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Inverting of General (K,K)-Pentadiagonal Matrices with Applications

Ghadeer Alshreef[*] *and Ahmed Tarabia*

Department of Mathematics, Faculty of Science, Damietta University, 34517, New Damietta City, Egypt

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Abstract: In this paper, an algorithm was proposed to calculate the inverse of general nonsingular (K,K)-pentadiagonal matrix. The efficiency of the proposed algorithm is shown via some numerical illustrative examples and solving a (K,K)-pentadiagonal system , where is nonsingular (K,K)-pentadiagonal matrix. All the numerical calculations are performed on a PC computer with the aid of MATLAB programs.

Keywords: Inverse Matrix - (K,K)-Pentadiagonal Matrix - LU Factorization - Linear Systems - Algorithm - MATLAB subclass: 15A15 - 15A23 - 68W30 - 11Y05 - 33F10

1 Introduction

Recently the concept of Pentadiagonal matrices have turned to be a mature topic in applied mathematics; specially when dealing with the systems of linear algebraic equations which often appear in solving partial differential equations in meteorology, oceanography, queuing networks and parallel computing as well as in numerical analysis fields, see for example Dongarra (1984) [4], Meier (1985) [10], Navon (1987) [11], Killingbeck , Jolicard (1992) [9], Golub , van Loan (1996) [7] and Asakar , Karawia (2015) [1]. Carlos M. et al. (2020) [2] consider a general (K,K) pentadiagonal matrix, they give the determinants by considering the toeplitz and imperfect toeplitz versions of this type of matrices, also they show that the inverse can be obtained as a product of an upper tridiagonal matrix with two super diagonals. Carlos M. et all again in October (2020) [3] consider pentadiagonal matrices by transforming it into tridiagonl matrix after multiplying them with suitable matrices and then transforming the last one into a diagonal matrix which can be easily calculate its determine, but they did not find the inverse of the pentadiagonal matrices.

The pentadiagonal linear systems (PLS) written as the form below

$$TX = \beta \qquad (1)$$

and X , β are the n-vectors

$X = (x_1, x_2, ..., x_n)^t$, $\beta = (\beta_1, \beta_2, ..., \beta_n)^t$ and

T is a $n \times n$ pentadiagonal matrix given in the form:

$$T = \begin{bmatrix} d_1 & a_1 & b_1 & 0 & & & \cdots & & 0 \\ c_1 & d_2 & a_2 & b_2 & 0 & & & & \\ e_1 & c_2 & d_3 & a_3 & b_3 & 0 & & & \vdots \\ 0 & e_2 & c_3 & d_4 & a_4 & b_4 & 0 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ \vdots & & 0 & e_{n-5} & c_{n-4} & d_{n-3} & a_{n-3} & b_{n-3} & 0 \\ & & & 0 & e_{n-4} & c_{n-3} & d_{n-2} & a_{n-2} & b_{n-2} \\ & & & & 0 & e_{n-3} & c_{n-2} & d_{n-1} & a_{n-1} \\ 0 & & & \cdots & & 0 & e_{n-2} & c_{n-1} & d_n \end{bmatrix} \qquad (2)$$

We define the general (K,K)-pentadiagonal matrix $T_n^{(k,k)}$, as the following form:

[*]Corresponding author e-mail: gh.alshreef@gmail.com

$$T_n^{(k,k)} =$$

$$\begin{bmatrix}
d_1 & 0 & \cdots & 0 & a_1 & 0 & \cdots & 0 & b_1 & 0 & \cdots & 0 \\
0 & d_2 & & & & a_2 & & & & b_2 & & \\
\vdots & & \ddots & & & & \ddots & & & & \ddots & \\
0 & & & \ddots & & & & \ddots & & & & b_{n-2k} \\
c_1 & & & & \ddots & & & & \ddots & & & 0 \\
0 & c_2 & & & & \ddots & & & & \ddots & & \vdots \\
\vdots & & \ddots & & & & \ddots & & & & \ddots & 0 \\
0 & & & \ddots & & & & \ddots & & & & a_{n-k} \\
e_1 & & & & \ddots & & & & \ddots & & & 0 \\
0 & e_2 & & & & \ddots & & & & \ddots & & \vdots \\
\vdots & \ddots & \ddots & & & & \ddots & & & & d_{n-1} & 0 \\
0 & \cdots & 0 & e_{n-2k} & 0 & \cdots & 0 & c_{n-k} & 0 & \cdots & 0 & d_n
\end{bmatrix} \quad (3)$$

Where $1 \le k \le \frac{n}{2}$.

