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Numerical treatment of the coupled fractional mKdV equations based on the Adomian decomposition technique

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Abstract: The present research implements the decomposition Adomian approach of the approximation solution for the nonlinear coupled modification Korteweg-De Vries (KdV) model in space time fractional order with appropriate initial values. This method yields a power series calculation for the solution. This process does not require linearization, the concept of weak nonlinear nature assumption, or perturbation theory. A mathematical software like Mathematica or Maple has been used to evaluate the Adomian formulas of the consequent series solution. This procedure might additionally be applied to resolve various types of fractional order nonlinear mathematical physics models. A graphic discussion is provided regarding the behavior of Adomian solutions and the varying changes in non integer order values and their effects. The approach is simple, clear and general enough to be used with other nonlinear fractional problems in mathematics and physics.

Keywords: Fractional nonlinear models, conformable fractional calculus, space time fractional coupled mKdV equation, Adomian decomposition method

1 Introduction

Recently, a growing number of nonlinear equations describing the solitary waves motion concentrated in a slight region of space have been proposed in numerous areas for example physics of plasma, hydrodynamics, optics, and so forth [1, 2]. It is fascinating and significant to look at the precise solutions to these nonlinear equations. The study of nonlinear equation solutions through a variety of techniques has been the focus of many authors over the last few decades [1-11]. These techniques include the Darboux and Backlund transformations, Inverse scattering technique, bilinear technique, the tanh procedure [9], the sine-cosine technique, and homogeneous balance procedure.

In several scientific areas such as physics, applied mathematics and engineering, the decomposition of Adomian approach [12– 14] has been used to solve an extensive variety of mechanistic and stochastic issues. Wazwaz [15] introduced the improved Adomian decomposition method directly, without requiring the formulas to be changed. The implementation of this approach ensures significant computation size savings furthermore offering the answer in a series that converges rapidly. The implementation of this enhanced Adomian decomposition method [13–16] has yielded reliable results, increasing its applicability in the management of assessment models.

A large variety of fractional order mechanistic or stochastic that are nonlinear or linear, partial or ordinary differential models have been demonstrated to be effectively, simply, and accurately solved using the decomposition approach, with approximations that converge quickly to precise solutions. These strategy is ideal for solving nonlinear physical models because it eliminates needless linearization, which can occasionally cause major perturbations. The current study aims to discuss the generated solutions by expanding the application of the decomposition Adomian approach for

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resolving fractional nonlinear models, for example, the coupled mKdV model of fractional order.

Non integer order calculus that is an extension of integer one, has been helpful in helping scientists define and model an extensive variety of phenomena within various engineering and scientific areas. The works that fall under this category include recent work on fluid flow [17], thermodynamics [18], optics [19, 20], signal processing [21], fractional kinetic equations [22, 23], and fractional differential equations (FDEs) [24, 25]. The literature commonly uses a few common approaches to find approximated or explicit solutions for fractional nonlinear differential models. For example, both of fractional linear and nonlinear diffusion and wave models can be resolved by means of the decomposition technique [26, 27]. The fractional power series convergence can be achieved through the application of differential transformation technique [28]. The space-fractional Burgers equations can be solved using the Variational iteration technique [29]. The fractional NNV system can be disentangled by means of the homotopy perturbation technique [30]. Ordinary fractional differential equations can be solved using the finite difference method [31]. while the perturbation-iteration algorithm (PIA) [32] can be applied to fractional differential equations.

