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Influence of Non-Homogeneity, Rotation, Magnetic Field, and Initial Stress on Radial Vibrations in Thermo-Viscoelastic Isotropic Media

Mohammed A. Balubaid

Department of Industrial Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia

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Abstract: This paper presents an innovative analytical approach to investigate the effects of rotation and initial stress on a magneto-thermo-viscoelastic non-homogeneous medium with a spherical cavity under periodic loading. The study focuses on analyzing the distribution of displacements, temperature, and stresses within the non-homogeneous medium using the generalized thermoelasticity framework based on the GL theory. The derived analytical solutions are extensively discussed, and the results are visually presented to illustrate the influences of rotation, initial stress, relaxation times, magnetic field, viscoelasticity, and non-homogeneity. Furthermore, comprehensive comparisons with prior research are conducted, encompassing scenarios with and without rotation, magnetic field, and initial stress. These comparisons contribute to establishing the significance of the findings and highlight the advancements made in this study in relation to existing knowledge in the field.

Keywords: Rotation, Viscoelasticity, Initial stress, Non-homogeneity, Magneto-thermo-elasticity.

1 Introduction

In the last decade, the scientific community has shown significant interest in the exploration of thermo-visco-elastic phenomena in the presence of magnetic fields. This research area has gained prominence due to its broad applicability across various domains. Notably, the field finds relevance in geophysics, where comprehending the interaction between Earth's magnetic field and seismic waves, as well as the attenuation of acoustic waves in a magnetic field, holds paramount importance. Moreover, the emission of electromagnetic radiation from nuclear devices, advancements in highly sensitive superconducting magnetometers, and applications in optics and electrical power engineering have further accentuated the significance of this field [1,2,3,4].

In recent years, several notable investigations have contributed to advancing our understanding in this area. Mahmoud et al. conducted comprehensive studies on the effects of rotation on plane vibrations in infinite hollow cylinders as transversely isotropic media. They also explored wave motion through cylindrical bores in micropolar porous cubic crystals under the influence of rotation, as well as the influence of magnetic fields on radial vibrations in non-homogeneous, rotating cylinders [5,6]. These investigations have shed light on key aspects of the complex interplay between rotation, magnetic fields, and various mechanical phenomena, paving the way for further advancements in this field. Abd-Alla et al. conducted a comprehensive investigation into the behavior of rotating non-homogeneous, infinite orthotropic cylinders. They examined the influence of rotation on radial vibrations in non-homogeneous orthotropic rotating elastic hollow spheres, as well as magneto-thermo-elastic problems in rotating non-homogeneous orthotropic cylinders under the hyperbolic heat conduction model [7,8,9,10]. Mahmoud conducted a study on the electrostatic potential solution for wave propagation in human wet bones.

Additionally, they explored the impact of generalized magneto-thermoelasticity on Rayleigh waves in granular rotating media under the influence of initial stress and gravity [11]. Abd-Alla and Mahmoud further investigates the magneto-thermoelastic behavior of a rotating, non-homogeneous orthotropic hollow cylindrical

* Corresponding author e-mail: mbalubaid@kau.edu.sa

structure under the hyperbolic heat conduction model, shedding light on its complex thermal and mechanical response. Balubaid et al. [12] study the dynamic behavior of orthotropic elastic materials is investigated through the application of an analytical solution, providing valuable insights into their mechanical response [13]. In a separate study, Abd-Alla et al. addressed various problems, including the propagation of S-waves in non-homogeneous, anisotropic, incompressible media under the influence of gravity, generalized magneto-thermoelastic Rayleigh waves in granular media under the influence of gravity and initial stress, and the transient coupled thermoelasticity of an annular fin [14, 15, 16].

Abd-Alla and colleagues [17, 18] conducted a comprehensive exploration of thermoelasticity and wave propagation modeling specifically in cylindrical structures. Similarly, Mukhopadhyay [19] investigated the intricate interactions of thermal relaxations and thermo-visco-elastic properties in unbounded bodies containing spherical cavities, with a focus on their response to periodic loading applied at the boundary. Roychoudhuri and Banerjee [20] conducted a study examining the effects of both periodic loading and thermal relaxations on thermoelastic interactions in unbounded bodies featuring spherical cavities or cylindrical holes. Building upon this research, Erbay et al. and Al-Basyouni et al. [21, 22] delved into the investigation of thermally induced vibrations in generalized thermoelastic solids, particularly those containing cavities. In a related vein, Mahmoud [23] explored the realm of elastodynamic orthotropic hollow spheres, providing an analytical solution for their free vibrations while considering the influence of rotation.