For this (k,k) pentadiagonal matrix, in case $k=1$, Askar and Karawia (2015) [1] solved pentadiagonal linear systems (PLS) using transformations. Their suggested methods are generalization to the algorithms which proposed by El-Mikkawy and Atlan (2014) [6] for k - tridiagonal system. Yalciner (2011) [13] gives the LU - factorization of a k - diagonal matrix. Also he finds eiginvalues of a k - tridiagonal Toeplitz matrix. Du and et. al. (2014) [5] discuss the general non-symmetric problem and propose an algorithm for solving nonsymmetric pentadiagonal Toeplitz linear systems. Also, an algorithm for solving a large system with a symmetric Toeplitz pentadiagonal coefficient matrix hasbeen given by Nemani (2010) [12]. Our motivation is to give a general algorithm which can be obtain and compute the invers of the general pentadiagonal matrix $T_n^{(k,k)}$ shown in Eq.(3) and hence solving any non-singular linear system in the form of Eq. (1) where $T = T_n^{(k,k)}$.

In addition, our formulas are generalization i.e. at $b_i = 0 = e_i$, $i = 1,2,.....,n-2k$, then $T_n^{(k,k)}$ defined as general (k,k) tridiagonal matrix and our formulas reduced to El-Mikkawy and Atlan's (2014) [6] formulas.

This paper organized as the following. In the next section, we will introduce the main results for the inverse of the general pentadiagonal matrix $T_n^{(k,k)}$. In Section sec. 3, we will state the algorithm for the inverse of the matrix $T_n^{(k,k)}$. Some numerical examples will be carried out to show the efficiency of the new algorithm in Section sec. 4.

2 Main Result

In this section we will define the following components

$$x_i = \begin{cases} d_i & i = 1, ..., k \\ d_i - z_{i-k} y_{i-k} & i = k-1, ..., 2k \\ d_i - z_{i-k} y_{i-k} - g_{i-2k} b_{i-2k} & i = 2k+1, ..., n. \end{cases} \quad (4)$$

$$y_i = \begin{cases} a_i & i = 1, ..., k \\ a_i - z_{i-k} b_{i-k} & i = k-1, ..., n-k \end{cases} \quad (5)$$

$$z_i = \begin{cases} \dfrac{c_i}{x_i} & i = 1, ..., k \\ \dfrac{c_i - g_{i-k} y_{i-k}}{x_i} & i = k+1, ..., n-k \end{cases} \quad (6)$$

$$g_i = \frac{e_i}{x_i} \qquad i = 1, ..., n-2k \quad (7)$$

Lemma 1. *Let* $T_n^{(k,k)}$ *be* (k,k) pentadiagonal matrix as given in Eq. (3), then the Doolittle LU factorization of $T_n^{(k,k)}$ is given by $T_n^{(k,k)} = L_n^{(k,k)} \times U_n^{(k,k)}$, where

$$L_n^{(k,k)} = \begin{pmatrix} 1 & 0 & & & & \cdots & & & & & 0 \\ 0 & \ddots & & & & & & & & & \\ \vdots & & \ddots & & & & & & & & \\ 0 & & & \ddots & & & & & & & \\ z_1 & & & & \ddots & & & & & & \vdots \\ 0 & \ddots & & & & \ddots & & & & & \\ \vdots & & \ddots & & & & \ddots & & & & \\ 0 & & & \ddots & & & & \ddots & & & \\ g_1 & & & & \ddots & & & & \ddots & & \\ 0 & \ddots & & & & \ddots & & & & \ddots & \\ \vdots & & \ddots & & & & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & g_{n-2k} & 0 & \cdots & 0 & z_{n-k} & 0 & \cdots & 0 \quad 1 \end{pmatrix},$$

