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The Solution of the Hierarchy of Quantum Kinetic Equations for Correlation Matrices with Delta function Potential

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Abstract: In paper, the evolution of N identical in mass and charge particles interacting with delta function potential is investigated. Using the semi-group theory, a solution of the chain of quantum kinetic equations for correlation matrices is defined

Keywords: statistical physics, BBGKY hierarchy, semigroup method

1 Introduction

In connection with the development of quantum information and quantum calculations, interest in the research of correlation matrices and their properties has risen

For detailed research of its properties, it is necessary to determine their explicit form at first. For this, it is needed to solve equations, describing the behavior of a quantum system of many interaction particles both in equilibrium and in non-equilibrium states. The fact that real physical quantum systems of interactive particles are in motion attracts interest in determining quantum correlation matrices, and solving kinetic equations describing the investigated system. As it is known from quantum physics, the dynamics of such a system is described by the equation of Liouville [1]. Unfortunately, the solution of equation of Liouville does not give information about the real physical process, which is described in Boltzmann and Vlasov equations. The most reasonable tool connecting the Liouville equation with Boltzmann and Vlasov equations is a chain of kinetic equations of Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) [2].

Quantum analog of classical BBGKY, describing dynamics of a quantum system of particles is a chain of quantum kinetic equations of BBGKY [3], [4]. It is a complicated system of interconnected integral-differential equations of density matrices of particles, that depends on interaction type of interaction potential between particles.

The present paper solves the Cauchy problem for the BBGKY chain for quantum kinetic equations, describing the dynamics of the quantum system of particles interacting with each other by the delta function potential. A chain of quantum kinetic equations for correlation matrices is defined based on the BBGKY chain for density matrices. Solution of the chain of equations for correlation matrices is defined using solutions of the Cauchy problem for the chain of quantum kinetic equations BBGKY for density matrices [5], [6], [7], [8], [9].

2 Formulation of the Problem

We consider the hierarchy Bogolubov-Born-Green-Kirkwood-Yvon (BBGKY) of quantum kinetic equations, which describes the evolution of a system of identical particles with mass m and charge q interacting via delta function potential $\delta(|x_i - x_j|)$ [10], which depends on the distance between particles $|x_i - x_j|$ in the length L for a one-dimensional case. We assume that the charge q is a real constant. In the present paper, the Cauchy problem is formulated for a quantum system of a finite number of particles contained in the length L. For this case, BBGKY has form [3], [4]:

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$$i\frac{\partial \rho_s^L(t,x_1,...,x_s;x'_1,...,x'_s)}{\partial t} = [H_s^L,\rho_s^L](t,x_1,...,x_s;x'_1,...,x'_s)$$

$$+\frac{N}{L}\left(1-\frac{s}{N}\right)Tr_{x_{s+1}}\sum_{1\leq i\leq s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$-\phi_{i,s+1}(|x'_i-x_{s+1}|)\rho_{s+1}^L(t,x_1,...,x_s,x_{s+1};x'_1,...,x'_s,x_{s+1}), \quad (1)$$

with the initial condition

$$\rho_s^L(t,x_1,...,x_s;x'_1,...,x'_s)|_{t=0} = \rho_s^L(0,x_1,...,x_s;x'_1,...,x'_s). \quad (2)$$

In the problem given by equation (1) and (2) x_i gives the position of ith particle in the 1-dimensional space R. In (1) $\hbar = 1$ is the Planck constant and $[,]$ denotes the Poisson bracket.

The reduced statistical operator of s particles is $\rho_s^L(x_1,..,x_s;x'_1,..,x'_s)$ related to the positive symmetric density matrix D of N particles by

$$\rho_s^L(x_1,.,x_s;x'_1,.,x'_s) =$$

$$L^s Tr_{x_{s+1},.,x_N} D_N^L(x_1,.,x_s,x_{s+1},.,x_N;x'_1,.,x'_s,x_{s+1},.,x_N),$$

where $s\in N$, N is the number of particles, and L the length of the system of particles. The trace is defined in terms of the kernel $\rho^L(x,x')$ by the formula

$$Tr_x\rho^L = \int_L \rho^L(x,x)dx.$$

The Hamiltonian of system is defined as

$$H_s^L(x_1,...,x_s) = \sum_{1\leq i\leq s}\left(-\frac{1}{2m}\triangle_{x_i}+u^L(x_i)\right)+$$

$$+\sum_{1\leq i<j\leq s}\phi_{i,j}(|x_i-x_j|),$$

where $\triangle_i$ is the Laplacian

$$\triangle_i = \frac{\partial^2}{\partial(x_i^1)^2}+\frac{\partial^2}{\partial(x_i^2)^2}+\frac{\partial^2}{\partial(x_i^3)^2},$$

