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Optical soliton solutions for Lakshmanan-Prosezain-Daniel model using three different methods

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Abstract: In Ithis paper the solution of lakshmanan-Prosezain-Daniel equation (LPDE) are obtained using three methods kudryashov method, $(\frac{G'(\eta)}{G(\eta)})$ expansion method and *tanh* expansion method. We use these methods to reduces the LPD equation to an ordinary differential equation (ODE). All solutions are investegated by introduced figures in 2D and 3D. Dark, singular and bright optical soliton solutions related with optical fibers are presented. We think that these solutions are very important in the field of optical fiber.These methods provide us with mathematical equipments which can be used to solve nonlinear partial differential equation in mathematical physics.

Keywords: Soliton solutions, optical fiber, Lakshmanan-Porsezian-Daniel equation, kurdyashove method, $\frac{G'(\eta)}{G(\eta)}$

1 Introduction

Soliton solutions of nonlinear partial differential equations (NPDES) is very important which has a lot of importance in many fields such as physics and mathematics due to its complicated in most of physical systems [1]. The (NPDES) such as Schrödinger equation [2], lakshmanan-Prosezain-Daniel equation [3, 7], Navier-Stokes equation [4], Korteweg-De Vries equation [5] and Burgers' equation [6] have been discussed in recent year.

The lakshmanan-Prosezain-Daniel equation (LPDE) is presented [3, 7] as this form:

$$b\psi_{\text{xt}} + p\psi\Omega|\psi|^2 + i\psi_t + a\psi_{\text{xx}} = \rho\psi_{\text{xxxx}} + \phi\psi_x^2\psi^* + \sigma\psi|\psi_x|^2 + c\psi_{\text{xx}}|\psi|^2 + \varphi\psi^2{\psi^*}_{\text{xx}} + \delta\psi|\psi|^4, \quad (1)$$

where $\psi(x,t)$ the complex valued wave function, $\psi^*(x,t)$ its complex conjugate, x, t are space and time variables respectively, p is constant, a shows group velocity of dispersion, ρ depicts the fourth order dispersion, the coefficient b is spatiotemporal dispersion (STD) of the model, φ, ϕ, σ, c represented the perturbation factor with nonlinear form of dispersion and δ depicts two-photon absorption. Ω is real valued algebraic function and source of non linearity. There are several methods solved lakshmanan-Prosezain-Daniel equation such as the modified auxiliary equation method [8], modified simple equation method [9], Sine-Gordon expansion method [10], trial equation method [11, 12], *tanh* method [13], unified method [14], collective variable method [15], improve $tan((\psi'(\eta)/2)$ expansion technique [16].

In this work, we solve (1) using three different methods:The Kudryashov method employed as one of the techniques to obtain the solutions [17] [20], [21], [22]. Another method used in this study is the $(\frac{G'(\eta)}{G(\eta)})$ expansion method. This method relies on expanding the solution of the equation in terms of the derivative of a function divided by the function itself [18] [23], [24], [25]. The third method employed the *tanh* expansion method. This method utilizes hyperbolic tangent functions to expand the solution of the equation [19] [26], [27]. To visualize and analyze the obtained solutions, We introduced figures in both two-dimensional (2D) and three-dimensional (3D)

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formats. These figures depict the behavior and characteristics of the solutions
The [LPDE] is a significant equation in the field of optical fiber, and finding its solutions is a great importance.The [LPDE] is considered an important types of Schrödinger equation that has great deals in optical materials.
This paper is organized as: Section 2 the descriptions of the methods are presented. In Section 3 present application of the methods. The graphical illustration for some solutions are introduced in Section 4. The conclusion of our work is present in Section 5.

2 Descriptions of the methods

In this section, the steps of all methods are showing to understand how to apply these methods on the proposed equation. Consider the partial differential equation.

$$W = \left(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \dots\right), \tag{2}$$

where: W represents a polynomial comprising the unknown function $\psi = \psi(x,t)$ as well as its different partial derivatives.

By using the transformation

$$\begin{aligned} &\xi = \gamma + \beta t + (-\alpha)x, \quad \eta = \theta_2 - \nu t + x, \\ &\psi(x,t) = \exp(i\xi)u(\eta), \end{aligned} \tag{3}$$

.

where: $\alpha, \beta, \nu, \gamma$ are arbitrary constants Substituting from (3) into (2),then (2) becomes ordinary differential equation as following:

$$S = \left(u', u'', u''', \dots\dots\dots\right), \tag{4}$$

where: S is a polynomial in $u(\eta)$ and its derivatives.

2.1 Kudryashov method

To use the Kudryashov method the following steps are applied.

Step 1:According to the method, suppose the solution of (4):

$$u(\eta) = \sum_{i=0}^{N} A_i (Q(\eta))^i, \tag{5}$$

where: $A_i, 0 \leq i \leq N$ are constants to be determined, $A_i \neq 0$, and $Q(\eta)$ satisfies the ordinary differential equation as following

$$Q'(\eta) = \sqrt{\alpha^2 Q(\eta)^2 (1 - \Omega Q(\eta)^2)}. \tag{6}$$

Then (6) gives the following solution:

$$Q(\eta) = \frac{4s\exp(-\alpha\eta)}{\Omega\exp(-2\alpha\eta) + 4s^2}. \tag{7}$$

Step 2:By substituting from (5), (6) into (4) and collect all terms have the same power of $Q(\eta)$ together, then taking all coefficients equal to zero. Thus, we get the system of algebraic equations by WOLFRAM MATHEMATIC 11.3.

Step 3:By using the mathematica program, we obtain the exact solution of (4) by solving the system of algebraic equations.

2.2 The $\frac{G'(\eta)}{G(\eta)}$ expansion method

The main steps of $\frac{G'(\eta)}{G(\eta)}$ expansion method are show as following:

Step 1:The method offers a solution to equation (4) as follows:

$$u(\eta) = \sum_{i=0}^{N} B_i \left(\frac{G'(\eta)}{G(\eta)}\right)^i, \tag{8}$$

where: $B_i, 0 \leq i \leq N, B_i \neq 0, G = G(\eta)$ satisfies the ordinary differential equation

$$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0. \tag{9}$$

Step 2:In (8) N is positive integral can be determined by the homogeneous balance principle.

Step 3:There are three possible solutions of (9).

1:Hyperbolic function solutions, when$\lambda^2 - 4\mu > 0$.

