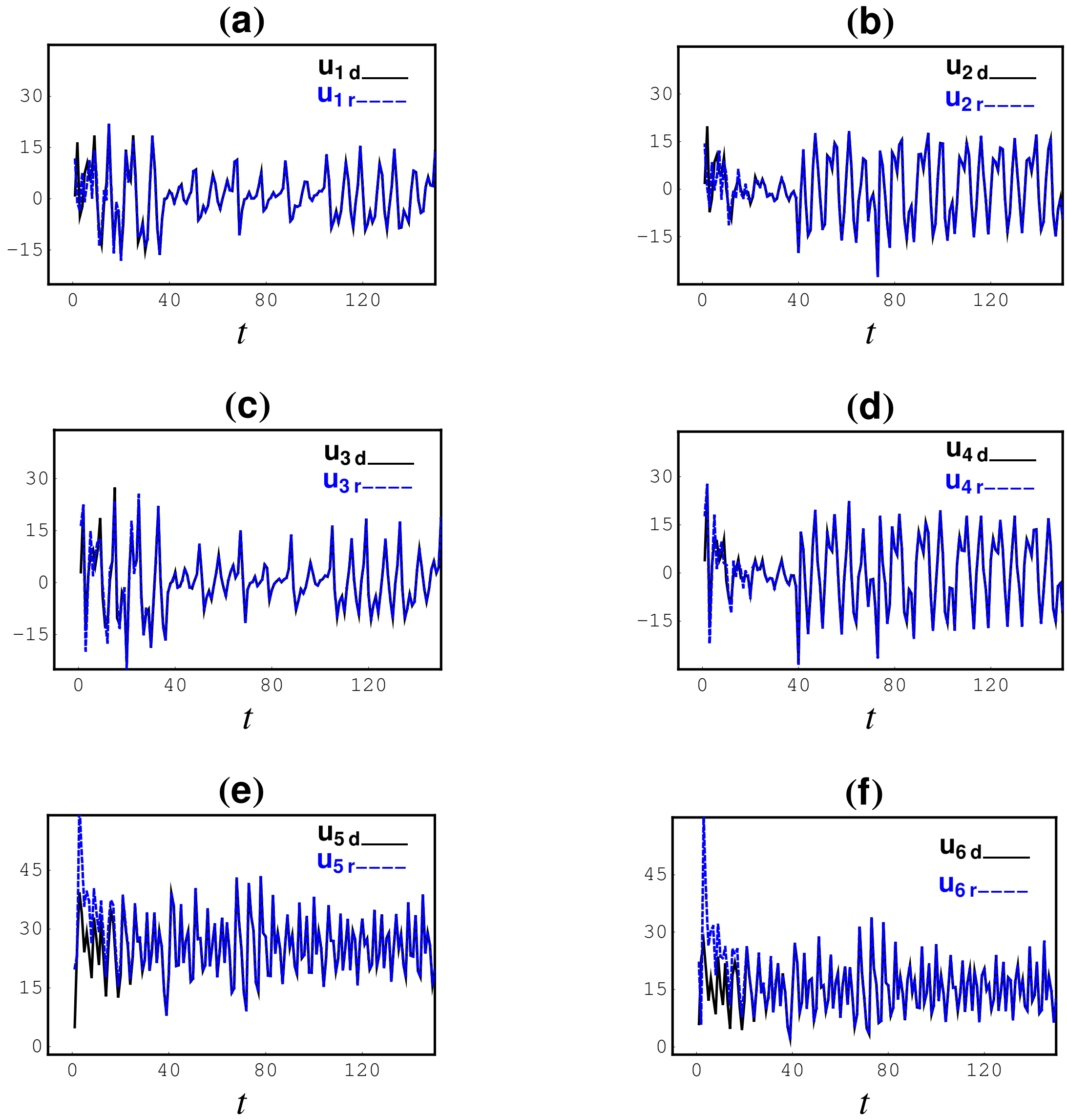

Fig. 1: Hyperchaos synchronizations of systems (4) and (5) with (12):
(a) $u_{1d}(t)$ and $u_{1r}(t)$ versus t, (b) $u_{2d}(t)$ and $u_{2r}(t)$ versus t, (c) $u_{3d}(t)$ and $u_{3r}(t)$ versus t, (d) $u_{4d}(t)$ and $u_{4r}(t)$ versus t, (e) $u_{5d}(t)$ and $u_{5r}(t)$ versus t (f) $u_{6d}(t)$ and $u_{6r}(t)$ versus t