This research paper is dedicated to the decomposition of equations pertaining to rotation, initial stress, and magneto-thermo-elasticity for a spherical cavity, transforming them into non-homogeneous equations with appropriate boundary conditions. The paramount objective is to comprehensively examine the impact of thermal relaxation times on wave propagation within magneto-thermo-viscoelastic media, employing the GL theory. The material properties of the spherical cavity are assumed to align with the Kelvin-Voigt model. Consequently, precise expressions are derived to characterize the transient response of displacement, stresses, and temperature within the spherical cavity. The investigation extends further through numerical calculations to explore the behavior of displacement, temperature, and stress components, with specific attention given to scenarios involving magnetic fields, non-homogeneity, initial stress, and rotation. The obtained numerical results are meticulously computed and graphically presented, facilitating a comprehensive analysis and in-depth discussion of the findings.

2 Formulation of The Problem

To properly analyze the problem at hand, it is essential to utilize spherical coordinates (r, θ, ϕ) to represent any given point. Additionally, it is assumed that the spherical cavity is subjected to a rapid temperature change denoted by T(r, t) and a magnetic field $\overline{H}(0,0,H_0)$, For the axisymmetric plane strain problem, the displacement components $\overline{u}=\overline{u}(u_r,u_\theta,u_\phi)$ can be expressed as follows $u_\theta=u_\phi=0,\ \ and\ \ u_r=u_r(r,t)$. It is important to note that we are considering an infinite isotropic non-homogeneous viscoelastic solid. The viscoelastic behavior of the material is described by the Voigt type of linear viscoelasticity. Within this medium, a spherical cavity with a radius of a is present. Additionally, Lorentz's force [18] can be expressed as:

$$f_r=\mu_e(\overline{J}\times\overline{H})=\mu_e H_0^2\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{2u_r}{r}\right), \tag{1}$$

$$\overline{h}=curl(\overline{u}\times\overline{H})=\left(0,0,-H_0\left(\frac{\partial u_r}{\partial r}+\frac{2u_r}{r}\right)\right),$$

$$\overline{J}=curl\ \overline{h}=\left(0,\frac{\partial h_\phi}{\partial r},0\right),$$

Let us denote $\overline{h}$, $\overline{E}$, $\overline{J}$, μ_e , $\overline{H}$ and $\overline{u}$ as the perturbed magnetic field, electric intensity, electric current density, magnetic permeability, constant primary magnetic field, and displacement vector, respectively. When considering the magneto-elastodynamic equation for the non-homogeneous spherical medium with radial displacement $u_r=u_r(r,t)$, it can be expressed as follows:

$$\begin{aligned}&\frac{\partial\sigma_{rr}}{\partial r}+\frac{2}{r}\sigma_{rr}-\frac{1}{r}\sigma_{\theta\theta}-\frac{1}{r}\sigma_{\varphi\varphi}+\mu_e\left(\overline{J}\times\overline{H}\right)\\&-\rho\left(\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{u}\right)_r-\rho\left(2\overleftarrow{\Omega}\times\overleftarrow{u}\right)_r=\rho\frac{\partial^2u_r}{\partial t^2}\end{aligned} \tag{2}$$

where $\overleftarrow{\Omega}=(0,0,\Omega)$, The cross product of the vector $\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{u}$ represents the centripetal acceleration, while the product of $2\overleftarrow{\Omega}\times\overleftarrow{u}$ denotes the Coriolis acceleration. For a system exhibiting spherical symmetry, the non-vanishing stress components can be expressed as follows:

$$\sigma_{rr}=\tau_m(\lambda+2\mu+P)\frac{\partial u_r}{\partial r}+(2\lambda+P)\tau_m\frac{u_r}{r}-\gamma(T+\tau_2\dot{T}),$$

$$\sigma_{\theta\theta}=2\tau_m(\lambda+\mu+P)\frac{u_r}{r}+(\lambda+P)\tau_m\frac{\partial u_r}{\partial r}-\gamma(T+\tau_2\dot{T}),$$

$$\sigma_{\varphi\varphi}=2\tau_m(\lambda+\mu)\frac{u_r}{r}+\lambda\tau_m\frac{\partial u_r}{\partial r}-\gamma(T+\tau_2\dot{T}),$$

$$\sigma_{r\varphi}=\sigma_{r\theta}=\sigma_{\theta\varphi}=0 \tag{3}$$

where σ_{rr} , $\sigma_{\theta\theta}\ and\ \sigma_{\varphi\varphi}$ are normal mechanical stresses, $\sigma_{r\varphi}$, $\sigma_{r\theta}$, $\sigma_{\theta\varphi}$ are shear mechanical stresses. $\tau_m=1+\tau_0\frac{\partial}{\partial t}$

and the parameter τ_0 represents the mechanical relaxation time arising from viscosity, P represents the initial stress.