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \; \frac{h_1 \sinh\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu}\right) + h_2 \cosh\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu}\right)}{h_2 \sinh\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu}\right) + h_1 \cosh\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu}\right)}. \tag{10}$$

2:Trigonometric function solutions, when$\lambda^2 - 4\mu < 0$.

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2} \; \frac{-h_1 \sin\left(\frac{1}{2}\eta\sqrt{4\mu - \lambda^2}\right) + h_2 \cos\left(\frac{1}{2}\eta\sqrt{4\mu - \lambda^2}\right)}{h_2 \sin\left(\frac{1}{2}\eta\sqrt{4\mu - \lambda^2}\right) + h_1 \cos\left(\frac{1}{2}\eta\sqrt{4\mu - \lambda^2}\right)}. \tag{11}$$

3:Rational function solutions, when $\lambda^2 - 4\mu = 0$.

$$\frac{G'(\eta)}{G(\eta)} = \frac{h_2}{h_1 + h_2\eta} - \frac{\lambda}{2} \tag{12}$$

Step 4:Substituting from (8),(9) into (4), then collect all term have the same power of $\frac{G'(\eta)}{G(\eta)}$ and equating all coefficients to zero, then solving the system by mathematica program to get the exact solution.

2.3 The extended tanh function method

The main steps of *tanh* expasion method are show as following:

Step 1:The solution of (4) by using extended *tanh* function method is giving by:

$$u(\eta)=\sum_{i=1}^{N}A_iQ(\eta)^i+\sum_{i=1}^{N}B_iQ(\eta)^{-i}+A_0, \qquad (13)$$

where: $Q=Q(\eta)$, satisfies the ordinary in the following form:

$$Q'(\eta)=Q(\eta)^2+\omega. \qquad (14)$$

Step 2:Applied the balance principle to determine the value of N.

step 3:There are three solutions of (14)

If $\omega>0$, then

$$Q(\eta)=\sqrt{\omega}\tan\left(\eta\sqrt{\omega}\right),\ Q(\eta)=-\sqrt{\omega}\cot\left(\eta\sqrt{\omega}\right). \qquad (15)$$

If $\omega<0$, then

$$\begin{aligned}Q(\eta)&=-\sqrt{-\omega}\tanh\left(\eta\sqrt{-\omega}\right),\\ Q(\eta)&=-\sqrt{-\omega}\coth\left(\eta\sqrt{-\omega}\right).\end{aligned} \qquad (16)$$

If $\omega=0$, then

$$Q(\eta)=\frac{1}{\eta}. \qquad (17)$$

Step 4:Substituting from(13) and (14) into (4), then collect all coefficient have the same power of $Q(\eta)$ and equating them to zero. Then, we get a system of algebraic equation, which can be solved by WOLFRAM MATHEMATIC 11.3.

3 Applications of the methods

By inserting (3) into (1) and equating the real and imaginary parts to zero,a real part is obtained as:

$$\begin{aligned}&-u''(\eta)(6\alpha^2\rho+a-b\nu)+u(\eta)(\alpha^4\rho+a\alpha^2-\alpha b\beta\\&+\beta+(\sigma+\emptyset)u'(\eta)^2)-u(\eta)^3(\alpha^2(c-\sigma+\varphi+\phi)+\\&p\Omega)+(c+\varphi)u(\eta)^2u''(\eta)+\rho u^{(4)}(\eta)+\delta u(\eta)^5=0,\end{aligned} \qquad (18)$$

and imaginary part giving by:

$$\begin{aligned}&u'(\eta)(4\alpha^3\rho+2a\alpha-b(\alpha\nu+\beta)-2\alpha u(\eta)^2(c-\varphi+\phi)+\\&\nu)-4\alpha\rho u^{(3)}(\eta)=0.\end{aligned} \qquad (19)$$

From equation (18), (19) the coefficients of linearly independent functions equal zero, as following:

$$c+\varphi=0. \qquad (20)$$

$$\sigma+\phi=0. \qquad (21)$$

$$c-\varphi+\phi=0. \qquad (22)$$

$$\rho=0. \qquad (23)$$

$$4\alpha^3\rho+2a\alpha-\alpha b\nu-b\beta+\nu=0,\quad \nu=\frac{b\beta-2a\alpha}{1-\alpha b}. \qquad (24)$$

Also substituting

$$\Omega(u)=su+du^2. \qquad (25)$$

We substitute from equation (20)- (23) into (18),(19), we get the single equation its solution are determined by the parabolic law nonlinearity in the form of equation (25), then we get the following equation:

$$\begin{aligned}&(a-b\nu)u''(\eta)+u(\eta)(-a\alpha^2+\alpha b\beta-\beta)+\\&u(\eta)^3(s-4\alpha^2c)+(d-\delta)u(\eta)^5=0.\end{aligned} \qquad (26)$$

By substituting

$$u(\eta)=\sqrt{y(\eta)},\nu=\frac{b\beta-2a\alpha}{1-\alpha b}. \qquad (27)$$

into (26), then becomes

$$\begin{aligned}&-4y(\eta)^2(a\alpha^2-\alpha b\beta+\beta)\\&+4y(\eta)^3(s-4\alpha^2c)+4(d-\delta)y(\eta)^4+\\&\frac{y'(\eta)^2(a\alpha b+a-\beta b^2)}{\alpha b-1}-\frac{2y(\eta)y''(\eta)(a\alpha b+a-\beta b^2)}{\alpha b-1}.\end{aligned} \qquad (28)$$

Balancing y^4 with yy'' in (28) we get the following relation

$$\Rightarrow 4N=N+2+N\Rightarrow N=1. \qquad (29)$$

3.1 Solution of The kurdyashove method

From (5) and (29) then, we can write the solution of (28) as the following form:

$$y(\eta)=A_0+A_1Q(\eta), \qquad (30)$$

By substituting equation (30) into equation (28) and equating the coefficients of each power of $Q(\eta)$ to zero, we obtain the ensuing system of equations:

$$-4a\alpha^2A_0^2+4\alpha A_0^2b\beta-4A_0^2\beta-16\alpha^2A_0^3c$$

$$+4A_0^4d-4A_0^4\delta+4A_0^3s=0,$$

$$-8a\alpha^2A_0A_1-\frac{2a\alpha^3A_0A_1b}{\alpha b-1}-\frac{2a\alpha^2A_0A_1}{\alpha b-1}$$

