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Dynamics of Mental Disorders Using ψ- Hilfer Fractional Derivative

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Abstract: Around the globe, fear and anxiety are being triggered by the COVID-19 pandemic and lockdown, the stress induced by this issue has resulted in a negative impact on people's mental health and psychosocial development. The repercussions of this epidemic on one's mental health have not received much focus in the society. In this paper, We formulate a mathematical model to investigate the dynamic behavior of mental disorders induced by the COVID-19 pandemic. The peculiar behavior of the transition between mental disorders, coexistence, and their influence on people's mental health are studied in the model. A fixed-point analysis is used to determine the existence and uniqueness of the solution with ψ-Hilfer fractional derivatives. Adomian decomposition method paired with Laplace integral transform is employed to obtain the solution. The numerical simulation of the fractional model is investigated through Caputo fractional derivative and the graphical representation are depicted for different fractional orders. Further, we compare our results with reported real data against the simulated data of the depressed population for the consecutive 7 years in India. This research is intended towards improving a strong knowledge about psychological disorders in an effort to enhance mental wellness.

Keywords: ψ- Hilfer fractional derivative, COVID-19, mental health, fixed point theory.

1 Introduction

The COVID-19 pandemic may have prompted significant changes in human life, including high mortality rates, financial challenges, and sustained social isolation [1]. The mental wellness of individuals was negatively impacted because of the COVID-19 outbreak and the accompanying economic downturn, and people who had mental health difficulties encountered additional challenges [2,3]. At this stage of COVID-19, it is reasonable to anticipate that government initiatives and efforts to curb the disease transmission will have an influence on mental health [4]. Unemployment, economic difficulty, and social isolation are all connected to mental health issues. [5]. As a result of physical distancing measures, unemployment has risen drastically in many countries, leading to significant financial strain [6].

An increase in anxiety and depression disorders, suicidal behaviour, and post-traumatic stress symptoms, have been observed among the people. In addition, quarantine-related emotions have contributed to increased feelings of depression, anxiety, insomnia, and social disconnect, partially due to the concern of contracting or transmitting the illness to family members [7,8]. Furthermore, closures of public buildings and commercial locations have disrupted many of the young generation's plans for education and career, while unemployment and financial debt have exacerbated their anxiety and depression [9].

National and international health organizations have highlighted the importance of mitigating the threats posed to psychological well-being by the COVID-19 pandemic [10]. Epidemics of mental health at a large scale across communities have minimal evidence of their acute phase mental health, when the symptoms of disorders begin to emerge. Current studies typically focus on individuals with the most direct contact with the disease [11]. As a result of lockdowns, false information, and the lack of knowledge, the public has suffered from depression and anxiety disorders.

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Despite this, there are no adequate studies available to contain or attenuate the mental health consequences of the COVID-19 pandemic [12,13].

A recent study found that certain inherited gene variations can contribute, to depression. Numerous studies have found that teenagers and young adults who are exposed to stressful conditions (relationship breakdowns, loss of jobs, the tragic passing of a parent or sibling) are still more prone to develop serious depression. This is particularly true for individuals who inherited a certain type of serotonin transporter gene that is responsible for brain cell signaling and communication. Further, functional magnetic resonance imaging scans showed that people with this gene variant were hyperactive in a brain region related to anxiety and fear [14].

There is an intriguing way in which depression and anxiety are contagious. It is possible to spread these emotions, including social comparison, emotional interpretation, and empathy [15].Comparison between yourself to others, especially those with negative thinking patterns, is social comparison. The emotional interpretation of information is when facts become negatively interpreted or interpreted differently. The ability to empathize involves understanding and sharing other people's feelings. Due to these factors, anxiety and depression are contagious [16].

The best way to study biological and engineering systems is to use fractional-order differential equations [17]. Modeling physical and biological systems with fractional-order derivatives is based on several fractional-order derivatives [18,19]. There are several reasons to use fractional order, but the essential one is that it helps to deal with the memory dynamics observed in numerous biological systems. Vanterler et al. [20] proposed a fractional differential operator for a function known as the ψ-Hilfer derivative. There are not many articles dealing with mental disorders from the fractional point of view. In [21], the author uses fractional differential equations to evaluate the mental health of college studentd. The goal of this study is to propose a fractional-order model that employs the ψ-Hilfer fractional derivative to describe the dynamics of mental disorders.

This paper is organized as follows: The preliminaries of the ψ-Hilfer fractional system is defined in section 1. The core idea to formulate this model that describes the transmission dynamics of mental disorders during COVID-19 pandemic are explained in section 2. Section 3 summarizes the existence and uniqueness of the fractional system solution. The analytical solution is obtained and the graphical results are shown in sections 4 and 5. We end with section 6 of conclusion and future research.

2 Preliminaries

We recall the fundamental definition of the ψ-Hilfer fractional derivative employed in the model [22,23].

