A Study of The Saudi Stock Market Using Some Statistical Models

Abdelgalal O. I. Abaker[1], Abdalla S. Mahmoud[2,*], Badawi O.Mohammed[3,4], Ali R. R. Alzahrani[5], Adil. O. Y. Mohamed[6], and Azhari A. Elhag[7]

[1]Applied College, Khamis Mushait, King Khalid University, Abha, Saudi Arabia
[2]Department of Mathematics, University College at Raniah, P.O. Box 11099, Taif University, Taif 21944, Saudi Arabia
[3]Department of MIS, Faculty of Business Administration, Albaha University, Albaha, Saudi Arabia
[4]Department of Statistics, Faculty of Economics and Political Science, Omdurman Islamic University, Sudan
[5]Department of Mathematics, Faculty of Sciences, Umm Al-Qura University, Makkah, 24382, Saudi Arabia
[6]Department of Computer Science, College of Computer, Qassim University, Buraydah 52571, Saudi Arabia
[7]Department of Mathematics and Statistics, College of Science, P.O. Box 11099, Taif University, Taif 21944, Saudi Arabia

Abstract: The objective of this paper is to estimate the diversification effects/benefits of an investment in a portfolio consisting of the South African Industrial (J520) and the Financial (J580) Indices using the Generalised Pareto Distributions (GPDs) with an extreme value Gumbel copula. The GPD is used as the marginal distribution to both assets to better characterize the extreme risk of returns in both Indices tails. The extreme value Gumbel copula captures the dependence structure (co-movement) of the financial assets in the portfolio. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) goodness of fit tests and the scatterplots indicate that the upper tail of the gains (the larger gains) risk and the losses tail (the larger losses) are best captured using the extreme value Gumbel copula. Monte Carlo simulation of an equally weighted portfolio of the two Indices is used to estimate the portfolio risk. The univariate marginal risks and the portfolio risks are used to calculate the diversification effects/benefits. The results show that there are benefits in diversification since the riskiness of the portfolio is less than the sum of the risk of the two financial assets. This implies that VaR, although not additive theoretically, is sub-additive in this practical situation. This property of sub-additivity represents the benefits of diversification for a portfolio. The implication is that investors investing in individual risky assets can benefit from constructing such a portfolio to reduce extreme risk. Due to high dependence and contagion between developed markets/Global markets, this is useful information for local and international investors seeking a portfolio which includes developing countries' market Indices, such as South African assets, which are less correlated with other Global markets, thereby reducing the risk of contagion.

Keywords: Machine Learning, GARCH models, Stock market, Statistical Model, ARIMA models, Volatility clustering, Leverage effect

1 Introduction

The problem of modeling and forecasting stock market volatility in literature has taken wide considerable attention around the world in both developing and developed countries. The models of univariate GARCH are used by Dana [1] in the Amman stock market to test the behavior of returns volatility. The models of ARCH/GARCH have presented a piece of strong evidence for the existence of each Leptokurtic and volatility clustering. Moreover, the results shown in Amman stock market returns that, there is no evidence for the presence of a leverage effect. Prashant Joshi [2] used three different models of GARCH to predict the daily volatility of the Bombay Stock Exchange's Sensex. According to their findings, the stock market shows volatility persistence and the existence of a leverage effect. Moreover, using a variety of macroeconomic variables, the predictive performance of linear and non-linear models are compared to the Johannesburg Stock Exchange by Michael Van et al. [3] when predicting financial returns. The authors used a variety of models such as Markov switching ARMA and Dynamic Regression, linear specifications, and univariate GARCH to capture conditional heteroscedasticity. According to their findings, the best in-sample fit is Markov switching models, while findings for the out-of-sample periods reveal the outperformance of linear models. Adolphus et al. [4]   attempted to determine the most efficient GARCH models to predict the volatility of the Nairobi stock exchange. The (SBC), (AIC), and Mean Squared Error were used to evaluate the models. The results showed that IGARCH models are the best models for modeling and forecasting the volatility of the Nairobi stock exchange.