$$+\frac{2\alpha^2 A_0 A_1 b^2\beta}{\alpha b-1}+8\alpha A_0 A_1 b\beta-$$
$$8A_0A_1\beta-48\alpha^2A_0^2A_1c+16A_0^3A_1d$$
$$-16A_0^3A_1\delta+12A_0^2A_1s=0,$$
$$-4a\alpha^2A_1^2-\frac{a\alpha^3A_1^2b}{\alpha b-1}-\frac{a\alpha^2A_1^2}{\alpha b-1}+\frac{\alpha^2A_1^2b^2\beta}{\alpha b-1}+4\alpha A_1^2b\beta-$$
$$4A_1^2\beta-48\alpha^2A_0A_1^2c+24A_0^2A_1^2d-24A_0^2A_1^2\delta+12A_0A_1^2s=0,$$
$$\frac{4a\alpha^3A_0A_1b\Omega}{\alpha b-1}+\frac{4a\alpha^2A_0A_1\Omega}{\alpha b-1}-\frac{4\alpha^2A_0A_1b^2\beta\Omega}{\alpha b-1}-$$
$$16\alpha^2A_1^3c+16A_0A_1^3d-16A_0A_1^3\delta+4A_1^3s=0,$$
$$\frac{3a\alpha^3A_1^2b\Omega}{\alpha b-1}+\frac{3a\alpha^2A_1^2\Omega}{\alpha b-1}-\frac{3\alpha^2A_1^2b^2\beta\Omega}{\alpha b-1}$$
$$+4A_1^4d-4A_1^4\delta=0.$$

Solve the previous system, we get the following sets of solutions:

Set 1:

$$A_0=0,$$
$$A_1=\mp\frac{\sqrt{3}\sqrt{5ab^2s^2\Omega-16ab\sqrt{c}s^{3/2}\Omega+16acs\Omega}}{\sqrt{25b^4ds^2-25b^4\delta s^2-96b^2cds+96b^2c\delta s+256c^2d-256c^2\delta}},$$
$$\beta=\frac{-\frac{25ab^3s^{5/2}}{\sqrt{c}}+50ab^2s^2+16ab\sqrt{c}s^{3/2}-96acs}{2\left(25b^4s^2-96b^2cs+256c^2\right)},$$
$$\alpha=-\frac{\sqrt{s}}{2\sqrt{c}}. \tag{31}$$

Set 2:

$$A_0=0,$$
$$A_1=\pm\frac{\sqrt{3}\sqrt{5ab^2s^2\Omega+16ab\sqrt{c}s^{3/2}\Omega+16acs\Omega}}{\sqrt{25b^4ds^2-25b^4\delta s^2-96b^2cds+96b^2c\delta s+256c^2d-256c^2\delta}},$$
$$\beta=\frac{\frac{25ab^3s^{5/2}}{\sqrt{c}}+50ab^2s^2-16ab\sqrt{c}s^{3/2}-96acs}{2\left(25b^4s^2-96b^2cs+256c^2\right)},$$
$$\alpha=\frac{\sqrt{s}}{2\sqrt{c}}. \tag{32}$$

By substituting from (31), (32) into (30) with (27),(7) and (3) we get the following solutions

$$\psi_{1,2,3,4}(x,t)=\exp(i\xi)\sqrt{\frac{A_1(4s\exp(-\alpha\eta))}{\Omega\exp(-2\alpha\eta)+4s^2}+A_0}. \tag{33}$$

where

$$\xi=\gamma+\beta t+(-\alpha)x,\quad \eta=\theta_2-\nu t+x. \tag{34}$$

3.2 Solution of $\frac{G'(\eta)}{G(\eta)}$ expansion method

From (8) and (29) then, the solution of (28) is giving by:

$$y(\eta)=\frac{B_1G'(\eta)}{G(\eta)}+B_0. \tag{35}$$

Substituting (35) into (28) and equating all terms of power $\frac{G'(\eta)}{G(\eta)}$ to zero, we get the next system:

$$-\frac{2aB_1B_0\lambda\mu}{\alpha b-1}-\frac{2a\alpha bB_1B_0\lambda\mu}{\alpha b-1}+\frac{aB_1^2\mu^2}{\alpha b-1}+\frac{a\alpha bB_1^2\mu^2}{\alpha b-1}-$$
$$4a\alpha^2B_0^2+\frac{2b^2\beta B_1B_0\lambda\mu}{\alpha b-1}-\frac{b^2\beta B_1^2\mu^2}{\alpha b-1}+4\alpha b\beta B_0^2-4\beta B_0^2-$$
$$16\alpha^2B_0^3c+4B_0^4d-4B_0^4\delta+4B_0^3s=0,$$
$$-\frac{2aB_1B_0\lambda^2}{\alpha b-1}-\frac{2a\alpha bB_1B_0\lambda^2}{\alpha b-1}-\frac{4aB_1B_0\mu}{\alpha b-1}-\frac{4a\alpha bB_1B_0\mu}{\alpha b-1}-$$
$$8a\alpha^2B_1B_0+\frac{2b^2\beta B_1B_0\lambda^2}{\alpha b-1}+\frac{4b^2\beta B_1B_0\mu}{\alpha b-1}+8\alpha b\beta B_1B_0-$$
$$8\beta B_1B_0-48\alpha^2B_1B_0^2c+16B_1B_0^3d$$
$$-16B_1B_0^3\delta+12B_1B_0^2s=0,$$
$$-\frac{a\alpha bB_1^2\lambda^2}{\alpha b-1}-\frac{aB_1^2\lambda^2}{\alpha b-1}-\frac{6a\alpha bB_0B_1\lambda}{\alpha b-1}-\frac{6aB_0B_1\lambda}{\alpha b-1}-$$
$$\frac{2a\alpha bB_1^2\mu}{\alpha b-1}-\frac{2aB_1^2\mu}{\alpha b-1}-4a\alpha^2B_1^2$$
$$+\frac{\beta b^2B_1^2\lambda^2}{\alpha b-1}+\frac{6\beta b^2B_0B_1\lambda}{\alpha b-1}+$$
$$\frac{2\beta b^2B_1^2\mu}{\alpha b-1}+4\alpha\beta bB_1^2-4\beta B_1^2-48\alpha^2B_0B_1^2c$$
$$+24B_0^2B_1^2d-24B_0^2B_1^2\delta+12B_0B_1^2s=0,$$
$$-\frac{4aB_1^2\lambda}{\alpha b-1}-\frac{4a\alpha bB_1^2\lambda}{\alpha b-1}-\frac{4aB_0B_1}{\alpha b-1}$$
$$-\frac{4a\alpha bB_0B_1}{\alpha b-1}+\frac{4b^2\beta B_1^2\lambda}{\alpha b-1}+$$
$$\frac{4b^2\beta B_0B_1}{\alpha b-1}-16\alpha^2B_1^3c+16B_0B_1^3d-16B_0B_1^3\delta+4B_1^3s=0,$$
$$-\frac{3aB_1^2}{\alpha b-1}-\frac{3a\alpha bB_1^2}{\alpha b-1}+\frac{3b^2\beta B_1^2}{\alpha b-1}+4B_1^4d-4B_1^4\delta=0.$$