Definition 1.1: Let a,b be a finite or an infinite real line interval and q be non-negative. Let $\psi(x)$ be a non-negative, increasing monotonic function with a continuous derivative on (a,b). Thus, on $[a,b]$, the left-sided fractional derivative of a function with respect to ψ is defined by

$$I^{q,\psi}f(x)=\frac{1}{\Gamma(q)}\int_a^x \psi'(t)(\psi(x)-\psi(t))^{q-1}f(t)dt$$

Definition 1.2: Let $\psi'(x)\neq 0, (-\infty\leq a<b\leq\infty)$ and $q>0,\ \eta\in N$. A function's Left-sided Riemann-Liouville derivative with regard to psi is defined as

$$D^{q;\psi}f(x)=\frac{1}{\Gamma(\eta-q)}\left(\frac{1}{\psi'(x)}\frac{d}{dx}\right)^{\eta}\int_a^x \psi'(t)(\psi(x)-\psi(t))^{(\eta-q-1)}f(t)dt \tag{2.1}$$

Definition 1.3: Let $\eta-1<q<\eta$ be an interval with $\eta\in N, I=[a,b]$ and $f,\psi\in B^{\eta}([a,b],R)$ two functions such that ψ is increasing and $\psi'(x)\neq 0,\forall x\in I$. As a result, the ψ-Hilfer fractional derivative of order q and type $0\leq p\leq 1$ is defined as

$$
{}^{H}D_{a+}^{q,p;\psi} f(x) = I_{a+}^{p(\eta-q);\psi} \left(\frac{1}{\psi'(x)} \frac{d}{dx} \right)^{\eta} I_{a+}^{(1-p)(\eta-q);\psi} f(x)
$$
$$
{}^{H}D_{b-}^{q,p;\psi} f(x) = I_{b-}^{p(\eta-q);\psi} \left(\frac{1}{\psi'(x)} \frac{d}{dx} \right)^{\eta} I_{b-}^{(1-p)(\eta-q);\psi} f(x)
$$

3 Mathematical model

As discussed in [24], Michael Daly et. al. have estimated that the pandemic has contributed to 27.6 percent more incidences of major depressive disorder and 25.6 percent more cases of anxiety disorder worldwide throughout 2020. In addition to isolation, another significant cause of stress is the inability to work, find support from family members and participate in the community. The WHO declared that ”Many studies have found that loneliness, fear of infection, pain, and death for oneself and one’s family, sadness following loss, and financial concerns all lead to anxiety and melancholy” [25]. Suicidal thoughts among health professionals tend to be driven by exhaustion. Yet, to the best of our knowledge, this factor has not been taken into account in studying the dynamics of mental disorders.

We propose a mathematical model based on the following assumptions to analyse and evaluate the dynamic behaviour of mental disorders triggered by the sudden emergence of COVID-19. In this model, three different psychological disorders are classified: Minor depression, Major depression, and Anxiety disorders. The total population $N(t)$ is divided into 7 compartments namely: people who are susceptible to depression amid the pandemic $S_p(t)$, people who have minor depressive disorder $D_1(t)$, people who have major depressive disorder $D_2(t)$, people who have anxiety disorder $A_D(t)$, people who have both major depression and anxiety disorders $M_A(t)$, people who overcome the disorders through treatment $R_1(t)$, people who overcome the disorders on their own $R_2(t)$.

In developing the model, the following assumptions are taken into account:

- People with major depressive disorder and anxiety disorders are inherited from family members are included in the model.
- Coexisting disorders are also common during the same time frame.
- Death from any other cause is also taken into consideration.
- Watchful waiting is defined as an assessment with scheduled follow-up in primary care with no medication and psychotherapy.
- Depressive people may benefit from watchful waiting , but this does not prevent major depression according to some guidelines.
- As we have numerous ways to get into depression and anxiety during the pandemic. But we have studied only two ways that major depression and anxiety occur by direct inheritance from family (genetic) and contact with someone suffering from major depressive disorder or anxiety disorder (environmental factors).
- People do not inherit the disease, they inherit susceptibility or they have a tendency toward depression. It can turn into depressive disorder with additional factors such as environmental factors.
- Recovery from depression and anxiety are not permanent.

We formulate a system of nonlinear differential equations as

$$
\begin{aligned}
\frac{dS_p}{dt} &= \Lambda[1-(k_1+k_2+k_3)] - \beta_1 S_p D_1 - \beta_2 S_p D_2 - \beta_3 S_p A_D - dS_p \\
\frac{dD_1}{dt} &= \beta_1 S_p D_1 + \beta_2 S_p D_2 + \beta_3 S_p A_D - (\alpha + d + r_1 + \eta + \varepsilon) D_1 \\
\frac{dD_2}{dt} &= \Lambda k_1 + (\alpha+\varepsilon) D_1 + \delta A_D - (r_1 + d + d') D_2 \\
\frac{dA_D}{dt} &= \Lambda k_2 - (r_1 + r_2 + d + d' + \delta) A_D \\
\frac{dM_A}{dt} &= \Lambda k_3 - (r_1 + d + d') M_A \\
\frac{dR_1}{dt} &= r_1 (D_1 + D_2 + A_D + M_A) - dR_1 \\
\frac{dR_2}{dt} &= \eta D_1 + r_2 A_D - dR_2
\end{aligned}
\tag{3.1}
$$

with the initial conditions, $S_p(0) \geq 0$, $D_1(0) \geq 0$, $D_2(0) \geq 0$, $A_D(0) \geq 0$, $M_A(0) \geq 0$, $R_1(0) \geq 0$, $R_2(0) \geq 0$.

Here, Λ is the recruitment rate of susceptible persons, k_1 is the rate of people who are susceptible to get major depressive disorder genetically, k_2 is the rate of people who are susceptible to get anxiety disorder genetically, k_3 is the rate of people who are susceptible to get both major depressive and anxiety disorder genetically, β_1 is the spread rate of stress from people having minor depressive disorder to susceptible individuals due to fear of getting infected to COVID-19, β_2 is the spread rate of depression from people having major depressive disorder to susceptible individuals, β_3 - spread rate of anxiety from people having anxiety disorder to susceptible people, d - natural death rate, α - rate of people having minor depressive episodes transitioning to major depression by feeling restless, agitated and not having enough sleep, δ - rate of untreated people with anxiety disorder ended up by major depressive disorder, r_1 - recovery rate of depressed people through proper treatment, r_2 - recovery rate of depressed people getting better on their own with positive reinforcement, ε - fraction of population with minor depression going to the next stage after clinically guided waiting period, η - fraction of people recovered from the depression after waiting period, d'- suicidal death rate.

