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Chaoticity of Low-Lying States of Even-Even Nuclei

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Abstract: The spacing distributions of 453 even-even nuclei are examined in this work. The range of nuclei considered spans from A = 22 to 250, with a minimum requirement of five unambiguous levels for each nucleus's spin-parity (J^{π}). The dataset is divided based on the spin-parity states, which range from 0 to 6 states. To assess the chaoticity parameter for each class, the Bayesian inference method is employed. The utilized model successfully interpolates from a Poisson (regular) to a Wigner (chaotic) distribution by varying the chaoticity parameter from 0 to 1 accordingly. Notably, regularity in the form of γ- and/or octupole-vibrations is observed for states 1+, 3+, and 1–. Conversely, other states exhibit an intermediate behavior that lies between the Wigner and Poisson distributions.

Keywords: nuclear chaos, random matrix theory, even-even nuclei.

1 Introduction

Classical mechanics and quantum mechanics are known to be integrable systems due to their reliance on differential equations, which allow for closed-range solutions. Once the initial conditions of these systems are known, their future behavior can be predicted with ease. Additionally, small perturbations in initial conditions do not affect the stability of solutions, despite slight changes in their time evolution. However, the majority of dynamical systems found in nature are not integrable. This is due to the difficulty in finding real solutions for systems involving forces and interactions, as well as the instability of these solutions when initial conditions are altered. As a result, such systems with unstable classical trajectories are referred to as chaotic. [1]

In the realm of chaotic systems, the progression of the system remains unaffected by its starting condition. It is observed that even a slight alteration in the initial state of the system can lead to utter chaos in its subsequent state. Consequently, attempting to accurately predict the future state of the system through calculations proves to be futile. Hence, it is advisable to approach chaotic systems from a statistical perspective by defining their statistical characteristics [2-6].

On the contrary, the uncertainty principle has led to a reevaluation of the significance of the path in quantum mechanics. Percival's observation, as cited in [7], regarding the semi-classical quantization rule (Bohr) and its relation to the bound state energy along the classical periodic orbit, which was further studied by Gutzwiller [8], suggests that the system can be classified as an integral or chaotic system through spectroscopy of its energy levels.

The random matrix theory (RMT) has proven to be a valuable tool in representing the energy levels of highly complex systems, such as atomic nuclei, through the eigenvalues of a matrix with a random distribution of its elements [9]. This theory has quickly gained popularity in the fields of physics and mathematics as a new form of statistical mechanics where the realization of the system is not significant, as noted by Dyson [10]. The efficiency of the random matrix lies in the fact that instead of having a set of states, we have a set of Hamiltonians. Therefore, ergodicity can be used instead of spectral averaging and the averaging over this ensemble.

Bohigas, Giannoni, and Schmit [11] proposed a conjecture based on the study of tile distribution of the eigenvalues in Sinai billiards. The conjecture suggests that the fluctuation characteristics of generic quantum systems, with or without time-reversal symmetry, that are fully chaotic in the classical limit coincide with those of the Gaussian orthogonal (unitary) ensemble, i.e. GOE (GUE). Studies of level spacing distributions and partial width distributions in nuclei are based on this approximation and generally recognize chaos in nuclei through the agreement with GOE statistics.

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The present manuscript endeavors to investigate the level of chaoticity present in even-even nuclei that adhere to specific regulations, while taking into account all their spin-parity states. The primary objective of this research is to determine the chaoticity parameter *f* through the application of Bayesian inference method on various classes of nuclei categorized based on their spin-parity states. This approach will enable us to gain insights into the structural development of the collective nuclear.

2 Data Set

National Nuclear Data Center [12] provides low-lying levels of even-even nuclei. First we considered nuclei ranging between A = 22 to 250 where spin-parity Jπ assignments of at least five sequential levels are unambiguous. Second if the spin-parity assignments were indeterminate and where the most possible value appeared in brackets, we recognized this value.

Sequence termination is due to arrival at a level with unassigned Jπ, or to an ambiguous assignment with spin-parity among several possibilities, e.g. $J^{\pi} = (2^{+}, 4^{+})$. However, an exception has been made when only one such level occurred and was trailed by numerous unambiguously assigned levels consisting at least two levels of the similar spin-parity, providing the ambiguous level in a similar spectrum position of a neighboring nucleus. Fortunately, less than 5% of the studied levels encounter this situation. In the following table (1) our data set is summarized.