2 Methodology

*Corresponding author e-mail: a.adaam@tu.edu.sa

In modeling and forecasting of stock market volatility, GARCH models are the most commonly used methodologies, see Hassan and Hassan [5]. The symmetric and asymmetric models present two types of volatility models. This paper employs one of the symmetric models known by the GARCH (1,1) model and two GARCH asymmetric models known by EGARCH (1,1) and TGARCH (1,1).

2.1 ARCH model

ARCH models are developed by Engle [6]. These models are used in modeling and predicting the volatility of the financial time series. The model is written as follows:

$$y_t = \varepsilon_t \tag{1}$$

$$\varepsilon_t \sim N(0, h_t) \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 \tag{3}$$

ARCH(q) is the name of this model. For q= 1 ARCH model is reduced to

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{4}$$

Where y_t denotes the observed time series, ε_t denotes the residual, α_0 is a constant, α_i denotes to the ARCH effect, q denotes the duration of ARCH lags and h_t denotes the conditional variance. α_0 and α_i are nonnegative normally values to satisfies positive conditional variance.

2.2 GRCH model

The GARCH model is capturing the volatility of the variance and volatility clustering of financial time series developed by Bollerslev [7] as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{5}$$

Where equation (5) denotes conditional variance equation with $\alpha_0 > 0, \alpha_i > 0, \beta_i > 0$

The model (5) referred to as GARCH(p ,q). Then, GARCH(1,1) can be specified by:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{6}$$

The shortcomings of GARCH model are that the asymmetric effect does not capture what characterizes the most stock market return data.

2.3 Exponential generalized auto regressive conditional heteroskedastic (E-GARCH) model

EGARCH model suggested by Nelson, which allows for asymmetric effects, see [8]. The model can be written as follows:

$$\sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma d_{t-1} \tag{7}$$

Where d_{t-1} is a dummy variable that takes the value 1 if $\varepsilon_{t-1} < 0$ (bad news) and the value 0 if $\varepsilon_{t-1} > 0$ (good news). γ represents the leverage term. $\varepsilon_{t-1} > 0$ indicates good news. The coefficient γ is the leverage term.

2.4 Time series analysis in statistics and Econometrics

 The model of Autoregressive Moving Average (ARMA) is generalized by Autoregressive Integrated Moving Average (ARIMA) model, see [9,10,11] and [12]. Therefore, the time series data is used under both these two models in order to better understand the data or to forecast it by the future points in the series [13] and [14].

Let Zt a time series data, which represents an integer index, and Zt be a real number, then

$$Z_t = Z_{t-1} + a_t \; ; \; a_t \sim WN(0, \sigma^2) \tag{8}$$

$$Z_t - Z_{t-1} = a_t \; ; \; a_t \sim WN(0, \sigma^2) \tag{9}$$

$$(1 - B)Z_t = a_t \tag{10}$$

$$Z_t = \frac{1}{(1 - B)} a_t \tag{11}$$

$$\psi(B) = \frac{1}{(1 - B)} \tag{12}$$

Where the weighed $\psi_1 \, \psi_2, \psi_3, \ldots \ldots \ldots \ldots$

$$\psi(B)(1 - B) = 1 \tag{13}$$

$$(\ 1 \ + \ \psi_1 B \ + \ \psi_2 \, B^2 \ + \ \psi_3 B^3 \ + \ ....)(\ 1 \ - \ B \ ) \ = \ 1 \tag{14}$$

$$B : \ \psi_1 \ - \ 1 = 0 \ \Rightarrow \ \psi_1 = 1 \tag{15}$$

$$B^2 : \ \psi_2 \ - \ \psi_1 = 0 \ \Rightarrow \ \psi_2 = \ \psi_1 = 1 \tag{16}$$
$$B^3 : \ \psi_3 \ - \ \psi_2 = 0 \ \Rightarrow \ \psi_3 = \ \psi_2 = 1 \tag{17}$$
$$B^j : \ \psi_j \ - \ \psi_{j-1} = 0 \ \Rightarrow \ \psi_j = \ \psi_{j-1} = 1 \tag{18}$$