The transmission model is as follows:

$$\begin{aligned}
{}^{H}D^{q,p;\psi}S_p &= \Lambda[1-(k_1+k_2+k_3)] - \beta_1 S_p D_1 - \beta_2 S_p D_2 - \beta_3 S_p A_D - dS_p \\
{}^{H}D^{q,p;\psi}D_1 &= \beta_1 S_p D_1 + \beta_2 S_p D_2 + \beta_3 S_p A_D - (\alpha + d + r_1 + \eta + \varepsilon)D_1 \\
{}^{H}D^{q,p;\psi}D_2 &= \Lambda k_1 + (\alpha+\varepsilon)D_1 + \delta A_D - (r_1 + d + d')D_2 \\
{}^{H}D^{q,p;\psi}A_D &= \Lambda k_2 - (r_1 + r_2 + d + d' + \delta)A_D \\
{}^{H}D^{q,p;\psi}M_A &= \Lambda k_3 - (r_1 + d + d')M_A \\
{}^{H}D^{q,p;\psi}R_1 &= r_1(D_1 + D_2 + A_D + M_A) - dR_1 \\
{}^{H}D^{q,p;\psi}R_2 &= \eta D_1 + r_2 A_D - dR_2
\end{aligned} \tag{3.2}$$

4 ψ - Hilfer fractional derivative

Using the fractional derivative theorem, the system (2.2) is transformed into

$$\begin{aligned}
S_p - S_p(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, S_p(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
D_1 - D_1(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, D_1(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
D_2 - D_2(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, D_2(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
A_D - A_D(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, A_D(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
M_A - M_A(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, M_A(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
R_1 - R_1(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, R_1(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho, \\
R_2 - R_2(0) &= \frac{1}{\Gamma(q)} \int_{t_0}^{t} g_1(\rho, R_2(\rho))(\psi(t)-\psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho,
\end{aligned} \tag{4.1}$$

Theorem 3.1: *If the kernel g_1 satisfy the Lipschitz condition and contracts,* $0 \leq (\beta_1 m_1 + \beta_2 m_2 + \beta_3 m_3 + d) < 1$ *hold.*

Proof: Consider for S_p and ${S_p}^*$,

$$\begin{aligned}
||\bar{g_1}(t,S_p) - \bar{g_1}(t,S_p*)|| =&|| - \beta_1 D_1(S_p - {S_p}^*) - \beta_2 D_2(S_p - {S_p}^*) \\
&- \beta_3 A_D(S_p - {S_p}^*) - d(S_p - {S_p}^*)||
\end{aligned}$$

$$\leq \beta_1||D_1||||S_p - S_p{}^*|| + \beta_2||D_2||||S_p - S_p{}^*|| + \beta_3||A_D||||S_p - S_p{}^*||$$
$$+ ||d||S_p - S_p{}^*||$$
$$\leq [\beta_1 m_1 + \beta_2 m_2 + \beta_3 m_3 + d]\,||S_p - S_p{}^*||$$

Hence, $h_1 = \beta_1 m_1 + \beta_2 m_2 + \beta_3 m_3 + d$, where $||D_1|| = m_1, ||D_2|| = m_2, ||A_D|| = m_3$ is bounded function. Hence,

$$||\bar{g_1}(t,S_p) - \bar{g_1}(t,S_p{}^*)|| \leq h_1||S_p - S_p{}^*|| \tag{4.2}$$

Therefore, if $0 \leq \beta_1 m_1 + \beta_2 m_2 + \beta_3 m_3 + d < 1$ then g_1 is contraction.

$$||\bar{g_2}(t,D_1) - \bar{g_2}(t,D_1{}^*)|| \leq h_2||D_1 - D_1{}^*||$$
$$||\bar{g_3}(t,D_2) - \bar{g_3}(t,D_2{}^*)|| \leq h_3||D_2 - D_2{}^*||$$
$$||\bar{g_4}(t,A_D) - \bar{g_4}(t,A_D{}^*)|| \leq h_4||A_D - A_D{}^*||$$
$$||\bar{g_5}(t,M_A) - \bar{g_5}(t,M_A{}^*)|| \leq h_5||M_A - M_A{}^*||$$
$$||\bar{g_6}(t,R_1) - \bar{g_6}(t,R_1{}^*)|| \leq h_6||R_1 - R_1{}^*||$$
$$||\bar{g_7}(t,R_2) - \bar{g_7}(t,R_2{}^*)|| \leq h_7||R_2 - R_2{}^*||$$

where, $h_2 = (\alpha + d + r_1 + \eta + \varepsilon), h_3 = (r_1 + d + d'), h_4 = (r_1 + r_2 + d + d' + \delta), h_5 = (r_1 + d + d'), h_6 = d, h_7 = d$. The recursive form of the system (3.2) by mathematical induction method is given by

$$A_{1r}(t) = S_{p_r}(t) - S_{p_{r-1}}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,S_{p_{r-1}}) - g_1(\rho,S_{p_{r-2}}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{2r}(t) = D_{1r}(t) - D_{1r-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,D_{1r-1}) - g_1(\rho,D_{1r-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{3r}(t) = D_{2r}(t) - D_{2r-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,D_{2r-1}) - g_1(\rho,D_{2r-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{4r}(t) = A_{Dr}(t) - A_{Dr-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,A_{Dr-1}) - g_1(\rho,A_{Dr-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{5r}(t) = M_{Ar}(t) - M_{Ar-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,M_{Ar-1}) - g_1(\rho,M_{Ar-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{6r}(t) = R_{1r}(t) - R_{1r-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,R_{1r-1}) - g_1(\rho,R_{1r-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho$$
$$A_{7r}(t) = R_{2r}(t) - R_{2r-1}(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,R_{2r-1}) - g_1(\rho,R_{2r-2}))(\psi(t) - \psi(\rho))^{q-1}\dot{\psi}(\rho)d\rho \tag{4.3}$$