Table 1. Data set classified according to their spin-parity states.

States	No. of nuclei	No. of energy levels
0^+	28	168
1^+	8	74
2^+	150	1132
3^+	9	82
4^+	61	434
5^+	3	25
6^+	6	35
8^+	1	5
1^-	6	63
2^-	3	29
3^-	25	188
4^-	6	42
5^-	7	51
6^-	5	30
7^-	1	6
All Parity +	167	1955
All parity –	31	409

3 Analysis methods

Normally, spectrum of unit mean level spacing is used in statistical studies using random matrix theory by appropriate theoretical expression of the number $N(E)$ When the levels of excitation energy are less than the energy E, which is called unfolding . Here we will use the theoretical formula for constant temperatures,

$$N(E) = N_0 + e^{\left(E-E_0/T\right)} \tag{1}$$

Where N_0 , E_0 and T acquired for each nucleus differ significantly with mass number. However, these parameters obviously show a propensity to inverse proportion with mass numbers variation. In this research, we will use the analysis method presented in the references Refs. [13, 14], and references there in.

Both space rotation and invariance under time reversal are used to characterize the nuclear states. This can be done by Gaussian orthogonal ensemble GOE of (RMT) Wigner's distribution [15], approximate nearest-neighbor-spacing distribution of levels of GOE.

$$P_w(s) = \frac{\pi}{2} s\, e^{\left(\frac{\pi}{4}s^2\right)} \tag{2}$$

Where s is the spacing of neighboring levels. It has mean level spacing units. Generally, Poisson distribution is often used to give nearest-neighbor-spacing distribution for integrable systems,

$$P_p(s) = e^{-s} \tag{3}$$

In seeking of finding appropriate analyzing in this work, for low-lying nuclear levels we use the method discussed in ref [16] to extract the chaoticity parameter(f). The mean value of chaoticity parameter $\bar{f}$ and its corresponding standard deviation σ of $P(f|s)$, are given by

$$\bar{f} = \int_0^1 f\, P(f|s)\mathrm{d}f, \text{ and } \sigma^2 = \int_0^1 \left(f - \bar{f}\right)^2 P(f|s)\mathrm{d}f \tag{4}$$

4 Results and Discussion

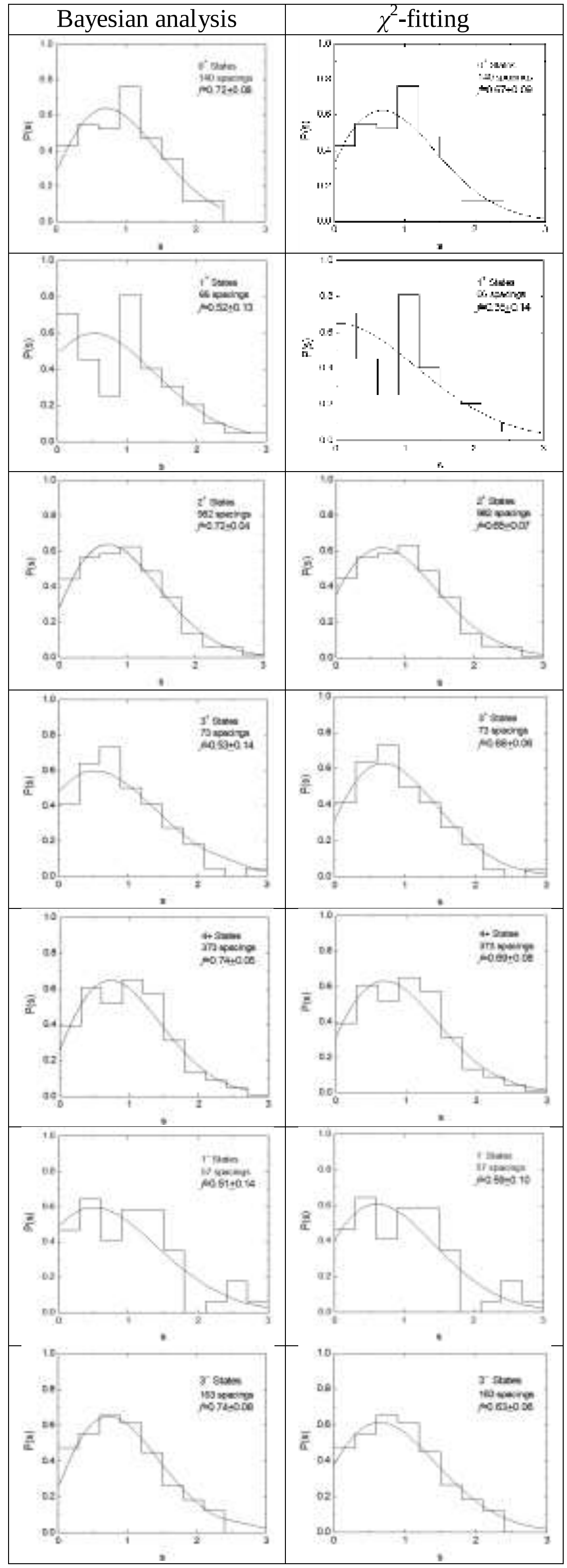
The nuclear shape is described by the collective model [17], which utilizes the parameters γ and β. According to this model, nuclear vibrations are identified by γ and β vibrations and oscillation. Despite the fixed values of γ and β, the stable shape undergoes rotation. This rotational motion gives the impression that the nucleons are engaged in collective oscillations. In the case of β vibrations, the spin projection of the phonon along the polar axis disappears, whereas for γ vibrations, it possesses a value. The energy of rotational states, which is influenced by these vibrations, can be determined using the given equation;