That is, the weights for the random walk model are

$$\psi_j \ = 1 \, , \ j \geq 1 \tag{19}$$

2.5 Some properties of the weighted function $\psi(B)$

The auto regressive moving average model of degree ARMA (P,q) can be written like that:

$$Z_t - \ \mu = \ \psi(B) \ a_t \, ; \ a_t \sim WN \, (0, \sigma^2) \tag{20}$$

This relationship could be written as follows [15]:

$$Z_t - \mu = a_t + \ \psi_1 \, a_{t-1} + \ \psi_2 \, a_{t-2} + \ .... \ = \ \sum_{j=0}^{\infty} \psi_j a_{j-1} \, ; \ \psi_0 \ = 1 \tag{21}$$

3. Data Analysis

The daily closing prices of Saudi Stock Exchange represents the data used in this article from January 1-1-2003 to June 18-6-2020[16]. The following formula was used to measure daily returns r_t:

$$r_t = \ \left(\frac{P_t}{P_{t-1}} \right) \tag{22}$$

where:

r_t is the index's daily return at time t

P_t denotes closing price index

The results in table (1) show a negative skewness coefficient which supports previous findings and confirms the fact that many financial data have a large left tail. The normality at the 1% level of significance of the high value of Jarque- Bbera is rejected. Furthermore, the LM test statistics of Engle (1982) suggest the presence of ARCH effects, the variance of the return series has indicated that the later series is time-varying.

**Table 1:** TASI returns descriptive statistics

Skewness	-0.944936
Kurtosis	14.08645
Jarque- Bera	10002.53
Probability of Jarque- Bera	0.00000
ARCH-LM (2)	8.017619
Probability of ARCH- LM	0.0182

In Fig. 1, we notice that periods of higher changes are pursued by periods of higher changes whereas periods of low changes are followed by periods of low changes. This pattern of stock return suggests that TASI returns show strong signs of volatility clustering. As a result, residuals are conditionally heteroscedastic, and ARCH and GARCH models can be used to describe it.

The distribution of TASI log returns is shown in Fig. 2, which clearly reveals a deviation from normality.
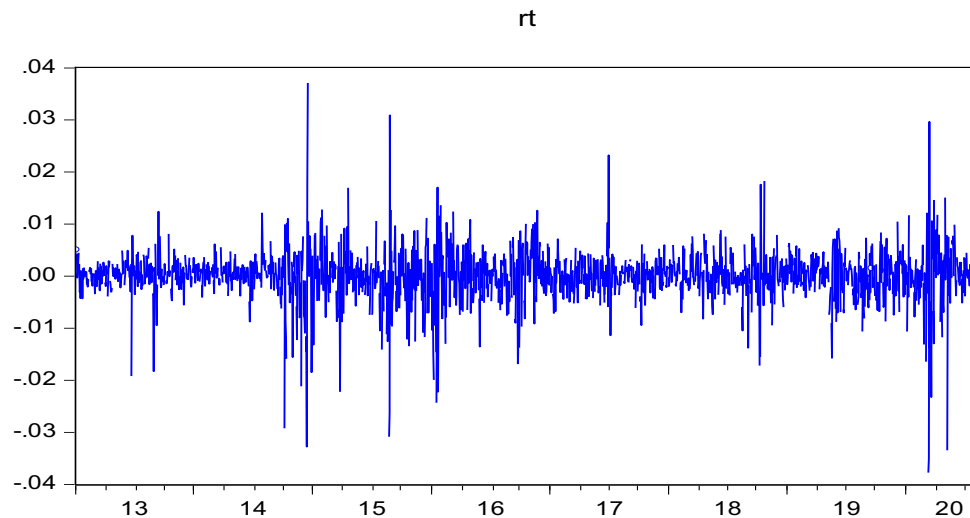
Fig 1: Daily Returns series
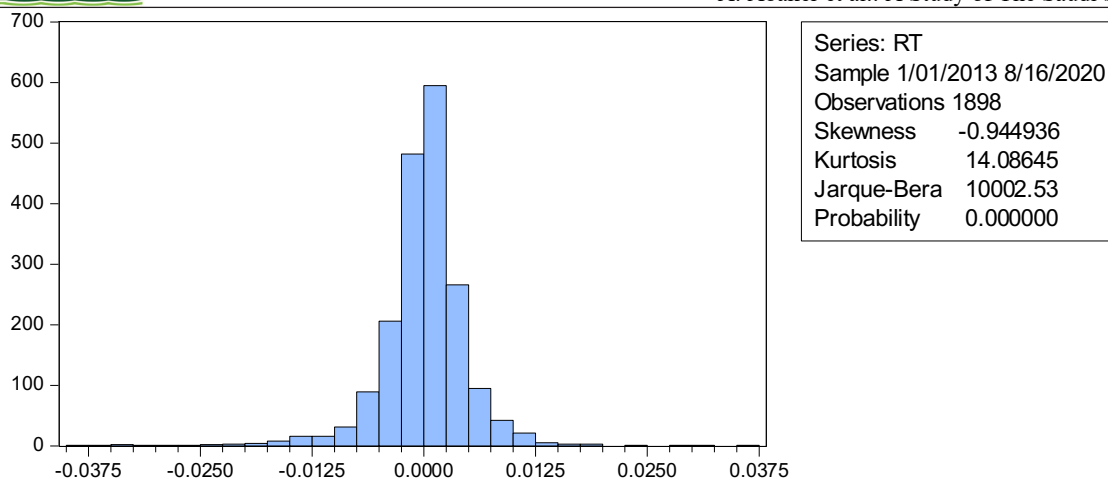
Fig 2: Daily Returns Distribution