$\phi_{i,j}(|x_i-x_j|) = \delta(|x_i-x_j|)$ and $u^L(x)$ is an external field which keeps the system in the region L: $u^L(x) = 0$ if $x\in L$ and $u^L(x) = +\infty$ if $x\notin L$. Here $\phi_{i,j}(|x_i-x_j|)$ is symmetric.

3 Solution of the Cauchy Problem for the BBGKY Hierarchy of Quantum Kinetic Equations with delta function potential

To obtain the solution of the Cauchy problem defined by (1) and (2) we use a semigroup method [5], [6], [7].

Let $L_2^s(L)$ be the Hilbert space of functions $\psi_s^L(x_1,...,x_s)$, $x_i\in R(L)$, and B_s^L be the Banach space of positive-definite, self-adjoint nuclear operators $\rho_s^L(x_1,...,x_s;x'_1,...,x'_s)$ on $L_2^s(L)$

$$(\rho_s^L\psi_s^L)(x_1,...,x_s) = \int_L \rho_s^L(x_1,...,x_s;x'_1,...,x'_s)\times$$

$$\times\psi_s^L(x'_1,...,x'_s)dx'_1...dx'_s,$$

with norm

$$|\rho_s^L|_1 = sup\sum_{1\leq i\leq\infty}|(\rho_s^L\psi_i^s,\tilde{\psi}_i^s)|,$$

where the upper bound is taken over all orthonormal systems of finite, twice differentiable functions with compact support $\{\psi_i^s\}$ in $L_2^s(L)$, $s\geq 1$. We'll suppose that the operators ρ_s^L and H_s^L act in the space $L_2^s(L)$ with zero boundary conditions.

Let B^L be the Banach space of sequences of nuclear operators

$$\rho^L = \{\rho_0^L,\rho_1^L(x_1;x'_1),...,\rho_s^L(x_1,...,x_s;x'_1,...,x'_s),...\},$$

where ρ_0^L are complex numbers, $\left|\rho_0^L\right|_1 = \left|\rho_0^L\right|$ and $\rho_s^L\subset B_s^L$,

$$\rho_s^L(x_1,...,x_s;x'_1,...,x'_s) = 0, \qquad when \qquad s>s_0,$$

where s_0 is finite and the norm is

$$|\rho^L|_1 = \sum_{s=0}^{\infty}|\rho_s^L|_1.$$

According [10] Bethe ansatz

$$\psi(x_1,\ldots,x_s) = \sum_P a(P)Pexp\left(i\sum_{i=1}^s k_{P_i}x_i\right)$$

satisfies

$$(-\sum_{i=1}^s\triangle_{x_i}+\sum_{1\leq i<j\leq s}\delta(|x_i-x_j|))\psi(x_1,\ldots,x_s) =$$

$$E\psi(x_1,\ldots,x_s) \quad (3)$$

in

$$0<x_1,x_2,...,x_s<L. \quad (4)$$

In (3) eigenvalue $E=\sum_{i=1}^s k_i^2$, where the summation is performed over all permutations P of the numbers $\{k\} = k_1,\ldots,k_s$ and $a(P)$ is a certain coefficient depending on P:

$$a(Q) = -a(P)exp(i\theta_{i,j}),$$

where $\theta_{i,j} = \theta(k_i-k_j)$, $\theta(r) = -2\arctan(r/c)$ and when r is a real value and $-\pi\leq\theta(r)\leq\pi$. Here all the $k's$ are real, distinct. Next, we adhere to the condition (4).