Fractional and non-integer calculus, a generalization of past mathematical discoveries, will be the calculus of the twenty-first century. Recent developments and applications in fractional calculus are a fascinating and in-demand field of study. Several situations from actual life are accurately and precisely represented by fractional differential models. Numerous methods have been used to define and establish fractional differentials. Kolwankar-Gangal, Caputo, Chen's fractal defensibilities, Riemann-Liouville, modified Riemann-Liouville, and Cresson's are a few examples [33-37], Khalil et al. [38] recently obtainable the conformable fractional differential (CFD) in their study [38] based on restrictions like

$$D^{\alpha}M(s) = \lim_{\varepsilon\to 0}\frac{M(s+\varepsilon s^{1-\alpha})-M(s)}{\varepsilon}, \quad \forall s>0,\ \alpha\in(0,1], \tag{1}$$

$$M^{(\alpha)}(0) = \lim_{\varepsilon\to 0^{+}} M^{(\alpha)}(s). \tag{2}$$

When $\alpha = 1$ is inserted in the final equations, the non-integer differential changes into the well-known integer differential. The CFD met the aforementioned axioms:

$$D^{\alpha}s^{n} = n s^{n-\alpha},\quad D^{\alpha}a = 0,\ \forall M(s) = a, \tag{3}$$

$$D^{\alpha}(aM+bN) = aD^{\alpha}M + bD^{\alpha}N,\ \forall\ \ a,b\in R, \tag{4}$$

$$D^{\alpha}(MN) = MD^{\alpha}N + ND^{\alpha}M, \tag{5}$$

$$D^{\alpha}\left(\frac{N}{M}\right) = \frac{MD^{\alpha}N - ND^{\alpha}M,}{M^{2}}, \tag{6}$$

$$D^{\alpha}M(N) = \frac{dM}{dN}D^{\alpha}N,\quad D^{\alpha}M(s) = s^{1-\alpha}\frac{dM}{ds}, \tag{7}$$

where s and α are two random constants and M and N are two α-differentiable functions of a dependent variable. Reference [38] provides an illustration of relations (5) to (7).

2 Explanation of the procedure

In order to resolve the coupled nonlinear fractional mKdV model, we assume that the partial differential space-time fractional system is expressed in the operator formula as

$$\begin{aligned}&\ell_{\alpha_t}u + \ell_{\alpha_x}u + f(u,v) = 0,\\ &\ell_{\alpha_t}v + \ell_{\alpha_x}v + g(u,v) = 0,\end{aligned} \tag{8}$$

where the nonlinear operators are denoted by the symbolizations $f(u,v), g(u,v)$ and the linear conformable fractional differential operators by the representations $\ell_{\alpha_t} = D_t^{\alpha}$ and $\ell_{\alpha_x} = D_x^{\alpha\alpha\alpha}$. Using system (8) and the inverse conformable fractional differential operator $\ell_{\alpha_t}^{-1} = \int_0^t(.)dt^{\alpha}$, we have

$$\begin{aligned}u(t^{\alpha},x^{\alpha}) &= f_1(x^{\alpha}) - \ell_{\alpha_t}^{-1}[\ell_{\alpha_x}u + f(u,v)],\\ v(t^{\alpha},x^{\alpha}) &= g_1(x^{\alpha}) - \ell_{\alpha_t}^{-1}[\ell_{\alpha_x}v + g(u,v)],\end{aligned} \tag{9}$$

with $u(0,x^{\alpha}) = f_1(x^{\alpha})$ and $v(0,x^{\alpha}) = g_1(x^{\alpha})$ functions for the initial conditions have been given. The decomposition technique makes the assumption that there is an infinite series for unknown functions $u(t^{\alpha},x^{\alpha})$ and $v(t^{\alpha},x^{\alpha})$ in the form

$$\begin{aligned}u(t^{\alpha},x^{\alpha}) &= \textstyle\sum_{j=0}^{\infty} u_j(t^{\alpha},x^{\alpha}),\\ v(t^{\alpha},x^{\alpha}) &= \textstyle\sum_{j=0}^{\infty} v_j(t^{\alpha},x^{\alpha}),\end{aligned} \tag{10}$$

additionally to nonlinear operators, the infinite sequence of Adomian polynomials that gives the expression for $f(v,u)$ and $g(v,u)$ are

$$\begin{aligned}f(u,v) &= \textstyle\sum_{j=0}^{\infty} M_j,\\ g(u,v) &= \textstyle\sum_{j=0}^{\infty} N_j,\end{aligned} \tag{11}$$

where the relevant Adomian polynomials,M_j and N_j are produced using the procedure found in [13].