The stationarity is determined under unit rule test of return series; the results, which are described in table (2), show that non-stationary returns were rejected, and we conclude that TASI returns are stationary in level.

Table 2: TASI daily return series, stationery test

Augmented Dickey- Fuller unit root test		-38.35
critical values	1%	-3.43
	5%	-2.86
	10%	-2.57

This section contains estimates for Saudi Exchange returns series from various GARCH and ARMA models. Also diagnostic test of the models are achieved

Table 3. The estimation values of TASI index under different GARCH models

parameters	GARCH (1,1)	EGARCH(1,1)	TGARCH(1,1)
The Equation of the Mean			
$\emptyset_0$	0.000333 (0.0000)	0.000262 (0.0000)	0.000285 (0.0000)
The Equation of the Variance			
$\emptyset_1$	6.21E-07 (0.0001)	-0.57554 (0.0000)	5.99E-07 (0.0000)
α	.0173918 (0.0000)	0.231682 (0.0000)	0.054040 (0.0082)
β	0.812225 (0.0000)	0.963812 (0.0000)	0.829462 (0.0000)
γ	-	-0.107144 (0.0000)	0.179816 (0.0000)
Log likelihood	7751.398	7785.570	7779.557
AIC	-8.163749	-8.161704	-8.192368
SIC	-8.152055	-8.184088	-8.177751
ARCH-LM(1) Test	0.14073 (0.9321)	0.171168 (0.9180)	0.181503 (0.9132)

The p-value is putted in parentheses

The results in Table 3 reveal that the constant coefficients of ARCH and GARCH in the conditional variance equation for GARCH (1,1) model are positive and statistically significant at 1%. The sum of parameters $(\alpha + \beta)$ of ARCH and GARCH is close to unity. Also, the results suggest that volatility shocks are quite persistent.

From Table 3, The EGARCH model results indicate that the parameters at 1% are statistically significant. The parameter γ has asymmetric effect captured by significant with negative sign, meaning that good news produce less volatility than bad news.

TGARCH is an alternative asymmetric model for investigating existence of leverage effect in TASI index. The results of

TGARCH model displayed in table (3) reveal that the parameter of the asymmetric term γ is positive and significant at the 1% level. This

The ARCH- LM test of order 1 shows that there is no ARCH effect remaining in the residuals of the variance equations for all GARCH models, as shown in table (3). This shows that the variance equations are correctly defined. The identified model in table (4) for this data is ARIMA (1, 0, 1) where it succeeded in estimated parameter significance test and it succeeded in residual analysis test.