$$U_n^{(k,k)} = \begin{pmatrix} x_1 & 0 & \cdots & 0 & y_1 & 0 & \cdots & 0 & b_1 & 0 & \cdots & 0 \\ 0 & \ddots & & & & \ddots & & & & \ddots & & \vdots \\ & & \ddots & & & & & & & & \ddots & 0 \\ & & & & & & & & & & & b_{n-2k} \\ & & & & & & & & & & & 0 \\ \vdots & & & & & & & & & & & \vdots \\ & & & & & & & & & & \ddots & 0 \\ & & & & & & & & & & & y_{n-k} \\ & & & & & & & & & & & 0 \\ & & & & & & & & & \ddots & & \vdots \\ & & & & & & & & & & \ddots & 0 \\ 0 & & & & & \cdots & & & & & 0 & x_n \end{pmatrix}$$

and $\det\left(U_n^{(k,k)}\right) = \prod_{i=1}^{n} x_i$, x_i defined in Eq.(4).

Now, we give the following result.

Theorem. *Let* $T_n^{(k,k)}$ *be* (k,k) pentadiagonal matrix as given in Eq. (3). Then

$\left(T_n^{(k,k)}\right)^{-1} = \left(\alpha_{ij}\right)_{i,j=1}^{n}$, where

$$\alpha_{ii} = \begin{cases} \dfrac{1}{x_i} & i = n, \ldots, n-k+1 \\ \dfrac{1}{x_i} + \dfrac{z_i y_i}{x_i}\alpha_{i+k\, i+k} & i = n-k, \ldots, n-2k+1 \\ \dfrac{1}{x_i} - z_i \alpha_{i\, i+k} - g_i \alpha_{i\, i+2k} & i = n-2k, \ldots, 1 \end{cases}, \quad (8)$$

and for $i < j$

$$\alpha_{ij} = \begin{cases} \dfrac{-y_i}{x_i}\alpha_{i+k\, j} & i = n, \ldots, n-k+1, \; j = i-k \\ \dfrac{-y_i}{x_i}\alpha_{i+k\, j} - \dfrac{b_i}{x_i}\alpha_{i+2k\, j} & i = n-2k, \ldots, 1, \; j = i+k, \; \forall\, j \equiv i \bmod(k) \\ 0 & \text{otherwise} \end{cases}, \quad (9)$$

and for $i > j$

$$\alpha_{ij} = \begin{cases} -z_j \alpha_{i\, j+k} & i = n, \ldots, n-k+1, \; j = i-k \\ -z_j \alpha_{i\, j+k} - g_j \alpha_{i\, j+2k} & i = n-k, \ldots, k+1, \; j = i-k, \; \forall\, i \equiv j \bmod(k) \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Proof. From Doolittle factorization method, $T_n^{(k,k)}$ can be decomposed into $L_n^{(k,k)}$ and $U_n^{(k,k)}$ as $T_n^{(k,k)} = L_n^{(k,k)} \times U_n^{(k,k)}$, where $L_n^{(k,k)}$ and $U_n^{(k,k)}$ are given in pervious lemma. The inverse of the pentadiagonal matrix $T_n^{(k,k)}$ can be found from the relation

$$\left(T_n^{(k,k)}\right)^{-1} = \left(L_n^{(k,k)} \times U_n^{(k,k)}\right)^{-1} = \left(U_n^{(k,k)}\right)^{-1} \times \left(L_n^{(k,k)}\right)^{-1}$$

where,

$$\left(U_n^{(k,k)}\right)^{-1} = \frac{1}{\det\left(U_n^{(k,k)}\right)} Adj\left(U_n^{(k,k)}\right)$$

and

$$\left(L_n^{(k,k)}\right)^{-1} = \frac{1}{\det\left(L_n^{(k,k)}\right)} Adj\left(L_n^{(k,k)}\right)$$