We use the nonlinear term $f(u,v) = \sum_{j=0}^{\infty} M_j$ of the general formulation for the Adomian polynomials M_j for the reader's convenience. These are described by

$$\begin{aligned}&M_j(u_0,...,u_j,v_0,...,v_j) = \frac{1}{j!}\frac{d^j}{d\lambda^j}\\ &\left[f\left(\textstyle\sum_{i=0}^{j}\lambda^i u_i, \sum_{i=0}^{j}\lambda^i v_i\right)\right]_{\lambda=0},\quad j>0,\end{aligned} \tag{12}$$

It is simple to use this formulation to tell the computer code to calculate as many polynomials as necessary for both the explicit and numerical solutions. We recommend the reader to [22,23] for a generic formula of Adomian polynomials and a full discussion of the Adomian decomposition technique.

The nonlinear system formula (8) is generated in an expression of the recursive relationship given by using the decomposition methodology

$$\begin{aligned} u_0(t^\alpha,x^\alpha) &= f_1(x^\alpha),\\ u_j(t^\alpha,x^\alpha) &= -\ell_{\alpha_t}^{-1}[\ell_{\alpha_x}u_j+M_j],\\ v_0(t^\alpha,x^\alpha) &= g_1(x^\alpha),\\ v_j(t^\alpha,x^\alpha) &= -\ell_{\alpha_t}^{-1}[\ell_{\alpha_x}v_j+N_j], \end{aligned} \tag{13}$$

when the initial conditions are in which functions $f_1(x^\alpha)$ and $g_1(x^\alpha)$ are derived. It is important to remember that the zeroth components, and, being more distinguish than the remainder components,$u_j(t^\alpha,x^\alpha)$ and $v_j(t^\alpha,x^\alpha)$ may be fully ascertained, meaning that every term can be computed by utilizing the terms that came before it.

Thus, the series solutions are fully established and components $u_0,u_1,u_2,...$ and $v_0,v_1,v_2,...$ are identified. Nonetheless, it is frequently possible to find the explicit solution in a closed form. We created the solutions $u(t^\alpha,x^\alpha)$ and $v(t^\alpha,x^\alpha)$ by means of numerical formulae in the form

$$\begin{aligned} u(t^\alpha,x^\alpha) &= \lim_{\varepsilon\to\infty}\textstyle\sum_{k=0}^{j}u_k(t^\alpha,x^\alpha),\\ v(t^\alpha,x^\alpha) &= \lim_{\varepsilon\to\infty}\textstyle\sum_{k=0}^{j}v_k(t^\alpha,x^\alpha), \end{aligned} \tag{14}$$

and the following is the recurring relationship: (13). Furthermore, in truly physical areas, the solutions of the decomposition series tend to converge relatively quickly. In the section that follows, we examine the space-time coupled fractional mKdV model to demonstrate the applicability of the previously discussed Adomian decomposition technique.

3 Application of the desired technique

Consider the following generalized space time coupled fractional mKdV model:

$$D_t^\alpha u+3u^2D_x^\alpha u-3D_x^\alpha(uv)=\varphi(t^\alpha,x^\alpha), \tag{15}$$

$$D_t^\alpha v-3u^2D_x^\alpha v-3vD_x^\alpha v-3D_x^\alpha uD_x^\alpha v=\psi(t^\alpha,x^\alpha), \tag{16}$$

under the initial condition

$$f(x^\alpha)=u(x^\alpha,0), \tag{17}$$

$$g(x^\alpha)=v(x^\alpha,0), \tag{18}$$

with $\varphi(t^\alpha,x^\alpha)$, $\psi(t^\alpha,x^\alpha)$, $f(x^\alpha)$ and $g(x^\alpha)$ are the given functions.