with the initial conditions. Consider

$$||A_{1r}(t)|| = ||S_{p_r}(t) - S_{p_{r-1}}(t)||$$
$$= ||\frac{1}{\Gamma(q)}\int_{t_0}^{t}(g_1(\rho,S_{p_{r-1}}) - g_1(\rho,S_{p_{r-2}}))(\psi(t) - \psi(\rho))^{q-1}d\rho||$$
$$\leq \frac{1}{\Gamma(q)}\int_{t_0}^{t}||g_1(\rho,S_{p_{r-1}}) - g_1(\rho,S_{p_{r-2}}))(\psi(t) - \psi(\rho))^{q-1}||d\rho$$

with the condition (3.3).

$$||A_{1r}|| \leq \frac{1}{\Gamma(q)}h_1\int_{t_0}^{t}||A_{1(r-1)}(\rho)||d\rho \tag{4.4}$$

Similarly we get,

$$
\begin{aligned}
||A_{2r}|| &\le \frac{1}{\Gamma(q)} h_2 \int_{t_0}^{t} ||A_{2(r-1)}(\rho)|| d\rho \\
||A_{3r}|| &\le \frac{1}{\Gamma(q)} h_3 \int_{t_0}^{t} ||A_{3(r-1)}(\rho)|| d\rho \\
||A_{4r}|| &\le \frac{1}{\Gamma(q)} h_4 \int_{t_0}^{t} ||A_{4(r-1)}(\rho)|| d\rho \\
||A_{5r}|| &\le \frac{1}{\Gamma(q)} h_5 \int_{t_0}^{t} ||A_{5(r-1)}(\rho)|| d\rho \\
||A_{6r}|| &\le \frac{1}{\Gamma(q)} h_6 \int_{t_0}^{t} ||A_{6(r-1)}(\rho)|| d\rho \\
||A_{7r}|| &\le \frac{1}{\Gamma(q)} h_7 \int_{t_0}^{t} ||A_{7(r-1)}(\rho)|| d\rho
\end{aligned} \tag{4.5}
$$

$$
\begin{aligned}
S_{p_r}(t) &= \sum_{k=1}^{n} A_{1k}(t), D_{1r}(t) = \sum_{k=1}^{n} A_{2k}(t), D_{2r}(t) = \sum_{k=1}^{n} A_{3k}(t), A_{Dr}(t) = \sum_{k=1}^{n} A_{4k}(t), \\
M_{Ar}(t) &= \sum_{k=1}^{n} A_{5k}(t), R_{1r}(t) = \sum_{k=1}^{n} A_{6k}(t), R_{2r}(t) = \sum_{k=1}^{n} A_{7k}(t)
\end{aligned}
$$

Theorem 3.2: *If t_1 exists, the system (2.2) yields the system of solutions such that $\frac{1}{\Gamma(q)} t_1 h_i < 1i$.*

Proof:

Using recursive technique, (3.4) is transformed to

$$
\begin{aligned}
&||A_{1r}|| \le ||S_{p_r}(0)|| \left[\frac{1}{\Gamma(q)} h_1 t\right]^r, ||A_{2r}|| \le ||D_{1r}(0)|| \left[\frac{1}{\Gamma(q)} h_2 t\right]^r \\
&||A_{3r}|| \le ||D_{2r}(0)|| \left[\frac{1}{\Gamma(q)} h_3 t\right]^r, ||A_{4r}|| \le ||A_{Dr}(0)|| \left[\frac{1}{\Gamma(q)} h_4 t\right]^r \\
&||A_{5r}|| \le ||M_{Ar}(0)|| \left[\frac{1}{\Gamma(q)} h_5 t\right]^r, ||A_{6r}|| \le ||R_{1r}(0)|| \left[\frac{1}{\Gamma(q)} h_6 t\right]^r \\
&||A_{7r}|| \le ||R_{2r}(0)|| \left[\frac{1}{\Gamma(q)} h_7 t\right]^r
\end{aligned} \tag{4.6}
$$

We assume

$$
\begin{aligned}
S_p - S_p(0) &= S_{p_r} - M_{1r}, D_1 - D_1(0) = D_{1r} - M_{2r}, \\
D_2 - D_2(0) &= D_{2r} - M_{3r}, A_D - A_D(0) = A_{Dr} - M_{4r}, \\
M_A - M_A(0) &= M_{Ar} - M_{5r}, R_1 - R_1(0) = R_{1r} - M_{6r}, \\
R_2 - R_2(0) &= R_{2r} - M_{6r}
\end{aligned}
$$

where

$$
\begin{aligned}
||M_{1r}|| =& || \frac{1}{\Gamma(q)} \int_{t_0}^{t} (g_1(\rho, S_p) - g_1(\rho, S_{p_{r-1}}))(\psi(t) - \psi(\rho))^{q-1} d\rho || \\
\le& \frac{1}{\Gamma(q)} h_1 ||S_p - S_{p_{r-1}}|| t
\end{aligned} \tag{4.7}
$$