Table 4: The identified model ARIMA (1, 0, 1)

Par.	Value	Met. St. error	95% Lower bound	95% upper bound	Asy. St. error	95% Lower bound	95% upper bound
AR(1)	-0.368	0.156	-0.674	-0.063	0.128	-0.619	-0.118
MA(1)	0.504	0.158	0.196	0.813	0.119	0.272	0.737

The GARCH and ARMA models were evaluated by using maximum log likelihood criteria, Akaike Information Criteria (AIC) and minimum Schwartz Bayesian Criteria (SBC) values. The results in table (5) show that the GARCH (1,1) outperforms the other models depending on the min AIC and SBC values. While the biggest log likelihood criteria reveal that EGARCH (1,1) model outperforms the other models. However, because of the asymmetric response to news, this study suggests that the EGARCH (1,1) model is the best model for predicting the volatility of Saudi stock market returns.

Table 5. The Criteria to evaluate Models

Criteria	GARCH (1,1)	TGARCH (1,1)	EGARCH (1,1)	ARIMA (1,0,1)
AIC	-8.299	-8.314	-8.319	-14.744
SBC	-8.284	-8.297	-8.302	-14.72
Log likelihood	7881	7896	7901	7375

4. Conclusion

  For academics and market participants, modelling the volatility of stock returns has become an important field of study in the financial markets. In this article ARIMA models and GARCH models were used to estimate and predict volatility returns of TASI series, from January 1, 2013 to August 16, 2020. Findings indicate that the daily returns of TASI are characterized by clustering characteristics of volatility, leptokurtosis and by the presence of heteroscedasticity in the residuals. Besides, the study has shown that the EGARCH model is superior to the other models in estimating and predicting the volatility returns of the Saudi stock exchange.

Acknowledgement:

References

[1] Alnajjar, D. Modelling and Estimating Volatility Using ARCH/GARCH Models in Jordan's Stock Market, Asian Journal of Finance and Accounting ,8(1),152-167(2016).

[2] Joshi, P.,& Vidyanagar, G. Forecasting Volatility of Bombay Stock Exchange, International Journal of Current Research and Academic Review, 2(7), 222-230(2014).

[3] Michael, V.G., Chun, S.H., & Ryan, K. The Performance of Linear Versus Non-Linear Models in Forecasting Returns on The Johannesburg Stock Exchange, International Business & Economics Research Journal, 12(8),985-994 (2013).

[4] Wagala, A., Nassiuma, D., Islam, A.,& Mwangi, J.W. Volatility Modelling of the Nairobi Securities Exchange Weekly Returns Using the Arch-Type Models, International Journal of Applied Science and Technology , 2(3),165-174 (2012).

[5] Ghassan, H.B., & Alhajhoj, H.R. Effect of Capital Market liberalization on Volatility of TASI", Journal of Development and Economic Policies ,14 (2),7-39 (2012).

[6] Engle, R.F. Autoregressive conditional heteroscedasticity with estimation of variance of United Kingdom inflation, Econometrica ,50(4),987-1007 (1982).

[7] Bollerslev, T. Generalized Autoregressive Conditional Heteroscedasticity with Estimate of the Variance of United Kingdom Inflation, Journal of Econometrics, 31, 307-327 (1986).

[8] Martinet, G.G & Micheal, M. On inevitability of EGARCH (p,q), Econometric Reviews, 37(8), 824-849,(2018).

[9] Miron, D., & Tudor, C. Asymmetric Conditional Volatility Models: Empirical Estimation and Comparison of Forecasting Accuracy, Romanian Journal of Economic Forecasting, 3, 74-91 (2010).

[10] Suliman, Z.S., &Winker, P. Modelling Stock Market Volatility Using Univariate GARCH Models: Evidence from Sudan and Egypt, International Journal of Economics and Finance, 4(8), 161176 (2012).

[11] Glosten, L. R., Jagannathan, R., & Runkle, D. E. On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, Journal of Finance, 48(5),1779-1801 (1993).

[12] Box, G. E., Jenkins, G. M., Reinsel, G. C., Ljung, G. M., 2015. John Wiley & Sons: Hoboken, NJ, USA.

[13] Fried, R., George, A. C., International Encyclopedia of Statistical Science, Springer Berlin Heidelberg (2014).

[14] Montgomery, D. C., Jennings, C. L., Kulahci, M. John Wiley & Sons, Hoboken, New Jersey, United States (2015).

[15] Touama, H.Y., Mathematical Theory and Modeling Journal, 4, 19-26 (2014).

[16] https://www.saudiexchange.sa/Resources/Reports/DailyFinancialIndicators_en.html