Let $\tilde{B}_s^L$ be a dense set of “good” elements of B_s^L of type $B_s^L\cap D(H_s^L)\bigotimes D(H_s^L)$, where $D(H_s^L)$ is the domain of the operator H_s^L [11] and $\bigotimes$ denote the algebraic tensor product.

We introduce the operators

$$(\Omega^L\rho^L)_s(x_1,..,x_s;x_1',..,x_s') =$$

$$= \frac{N}{L}(1-\frac{s}{N})\int_L \sum_i \rho_{s+1}^L(x_1,..,x_s,x_{s+1};x_1',..,x_s',x_{s+1})\times$$

$$g_i^1(x_{s+1})\tilde{g}_i^1(x_{s+1})dx_{s+1}, \tag{5}$$

$$U^L(t)\rho_s^L(x_1,..,x_s;x_1',..,x_s') =$$

$$(e^{\Omega(L)}e^{-iH^Lt}e^{-\Omega(L)}\rho^L e^{iH^Lt})_s(x_1,..,x_s;x_1',..,x_s').$$

In (5) $g_i^1(x_{s+1})$ is a complete orthonormal system of vectors in the one-particle space $L_2(L)$.

Let

$$(\tilde{\mathscr{H}}^L\rho^L)_s(x_1,..,x_s;x_1',..,x_s') = [H_s^L,\rho_s^L](x_1,..,x_s;x_1',..,x_s')+$$

$$\frac{N}{L}(1-\frac{s}{N})Tr_{x_{s+1}}\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$-\phi_{i,s+1}(|x_i'-x_{s+1}|))\rho_{s+1}^L(x_1,..,x_{s+1};x_1',..,x_{s+1}).$$

Theorem. *If potential $\delta(|x_i-x_j|)$ is delta function potential, the operator $U^L(t)$ generates a strongly continuous semigroup of bounded operators on B^L, whose generators coincide with the operator $-i\tilde{\mathscr{H}}^L$ on $\tilde{B}^L$ everywhere dense in B^L.*

*Proof.*According to the general theory of groups of bounded strongly continuous operators, there always exists an infinitesimal generator of the group $U^L(t)$ given by the formula $lim_{t\longrightarrow 0}\frac{U^L(t)\rho^L-\rho^L}{t}$ in the sense of convergence in norm in the space B^L for ρ^L that belong to a certain set $D(\tilde{\mathscr{H}}^L)$ everywhere dense in B^L [12]. Therefore, since $U^L(t)$ is a strongly continuous semigroup on B^L with generator $-i\tilde{\mathscr{H}}^L$ on the right-hand side of the BBGKY hierarchy of quantum kinetic equations on $\tilde{B}_s^L$ which is dense in B_s^L [5], [6] the abstract Cauchy problem (1)-(2) has the unique solution

$$\rho_s^L(t,x_1,...,x_s;x_1',...,x_s') = (U^L(t)\rho^L)_s(x_1,...,x_s;x_1',...,x_s')$$

$$= (e^{\Omega(L)}e^{-iH^Lt}e^{-\Omega(L)}\rho^L e^{iH^Lt})_s(x_1,...,x_s;x_1',...,x_s') \tag{6}$$

for each $\rho_s^L(x_1,...,x_s;x_1',...,x_s') \subset \tilde{B}_s^L$. For the initial data ρ_s^L belonging to a certain subset of B_s^L (to the domain of definition of $D(-i\tilde{\mathscr{H}}^L)$), which is everywhere dense in B_s^L, (6) is strong solution of Cauchy problem (1)-(2).

This proves the Theorem.

4 Derivation of Hierarchy of Kinetic Equations for Correlation Matrices with delta function potential and its Solution

Introducing the notation

$$\left(\mathscr{H}^L\rho^L\right)_s(t,x_1,...,x_s;x_1',...,x_s') =$$

$$= \left[H_s^L,\rho_s^L\right](t,x_1,...,x_s;x_1',...,x_s');$$

$$\left(\mathscr{D}_{x_{s+1}}^L\rho^L\right)_s(x_1,\cdots,x_s;x_1',\cdots,x_s') =$$