The magneto-thermo-elastodynamic equations governing the behavior of the non-homogeneous sphere can be expressed as follows:

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{2}{r}\sigma_{rr}-\frac{1}{r}\sigma_{\theta\theta}-\frac{1}{r}\sigma_{\varphi\varphi}+\mu_e H_0^2\frac{\partial}{\partial r}(\frac{\partial u_r}{\partial r}+\frac{2u_r}{r})+\rho\Omega^2 u_r=\rho\frac{\partial^2 u_r}{\partial t^2}, \tag{4}$$

$$K(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r})=\rho c_v(\frac{\partial T}{\partial t}+\tau_1\frac{\partial^2 T}{\partial t^2})+\gamma T_0[\frac{\partial}{\partial r}+\frac{2}{r}]u_r. \tag{5}$$

In the context of the discussed parameters, thermal conductivity (K) plays a significant role in the heat transfer process. The coefficient γ is defined as the product of $\gamma=\alpha_t(3\lambda+2\mu)$, where α_t represents the coefficient of linear thermal expansion, and λ and μ denote the Lame elastic constants.

Other essential parameters include Ω, which represents the rotation, ρ for material density, c_v for the specific heat of the material per unit mass, τ_1,τ_2 as thermal relaxation parameters. Additionally, T represents the temperature difference $(?-T_0)$, ? as the absolute temperature, and T_0 as the reference temperature of the solid and τ_0 accounts for the mechanical relaxation time resulting from viscosity. It is noteworthy to mention a special law that characterizes the non-homogeneity of the material. This law embodies specific characteristics or properties related to the material's variation in composition or structure, which significantly impact its behavior and response under the given conditions.

$$\lambda=\lambda_0 r^{2n},\quad \mu=\mu_0 r^{2n},\quad P=P_0 r^{2n},\quad \rho=\rho_0 r^{2n},\quad \mu_e=\mu_e^0 r^{2n},\quad \gamma=\gamma_0 r^{2n}, \tag{6}$$

In the equations provided, λ_0 *and* μ_0 represent Lame's constants, P_0 represents the intial stress, γ represents the shear modulus, ρ_0 represents the mass density, pressure represents the pressure, and μ_e^0 represents the magnetic permeability coefficient of the homogeneous material. The variable n is an arbitrary real number that represents the non-homogeneous exponent of the material. By substituting equation (5) into equations (3), we obtain the following results.

$$\sigma_{rr}=r^{2n}[\tau_m(\lambda_0+2\mu_0+P_0)\frac{\partial u_r}{\partial r}+2(\lambda_0+P_0)\tau_m\frac{u_r}{r}-\gamma_0(T+\tau_2 T)],$$

$$\sigma_{\theta\theta}=r^{2n}[2\tau_m(\lambda_0+\mu_0+P_0)\frac{u_r}{r}+(\lambda_0+P_0)\tau_m\frac{\partial u_r}{\partial r}-\gamma_0(T+\tau_2 T)],$$

$$\sigma_{\varphi\varphi}=r^{2n}[2\tau_m(\lambda_0+\mu_0)\frac{u_r}{r}+\lambda_0\tau_m\frac{\partial u_r}{\partial r}-\gamma_0(T+\tau_2 T)],$$

$$\sigma_{r\varphi}=\tau_{r\theta}=\tau_{\theta\varphi}=0$$

$$\sigma_{rr}^*=\mu_e^0 H_0^2\ r^{2n}(\frac{\partial u_r}{\partial r}+\frac{2u_r}{r}), \tag{7}$$

By substituting the expressions of σ_{rr}^* as the Maxwell stress tensor from equations (6) and (7) into equation (2), we obtain

$$[\tau_m+\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}]\frac{\partial^2 u_r}{\partial r^2}+[2(n+1)\tau_m+\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}]\times\frac{1}{r}\frac{\partial u_r}{\partial r}+[\frac{4n\lambda_0\tau_m}{(\lambda_0+2\mu_0+P_0)}-2\tau_m-\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}]\frac{u_r}{r^2}-[\frac{2n}{r}+\frac{\partial}{\partial r}]\frac{\gamma_0}{(\lambda_0+2\mu_0+P_0)}(T+\tau_2\dot{T})+\rho_0 r^{2n}\Omega^2 u=\frac{\rho_0}{(\lambda_0+2\mu_0+P_0)}\frac{\partial^2 u}{\partial t^2}. \tag{8}$$

Subsequently, the elastodynamic equation (7) can be reformulated as follows:

$$\left(\tau_m+\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}\right)\frac{\partial^2 u_r}{\partial r^2}+\left[2(n+1)\tau_m+\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}\right]\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{\gamma_0}{(\lambda_0+2\mu_0+P_0)}\times\left[\frac{2n}{r}+\frac{\partial}{\partial r}\right](T+\tau_2\dot{T})+\left[4nc_0\tau_m-2\tau_m-\frac{\mu_e^0 H_\phi^2}{(\lambda_0+2\mu_0+P_0)}\right]\frac{u_r}{r^2}+\rho_0\Omega^2 u_r=\frac{1}{c_v^2}\frac{\partial^2 u_r}{\partial t^2}. \tag{9}$$