We redefine Equations (15) and (16) in an operator formulae in order to solve them using the Adomian decomposition approach.

$$\ell_{\alpha_t}u=\varphi(t^\alpha,x^\alpha)+3M(u,v)-3F(u,v), \tag{19}$$

$$\ell_{\alpha_t}v=\psi(t^\alpha,x^\alpha)-3[H(u,v)+G(u,v)+N(u,v)], \tag{20}$$

where the linear fractional differential operator is represented by $\ell_{\alpha_t}=D_t^\alpha$ and the inverse fractional operator $\ell_{\alpha_t}^{-1}$ is provided by

$$\ell_{\alpha_t}^{-1}=\int_0^t(.)dt^\alpha, \tag{21}$$

Operating with $\ell_{\alpha_t}^{-1}$ on the both sides of Eqs. (19) and (20), give

$$\begin{aligned} &u(t^\alpha,x^\alpha)=u(0,x^\alpha)\\ &+\ell_{\alpha_t}^{-1}\left[\varphi(t^\alpha,x^\alpha)+3M(u,v)-3F(u,v)\right], \end{aligned} \tag{22}$$

$$\begin{aligned} &v(t^\alpha,x^\alpha)=v(0,x^\alpha)\\ &+\ell_{\alpha_t}^{-1}\left[\psi(t^\alpha,x^\alpha)-3[H(u,v)+G(u,v)+N(u,v)]\right], \end{aligned} \tag{23}$$

The Adomian decomposition approach makes the assumption that an infinite series can be used to describe the two unknown functions, $u(t^\alpha,x^\alpha)$ and $v(t^\alpha,x^\alpha)$ as

$$u(t^\alpha,x^\alpha)=\sum_{j=0}^{\infty}u_j(t^\alpha,x^\alpha), \tag{24}$$

$$v(t^\alpha,x^\alpha)=\sum_{j=0}^{\infty}v_j(t^\alpha,x^\alpha), \tag{25}$$

Equations (22) and (23) can be substituted with Eqs. (24) and (25) to get

$$\begin{aligned} &u_{j+1}(t^\alpha,x^\alpha)=u(0,x^\alpha)\\ &+\ell_{\alpha_t}^{-1}\left[\varphi(t^\alpha,x^\alpha)+3M(u,v)-3F(u,v)\right], \end{aligned} \tag{26}$$

$$\begin{aligned} &v_{j+1}(t^\alpha,x^\alpha)=v(0,x^\alpha)\\ &+\ell_{\alpha_t}^{-1}\left[\psi(t^\alpha,x^\alpha)-3[H(u,v)+G(u,v)+N(u,v)]\right], \end{aligned} \tag{27}$$

where the functions $F(u,v)=u^2D_x^\alpha u$, $G(u,v)=vD_x^\alpha u$, $M(u,v)=D_x^\alpha(uv)$, $H(u,v)=D_x^\alpha uD_x^\alpha v$, and $N(u,v)=u^2D_x^\alpha v$, are associated with the nonlinear term and have the following expressions in terms of the Adomian polynomials: $F(u,v)=\sum_{j=0}^{\infty}F_j$, $G(u,v)=\sum_{j=0}^{\infty}G_j$, $M(u,v)=\sum_{j=0}^{\infty}M_j$, $H(u,v)=\sum_{j=0}^{\infty}H_j$, and $N(u,v)=\sum_{j=0}^{\infty}N_j$, where it is possible to compute the components F_j,G_j,M_j,H_j, and N_j using the formula

$$\begin{aligned} &F(u,v)=\textstyle\sum_{j=0}^{\infty}F_j=u^2D_x^\alpha u\\ &=(u_0+u_1\lambda+u_2\lambda^2+...)^2D_x^\alpha(u_0+u_1\lambda+u_2\lambda^2+...) \end{aligned} \tag{28}$$