By solve the previous system, then we have the following cases of solutions:

Case 1:

$$B_0=\frac{1}{2}B_1\left(\lambda-\sqrt{\lambda^2-4\mu}\right),$$
$$a=-\frac{B_1\left(s-4\alpha^2c\right)\left(b^2\left(4\alpha^2+\lambda^2-4\mu\right)-8\alpha b+4\right)}{4\sqrt{\lambda^2-4\mu}},$$
$$\beta=-\frac{B_1\left(s-4\alpha^2c\right)\left(-4\alpha^2+4\alpha^3b+\alpha b\left(\lambda^2-4\mu\right)+\lambda^2-4\mu\right)}{4\sqrt{\lambda^2-4\mu}},$$
$$d=\frac{3\left(s-4\alpha^2c\right)}{4B_1\sqrt{\lambda^2-4\mu}}+\delta. \tag{36}$$

Case 2:

$$B_0 = \frac{1}{2}B_1\left(\sqrt{\lambda^2-4\mu}+\lambda\right),$$
$$a = \frac{B_1\left(s-4\alpha^2 c\right)\left(b^2\left(4\alpha^2+\lambda^2-4\mu\right)-8\alpha b+4\right)}{4\sqrt{\lambda^2-4\mu}},$$
$$\beta = \frac{B_1\left(s-4\alpha^2 c\right)\left(-4\alpha^2+4\alpha^3 b+\alpha b\left(\lambda^2-4\mu\right)+\lambda^2-4\mu\right)}{4\sqrt{\lambda^2-4\mu}},$$
$$d = \delta - \frac{3\left(s-4\alpha^2 c\right)}{4B_1\sqrt{\lambda^2-4\mu}}. \tag{37}$$

Using (3),(10), (11), (27),(35), (36), (37), then get the next solutions.

Hyperbolic function solutions, when$\lambda^2-4\mu>0$

$$\psi_{5,6}(x,t) = \exp(i\xi)\sqrt{B_1\left(\frac{\sqrt{\lambda^2-4\mu}\left(h_1\sinh\left(\frac{1}{2}\eta\sqrt{\lambda^2-4\mu}\right)+h_2\cosh\left(\frac{1}{2}\eta\sqrt{\lambda^2-4\mu}\right)\right)}{2\left(h_2\sinh\frac{1}{2}\eta\sqrt{\lambda^2-4\mu}+h_1\cosh\left(\frac{1}{2}\eta\sqrt{\lambda^2-4\mu}\right)\right)}-\frac{\lambda}{2}\right)+B_0}. \tag{38}$$

Trigonometric function solutions, when$\lambda^2-4\mu<0$

$$\psi_{7,8}(x,t) = \exp(i\xi)\sqrt{B_1\left(\frac{\sqrt{4\mu-\lambda^2}\left(-h_1\sin\left(\frac{1}{2}\eta\sqrt{4\mu-\lambda^2}\right)+h_2\cos\left(\frac{1}{2}\eta\sqrt{4\mu-\lambda^2}\right)\right)}{2\left(h_2\sin\left(\frac{1}{2}\eta\sqrt{4\mu-\lambda^2}\right)+h_1\cos\left(\frac{1}{2}\eta\sqrt{4\mu-\lambda^2}\right)\right)}-\frac{\lambda}{2}\right)+B_0}. \tag{39}$$

where

$$\xi = \gamma+\beta t+(-\alpha)x, \quad \eta = \theta_2-\nu t+x. \tag{40}$$

3.3 solution of tanh expansion method

From (13) and (29), thus the solution (28) takes the form:

$$y(\eta) = A_1 Q(\eta)+A_0+\frac{B_1}{Q(\eta)}. \tag{41}$$

Substituting (41) into (28), we get the following system:

$$-4a\alpha^2 A_0^2+\frac{aA_1^2\omega^2}{\alpha b-1}+\frac{a\alpha A_1^2 b\omega^2}{\alpha b-1}-\frac{12aA_1B_1\omega}{\alpha b-1}-$$
$$\frac{12a\alpha A_1 bB_1\omega}{\alpha b-1}-8a\alpha^2 A_1B_1+\frac{aB_1^2}{\alpha b-1}+\frac{a\alpha bB_1^2}{\alpha b-1}-$$
$$\frac{A_1^2 b^2\beta\omega^2}{\alpha b-1}+\frac{12A_1b^2\beta B_1\omega}{\alpha b-1}+4\alpha A_0^2 b\beta$$
$$+8\alpha A_1 b\beta B_1-4A_0^2\beta-$$
$$8A_1\beta B_1-96\alpha^2 A_1A_0B_1c+48A_1A_0^2B_1d-48A_1A_0^2B_1\delta+$$
$$24A_1^2B_1^2d-24A_1^2B_1^2\delta+24A_1A_0B_1s-16\alpha^2A_0^3c+$$
$$4A_0^4d-4A_0^4\delta+4A_0^3s-\frac{b^2\beta B_1^2}{\alpha b-1}=0,$$
$$-\frac{4aA_0A_1}{\alpha b-1}-\frac{4a\alpha A_0A_1b}{\alpha b-1}+\frac{4A_0A_1b^2\beta}{\alpha b-1}$$
$$-16\alpha^2A_1^3c+16A_0A_1^3d-$$
$$16A_0A_1^3\delta+4A_1^3s=0,$$
$$\frac{3aA_1^2}{\alpha b-1}-\frac{3a\alpha A_1^2b}{\alpha b-1}+\frac{3A_1^2b^2\beta}{\alpha b-1}+4A_1^4d-4A_1^4\delta=0,$$
$$-8a\alpha^2A_1A_0-\frac{4aA_1A_0\omega}{\alpha b-1}-\frac{4a\alpha A_1A_0b\omega}{\alpha b-1}+\frac{4A_1A_0b^2\beta\omega}{\alpha b-1}+$$
$$8\alpha A_1A_0b\beta-8A_1A_0\beta-48\alpha^2A_1^2B_1c+48A_1^2A_0B_1d-$$
$$48A_1^2A_0B_1\delta+12A_1^2B_1s-48\alpha^2A_1A_0^2c+$$
$$16A_1A_0^3d-16A_1A_0^3\delta+12A_1A_0^2s=0,$$
$$-4a\alpha^2A_1^2-\frac{2aA_1^2\omega}{\alpha b-1}-\frac{2a\alpha A_1^2b\omega}{\alpha b-1}$$
$$-\frac{6aA_1B_1}{\alpha b-1}-\frac{6a\alpha A_1bB_1}{\alpha b-1}+$$
$$\frac{2A_1^2b^2\beta\omega}{\alpha b-1}+\frac{6A_1b^2\beta B_1}{\alpha b-1}+4\alpha A_1^2b\beta-4A_1^2\beta+$$
$$16A_1^3B_1d-16A_1^3B_1\delta-48\alpha^2A_0A_1^2c+24A_0^2A_1^2d-$$
$$24A_0^2A_1^2\delta+12A_0A_1^2s=0,$$
$$-\frac{4aA_0B_1\omega}{\alpha b-1}-\frac{4a\alpha A_0bB_1\omega}{\alpha b-1}-8a\alpha^2A_0B_1+\frac{4A_0b^2\beta B_1\omega}{\alpha b-1}+$$
$$8\alpha A_0b\beta B_1-8A_0\beta B_1-48\alpha^2A_0^2B_1c-48\alpha^2A_1B_1^2c+$$
$$16A_0^3B_1d-16A_0^3B_1\delta+48A_1A_0B_1^2d-48A_1A_0B_1^2\delta+$$
$$12A_0^2B_1s+12A_1B_1^2s=0,$$
$$-\frac{4aA_0B_1\omega^2}{\alpha b-1}-\frac{4a\alpha A_0bB_1\omega^2}{\alpha b-1}+\frac{4A_0b^2\beta B_1\omega^2}{\alpha b-1}+$$
$$16A_0B_1^3d-16A_0B_1^3\delta-16\alpha^2B_1^3c+4B_1^3s=0,$$
$$-\frac{6aA_1B_1\omega^2}{\alpha b-1}-\frac{6a\alpha A_1bB_1\omega^2}{\alpha b-1}-\frac{2aB_1^2\omega}{\alpha b-1}-\frac{2a\alpha bB_1^2\omega}{\alpha b-1}-$$
$$4a\alpha^2B_1^2+\frac{6A_1b^2\beta B_1\omega^2}{\alpha b-1}-48\alpha^2A_0B_1^2c+16A_1B_1^3d$$
$$-16A_1B_1^3\delta+24A_0^2B_1^2d-24A_0^2B_1^2\delta+12A_0B_1^2s$$
$$+\frac{2b^2\beta B_1^2\omega}{\alpha b-1}+4\alpha b\beta B_1^2-4\beta B_1^2=0,$$
$$-\frac{3aB_1^2\omega^2}{\alpha b-1}-\frac{3a\alpha bB_1^2\omega^2}{\alpha b-1}+\frac{3b^2\beta B_1^2\omega^2}{\alpha b-1}+4B_1^4d-4B_1^4\delta=0.$$

Solve the above system by mathematica then, the subsequent sets of solutions are obtained as:
set 1:

$$\begin{aligned} A_1 &= \pm\frac{2a\sqrt{\omega}}{\sqrt{\left((\alpha b-1)^2-b^2\omega\right)^2\left(-(s-4\alpha^2 c)^2\right)}},\\ A_0 &= -\frac{2a\omega}{\left((\alpha b-1)^2-b^2\omega\right)(s-4\alpha^2 c)}, B_1=0,\\ \beta &= \frac{a\alpha^2(\alpha b-1)-a\omega(\alpha b+1)}{(\alpha b-1)^2-b^2\omega},\\ \delta &= d-\frac{3\left((\alpha b-1)^2-b^2\omega\right)(s-4\alpha^2 c)^2}{16a\omega}. \end{aligned} \tag{42}$$

set 2:

$$\begin{aligned} A_1 &= 0, A_0 = -\frac{2a\omega}{\left((\alpha b-1)^2-b^2\omega\right)(s-4\alpha^2 c)},\\ B_1 &= \mp\frac{2a\omega^{3/2}}{\sqrt{\left((\alpha b-1)^2-b^2\omega\right)^2\left(-(s-4\alpha^2 c)^2\right)}},\\ \beta &= \frac{a\alpha^2(\alpha b-1)-a\omega(\alpha b+1)}{(\alpha b-1)^2-b^2\omega},\\ \delta &= d-\frac{3\left((\alpha b-1)^2-b^2\omega\right)(s-4\alpha^2 c)^2}{16a\omega}. \end{aligned} \tag{43}$$

set 3 :

$$\begin{aligned} A_1 &= \mp\\ &\frac{4\sqrt{\frac{a^2\omega(5b^4\omega^2+11b^2\omega(\alpha b-1)^2+2(\alpha b-1)^4)}{4b^2\omega-(\alpha b-1)^2}}}{\sqrt{\left(-20b^6\omega^3-39b^4\omega^2(\alpha b-1)^2+3b^2\omega(\alpha b-1)^4+2(\alpha b-1)^6\right)(s-4\alpha^2 c)^2}},\\ A_0 &= -\frac{8a\omega}{\left((\alpha b-1)^2-4b^2\omega\right)(s-4\alpha^2 c)},\\ B_1 &= \pm\\ &\frac{4\omega\sqrt{\frac{a^2\omega(5b^4\omega^2+11b^2\omega(\alpha b-1)^2+2(\alpha b-1)^4)}{4b^2\omega-(\alpha b-1)^2}}}{\sqrt{\left(-20b^6\omega^3-39b^4\omega^2(\alpha b-1)^2+3b^2\omega(\alpha b-1)^4+2(\alpha b-1)^6\right)(s-4\alpha^2 c)^2}},\\ \beta &= \frac{a\alpha^2(\alpha b-1)-4a\omega(\alpha b+1)}{(\alpha b-1)^2-4b^2\omega},\\ \delta &= d-\frac{3\left((\alpha b-1)^2-4b^2\omega\right)(s-4\alpha^2 c)^2}{64a\omega}. \end{aligned} \tag{44}$$

set 4:

$$\begin{aligned} A_1 &= \pm\\ &\frac{4\sqrt{\frac{a^2\omega(-4b^4\omega^2-7b^2\omega(\alpha b-1)^2+2(\alpha b-1)^4)}{5b^2\omega+(\alpha b-1)^2}}}{\sqrt{\left(-20b^6\omega^3-39b^4\omega^2(\alpha b-1)^2+3b^2\omega(\alpha b-1)^4+2(\alpha b-1)^6\right)(s-4\alpha^2 c)^2}},\\ A_0 &= \frac{8a\omega}{\left(5b^2\omega+(\alpha b-1)^2\right)(s-4\alpha^2 c)},\\ B_1 &= \pm\\ &\frac{4\omega\sqrt{\frac{a^2\omega(-4b^4\omega^2-7b^2\omega(\alpha b-1)^2+2(\alpha b-1)^4)}{5b^2\omega+(\alpha b-1)^2}}}{\sqrt{\left(-20b^6\omega^3-39b^4\omega^2(\alpha b-1)^2+3b^2\omega(\alpha b-1)^4+2(\alpha b-1)^6\right)(s-4\alpha^2 c)^2}},\\ \beta &= \frac{a\alpha^2(\alpha b-1)+5a\omega(\alpha b+1)}{5b^2\omega+(\alpha b-1)^2},\\ \delta &= \frac{3\left(5b^2\omega+(\alpha b-1)^2\right)(s-4\alpha^2 c)^2}{64a\omega}+d. \end{aligned} \tag{45}$$

By using (42)-(45) into (41) with (27), (15), (16) and (3) we get the following solution of (1):
If $\omega < 0$ then, we have

$$\begin{aligned} &\psi_{9-16}(x,t) = \exp(i\xi)\\ &\sqrt{A_1\left(-\sqrt{-\omega}\tanh\left(\eta\sqrt{-\omega}\right)\right)+A_0-\frac{B_1}{\sqrt{-\omega}\tanh\left(\eta\sqrt{-\omega}\right)}}. \end{aligned} \tag{46}$$

$$\begin{aligned} &\psi_{17-24}(x,t) = \exp(i\xi)\\ &\sqrt{A_1\left(-\sqrt{-\omega}\coth\left(\eta\sqrt{-\omega}\right)\right)+A_0-\frac{B_1}{\sqrt{-\omega}\coth\left(\eta\sqrt{-\omega}\right)}}. \end{aligned} \tag{47}$$

If $\omega > 0$, then

$$\begin{aligned} &\psi_{25-32}(x,t) = \exp(i\xi)\\ &\sqrt{A_1\left(\sqrt{\omega}\tan\left(\eta\sqrt{\omega}\right)\right)+A_0+\frac{B_1}{\sqrt{\omega}\tan\left(\eta\sqrt{\omega}\right)}}. \end{aligned} \tag{48}$$

$$\begin{aligned} &\psi_{33-40}(x,t) = \exp(i\xi)\\ &\sqrt{A_1\left(-\sqrt{\omega}\cot\left(\eta\sqrt{\omega}\right)\right)+A_0-\frac{B_1}{\sqrt{\omega}\cot\left(\eta\sqrt{\omega}\right)}} \end{aligned} \tag{49}$$

where

$$\xi = \gamma+\beta t+(-\alpha)x, \quad \eta = \theta 2-\nu t+x. \tag{50}$$

4 Graphical illustrating

In this section, we introduce some figures in the two-dimensional and three dimensional about some the solution of LPDE by three method : kurdyashove method, $(\frac{G'(\eta)}{G(\eta)})$ expansion method and *tanh* expansion method and show what is the typices of soliton solution. In figure 1, we plot the graph of set 1 (31) with $\psi_1(x,t)$ (33) by using kudryashov method at constants $a = 0.2, b = 0.1, c = 0.1, \gamma = 0.3, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.2$, we use kudryashov method in figure 2 with constant $\gamma = 0.2, a = 0.7, b = 0.3, c = 0.2, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.2$, we plot the graph in figure 3,4 of set 2 (32) with $\psi_3(x,t)$ (33) and set 2 (32) with $\psi_4(x,t)$ (33) use kudryashov method with constants $\gamma = 0.2, a = 0.7, b = 0.3, c = 0.2, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.2$ and $a = 0.1, b = 0.2, c = 0.1, \gamma = 0.3, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.1$. respectively. Use $(\frac{G'(\eta)}{G(\eta)})$ expanded method, we plot the graph in Case (1) (36) figure 5 with $\psi_5(x,t)$ (38) at constants $\gamma = 0.1, a = 0.32, \alpha = 0.2, b = 0.2, B_1 = 0.4, c = 0.2, d = 0.1, \delta = 0.2, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.5, \mu = 0.01, s = 0.3$, the optical soliton solution of Case (2) (37) with $\psi_6(x,t)$ (38) in figure 6 at constants $a = 0.4, \alpha = 0.2, b = 0.4, B_1 = 0.4, c = 0.2, \gamma = 0.3, d = 0.1, \delta = 0.4, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda =$

$0.5, \mu = 0.01, s = 0.3$, in figure 7,8, we use $(\frac{G'(\eta)}{G(\eta)})$ expansion method of Case (1) (36) with $\psi_7(x,t)$ (39) and Case (2) (37) with $\psi_8(x,t)$ (39) at constants $a = 0.1, \alpha = 0.2, b = 0.1, B_1 = 0.2, c = 0.2, \gamma = 0.5, d = 0.1, \delta = 0.4, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.0005, \mu = 0.01, s = 0.1$ and $a = -0.1, \alpha = 0.1, b = 0.1, B_1 = 0.4, c = 0.1, \gamma = 0.1, d = 0.1, \delta = 0.2, h_1 = 0.4, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.0005, \mu = 0.01, s = 0.1$. respectively. Also show the soliton solutions of *tanh* expansion method in figure 9 of set 1 (42) with $\psi_9(x,t)$ at constants $a = 1, \alpha = 0.1, b = 1, \gamma = 0, c = 0.2, d = 0.5, \theta 2 = 0, s = 0.3, \omega = -0.9$, The optical soliton solutions of set 1 (42) with $\psi_{10}(x,t)$ (46) by using *tanh* expanded method with constants $a = 1, \alpha = 0.1, b = 1, \gamma = 0, c = 0.2, d = 0.5, \theta 2 = 0, s = 0.3, \omega = -0.9$ in figure 10. We plot figure 11, 12 by *tanh* expanded method set 1 (42) with $\psi_{26}(x,t)$ (48) and set 2 (42) with $\psi_{26}(x,t)$ (48) at constants $\omega = 0.001, a = 0.1, \alpha = 0.6, b = 0.7, \gamma = -2, c = 0.5, d = 0.2, \theta 2 = -1, s = 0.2$ and $\omega = 0.001, a = 0.1, \alpha = 0.6, b = 0.7, \gamma = -2, c = 0.5, d = 0.2, \theta 2 = -1, s = 0.2$ respectively.