Hence,

$$
||M_{1r}|| \le \left[\frac{1}{\Gamma(q)} t\right]^{n+1} h_1^{n+1} b
$$

$$||M_{1r}|| \leq \left[\frac{1}{\Gamma(q)}t_1\right]^{n+1} h_1^{n+1} b$$

We obtain $||M_{1r}|| \to 0$ as r tends to ∞. Assume the system (2.2) has another solution $S_{p1}, D_{11}, D_{21}, A_{D1}, M_{A1}, R_{11}, R_{21}$. We have

$$S_p - S_{p1} = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (g_1(\rho, S_p) - g_1(\rho, S_{p1}))(\psi(t) - \psi(\rho))^{q-1} d\rho$$

we have

$$||S_p - S_{p1}|| = \frac{1}{\Gamma(q)} \int_{t_0}^{t} ||(g_1(\rho, S_p) - g_1(\rho, S_{p1}))(\psi(t) - \psi(\rho))^{q-1}|| d\rho$$

$$||S_p - S_{p1}|| \leq \frac{1}{\Gamma(q)} v_1 t ||S_p - S_{p1}||$$

Therefore,

$$||S_p - S_{p1}|| \left(1 - \frac{1}{\Gamma(q) v_1 t}\right) \leq 0 \tag{4.8}$$

Hence, $||S_p - S_{p1}|| = 0$. Therefore, $S_p = S_{p1}$, this result is true for all the relations.

5 Numerical Simulations

In this section, we propose the numerical simulation of the model (3.1) in the sense of Caputo.

$$\begin{aligned}
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n S_p &= \Lambda[1 - (k_1 + k_2 + k_3)] - \beta_1 S_p D_1 - \beta_2 S_p D_2 - \beta_3 S_p A_D - d S_p \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n D_1 &= \beta_1 S_p D_1 + \beta_2 S_p D_2 + \beta_3 S_p A_D - (\alpha + d + r_1 + \eta + \varepsilon) D_1 \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n D_2 &= \Lambda k_1 + (\alpha + \varepsilon) D_1 + \delta A_D - (r_1 + d + d') D_2 \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n A_D &= \Lambda k_2 - (r_1 + r_2 + d + d' + \delta) A_D \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n M_A &= \Lambda k_3 - (r_1 + d + d') M_A \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n R_1 &= r_1 (D_1 + D_2 + A_D + M_A) - d R_1 \\
\frac{1}{\lambda^{n-1}} {}^{CF}D_t^n R_2 &= \eta D_1 + r_2 A_D - d R_2
\end{aligned} \tag{5.1}$$

To obtain the results we use the methods as in [26,27] for the system (4.1), we have

$$\begin{aligned}
L[S_p]-S_p(0) &= \frac{s+p(1-s)}{s}L[\Lambda[1-(k_1+k_2+k_3)]-\beta_1 S_p D_1-\beta_2 S_p D_2-\beta_3 S_p A_D-dS_p]\\
L[D_1]-D_1(0) &= \frac{s+p(1-s)}{s}L[\beta_1 S_p D_1+\beta_2 S_p D_2+\beta_3 S_p A_D-(\alpha+d+r_1+\eta+\varepsilon)D_1]\\
L[D_2]-D_2(0) &= \frac{s+p(1-s)}{s}L[\Lambda k_1+(\alpha+\varepsilon)D_1+\delta A_D-(r_1+d+d')D_2]\\
L[A_D]-A_D(0) &= \frac{s+p(1-s)}{s}L[\Lambda k_2-(r_1+r_2+d+d'+\delta)A_D]\\
L[M_A]-M_A(0) &= \frac{s+p(1-s)}{s}L[\Lambda k_3-(r_1+d+d')M_A]\\
L[R_1]-R_1(0) &= \frac{s+p(1-s)}{s}L[r_1(D_1+D_2+A_D+M_A)-dR_1]\\
L[R_2]-R_2(0) &= \frac{s+p(1-s)}{s}L[\eta D_1+r_2 A_D-dR_2]
\end{aligned} \tag{5.2}$$

The series of the solution is considered as [28],

$$\begin{aligned}
S_p &= \sum_{l=0}^{\infty} S_{pl}, D_1=\sum_{l=0}^{\infty} D_{1l}, D_2=\sum_{l=0}^{\infty} D_{2l}\\
A_D &= \sum_{l=0}^{\infty} A_{Dl}, M_A=\sum_{l=0}^{\infty} M_{Al}, R_1=\sum_{l=0}^{\infty} R_{1l}\\
R_2 &= \sum_{l=0}^{\infty} R_{2l}
\end{aligned}$$

$$S_pD_1 = \sum_{l=0}^{\infty} A_l(S_p,D_1), S_pD_2=\sum_{l=0}^{\infty} B_l(S_p,D_2), S_pA_D=\sum_{l=0}^{\infty} C_l(S_p,A_D)$$

where $A_l(S_p,D_1), B_l(S_p,D_2), C_l(S_p,A_D)$ is referred to as an Adomian polynomial. After some manipulation, we get After some manipulation, we get

$$\mathscr{L}[S_{pl+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\Lambda[1-(k_1+k_2+k_3)]-\beta_1 A_l(S_p,D_1)-\beta_2 B_l(S_p,D_2) - \beta_3 C_l(S_p,A_D)-dS_{pl}] \tag{5.3}$$

$$\mathscr{L}[D_{1l+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\beta_1 A_l(S_p,D_1)+\beta_2 B_l(S_p,D_2)+\beta_3 C_l(S_p,A_D) - (\alpha+d+r_1+\eta+\varepsilon)D_{1l}] \tag{5.4}$$

$$\mathscr{L}[D_{2l+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\Lambda k_1+(\alpha+\varepsilon)D_{1l}+\delta A_{Dl}-(r_1+d+d')D_{2l}] \tag{5.5}$$

$$\mathscr{L}[A_{Dl+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\Lambda k_2-(r_1+r_2+d+d'+\delta)A_{Dl}] \tag{5.6}$$

$$\mathscr{L}[M_{Al+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\Lambda k_3-(r_1+d+d')M_{Al}] \tag{5.7}$$

$$\mathscr{L}[R_{1l+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[r_1(D_{1l}+D_{2l}+A_{Dl}+M_{Al})-dR_{1l}] \tag{5.8}$$

$$\mathscr{L}[R_{2l+1}] = \frac{s+p(1-s)}{s}\mathscr{L}[\eta D_{1l}+r_2 A_{Dl}-dR_{2l}] \tag{5.9}$$