Furthermore, the heat conduction equation is considered in the current study to

$$K(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r})=\rho_0 r^{2n}\sqrt{\frac{(\lambda_0+2\mu_0+P_0)}{\rho_0}}\times(\frac{\partial T}{\partial t}+\tau_1\frac{\partial^2 T}{\partial t^2})+\gamma_0 r^{2n}T_0[\frac{\partial}{\partial r}+\frac{2}{r}]u_r. \tag{10}$$

The following dimensionless quantities are employed in our analysis:

$$u_r'=\frac{u_r}{a}, l'=\frac{K}{\rho_0\,c_v}, t'=\frac{kc_v}{a}t, T'=\frac{T}{T_0}, \tau'_0=\frac{c_v}{a}\tau_0, \tau'_1=\frac{c_v}{a}\tau_1, \tau'_2=\frac{kc_v}{a}\tau_2, r'=\frac{r}{a}, \Omega'=\frac{\Omega}{a}$$

$$\sigma'_{rr}=\frac{\sigma_{rr}}{(\lambda_0+2\mu_0+P_0)}, \sigma'_{\theta\theta}=\frac{\sigma_{\theta\theta}}{(\lambda_0+2\mu_{0+P_0})}.$$

In the ensuing discourse, the prime symbols ('), denoting differentiation with respect to a particular variable, are omitted for brevity and clarity. The equations governing the normal stresses can be suitably formulated in dimensionless terms as presented below:

$$\sigma_{rr}=(ar)^{2n}[(1+\tau_0\frac{\partial}{\partial t})\frac{\partial u_r}{\partial r}+2c_1(1+\tau_0\frac{\partial}{\partial t})\frac{u_r}{r}-T_0c_2(1+\tau_2\frac{\partial}{\partial t})T]$$

$$\sigma_{\Theta\Theta}=(ar)^{2n}[2(1+\tau_0\frac{\partial}{\partial t})\frac{u_r}{r}+c_4(1+\tau_0\frac{\partial}{\partial t})\frac{\partial u_r}{\partial r}-T_0c_5(1+\tau_2\frac{\partial}{\partial t})T],$$

$$\sigma^*_{rr}=\mu_e^0H_\phi^2(ar)^{2n}\left(\frac{\partial u_r}{\partial r}+\frac{2u_r}{r}\right),$$

$$f_r=\frac{\mu_e^0H_\phi^2(ar)^{2n}}{a}\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{2}{r}\frac{\partial u_r}{\partial r}-\frac{2u_r}{r^2}\right), \quad (11)$$

where

$$c_0=\frac{\lambda_0}{(\lambda_0+2\mu_0+P_0)},\quad c_2=\frac{\gamma_0}{(\lambda_0+2\mu_0+P_0)},$$

$$c_3=\frac{\mu_e^0H_\phi^2}{(\lambda_0+2\mu_0+P_0)},\quad c_4=\frac{\lambda_0}{(\lambda_0+2\mu_0)},$$

$$c_5=\frac{\gamma_0}{(\lambda_0+2\mu_0)},\quad c_v=\sqrt{\frac{(\lambda_0+2\mu_0+P_0)}{\rho_0}}$$

By substituting equation (11) into equations (9) and (10), we obtain the non-dimensional form of the displacement equation for the non-homogeneous spherical medium as follows:

$$[(1+\tau_0\frac{\partial}{\partial t})+c_3]\frac{\partial^2 u_r}{\partial r^2}+[2(n+1)(1+\tau_0\frac{\partial}{\partial t})+c_3]\frac{1}{r}\frac{\partial u_r}{\partial r}+[(4nc_1-2)(1+\tau_0\frac{\partial}{\partial t})-c_3]\frac{u_r}{r^2}-c_2T_0\left[1+\tau_2\frac{\partial}{\partial t}\right]\times\left(\frac{2l}{r}+\frac{1}{c_L^2}\frac{\partial}{\partial r}\right)T+\rho_0ac_1^2\Omega^2u_r=l^2\frac{\partial^2 u_r}{\partial t^2}. \quad (12)$$

The heat conduction equation is expressed in non-dimensional forms as follows:

$$\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right)=\frac{a\sqrt{\frac{(\lambda_0+2\mu_0+P_0)}{\rho_0}}}{l}(1+\tau_1\frac{\partial}{\partial t})\frac{\partial T}{\partial t}+\frac{a\gamma_0}{\rho_0}[\frac{\partial}{\partial r}+\frac{2}{r}]\frac{\partial u_r}{\partial t}, \quad (13)$$

3 The Problem Solution

Our objective is to obtain the general solution for the fundamental equation (12) governing the harmonic vibration of magneto-thermo-elastic motion.