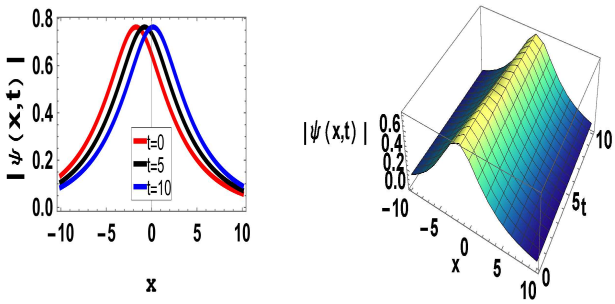

Fig. 1: Optical rational soliton solution of set 1 (31) with $\psi_1(x,t)$ (33) using kudryashov method with constants $a = 0.2, b = 0.1, c = 0.1, \gamma = 0.3, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.2$.

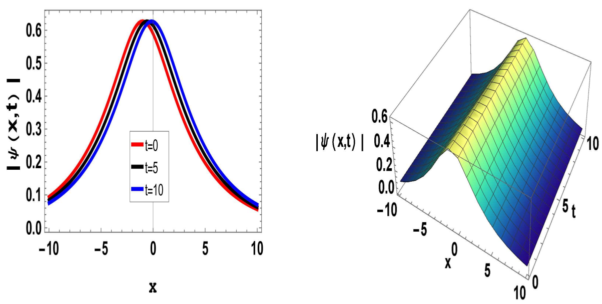

Fig. 2: Optical rational soliton solution of set 1 (31) with $\psi_2(x,t)$ (33) using kudryashov method with constants $a = 0.2, b = 0.1, c = 0.8, \delta = 1, \theta 2 = 0.3, \nu = 0.1, s = 0.1, \sigma = 0.2, \Omega$=0.1.

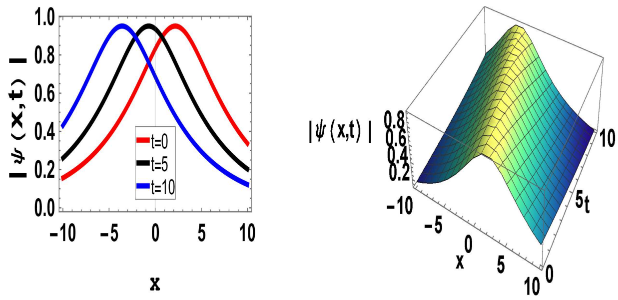

Fig. 3: Optical rational soliton solution of set 2 (32) with $\psi_3(x,t)$ (33) using kudryashov method plots with constants $\gamma = 0.2, a = 0.7, b = 0.3, c = 0.2, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.2$.

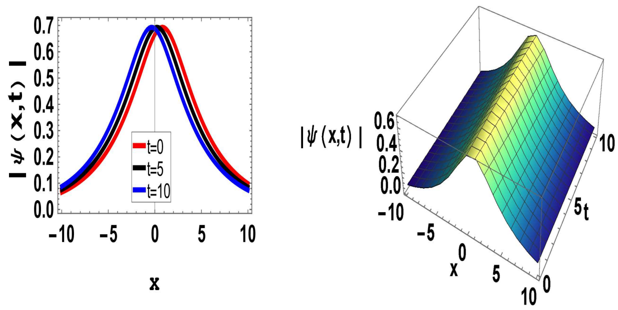

Fig. 4: Optical rational soliton solution of set 2 (32) with $\psi_4(x,t)$ (33) using kudryashov method plots with constants $a = 0.1, b = 0.2, c = 0.1, \gamma = 0.3, \delta = 0.2, d = 0.1, \theta 2 = 0.1, s = 0.1, \Omega = 0.1$.

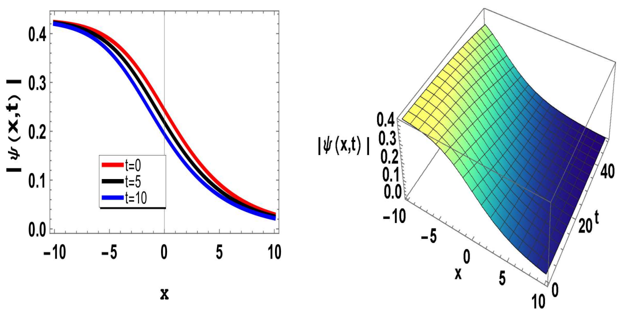

Fig. 5: Optical bright-singular soliton solution of Case (1) (36) with $\psi_5(x,t)$ (38) using$(\frac{G'(\eta)}{G(\eta)})$ expanded method with constants $\gamma = 0.1, a = 0.32, \alpha = 0.2, b = 0.2, B_1 = 0.4, c = 0.2, d = 0.1, \delta = 0.2, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.5, \mu = 0.01, s = 0.3$.

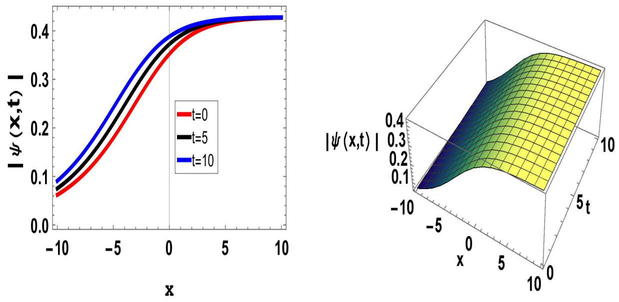

Fig. 6: Optical bright-singular soliton solution of Case (2) (37) with $\psi_6(x,t)$ (38) using $(\frac{G'(\eta)}{G(\eta)})$ expanded method with constants $a = 0.4, \alpha = 0.2, b = 0.4, B_1 = 0.4, c = 0.2, \gamma = 0.3, d = 0.1, \delta = 0.4, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.5, \mu = 0.01, s = 0.3$.