$$E = \frac{\hbar^2}{2\Phi}[J(J+1) - K^2], \quad (5)$$

where K is the angular momentum projection on the symmetry axis, J is angular momentum and Φ is the moment of inertia perpendicular to the symmetry axis. For β vibrations ($\lambda = 2$, $K = 0$) the values of J^π are 0^+, 2^+, 4^+, …; for γ vibrations ($\lambda = 2$, $K = 2$) the spin parity sequence is 2^+, 3^+ 4^+,…; for octupole vibrations ($\lambda = 3$, $K = 1$) the sequence is 1^-, 3^-, 5^-,…[18]. Table (2), and figure (1) summarize the results. an apparent regularity has been observed for states 1^+, 1^-, and 3^+. We may refer to the second state as octupole-vibrations, while the third state may be described as γ-vibrations respectively. The rest of the states show an intermediate behavior between Wigner and Poisson distribution as expected.

Table 2. Comparison between the Bayesian inference and χ^2-fitting methods in determining the chaoticity parameter f for each spin-parity state.

states	(f) by Bayesian analysis	(f) by χ^2-fitting	No. of spacings
0^+	0.72 ± 0.08	0.67 ± 0.09	140
1^+	0.52 ± 0.13	0.35 ± 0.14	66
2^+	0.72 ± 0.04	0.65 ± 0.07	982
3^+	0.53 ± 0.14	0.68 ± 0.07	73
4^+	0.74 ± 0.05	0.69 ± 0.08	373
5^+	--	--	22
6^+	--	--	29
8^+	--	--	4
1^-	0.51 ± 0.14	0.59 ± 0.10	57
2^-	--	--	26
3^-	0.74 ± 0.08	0.63 ± 0.06	163
4^-	--	--	36
5^-	--	--	44
6^-	--	--	25
7^-	--	--	5
All parity +	0.70 ± 0.03	0.64 ± 0.07	1689
All parity -	0.66 ± 0.06	0.60 ± 0.05	356



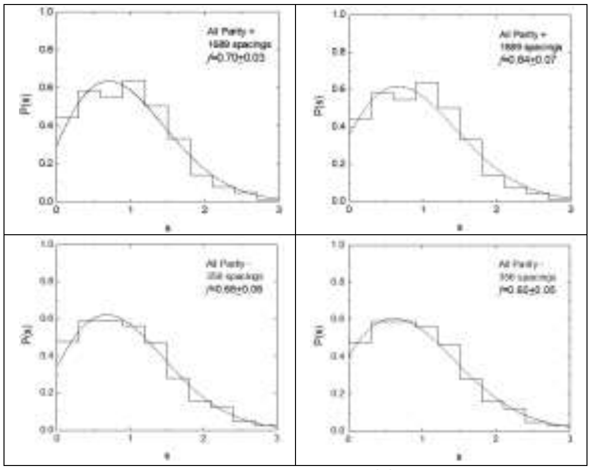


Fig. 1. The chaoticity parameter *f* for each spin-parity state of even-even nuclei.