After some manipulation, we have

$$S_{p0}(t) = S_{p0}, \quad D_{10}(t) = D_{10}, \quad D_{20}(t) = D_{20}, \quad A_{D0}(t) = A_{D0},$$
$$M_{A0}(t) = M_{A0}, \quad R_{10}(t) = R_{10}, \quad R_{20}(t) = R_{20} \tag{5.10}$$

$$S_{p1} = [\Lambda[1-(k_1+k_2+k_3)] - \beta_1 S_{p0} D_{10} - \beta_2 S_{p0} D_{20} - \beta_3 S_{p0} A_{D0} \tag{5.11}$$
$$- dS_{p0}](1+p(t-1)),$$
$$D_{11} = [\beta_1 S_{p0} D_{10} + \beta_2 S_{p0} D_{20} + \beta_3 S_{p0} A_{D0} - (\alpha + d + r_1 + \eta + \varepsilon) \tag{5.12}$$
$$D_{10}](1+p(t-1)),$$
$$D_{21} = [\Lambda k_1 + (\alpha+\varepsilon) D_{10} + \delta A_{D0} - (r_1 + d + d') D_{20}](1+p(t-1)),$$
$$A_{D1} = [\Lambda k_2 - (r_1 + r_2 + d + d' + \delta) A_{D0}](1+p(t-1)),$$
$$M_{A1} = [\Lambda k_3 - (r_1 + d + d') M_{A0}](1+p(t-1)),$$
$$R_{11} = [r_1(D_{10} + D_{20} + A_{D0} + M_{A0}) - dR_{10}](1+p(t-1))$$
$$R_{21} = [\eta D_{10} + r_2 A_{D0} - dR_{20}](1+p(t-1))$$

$$S_{p2} = [\Lambda[1-(k_1+k_2+k_3)] - \beta_1 S_{p1} D_{11} - \beta_2 S_{p1} D_{21} - \beta_3 S_{p1} A_{D1} \tag{5.13}$$
$$- dS_{p1}](1+p(t-1)),$$
$$D_{12} = [\beta_1 S_{p1} D_{11} + \beta_2 S_{p1} D_{21} + \beta_3 S_{p1} A_{D1} - (\alpha + d + r_1 + \eta + \varepsilon) \tag{5.14}$$
$$D_{11}](1+p(t-1)),$$
$$D_{22} = [\Lambda k_1 + (\alpha+\varepsilon) D_{11} + \delta A_{D1} - (r_1 + d + d') D_{21}](1+p(t-1)),$$
$$A_{D2} = [\Lambda k_2 - (r_1 + r_2 + d + d' + \delta) A_{D1}](1+p(t-1)),$$
$$M_{A2} = [\Lambda k_3 - (r_1 + d + d') M_{A1}](1+p(t-1)),$$
$$R_{12} = [r_1(D_{11} + D_{21} + A_{D1} + M_{A1}) - dR_{11}](1+p(t-1))$$
$$R_{22} = [\eta D_{11} + r_2 A_{D1} - dR_{21}](1+p(t-1))$$

Therefore the solution is

$$S_p = \sum_{v=0}^{\infty} S_{p_v}, D_1 = \sum_{v=0}^{\infty} D_{1v}, D_2 = \sum_{v=0}^{\infty} D_{2v}$$
$$A_D = \sum_{v=0}^{\infty} A_{Dv}, M_A = \sum_{v=0}^{\infty} M_{Av}, R_1 = \sum_{v=0}^{\infty} R_{1v}$$
$$R_2 = \sum_{v=0}^{\infty} R_{2v}$$

6 Graphical Discussion

In this section, graphical representation were performed and implemented using matlab. Fig. 1 and Fig. 2 illustrate the decreasing densities of susceptible and minor depressed populations according to the corresponding fractional orders. As shown in Figures 3 and 4, infection decreases the density of the major depressed people and anxiety people. Figure 5, shows the decline in depression co-exists with the anxiety population. It is evident from Figure 6-7 that the number of people receiving treatment and those without treatment is on the rise since many have cured themselves through proper

Parameter	Values	Source
Λ	0.0003	[29]
β_1	0.18	[30]
β_2	0.337	[31]
β_3	0.319	[31]
k_1	0.1	Assumed
k_2	0.0177	[32]
k_3	0.00835	[33]
d	0.0104	[34]
α	0.04	Assumed
r_1	0.8	[35]
ε	0.0007	Assumed
d'	0.09	[36]
r_2	0.1	Assumed
δ	0.04	Assumed

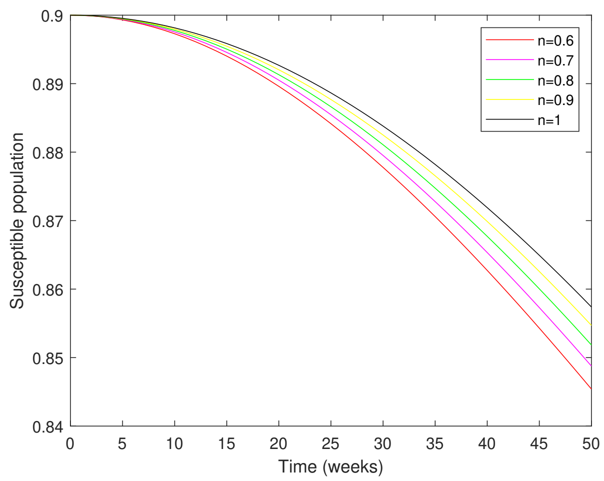

Fig. 1: Schematic illustration of susceptible population according to fractional order