and after some analytic calculation we found

$$\left(U_n^{(k,k)}\right)^{-1} = \frac{1}{\prod_{t=1}^{n} x_t} \times$$

$$\begin{pmatrix}
\prod_{t=2}^{n} x_t & 0 & \cdots & 0 & y_1 \prod_{\substack{t=1 \\ t\neq 1,k+1}}^{n} x_t & 0 & \cdots & 0 & (y_1 y_4 - b_1 x_4) \prod_{\substack{t=2 \\ t\neq k+1,2k+1}}^{n} x_t & 0 & \cdots & & \cdots & 0 \\
 & \prod_{\substack{t=1 \\ t\neq 2}}^{n} x_t & & & & y_2 \prod_{\substack{t=1 \\ t\neq 2,k+2}}^{n} x_t & & & & \ddots & & & & \vdots \\
 & & & & & & & & & & & & & 0 \\
 & & & \ddots & & & & \ddots & & & & (y_{n-2k} y_{n-k} - b_{n-2k} x_{n-k}) & & \prod_{\substack{t=1 \\ t\neq n-2k,n-k}}^{n} x_t \\
 & & & & & & & & & & & 0 & & \\
 & & & & & & & & & \ddots & & \vdots & & \\
 & & & & & & & & & & & 0 & & \\
 & & & & & & & & & & & y_{n-k} \prod_{\substack{t=1 \\ t\neq n-k,n}}^{n} x_t & & \\
 & & & & & & & & & & & 0 & & \\
 & & & & & & & & & \ddots & & \vdots & & \\
 & & & & & & & & & & & 0 & & \\
 & & & & & & & & & & & \prod_{t=1}^{n-1} x_t & &
\end{pmatrix}$$

and

$$\left(L_n^{(k,k)}\right)^{-1} = \begin{pmatrix}
1 & 0 & & & \cdots & & & & 0 \\
0 & 1 & & & & & & & \\
\vdots & & & & & & & & \\
0 & & & & & & & & \\
z_1 & & & & & & & & \vdots \\
0 & & & & & & & & \\
\vdots & z_2 & & & & & & & \\
0 & & \ddots & & & & & & \\
z_1 z_{k+1} - g_1 & & & & \ddots & & & & \\
0 & \ddots & & & & & & & \\
\vdots & & \ddots & & & \ddots & & \ddots & 0 \\
0 & \cdots & 0 & z_{n-2k} z_{n-k} - g_{n-2k} & & 0 \cdots 0 & z_{n-k} & 0 \cdots 0 & 1
\end{pmatrix}$$

The inverse $\left(T_n^{(k,k)}\right)^{-1}$ follows immediately from multiplying $\left(U_n^{(k,k)}\right)^{-1} \times \left(L_n^{(k,k)}\right)^{-1}$ and the proof is completed.

3 Algorithm for the inverse of general (k, k) pentadiagonal matrix

Algorithm

INPUT:
The matrix order n
The value of k **and**
The values

$a_i, c_i,\ i = 1,\ldots,n-k$

$b_i, e_i,\ i = 1,\ldots,n-2k$

$d_i,\ \ i = 1,\ldots,n$

OUTPUT:

The inverse matrix, $\left(T_n^{(k,k)}\right)^{-1} = \left(\alpha_{ij}\right)_{i,j=1}^{n}$

Step 1:
for $i = 1$ to k
do

$x_i = d_i$

$y_i = a_i$

$z_i = \dfrac{c_i}{x_i}$

$g_i = \dfrac{e_i}{x_i}$

end do
for $i = k+1$ to $2k$
do

$x_i = d_i - z_{i-k} y_{i-k}$

$y_i = a_i - z_{i-k} b_{i-k}$

$z_i = \left(c_i - g_{i-k} y_{i-k}\right)/x_i$

$g_i = e_i / x_i$

end do
for $i = 2k+1$ to $n-k$
do

$x_i = d_{i-k} - z_{i-k} y_{i-k} - g_{i-2k} b_{i-2k}$

$y_i = a_i - z_{i-k} b_{i-k}$

$z_i = \left(c_i - g_{i-k} y_{i-k}\right)/x_i$

$g_i = e_i / x_i$

end do
for $i = n-k+1$ to n
do

$x_i = d_i - z_{i-k} y_{i-k} - g_{i-2k} b_{i-2k}$

end do

Step 2:
for $i = n$ to
do step -1

$\alpha_{ii} = 1/x_i$

end do
for $i = n-k$ to $i = n-2k+1$
do step -1

$\alpha_{ii} = \dfrac{1}{x_i} + \dfrac{z_i y_i}{x_i} \alpha_{i+k\,i+k}$

end do
for $i = n-k$ to $i = n-2k+1$
do step -1

$\alpha_{i\,i+k} = -\dfrac{y_i}{x_i} \alpha_{i+k\,i+k}$

end do
for $i = n$ to $i = n-k+1$
do step -1

$\alpha_{i\,i-k} = -z_{i-k} \alpha_{i\,i}$

end do
for $i = n-2k$ to 1 step -1
for $j = i+k$ to n step k

$\alpha_{i\,j} = -\dfrac{y_i}{x_i} \alpha_{i+k\,j} - \dfrac{b_i}{x_i} \alpha_{i+2k\,j}$