Since

$$\begin{aligned} &F_j=\frac{1}{j!}\frac{d^j}{d\lambda^j}\\ &\left[(u_0+u_1\lambda+u_2\lambda^2+...)^2D_x^\alpha(u_0+u_1\lambda+u_2\lambda^2+...)\right]_{\lambda=0}, \end{aligned} \tag{29}$$

The first four terms can be written as

$$\begin{aligned} &F_0 = u_0^2 D_x^\alpha u_0, \\ &F_1 = u_0^2 D_x^\alpha u_1 + 2u_0 u_1 D_x^\alpha u_0, \\ &F_2 = (u_1^2 + 2u_0 u_2) D_x^\alpha u_0 + u_0^2 D_x^\alpha u_2 + 2u_0 u_1 D_x^\alpha u_1, \\ &F_3 = (u_1^2 + 2u_0 u_2) D_x^\alpha u_0 + u_0^2 D_x^\alpha u_3 + 2u_0 u_1 D_x^\alpha u_2 \\ &+2(u_0 u_3 + u_1 u_2) D_x^\alpha u_1, \end{aligned} \tag{30}$$

$$\begin{aligned} &G(u,v) = \sum_{j=0}^{\infty} G_j = v D_x^\alpha v \\ &= (v_0 + v_1\lambda + v_2\lambda^2 + ...) D_x^\alpha (v_0 + v_1\lambda + v_2\lambda^2 + ...) \end{aligned} \tag{31}$$

Since

$$\begin{aligned} &G_j = \frac{1}{j!}\frac{d^j}{d\lambda^j} \\ &\left[(v_0 + v_1\lambda + v_2\lambda^2 + ...) D_x^\alpha (v_0 + v_1\lambda + v_2\lambda^2 + ...)\right]_{\lambda=0}, \end{aligned} \tag{32}$$

The first four terms can be written as

$$\begin{aligned} &G_0 = 2v_0 D_x^\alpha v_0, \\ &G_1 = 2v_0 D_x^\alpha v_1 + 2v_1 D_x^\alpha v_0, \\ &G_2 = 2v_0 D_x^\alpha v_2 + 2v_1 D_x^\alpha v_1 + 2v_2 D_x^\alpha v_0, \\ &G_3 = 2v_0 D_x^\alpha v_3 + 2v_1 D_x^\alpha v_2 + 2v_2 D_x^\alpha v_1 + 2v_3 D_x^\alpha v_0, \end{aligned} \tag{33}$$

$$\begin{aligned} &M(u,v) = \sum_{j=0}^{\infty} M_j = D_x^\alpha (uv) \\ &= D_x^\alpha \left((u_0 + u_1\lambda + u_2\lambda^2 + ...)(v_0 + v_1\lambda + v_2\lambda^2 + ...)\right) \end{aligned} \tag{34}$$

Since

$$\begin{aligned} &M_j = \frac{1}{j!}\frac{d^j}{d\lambda^j} \\ &\left[D_x^\alpha \left((u_0 + u_1\lambda + u_2\lambda^2 + ...)(v_0 + v_1\lambda + v_2\lambda^2 + ...)\right)\right]_{\lambda=0}, \end{aligned} \tag{35}$$

The first four terms can be written as

$$\begin{aligned} &M_0 = u_0 D_x^\alpha v_0 + v_0 D_x^\alpha u_0, \\ &M_1 = u_0 D_x^\alpha v_1 + v_1 D_x^\alpha u_0 + v_0 D_x^\alpha u_1 + u_1 D_x^\alpha v_0, \\ &M_2 = u_0 D_x^\alpha v_2 + v_2 D_x^\alpha u_0 + u_1 D_x^\alpha v_1 + v_1 D_x^\alpha u_1 \\ &\quad + u_2 D_x^\alpha v_0 + v_0 D_x^\alpha u_2, \\ &M_3 = u_0 D_x^\alpha v_3 + v_3 D_x^\alpha u_0 + u_1 D_x^\alpha v_2 + v_2 D_x^\alpha u_1 \\ &\quad + u_2 D_x^\alpha v_1 + v_1 D_x^\alpha u_2 + u_3 D_x^\alpha v_0 + v_0 D_x^\alpha u_3, \end{aligned} \tag{36}$$