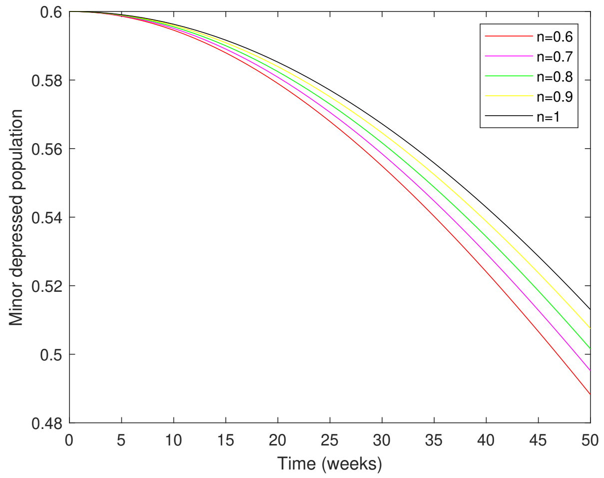

Fig. 2: Schematic illustration of minor depressed population according to fractional order

treatment and following a healthy lifestyle, resulting in an increased number of people recovering. Figure 8 compares our results with available real data for the depressed population that describes the accuracy of the model.

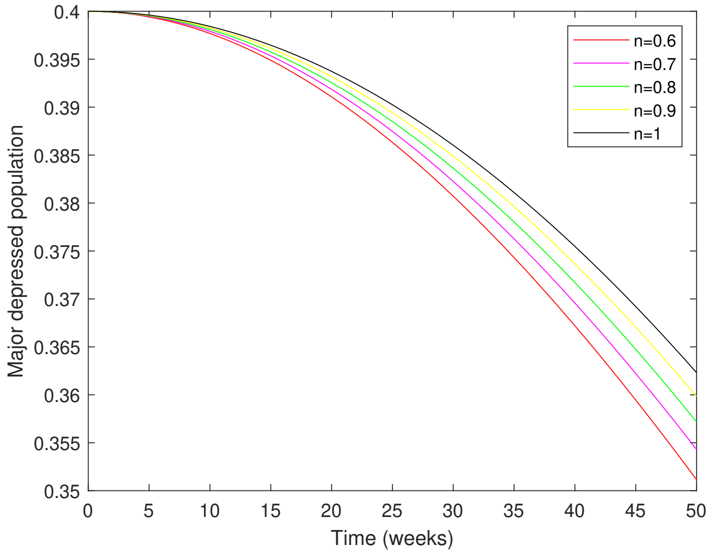

Fig. 3: Schematic illustration of major depressed population according to fractional order

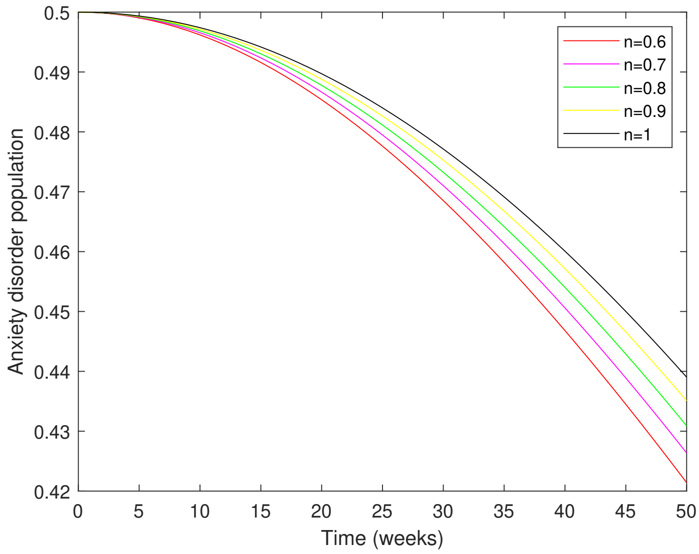

Fig. 4: Schematic illustration of anxiety disorder population according to fractional order

7 Conclusion

In this paper, we have formulated a mathematical model that describes the dynamics of mental health in people with depressive disorders caused by the pandemic. Various types of mental disorders are taken into account in order to predict mental health's dynamic behavior. There exists a unique solution to the mental disorders model with ψ-Hilfer fractional derivative. We show that the fractional system has a unique solution using fixed point theory. The Adomian approach paired with a Laplace integral transform can be used to approximate the proposed model. As a result of presenting the

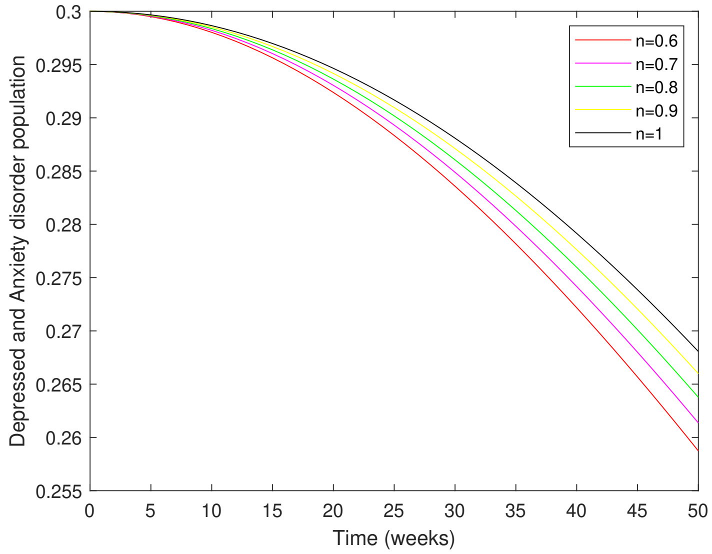

Fig. 5: Schematic illustration of depressed and anxiety disorder population according to fractional order