$$u_r(r,t)=U(r)e^{i\omega t}, \quad (14)$$

$$T(r,t)=T_1(r)e^{i\omega t}. \quad (15)$$

By substituting the values from equation (14) into the equation of motion, the resulting equation can be expressed as:

$$[(1+i\omega\tau_0)+c_3]\frac{d^2U}{dr^2}+[2(n+1)(1+i\omega\tau_0)+c_3]\frac{1}{r}\frac{dU}{dr}+[(4nc_1-2)(1+i\omega\tau_0)-c_3]\frac{U}{r^2}+\rho_0ac_1^2\Omega^2U=-k^2\omega^2U+c_2T_0\gamma'(\frac{2n}{r}+\frac{d}{dr})T_1, \quad (16)$$

where $\gamma'=(1+i\tau_2\omega)$. or in the form

$$\frac{d^2U}{dr^2}+\left(\frac{(2n+1)(1+i\omega\tau_0)}{(1+i\omega\tau_0)+c_3}+1\right)\frac{1}{r}\frac{dU}{dr}+\eta_2\frac{U}{r^2}+\rho_0ac_1^2\,\Omega^2U=-m_1^2U+\varepsilon(\frac{2n}{r}+\frac{d}{dr})T$$

Let

$$\eta_1=\frac{(2n+1)(1+i\omega\tau_0)}{(1+i\omega\tau_0)+c_3}+1,$$

$$\eta_2=\frac{(4nc_1-1)(1+i\omega\tau_0)}{(1+i\omega\tau_0)+c_3}-1,$$

$$\varepsilon=\frac{c_2T_0\gamma'}{(1+i\omega\tau_0)+c_3},\quad m_1^2=\frac{k^2\omega^2}{(1+i\omega\tau_0)+c_3},$$

$$c_1=\sqrt{\frac{\lambda+2\mu}{\rho_0}}$$

Furthermore, the heat conduction equation can be expressed as follows:

$$(\nabla^2+l_1(\omega^2\tau_1-i\omega))T_1=i\omega\, l_2[\frac{d}{dr}+\frac{2}{r}]U, \quad (17)$$

where $\nabla^2=\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}$ $\quad l_1=\frac{ac_v}{l},\quad l_2=\frac{a\gamma_0}{\rho_0}$

In order to obtain solutions for equations (16) and (17), we utilize the approach of

$$U(r)=\frac{d\xi(r)}{dr}, \quad (18)$$

$$\frac{d}{dr}\left[\frac{d^2\xi(r)}{dr^2}+\frac{\eta_1}{r}\frac{d\xi(r)}{dr}+\frac{\eta_2}{r^2}\xi(r)\right]+f_1\Omega^2\frac{d\xi(r)}{dr}=-m_1^2\frac{d\xi(r)}{dr}+\varepsilon\left(\frac{2n}{r}+\frac{d}{dr}\right)T_1. \quad (19)$$

By performing a comparison of the coefficient of the derivative with respect to r in equation (19), we are able to derive insightful conclusions.

$$\frac{d^2\xi(r)}{dr^2}+\frac{\eta_1}{r}\frac{d\xi(r)}{dr}+[\frac{\eta_2}{r^2}+m_1^2]\xi(r)=\varepsilon T_1. \quad (20)$$

The fundamental equation that governs the process of heat conduction can be mathematically represented as follows:

$$(\nabla^2+\gamma_1)T_1=i\omega\, l_2\nabla^2\xi(r). \tag{21}$$

Where $\gamma_1=l_1\left(\omega^2\tau_1-i\omega\right)$ By referring to equations (20) and (21), we can deduce

$$\frac{d^4\xi(r)}{dr^4}+[\eta_1+1]\frac{1}{r}\frac{d^3\xi(r)}{dr^3}+[\Gamma_1+\frac{\eta_2-\eta_1}{r^2}]\frac{d^2\xi(r)}{dr^2} +[\frac{\eta_2+\eta_1}{r^3}+\frac{\Gamma_2}{r}]\frac{d\xi(r)}{dr}+\gamma_1N^2\xi+f_1\Omega^2\frac{d\xi(r)}{dr}=0 \tag{22}$$

where

$$\Gamma_1=m_1^2+\gamma_1-\varepsilon\gamma_2,\ \Gamma_2=m_1^2+\gamma_1\eta_1-\varepsilon\gamma_2.,\ \gamma_2=i\omega\, l_2$$

By decoupling equations (20) and (21), we arrive at the following expressions:

$$(\nabla^2+{\chi_1}^2)(\nabla^2+{\chi_2}^2)(\xi,T_1)=0, \tag{23}$$

where $\gamma_4=\frac{\gamma_1}{l}$, χ_1^2 *and* χ_2^2 are the biquadratic equation possesses roots with positive real parts.

$$\chi^4+({m_1}^2+{\gamma_4}^2-\eta_1\eta_2)\chi^2+{m_1}^2{\gamma_4}^2=0. \tag{24}$$