$\alpha_{j\,i} = -z_i \alpha_{j\,i+k} - g_i \alpha_{j\,i+2k}$

end do

$\alpha_{i\,i} = \dfrac{1}{x_i} - z_i \alpha_{i\,i+k} - g_i \alpha_{i\,i+2k}$

end do

step 3: Show the inverse.

4 Illustrative Numerical Examples

Some numerical examples for illustration are given in this section.

Example 1 Calculate the inverse of the 10×10 following pentadiagonal matrix

$$T_{10}^{(2,2)}=\begin{pmatrix} 1&0&5&0&2&0&0&0&0&0\\ 0&3&0&5&0&2&0&0&0&0\\ 8&0&2&0&4&0&3&0&0&0\\ 0&-1&0&2&0&3&0&2&0&0\\ 1&0&2&0&3&0&5&0&3&0\\ 0&7&0&4&0&1&0&2&0&3\\ 0&0&2&0&4&0&2&0&-1&0\\ 0&0&0&1&0&5&0&1&0&9\\ 0&0&0&0&3&0&2&0&4&0\\ 0&0&0&0&0&1&0&8&0&5 \end{pmatrix}$$

By applying the proposed algorithm, we get

$$\left(T_{10}^{(2,2)}\right)^{-1}=\begin{pmatrix} 0.0046&0&0.1289&0&-0.0356&0&-0.1048&0&0.0005&0\\ 0&-0.3323&0&0.2155&0&0.3160&0&-0.0339&0&-0.1256\\ 0.2102&0&-0.0365&0&0.0818&0&-0.0707&0&-0.0790&0\\ 0&0.5813&0&-0.3575&0&-0.3002&0&0.0094&0&0.1632\\ -0.0277&0&0.0268&0&0.1866&0&0.2291&0&0.1972&0\\ 0&-0.4548&0&0.5705&0&0.2764&0&0.0274&0&-0.2151\\ -0.1155&0&-0.0217&0&0.2891&0&0.0212&0&-0.2115&0\\ 0&-0.0652&0&0.1095&0&0.0436&0&-0.0674&0&0.0951\\ 0.0785&0&-0.0092&0&-0.0046&0&-0.1824&0&0.2079&0\\ 0&0.1953&0&-0.2894&0&-0.1250&0&0.1023&0&0.0908 \end{pmatrix}$$

Example 2 To solve the following system

$$2x_1+2x_5+x_9=1$$
$$2x_2+2x_6+x_{10}=1$$
$$2x_3+2x_7+x_{11}=0$$
$$2x_4+2x_8+x_{12}=2$$
$$-x_1+2x_5+2x_9+x_{13}=-1$$
$$-x_2+2x_6+2x_{10}+x_{14}=5$$
$$-x_3+2x_7+2x_{11}=3$$
$$-x_4+2x_8+2x_{12}=1$$
$$3x_1-x_5+2x_9+2x_{13}=2$$
$$3x_2-x_6+2x_{10}+2x_{14}=1$$
$$3x_3-x_7+2x_{11}=1$$
$$3x_4-x_8+2x_{12}=-1$$
$$3x_5-x_9+2x_{13}=0$$
$$3x_6-x_{10}+2x_{14}=2$$