$$\begin{aligned} &H(u,v) = \sum_{j=0}^{\infty} H_j = D_x^\alpha u D_x^\alpha v \\ &= D_x^\alpha \left(u_0 + u_1\lambda + u_2\lambda^2 + ...\right) D_x^\alpha \left(v_0 + v_1\lambda + v_2\lambda^2 + ...\right) \end{aligned} \tag{37}$$

Since

$$\begin{aligned} &H_j = \frac{1}{j!}\frac{d^j}{d\lambda^j} \\ &\left[D_x^\alpha \left(u_0 + u_1\lambda + u_2\lambda^2 + ...\right) D_x^\alpha \left(v_0 + v_1\lambda + v_2\lambda^2 + ...\right)\right]_{\lambda=0}, \end{aligned} \tag{38}$$

The first four terms can be written as

$$\begin{aligned} &H_0 = D_x^\alpha u_0 D_x^\alpha v_0, \\ &H_1 = D_x^\alpha u_0 D_x^\alpha v_1 + D_x^\alpha u_1 D_x^\alpha v_0, \\ &H_2 = D_x^\alpha u_0 D_x^\alpha v_2 + D_x^\alpha u_1 D_x^\alpha v_1 + D_x^\alpha u_2 D_x^\alpha v_0, \\ &H_3 = D_x^\alpha u_0 D_x^\alpha v_3 + D_x^\alpha u_0 D_x^\alpha v_2 + D_x^\alpha u_2 D_x^\alpha v_1 \\ &+ D_x^\alpha u_3 D_x^\alpha v_0, \end{aligned} \tag{39}$$

$$\begin{aligned} &N(u,v) = \sum_{j=0}^{\infty} N_j = u^2 D_x^\alpha v \\ &= \left(u_0 + u_1\lambda + u_2\lambda^2 + ...\right)^2 D_x^\alpha \left(v_0 + v_1\lambda + v_2\lambda^2 + ...\right) \end{aligned} \tag{40}$$

Since

$$\begin{aligned} &N_j = \frac{1}{j!}\frac{d^j}{d\lambda^j} \\ &\left[\left(u_0 + u_1\lambda + u_2\lambda^2 + ...\right)^2 D_x^\alpha \left(v_0 + v_1\lambda + v_2\lambda^2 + ...\right)\right]_{\lambda=0}, \end{aligned} \tag{41}$$

The first four terms can be written as

$$\begin{aligned} &N_0 = u_0^2 D_x^\alpha v_0, \\ &N_1 = u_0^2 D_x^\alpha v_1 + 2u_0 u_1 D_x^\alpha v_0, \\ &N_2 = (u_1^2 + 2u_0 u_2) D_x^\alpha v_0 + u_0^2 D_x^\alpha v_2 + 2u_0 u_1 D_x^\alpha v_1, \\ &N_3 = (u_1^2 + 2u_0 u_2) D_x^\alpha v_0 + u_0^2 D_x^\alpha v_3 + 2u_0 u_1 D_x^\alpha v_2 \\ &+2(u_0 u_3 + u_1 u_2) D_x^\alpha v_1, \end{aligned} \tag{42}$$