$$= \rho_{s+1}^L\left(x_1,\cdots x_s,x_{s+1};x_1',\cdots,x_s',x_{s+1}\right);$$

$$(\mathscr{A}_{x_{s+1}}^L\rho^L)_s(t,x_1,...,x_s;x_1',...,x_s') =$$

$$= \frac{N}{L}(1-\frac{s}{N})\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$-\phi_{i,s+1}(|x_i'-x_{s+1}|))\rho_s^L(t,x_1,...,x_s;x_1',...,x_s');$$

$$\rho^L(t) = \{\rho_1^L(t,x_1;x_1'),...,\rho_s^L(t,x_1,...,x_s:x_1',...,x_s'),...\}, \tag{7}$$

$$s=1,2,\cdots,$$

we can cast (1) and (2) in the form

$$i\frac{\partial}{\partial t}\rho_s^L(t,x_1,...,x_s;x_1',...,x_s') =$$

$$\left(\mathscr{H}^L\rho^L\right)_s(t,x_1,...,x_s;x_1',...,x_s')$$

$$+\int_L\left(\mathscr{A}_{x_{s+1}}^L\mathscr{D}_{x_{s+1}}^L\rho^L\right)_s(t,x_1,...,x_s;x_1',...,x_s')dx_{s+1},$$

$$\rho_s^L(t,x_1,...,x_s;x_1',...,x_s')|_{t=0} = \rho_s^L(x_1,...,x_s;x_1',...,x_s').$$

For sequences (7) this problem can formulated as

$$i\frac{\partial}{\partial t}\rho^L(t) = \left(\mathscr{H}^L\rho^L\right)(t)+\int_L\mathscr{A}_x^L\mathscr{D}_x^L\rho^L(t)dx, \tag{8}$$

$$\rho^L(t)|_{t=0} = \rho^L(0). \tag{9}$$

Proposition *For sequence of correlation matrices*

$$\varphi = \{\varphi_0,\varphi_1(x_1;x_1'),...,\varphi_s(x_1,...,x_s;x_1',...,x_s'),...\},$$

the hierarchy of kinetic equations has the form:

$$i\frac{\partial}{\partial t}\varphi(t) = \mathscr{H}\varphi(t)+\frac{1}{2}\mathscr{W}(\varphi(t),\varphi(t))+\int_L\mathscr{A}_x\mathscr{D}_x\varphi(t)dx+$$

$$+\int_L(\mathscr{A}_x\varphi\star\mathscr{D}_x\varphi)(t)dx, \tag{10}$$

$$\varphi(t)|_{t=0} = \varphi(0). \tag{11}$$

In (8) relation between density matrices and correlation matrices [13],[14] is:

$$\rho(t)=\Gamma\varphi(t)=I+\varphi(t)+\frac{\varphi(t)\star\varphi(t)}{2!}+\cdots\frac{(\star\varphi(t))^s}{s!}+\cdots,$$

where:

$$(\varphi\star\varphi)(X)=\sum_{YCX}\varphi(Y)\varphi(X\setminus Y),$$

$$I\star\varphi=\varphi,\quad (\star\varphi)^s=\underbrace{\varphi\star\varphi\star\cdots\star\varphi}\text{ s times;}$$

$$X=(x_1,\cdots,x_s;x'_1,\cdots,x'_s),$$

$$Y=(x_1,\cdots,x_s;x'_1,\cdots,x'_{s'}),\quad s'\in s,\quad s=1,2,\cdots;$$

$$\mathscr{W}(\varphi,\varphi)(X)=\sum_{YCX}\mathscr{U}(Y;X\setminus Y)\varphi(Y)\varphi(X\setminus Y),$$

$$(\mathscr{U}\varphi)(X)=\left[\sum_{1\le i<j\le s}\phi(x_i-x_j),\varphi\right](X).$$

The prove of the proposition is analogically to [14], [15], [16].