5 Conclusions

The present manuscript investigates the disorderliness of each spin-parity state in even-even nuclei. Notably, states 1+, 1–, and 3+ exhibit a discernible pattern. The second state can be characterized as octupole-vibrations, whereas the third state can be described as γ-vibrations. Conversely, the remaining states display an intermediate behavior that lies between the Wigner and Poisson distributions, as anticipated. However, it is important to note that the statistical analysis conducted in this study was constrained, thereby preventing definitive conclusions from being drawn.

References

[1] M. L. Mehta, Random Matrices. Elsevier, 2014.
[2] H. A. Weidenmüller and G. E. Mitchell, “Random matrices and chaos in nuclear physics: Nuclear structure,” Reviews of Modern Physics, vol. 81, no. 2, pp. 539–589, May 2009.
[3] H. Sabri, S. K. M. Mobarakeh, A. J. Majarshin, Y.-A. Luo, and F. Pan, “Partial dynamical symmetry versus quasi dynamical symmetry examination within a quantum chaos analyses of spectral data for even–even nuclei,” Scientific Reports, vol. 11, no. 1, Aug. 2021.
[4] J. M. G. Gómez, E. Faleiro, L. Muñoz, R. A. Molina, and A. Relaño, “Shell-Model studies of chaos and statistical properties in nuclei,” Journal of Physics: Conference Series, vol. 580, p. 012045, Feb. 2015.
[5] L.-J. Wang, F.-Q. Chen, and Y. Sun, “Basis-dependent measures and analysis uncertainties in nuclear chaoticity,” Physics Letters B, vol. 808, p. 135676, Sep. 2020.
[6] L. Muñoz, R. A. Molina, and J. M. G. Gómez, “Chaos in nuclei: Theory and experiment,” Journal of Physics: Conference Series, vol. 1023, p. 012011, May 2018.
[7] I. C. Percival, “Regular and irregular spectra,” Journal of Physics B: Atomic and Molecular Physics, vol. 6, no. 9, pp. L229–L232, Sep. 1973.
[8] M. C. Gutzwiller, Chaos in Classical and Quantum Mechanics. Springer Science & Business Media, 2013.
[9] O. Bohigas and H. A. Weidenmuller, “Aspects of Chaos in Nuclear Physics,” Annual Review of Nuclear and Particle Science, vol. 38, no. 1, pp. 421–453, Dec. 1988.
[10] F. J. Dyson, “Statistical Theory of the Energy Levels of Complex Systems. I,” Journal of Mathematical Physics, vol. 3, no. 1, pp. 140–156, Jan. 1962.
[11] O. Bohigas, M. J. Giannoni, and C. Schmit, “Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws,” Physical Review Letters, vol. 52, no. 1, pp. 1–4, Jan. 1984.
[12] “NNDC | National Nuclear Data Center.” [Online]. Available: https://www.nndc.bnl.gov/.
[13] A. Y. Abul-Magd, H. L. Harney, M. H. Simbel, and H. A. Weidenmüller, “Statistics of 2+ levels in even–even nuclei,” Physics Letters B, vol. 579, no. 3–4, pp. 278–284, Jan. 2004.
[14]’ A. Y. Abul-Magd, H. L. Harney, M. H. Simbel, and H. A. Weidenmüller, “Statistical analysis of composite spectra,” Annals of Physics, vol. 321, no. 3, pp. 560–580, Mar. 2006.
[15] E. P. Wigner, "Conference on Neutron Physics by Time of Flight" , Oak Ridge National Lab. Report ORNL-2309, 1957.
[16] A. Y. Abul-Magd and M. H. Simbel, “Nearest-neighbor-spacing distribution of a system with many degrees of freedom, some regular and some chaotic,” Physical Review E, vol. 54, no. 4, pp. 3293–3299, Oct. 1996.
[17] A. Bohr, and B.R. Mottelson, Nuclear structure, Vol. II, Nuclear Deformations, Benjamin, New York, 1975.
[18] G. Friedlander, J.W. Kennedy, E.S. Macias, and J.M. Miller, Nuclear and Radiochemistry, John Wiley & Sons. Inc., 3rd Ed. 1981.