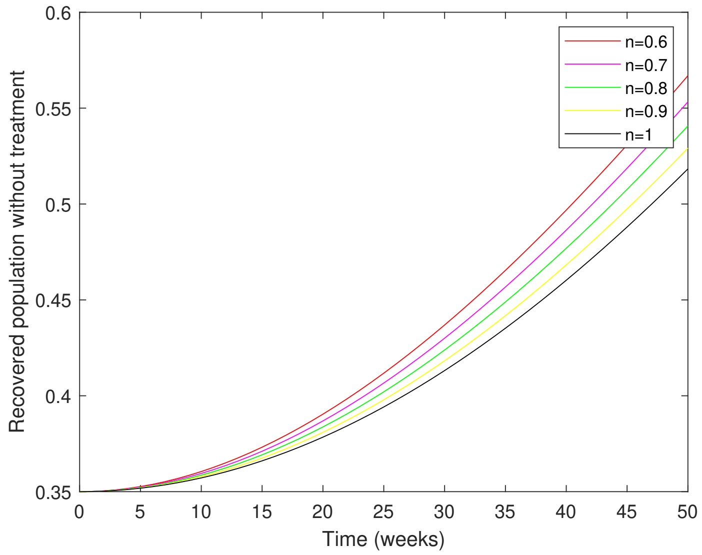

Fig. 6: Schematic illustration of recovered population without treatment according to fractional order

dynamics graphically, the proposed technique is better understood, and its convergence speed is favorable. Additionally, we compared our simulated results with real-world data collected from depressed populations. In the future, the stability theory and bifurcation analysis for a ψ-Hilfer fractional system can be obtained to gain an in-depth understanding of the range of conditions required for a system to be stable.

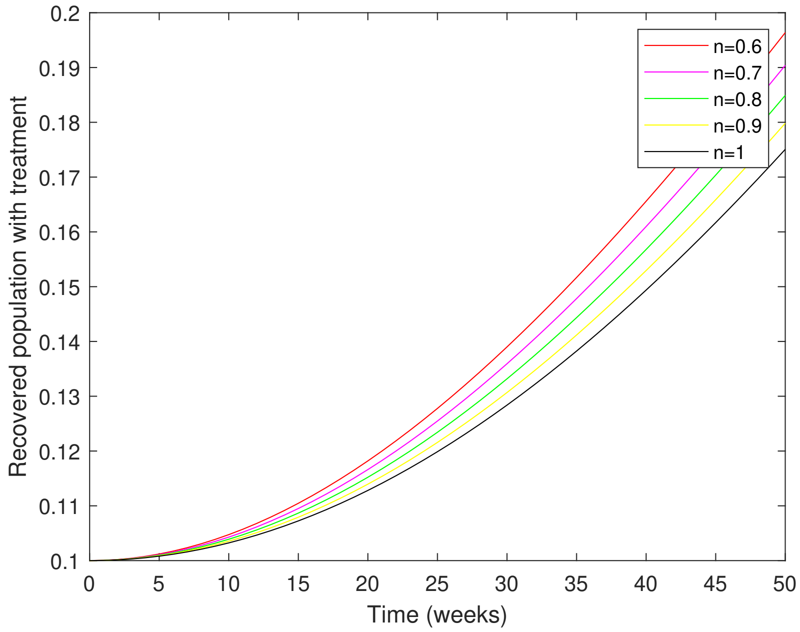

Fig. 7: Schematic illustration of recovered population with treatment according to fractional order

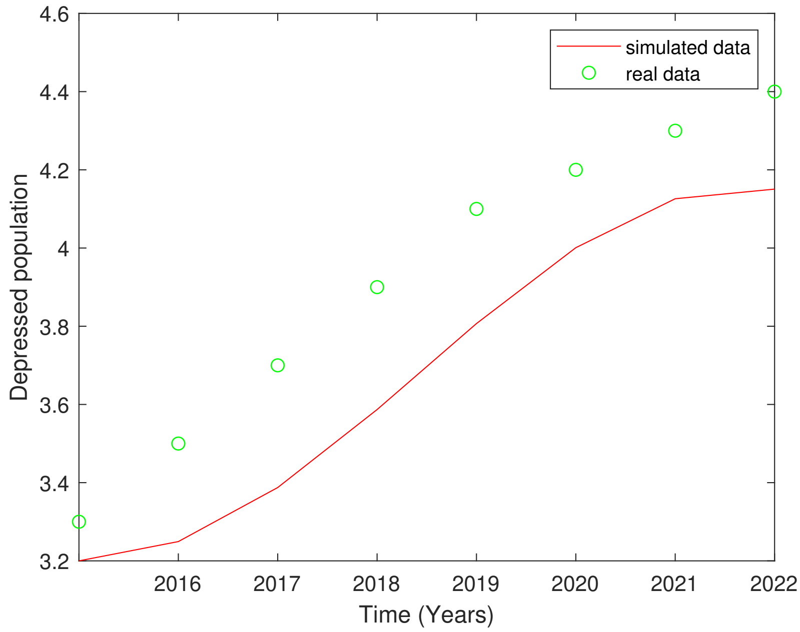

Fig. 8: Comparison of depressed population with real data

8 Acknowledgement

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