This system can be expressed in matrix form as:

$$\begin{pmatrix} 2&0&0&0&2&0&0&0&1&0&0&0&0&0\\ 0&2&0&0&0&2&0&0&0&1&0&0&0&0\\ 0&0&2&0&0&0&2&0&0&0&1&0&0&0\\ 0&0&0&2&0&0&0&2&0&0&0&1&0&0\\ -1&0&0&0&2&0&0&0&2&0&0&0&1&0\\ 0&-1&0&0&0&2&0&0&0&2&0&0&0&1\\ 0&0&-1&0&0&0&2&0&0&0&2&0&0&0\\ 0&0&0&-1&0&0&0&2&0&0&0&2&0&0\\ 3&0&0&0&-1&0&0&0&2&0&0&0&2&0\\ 0&3&0&0&0&-1&0&0&0&2&0&0&0&2\\ 0&0&3&0&0&0&-1&0&0&0&2&0&0&0\\ 0&0&0&3&0&0&0&-1&0&0&0&2&0&0\\ 0&0&0&0&3&0&0&0&-1&0&0&0&2&0\\ 0&0&0&0&0&3&0&0&0&-1&0&0&0&2 \end{pmatrix}\begin{pmatrix} x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\\x_9\\x_{10}\\x_{11}\\x_{12}\\x_{13}\\x_{14}\end{pmatrix}=\begin{pmatrix}1\\1\\0\\2\\-1\\5\\3\\1\\2\\1\\1\\-1\\0\\2\end{pmatrix}$$

The coefficient matrix is 14×14 pentadigonal matrix with $k=4$.

Firstly, finding the inverse of the coefficient matrix using our proposed algorithm, we have

$$\left(T_{14}^{(4,4)}\right)^{-1} =$$

$$\begin{pmatrix}
0.2473 & 0 & 0 & 0 & -0.2151 & 0 & 0 & 0 & 0.0968 & 0 & 0 & 0 & 0.0108 & 0 \\
0 & 0.2473 & 0 & 0 & 0 & -0.2151 & 0 & 0 & 0 & 0.0968 & 0 & 0 & 0 & 0.0108 \\
0 & 0 & 0.2609 & 0 & 0 & 0 & -0.2174 & 0 & 0 & 0 & 0.0870 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2609 & 0 & 0 & 0 & -0.2174 & 0 & 0 & 0 & 0.0870 & 0 & 0 \\
0.2258 & 0 & 0 & 0 & 0.0645 & 0 & 0 & 0 & -0.1290 & 0 & 0 & 0 & 0.0968 & 0 \\
0 & 0.2258 & 0 & 0 & 0 & 0.0645 & 0 & 0 & 0 & -0.1290 & 0 & 0 & 0 & 0.0968 \\
0 & 0 & 0.3478 & 0 & 0 & 0 & 0.0435 & 0 & 0 & 0 & -0.2174 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.3478 & 0 & 0 & 0 & 0.0435 & 0 & 0 & 0 & -0.2174 & 0 & 0 \\
0.0538 & 0 & 0 & 0 & 0.3011 & 0 & 0 & 0 & 0.0645 & 0 & 0 & 0 & -0.2151 & 0 \\
0 & 0.0538 & 0 & 0 & 0 & 0.3011 & 0 & 0 & 0 & 0.0645 & 0 & 0 & 0 & -0.2151 \\
0 & 0 & 0.0538 & 0 & 0 & 0 & 0.3011 & 0 & 0 & 0 & 0.2609 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.2174 & 0 & 0 & 0 & 0.3478 & 0 & 0 & 0 & 0.2609 & 0 & 0 \\
-0.3118 & 0 & 0 & 0 & 0.0538 & 0 & 0 & 0 & 0.2258 & 0 & 0 & 0 & 0.2473 & 0 \\
0 & -0.3118 & 0 & 0 & 0 & 0.0538 & 0 & 0 & 0 & 0.2258 & 0 & 0 & 0 & 0.2473
\end{pmatrix}$$

and finally the solution of the system can be easily calculated as

$$X = \left(T_{14}^{(4,4)}\right)^{-1} \beta$$

then,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \end{pmatrix} = \begin{pmatrix} 0.6559 \\ -0.7097 \\ -0.5652 \\ 0.2174 \\ -0.0968 \\ 0.6129 \\ -0.0870 \\ 0.9565 \\ -0.1183 \\ 1.1935 \\ 1.3043 \\ -0.3478 \\ 0.0860 \\ 0.6774 \end{pmatrix}$$

5 Conclusion

Based on the results, it can be concluded that our new algorithm for inverting a (k,k) pentadiagonal matrix confirms its effectiveness and validity. In addition, our formulas are generalization for El-Mikkawy et al. (2014) [6] i.e. at $b_i = 0 = e_i$, $i = 1, 2, ..., n-2k$ our formulas reduced to their formulas.

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