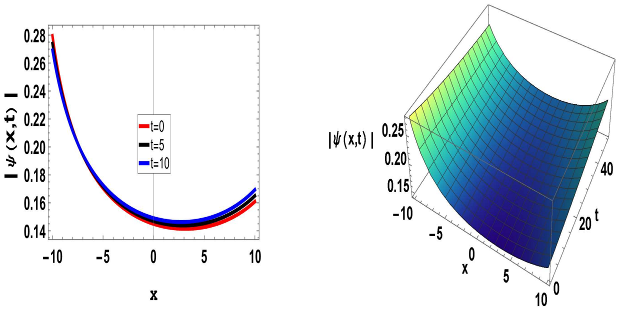

Fig. 7: Optical bright-singular soliton solution of Case (1) (36) with $\psi_7(x,t)$ (39) using$(\frac{G'(\eta)}{G(\eta)})$ expanded method with constants $a = 0.1, \alpha = 0.2, b = 0.1, B_1 = 0.2, c = 0.2, \gamma = 0.5, d = 0.1, \delta = 0.4, h_1 = 0.3, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.0005, \mu = 0.01, s = 0.1$.

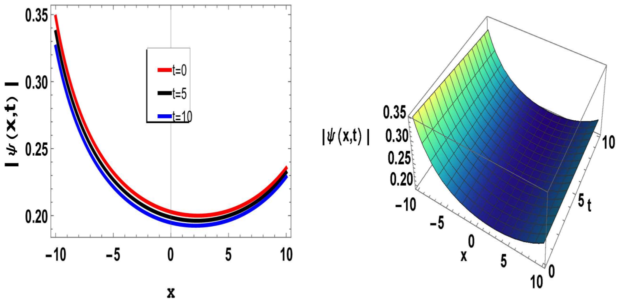

Fig. 8: Optical bright-singular soliton solution of Case (2) (37) with with $\psi_8(x,t)$ (39) using$(\frac{G'(\eta)}{G(\eta)})$ expanded method with constants $a = 0.1, \alpha = 0.1, b = 0.1, B_1 = 0.4, c = 0.1, \gamma = 0.1, d = 0.1, \delta = 0.2, h_1 = 0.4, h_2 = 0.1, \theta 2 = 0.1, \lambda = 0.0005, \mu = 0.01, s = 0.1$.

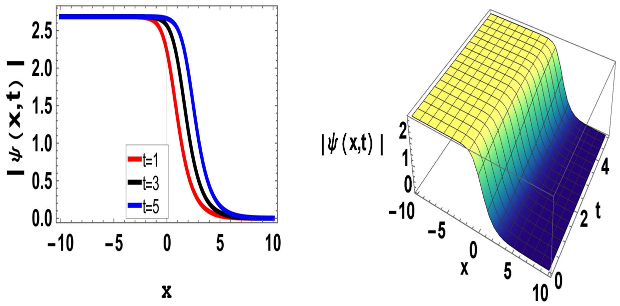

Fig. 9: Optical bright soliton solution of set 1 (42) with $\psi_9(x,t)$ (46) by using *tanh* expanded method with constants $a = 1, \alpha = 0.1, b = 1, \gamma = 0, c = 0.2, d = 0.5, \theta 2 = 0, s = 0.3, \omega = -0.9$.

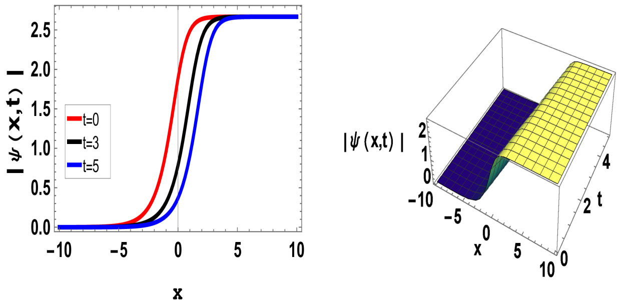

Fig. 10: Optical bright soliton solution of set 1 (42) with $\psi_{10}(x,t)$ (46) by using *tanh* expanded method with constants $a = 1, \alpha = 0.1, b = 1, \gamma = 0, c = 0.2, d = 0.5, \theta 2 = 0, s = 0.3, \omega = -0.9$.

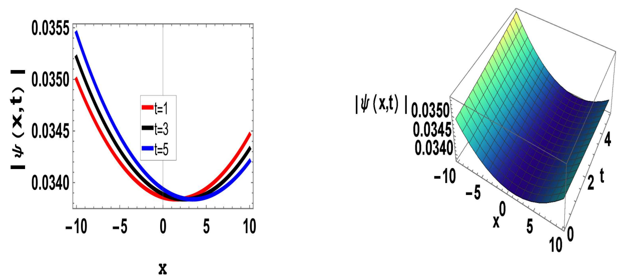

Fig. 11: Optical dark soliton solution of set 1 (42) with $\psi_{25}(x,t)$ (48) by using *tanh* expanded method with constants $\omega = 0.001, a = 0.1, \alpha = 0.6, b = 0.7, \gamma = -2, c = 0.5, d = 0.2, \theta 2 = -1, s = 0.2$.

5 Conclusion

In conclusion, this paper presented an effective and simple methods to solve the Lakshmanan-Porsezian-Daniel equation (LPDE) by using three analytical method, namely the Kudryashov method,

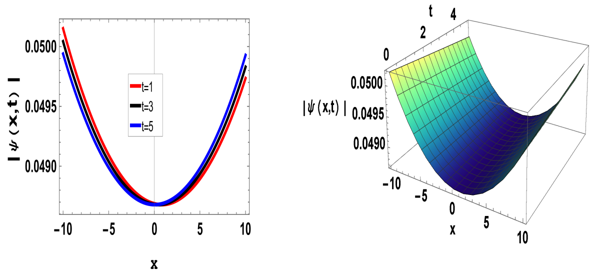

Fig. 12: Optical dark soliton solutions of set 1 (42) with $\psi_{26}(x,t)$ (48) by using *tanh* expanded method with constants $\omega = 0.001, a = 0.1, \alpha = 0.6, b = 0.7, \gamma = -2, c = 0.5, d = 0.2, \theta 2 = -1, s = 0.2$.

the $(\frac{G'(\eta)}{G(\eta)})$ expanded method, and the tanh expanded method. The solutions derived through these methods were further illustrated through figures presented in both 2D and 3D format. Types of solutions dark , bright and singular are shown. The solutions of [LPDE] play great role in field of mathematics and physics, as they contribute to our understanding of various physical phenomena governed by the LPDE. Finding solutions of [LPDE] considered very benefit in the field of optical fiber.

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