By assuming the regularity conditions for ξ *and* T_1, the solutions of equation (23) can be derived using spherical Hankel's functions. Equation (23) represents a fourth-order ordinary differential equation with variable coefficients. Solving this equation allows us to determine the component of displacement, U(r), and the temperature, T, which in turn enables us to determine the components of stress

$$\xi(r)=K_1{h_0}^{(2)}(\chi_1 r)+K_2{h_0}^{(2)}(\chi_2 r).$$

$$U(r)=\frac{d}{dr}\left(K_1{h_0}^{(2)}(\chi_1 r)+K_2{h_0}^{(2)}(\chi_2 r)\right), \tag{25}$$

$$T_1=K_1{h_0}^{(2)}(\chi_1 r)+K_2{h_0}^{(2)}(\chi_2 r), \tag{26}$$

Here, K1 and K2 represent arbitrary constants, while ${h_0}^2$ denotes Hankel's function of order zero and second kind. By analyzing equations (14), (18), (16), (25) and (26), the solutions for displacement, temperature, radial stress, and hoop stress can be expressed as follows:

$$u_r=\left\{C_1{h_1}^{(2)}(\chi_1 r)+C_2{h_1}^{(2)}(\chi_2 r)\right\}e^{i\omega t'},$$

$$T=\left\{K_1{h_0}^{(2)}(\chi_1 r)+K_2{h_0}^{\ (2)}(\chi_2 r)\right\}e^{i\omega t'},$$

$$\sigma_{rr}=\left\{S_1{h_0}^{(2)}(\chi_1 r)+\frac{S_2}{r}{h_1}^{(2)}(\chi_1 r)\right\}C_1e^{i\omega t'} +\left\{S_3{h_0}^{(2)}(\chi_2 r)+\frac{S_2}{r}{h_1}^{(2)}(\chi_2 r)\right\}C_2e^{i\omega t'},$$

$$\sigma_{\theta\theta}=\left\{S_4{h_0}^{(2)}(\chi_1 r)+\frac{S_5}{r}{h_1}^{(2)}(\chi_1 r)\right\}C_1e^{i\omega t'} +\left\{S_6{h_0}^{(2)}(\chi_2 r)+\frac{S_5}{r}{h_1}^{(2)}(\chi_2 r)\right\}C_2e^{i\omega t'}.$$

4 Boundary Conditions

Let us denote the corresponding boundary conditions as follows: $u_r(1,t)=0,\ r=1\ \ \sigma_{rr}+\sigma_{rr}^*=-\sigma_0e^{i\omega t}\ ,\ r=1$ where σ_0 is a constant we get $z_i=\frac{{\chi_i}^2-{m_1}^2}{t'\chi_i}$, $K_i=z_iA_i,\quad i=1,2$

$$A_1=-\frac{\sigma'_0\,h_0{}^{(2)}(\chi_2)}{d_1},$$

$$A_2=\frac{\sigma'_0z_1h_0{}^{(2)}(\chi_1)}{d_1},$$

$$\sigma_0{}'=\frac{\sigma_0}{\gamma T_0}.$$

$$d_1=z_2{h_0}^{(2)}(\chi_2)\left\{S_1{h_0}^{(2)}(\chi_1)+S_2{h_1}^{(2)}(\chi_1)\right\} -z_1{h_0}^{(2)}(\chi_1)\left\{S_3{h_0}^{(2)}(\chi_2)+S_2{h_1}^{(2)}(\chi_2)\right\},$$

$$S_1=q\chi_1-q_1z_1,\qquad S_2=q(2\lambda_e-2),$$

$$S_3=q\chi_2-q_1z_2,\qquad S_4=q\chi_1-q_1z_1,$$

$$S_5=q(1-\lambda_e),\qquad S_6=\lambda_eq\chi_2-q_1z_2,$$

$${m_1}^2=\frac{k^2w^2}{qa^2c^2},\qquad \lambda_e=\frac{\lambda}{\lambda+2\mu+P},\qquad q=\left(1+i\omega{\tau_0}'+{\gamma_4}^2\right),$$

$$q_1=\left(1+i\omega{\tau_2}'\right),\qquad {\gamma_4}^2=\frac{\mu_e^0H_\phi^2}{\rho c^2},$$

This research paper introduces a novel solution addressing a pertinent issue concerning a non-homogeneous isotropic viscoelastic unbounded body containing a spherical cavity. Significantly, this solution takes into account the impact of crucial factors such as the magnetic field, rotation, and initial stress. Importantly, the findings of this study align with and build upon the outcomes presented in the preceding publication, providing further insights into the problem at hand.