The problems (8), (9) for the system of s particles in volume V have the form:

$$i\frac{\partial}{\partial t}\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)=\mathscr{H}^L\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)+$$
$$+\frac{1}{2}\mathscr{W}^L(\varphi^L,\varphi^L)_s(t,x_1,...,x_s;x'_1,...,x'_s)+$$
$$+\int_L\mathscr{A}^L_{x_{s+1}}\mathscr{D}^L_{x_{s+1}}\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)dx_{s+1}+$$
$$+\int_L\left(\mathscr{A}^L_{x_{s+1}}\varphi^L\star\mathscr{D}^L_{x_{s+1}}\varphi^L\right)_s(t,x_1,...,x_s;x'_1,...,x'_s)dx_{s+1},\tag{12}$$
$$\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)|_{t=0}=\varphi_s^L(0,x_1,...,x_s;x'_1,...,x'_s).$$

To determine the solution to equation (12) we use the relation [17]:

$$\Gamma e^{\Omega}\varphi=e^{\Omega}\Gamma\varphi.\tag{13}$$

You can verify this by series expansion on the right $\Omega^k(\star\varphi)^m$ $(\frac{\Omega^k}{k!})\frac{(\star\varphi)^m}{m!}$ and the left side of the relation (13)

$$\sum_{l\ge0}\frac{1}{l!}(\star\sum_{n\ge0}\frac{\Omega^n}{n!}\varphi^l=\sum_{l\ge0}\frac{1}{l!}\sum_{n_1\ge0,...,n_l\ge0}\frac{\Omega^{n_1}\varphi\star...\star\Omega^{n_l}\varphi}{n_1!...n_l!},$$

$$\frac{1}{m}\sum_{n_1+...+n_m=k}\frac{\Omega^{n_1}\varphi\star...\star\Omega^{n_m}\varphi}{n_1!...n_m!}$$

and considering

$$\Omega(\varphi\star\psi)=\psi\star\Omega\varphi+\varphi\star\Omega\psi$$

in both parts (13).

Using (13), you can rewrite the formula (6) as:

$$U'^L(t)\varphi_s^L(0,x_1,...,x_s;x'_1,...,x'_s)=$$
$$=\Gamma exp(\Omega^L)\Gamma^{-1}[exp(iH^Lt)\Gamma(exp(-\Omega^L)\Gamma^{-1}\Gamma\times$$
$$\times\varphi_s(0,x_1,...,x_s;x'_1,...,x'_s))exp(-iH^Lt)].$$

Using (13) in (6) and $\Gamma^{-1}\Gamma\varphi(t)=\varphi(t)$ we obtain:

$$\rho_s^L(t,x_1,...,x_s;x'_1,...,x'_s)=\Gamma\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)=$$
$$=\Gamma exp(\Omega^L)\Gamma^{-1}[exp(iH^Lt)\Gamma(exp(-\Omega^L)\Gamma^{-1}\times$$
$$\times\Gamma\varphi_s^L(0,x_1,...,x_s;x'_1,...,x'_s)exp(-iH^Lt)]=$$
$$=\Gamma exp(\Omega^L)\Gamma^{-1}[exp(iH^Lt)\Gamma(exp(-\Omega^L)\times$$
$$\times\varphi_s^L(0,x_1,...,x_s;x'_1,...,x'_s)exp(-iH^Lt)].\tag{14}$$

Acting to (14) by Γ^{-1} we receive:

$$\varphi_s^L(t,x_1,...,x_s;x'_1,...,x'_s)=$$
$$=U'^L(t)\varphi_s^L(0,x_1,...,x_s;x'_1,...,x'_s)=$$
$$=exp(\Omega^L)\Gamma^{-1}[exp(iH^Lt)\Gamma(exp(-\Omega^L)\times$$
$$\times\varphi_s(0,x_1,...,x_s;x'_1,...,x'_s)exp(-iH^Lt)].\tag{15}$$

The generator of the semigroup $U'^{\Lambda}(t)$ coincides with

$$-i(\mathscr{H}^L+\frac{1}{2}\mathscr{W}^L+\int_L\mathscr{A}^L_{x_{s+1}}\mathscr{D}^L_{x_{s+1}}dx_{s+1}+$$
$$+\int_L\mathscr{A}^L_{x_{s+1}}\star\mathscr{D}^L_{x_{s+1}}dx_{s+1}),$$

on the set.

So, (15) the unique solution of the Cauchy hierarchy of kinetics equations for correlation matrices with delta function potential (10),(11).

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