For positive parameters γ, α and β, one defines a Lyapunov function by the following quantity:

$$V(t) = 1/2\sum_{i=1}^{6} e_{u_i}^2. \tag{10}$$

The derivative of $V(t)$ along the solution of system (8) is :

$$\begin{aligned}\dot{V}(t) = {} & e_{u_1}[\alpha(e_{u_3} - e_{u_1})] \\ & + e_{u_2}[\alpha(e_{u_4} - e_{u_2})] \\ & + e_{u_3}[-\alpha e_{u_1} + \gamma(e_{u_1} + e_{u_3}) \\ & - e_{u_5}u_{1r} - u_{5d}e_{u_1} + e_{u_6}] \\ & + e_{u_4}[-\alpha e_{u_2} + \gamma(e_{u_2} + e_{u_4}) \\ & - e_{u_5}u_{2r} - u_{5d}e_{u_2}] \\ & + e_{u_5}[-\beta e_{u_5} + u_{1d}e_{u_3} + u_{3r}e_{u_1} \\ & + u_{2d}e_{u_4} + u_{4r}e_{u_2} + e_{u_6}] \\ & + e_{u_6}[-de_{u_6} + u_{1d}e_{u_3} + u_{3r}e_{u_1} \\ & + u_{2d}e_{u_4} + u_{4r}e_{u_2}] + \Sigma_{i=1}^{6} v_i e_{u_i}.\end{aligned} \tag{11}$$