5 Discussion and Numerical Results

The findings presented in this study are expected to be of great value to researchers in the fields of material science and low-temperature physics, as well as those involved in advancing the magneto-thermo-viscoelastic theory. Numerical evaluations are conducted using copper as the material of interest, with the relevant material constants provided [24]. The numerical method outlined in this research is employed to obtain radial displacement, radial stress, hoop stress, and temperature profiles within the material. The resulting distributions are depicted in Figures 1-4, respectively. Significant observations emerge from the computational analyses. For large time values, the coupled and generalized theories exhibit similar behavior. However, notable differences arise when considering small time intervals. In the case of the coupled theory, infinite wave propagation speeds are predicted. This is evident from the non-zero nature of the obtained solutions, which gradually diminish to very small values as one moves away from the surface. Conversely, solutions obtained using the generalized theory within the framework of the GL theory demonstrate finite wave propagation speeds. The computations are performed using specific parameter values, namely a thermal relaxation time $\tau_1 = 0.7$, a magnetic field strength $H_0 = 1x10^2$, a mechanical relaxation time $\tau_0 = 0.6$,, and a frequency $\omega = 2x10^2$. Due to brevity constraints, not all computational results are presented in this paper.

Figures 1-4 depict the solutions obtained for a non-homogeneous material (m=0.6) subjected to rotation, initial stress, thermal relaxation times, and magnetic field. Figure 1 showcases the temperature distribution subjected to rotation, thermal relaxation times, and magnetic field and the solution incorporating the effect of initial stress. In Figure 2, the depicted results exhibit the radial displacement within a generalized thermoelastic non-homogeneous medium under the influence of rotation, thermal relaxation times, and magnetic field. The figures unmistakably portray the occurrence of expansion deformation along the radius r as a consequence of these factors. It is worth noting that the radial displacement exhibits a direct correlation with both the initial stress and the radius r. Moving to Figure 3, it showcases the radial stress in the same generalized thermoelastic non-homogeneous medium, subjected to rotation, thermal relaxation times, and magnetic field. Interestingly, an intriguing observation can be made regarding the relationship between the radial stress and the initial stress when the radius r is smaller than 2. Specifically, the radial stress demonstrates a declining trend with increasing initial stress. Furthermore, Figure 3 also presents the solution incorporating the influence of the initial stress parameter H0.

Lastly, Figure 4 provides an insightful depiction of the hoop stress within the generalized thermoelastic non-homogeneous medium under the influence of rotation, thermal relaxation times, and magnetic field. It becomes apparent from this figure that the hoop stress exhibits a diminishing pattern as the radius r increases. Figure 4 depicts both the initial stress effect and the influence of the magnetic field H0 on the hoop stress. Notably, the generalized and coupled theories yield similar results near the surface cavity, while distinct solutions arise within the sphere. This discrepancy stems from the fact that thermal waves in the coupled theory exhibit non-zero behavior, albeit small, during small time intervals. Comparisons between these results and those in [24] indicate that the behavior of the variable u in both media is consistent. However, the values of u in the generalized thermoelastic medium are larger compared to those in the thermoelastic medium. The same observation applies to the radial stress σ_{rr} when comparing, the figures are done. This discrepancy can be attributed to the influence of relaxation time, magnetic field, and frequency.

The topic under consideration has been extensively explored in the existing literature, with numerous noteworthy investigations being conducted. Due to the limitations of space, we can only mention a select few recent and compelling studies, which can be found in the references provided [25,26,27,28,29,30,31,32,33]; the phenomena of rotation, viscoelasticity, initial stress, non-homogeneity, and magneto-thermo-elasticity are fundamental to the present study, contributing significantly to the overall comprehension of the behavior and response exhibited by the investigated system. Rotation: The investigation focuses on exploring the influence of rotation on the magneto-thermo-viscoelastic non-homogeneous medium featuring a spherical cavity. Rotation introduces intricacies that impact the distribution of displacements, temperature and stresses within the system, further enriching our understanding. Viscoelasticity: The study takes into account the viscoelastic nature of the medium, acknowledging its time-dependent response to applied loads. This characteristic plays a pivotal role in shaping the overall mechanical behavior and deformation patterns of the material under examination. Initial Stress: The presence of initial stress conditions is duly considered, recognizing their significance. Initial stress represents pre-existing internal forces within the medium, exerting a substantial influence on its response to external loading conditions. Non-homogeneity: The non-homogeneous nature of the medium forms a crucial aspect of the investigation. The spatial variations in material properties across the system give rise to complex variations, impacting the overall response and distribution of displacements, temperature, and stresses.

Magneto-thermo-elasticity: The study incorporates the coupled effects of magneto-thermo-elasticity, where the magnetic field, temperature, and mechanical response interact. This coupling enables a comprehensive analysis of the intricate interplay between these parameters and their combined impact on the behavior of the medium. Gaining a deep understanding of the interrelationships and influences of rotation, viscoelasticity, initial stress, non-homogeneity, and magneto-thermo-elasticity is crucial for comprehending the intricate behavior of the magneto-thermo-viscoelastic non-homogeneous medium and its response to periodic loading. By thoroughly investigating these factors, we can attain a more precise and comprehensive understanding of the system's behavior.