For a given

$$\varphi(t^\alpha, x^\alpha) = \frac{1}{2} D_x^{\alpha\alpha\alpha} u + \frac{3}{2} D_x^{\alpha\alpha} v - 3a D_x^\alpha u, \tag{43}$$

$$\psi(t^\alpha, x^\alpha) = -D_x^{\alpha\alpha\alpha} v + 3a D_x^\alpha v, \tag{44}$$

$$u(0, x^\alpha) = f(x^\alpha) = \frac{b_1}{2k} + k \tanh\left(\frac{k x^\alpha}{\alpha}\right), \tag{45}$$

$$v(0, x^\alpha) = g(x^\alpha) = \frac{a}{2}\left(1 + \frac{k}{b_1}\right) + b_1 \tanh\left(\frac{k x^\alpha}{\alpha}\right), \tag{46}$$

the remainder components $u_j(t^\alpha, x^\alpha)$ and $v_j(t^\alpha, x^\alpha)$, $j > 0$ can be found by means of the recursive relations with constant values of $a = k = b_1 = 1$ in the following manner, keeping in mind the theoretical components of Eqs. (27) and (28).

$$u_0 = \frac{1}{2} + \tanh\left(\frac{x^\alpha}{\alpha}\right), \tag{47}$$

$$v_0 = 1 + \tanh\left(\frac{x^\alpha}{\alpha}\right), \tag{48}$$

$$u_1 = \frac{t^\alpha}{4\alpha}\left[-1 + \tanh^2\left(\frac{x^\alpha}{\alpha}\right)\right], \tag{49}$$

$$v_1 = \frac{t^\alpha}{4\alpha}\left[-1 + \tanh^2\left(\frac{x^\alpha}{\alpha}\right)\right], \tag{50}$$

$$u_2 = \left(\frac{t^\alpha}{4\alpha}\right)^2 \left[-\tanh\left(\frac{x^\alpha}{\alpha}\right) + \tanh^3\left(\frac{x^\alpha}{\alpha}\right)\right], \tag{51}$$

$$v_2 = \left(\frac{t^\alpha}{4\alpha}\right)^2 \left[-\tanh\left(\frac{x^\alpha}{\alpha}\right) + \tanh^3\left(\frac{x^\alpha}{\alpha}\right)\right], \tag{52}$$

and so forth. It is possible to fully determine the remainder components $u_j(t^\alpha, x^\alpha)$ and $v_j(t^\alpha, x^\alpha)$, $j > 0$ so that each phrase is determined using the preceding term. The decomposition of Adomian solutions of $u(t^\alpha, x^\alpha)$ and $v(t^\alpha, x^\alpha)$ are obtained in power series form

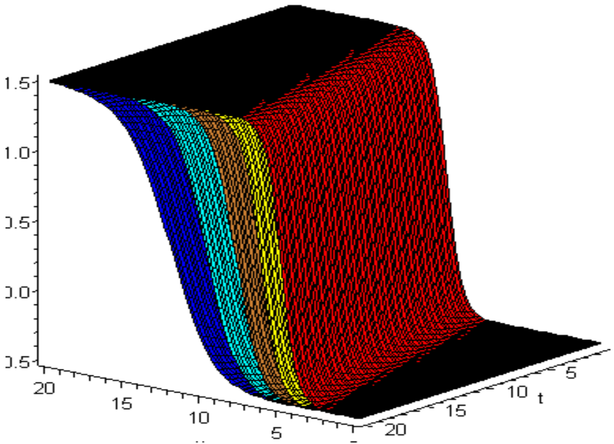

Fig. 1: The evolution behavior of the function u when $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ and $\delta = -10$ The layer red $\alpha = 1$, yellow $\alpha = 0.9$, gold $\alpha = 0.8$, cyan $\alpha = 0.7$, blue $\alpha = 0.6$