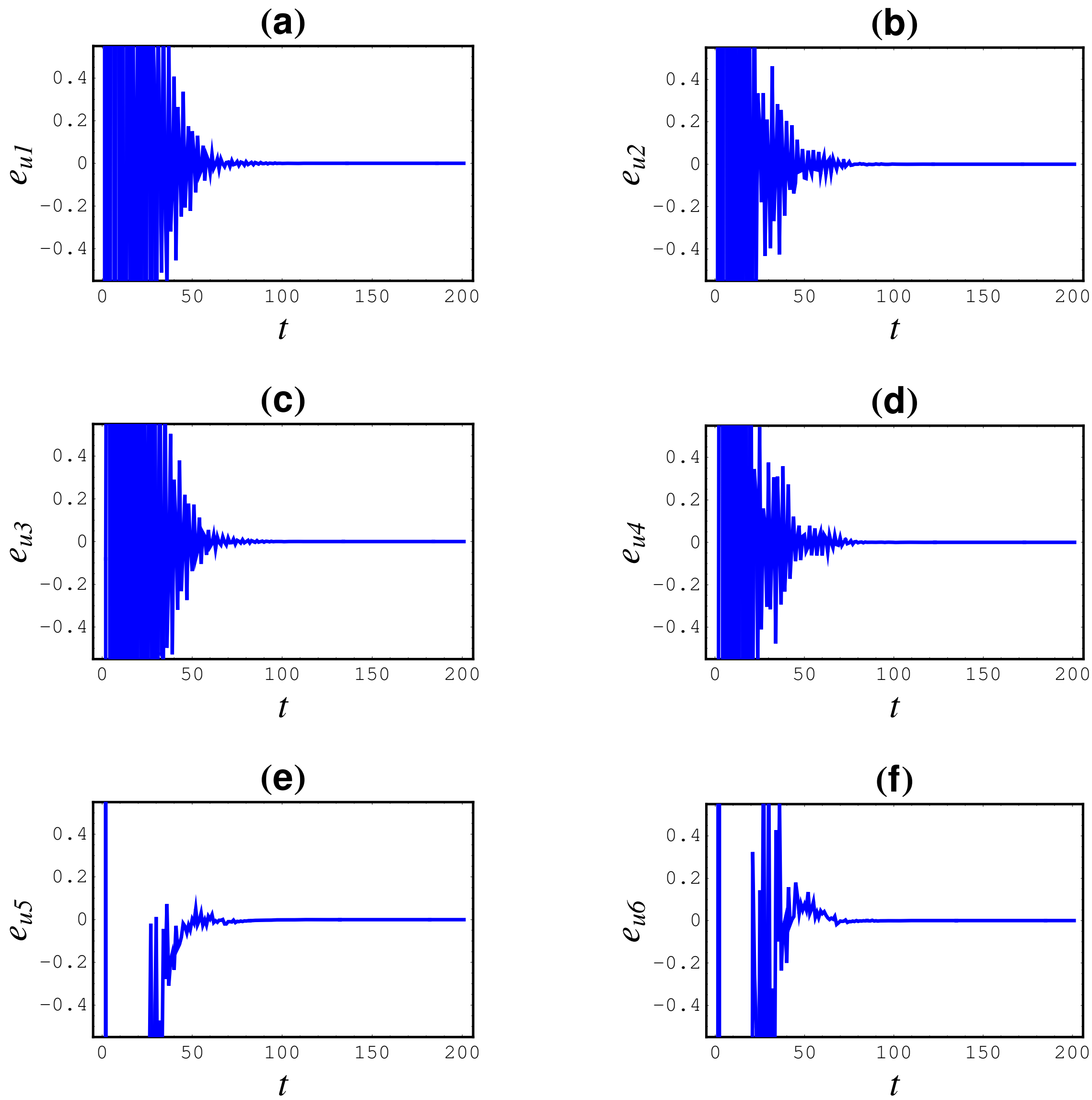

Fig. 2: Hyperchaos synchronizations errors (solutions of system (9))
(a) (e_{u_1},t) diagram, (b) (e_{u_2},t) diagram, (c) (e_{u_3},t) diagram, (d) (e_{u_4},t) diagram, (e) (e_{u_5},t) diagram, (f) (e_{u_6},t) diagram

If we choose the active control function v_i such that:

$$\begin{aligned} v_1 &= (\alpha-1)e_{u_1}, \\ v_2 &= (\alpha-1)e_{u_2}, \\ v_3 &= -(\gamma+1)e_{u_3}+e_{u_1}e_{u_5} \\ &\quad +(u_{5d}-\gamma)e_{u_1}-e_{u_6}, \\ v_4 &= -(\gamma+1)e_{u_4}+e_{u_2}e_{u_5}+(u_{5d}-\gamma)e_{u_2}, \\ v_5 &= (\beta-1)e_{u_5}-u_{3r}e_{u_1}-u_{4r}e_{u_2}-e_{u_6}, \\ v_6 &= (d-1)e_{u_6}-u_{3r}e_{u_1}-u_{4r}e_{u_2}. \end{aligned} \quad (12)$$

Equation (11) becomes:

$$\dot{V}(t) = -(e_{u_1}^2+e_{u_2}^2+e_{u_3}^2+e_{u_4}^2+e_{u_5}^2+e_{u_6}^2) < 0. \quad (13)$$

Since $V(t)$ is a positive function and $\dot{V}(t)$ is negative function, it follows that the equilibrium points ($e_{u_1}=0$, $e_{u_2}=0$, $e_{u_3}=0$, $e_{u_4}=0$, $e_{u_5}=0$, $e_{u_6}=0$) of the system (9) is asymptotically stable, which means that the error states $e_{u_1}, e_{u_2}, e_{u_3}, e_{u_4}$, e_{u_5} and e_{u_6} are converged to zero as time t tends to infinity and hence the nonlinear control technique of two identical chaotic systems is achieved.

2.2 Numerical results

Systems (2) and (3) with (12) are solved numerically (using e.g. Mathematica 7 software) for $\alpha = 32$, $\beta = 4$, $\gamma = 25$ and $d = 5$ with the initial conditions $t_0 = 0$, $u_{1d}(0) = 1$, $u_{2d}(0) = 2$, $u_{3d}(0) = 3$, $u_{4d}(0) = 4$, $u_{5d}(0) = 5, u_{6d}(0) = 6$ and $u_{1r}(0) = -13$, $u_{2r}(0) = -12$, $u_{3r}(0) = -13$, $u_{4r}(0) = -14$, $u_{5r}(0) = 40, u_{6r}(0) = 30$. The simulation results are illustrated in Figures 1 and 2. In Figure 1 the solutions of (2) and (3) are plotted subject to different initial conditions. It shows that the hyperchaos synchronization is achieved after very small values of t. In Figure 2 it can be seen that the synchronization errors e_{u_j} $j = 1, 2, \ldots, 6$ will converge to zero.

3 Conclusions

Our main goal in this paper is to investigate and study the synchronization of two identical hyperchaotic complex Chen systems by using nonlinear control technique. This work demonstrates that hyperchaos synchronization between two identical hyperchaotic systems using nonlinear control technique is achieved. Figures 1 shows the chaos synchronizations is achieved after small values of t using the technique of this paper. In Figures 2 It's clear that the synchronization errors e_{u_j} will converge to zero. Numerical results show that this technique is very effective.

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