6 Concluding

The elastodynamic equations governing the generalized thermo-visco-elastic theory in the presence of non-homogeneous material, rotation, relaxation times, initial stress, and magnetic fields exhibit a complex nature. In this study, a highly effective methodology has been employed to tackle these intricate problems. Analytical expressions for the displacement, temperature, and stress components have been derived, employing a transformative approach based on Hankel's transform domain, which aligns with the governing equations of the investigated problem.

The obtained analytical solution offers precise results, and numerical computations have been carried out to evaluate and discuss the findings. Additionally, the results have been visually presented through informative graphical representations, enabling a comprehensive interpretation and analysis of the data.

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Conflicts of Interest:
The authors declare no conflict of interest.

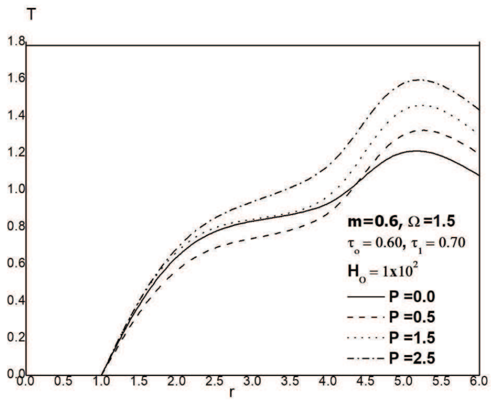

Fig. 1: illustrates the variation of temperature T with respect to the radius (r) at different intial stress values, with $\tau_1 = 0.7, \tau_0 = 0.6, w = 2\times^3, m = 0.6, h_0 = 1\times 10^2, \Omega = 1.5$

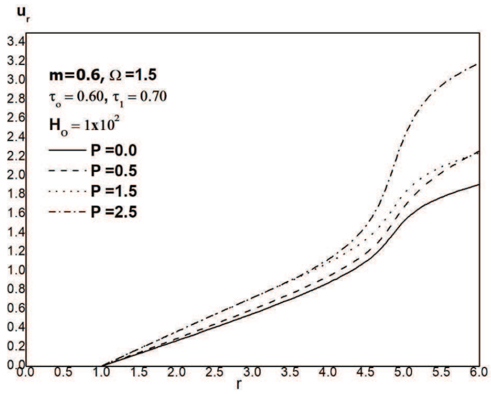

Fig. 2: illustrates the variation of radial displacement u_r with respect to the radius (r) at different intial stress values, with $\tau_1 = 0.7, \tau_0 = 0.6, w = 2\times^3, m = 0.6, h_0 = 1\times 10^2, \Omega = 1.5$

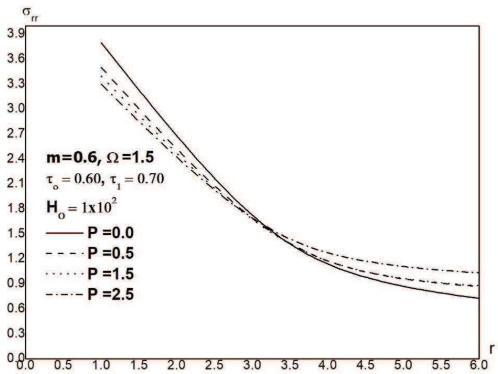

Fig. 3: illustrates the variation of radial stress with respect to the radius (r) at different intial stress values, with $\tau_1 = 0.7, \tau_0 = 0.6, w = 2\times^3, m = 0.6, h_0 = 1\times 10^2, \Omega = 1.5$

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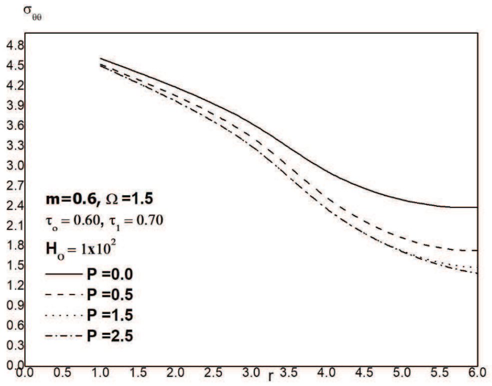

Fig. 4: illustrates the variation of hoop stress with respect to the radius (r) at different intial stress values, with $\tau_1 = 0.7, \tau_0 = 0.6, w = 2\times^3, m = 0.6, h_0 = 1\times10^2, \Omega = 1.5$

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Mohammed A. Balubaid received my MSc degree in 2002 from Warwick University, UK, and my PhD in 2007 from Manchester University, UK. He has been an Assistant professor at King Abdulaziz University since 2008. He has had several research papers published in reputed international journals and conferences. Among his research interests are Mathematical Engineering, Industrial Engineering, Mechanical Engineering, and numerical methods.