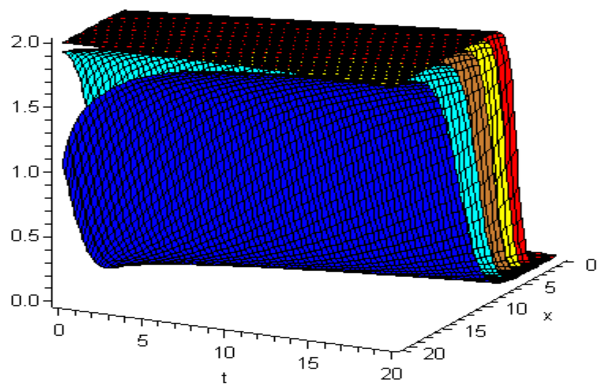

Fig. 2: The evolution behavior of the function v when $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ and $\delta = -10$ The layer red $\alpha = 1$, yellow $\alpha = 0.9$, gold $\alpha = 0.8$, cyan $\alpha = 0.7$, blue $\alpha = 0.6$

by substituting the expressions $v_0, v_1, v_2, \ldots$ and $u_0, u_1, u_2, \ldots$ into the summation $\sum_{j=0}^{\infty} u_j(t^\alpha, x^\alpha)$ and $\sum_{j=0}^{\infty} v_j(t^\alpha, x^\alpha)$

$$u(t^\alpha, x^\alpha) = \tfrac{1}{2} + \tanh\left(\tfrac{x^\alpha}{\alpha}\right) + \tfrac{t^\alpha}{4\alpha}\left[-1 + \tanh^2\left(\tfrac{x^\alpha}{\alpha}\right)\right] + \left(\tfrac{t^\alpha}{4\alpha}\right)^2\left[-\tanh\left(\tfrac{x^\alpha}{\alpha}\right) + \tanh^3\left(\tfrac{x^\alpha}{\alpha}\right)\right] + \ldots, \tag{53}$$

$$v(t^\alpha, x^\alpha) = 1 + \tanh\left(\tfrac{x^\alpha}{\alpha}\right) + \tfrac{t^\alpha}{4\alpha}\left[-1 + \tanh^2\left(\tfrac{x^\alpha}{\alpha}\right)\right] + \left(\tfrac{t^\alpha}{4\alpha}\right)^2\left[-\tanh\left(\tfrac{x^\alpha}{\alpha}\right) + \tanh^3\left(\tfrac{x^\alpha}{\alpha}\right)\right] + \ldots, \tag{54}$$

This is compactly expressed as

$$u(t^\alpha, x^\alpha) = \frac{1}{2} + \tanh\left(\delta + \frac{t^\alpha}{4\alpha} + \frac{x^\alpha}{\alpha}\right), \tag{55}$$

$$v(t^\alpha, x^\alpha) = 1 + \tanh\left(\delta + \frac{t^\alpha}{4\alpha} + \frac{x^\alpha}{\alpha}\right), \tag{56}$$

where δ is an arbitrary constant called the phase shift. The Adomian decomposition procedure's evolutionary behavior of the two solutions with varying fractional order values $\alpha = 1, 0.9, 0.8, 0.7$ and 0.6 with $\delta = -10$ as shown in Figs. 1 and 2.
Remark that all the results obtained in [39] are recovered when $\alpha = 1$.

4 Discussion and Summary

It has been discovered that non-classical calculus techniques, such as fractional calculus and non-integer order calculus, are helpful in explaining significant physical phenomena. This is partly because of the rapid development of advanced applied sciences. The use of the Adomian decomposition approach is discussed in this work, which also offers a possible analytical tool for investigating these models. The symbolic computation of the Adomian decomposition technique in non integer calculus, maple packages, additional numerical techniques derived from the Adomian decomposition technique, and other related studies are still to be taken into consideration. The Adomian decomposition approach may eventually become as important as classical calculus, according to the authors. In the future work, we want to create a software application for Matlab or Maple that solves fractional differential models by applying the Adomian decomposition technique.

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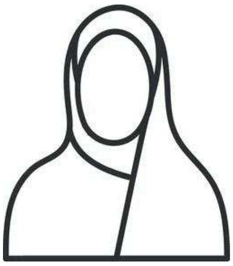

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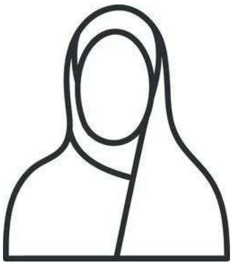

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