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Statistical Inference of Weighted Exponential Distribution Under Partially Constant-Stress Accelerated Life Tests with Type-II Censoring Scheme

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Abstract: This paper aims to adopt partially accelerated life tests model, under Type-II censoring scheme. The model parameters and the parameter of life are estimated by maximum likelihood, and Bayes methods. Approximate confidence interval under asymptotic distribution of maximum likelihood estimate, bootstrap confidence and Bayesian credible intervals are estimated. The developed model and its results are assessed and compared by Monte Carlo simulation study. Finally, a numerical discussion is considered in conclusion section.

Keywords: Weighted exponential distribution; Type-II censoring scheme; Partially accelerated life tests; Maximum likelihood estimation; Bootstrap confidence intervals; Bayes estimation.

1 Introduction

he weighted exponential distribution (WED) introduces a shape parameter for the exponential distribution [1]. It has more attractive and fascinating properties as compared with gamma, Weibull, or generalized exponential distribution for fitting the survival time data. The WED presents statistical model with non-constant failure rate function for modeling the lifetime data.

The random variable X has the two-parameter WED and its probability density function (PDF) is formulated as follows:

$$f(x) = \frac{\beta+1}{\beta}\theta e^{-\theta x}(1-e^{-\beta\theta x}),\ x>0,\ \beta,\ \theta>0, \quad (1)$$

where β and θ are shape and scale parameters respectively. It is noted that WED reduces to exponential distribution with parameter θ as $\beta \to \infty$. Also, it reduces to $\Gamma(2,\ \theta)$ when $\beta \to 0$. The WED is uni-modal at $\log(1+\beta)/\beta$. The random variable X with WED (1) has the cumulative distribution function (CDF) and the hazerd failure rate function given as follows:

$$F(x) = 1-\frac{1}{\beta}e^{-\theta x}(\beta+1-e^{-\beta\theta x}), \quad (2)$$

$$H(t) = (\beta+1)\theta\frac{1-e^{-\beta\theta t}}{\beta+1-e^{-\beta\theta t}}. \quad (3)$$

The failure rate function WED is in increasing for $0<t<\infty$. Different methods of estimation of WED and family of two-parameter weighted exponential distributions have been discussed in [2,3].

It is known that, if some units of a product are put under life testing experiment, the failure times of all units under the test composed a complete sample, and the failure times of some but not all units under test composed the censoring sample. In literature, censoring is available under several types, the commonly used types are called Type-I and Type-II censoring schemes (Type-I and Type-II CSs). For the experiment running in Type-I CS, the ideal test time is proposed prior and number of failure is taken at random, while in Type-II CS number of failure is proposed prior and the test time is taken at random. In both types, we don't have the flexibility of removing unit, other than the final stage. But this flexibility of removing unit is available in progressive censoring scheme, see [4].

In modern technology, the problem of statistical inference of a high reliable product is difficult, and obtaining a sufficient number of failures in a small period

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becomes more serious under normal stress level. Authors solved this problem by applying accelerated life test (ALT) model. For more details and applications of ALT models, see [5]. In literature, different schemes of ALT model are available. For example, constant-stress ALT model tests units input under constant stress until removing the test, see [6]. Another is the step-stress ALT model, in which stress level is changed with respect to constant period of time or number of failures, see [7] and [8]. Finally, the progressive-stress ALT model, in which the stress level continuously increases through the experiment, see [9] and [10]. Modeling some cases in ALT, we test some units under normal stress level and other units under stress level which is known by partially ALT model. In the mechanism of partially constant-stress ALT model, units are tested under normal stress, and stress conditions at the same time. But, in partially step-stress ALT model all units are put in normal stress conditions until constant period of time or number of failures, and the survival units are put under stress level.

The goal of this paper is to study the reliability of a product under population characteristics for given Type-II lifetime data of product with WED. We formulate the estimations of unknown quantities in the population. The material or engineering products which have WED are tested under partially constant ALT model. The model parameter and the corresponding parameters of life (reliability and hazer failure rate) under normal stress level are estimated. The proposed model and different estimation methods are tested under Monte Carlo simulation study and data analysis.

The outlines of paper are summarized as follows: Model formulation and its assumptions are discussed in Section 2. The model parameters are estimated with maximum likelihood and Bayes methods in Section 3. Interval estimation under asymptotic distribution of maximum likelihood estimates, bootstrap confidence interval and credible interval is discussed in Section 4. Theoretical results are assessed and compared through Monte Carlo simulation study in Section 5. The discussion of results is summarized in Section 6.

2 Model and Assumptions

Modeling under partially constant-stress ALT reflects the effect of environment on the life product. This effect can be described by hazard failure rate model $H_2(.) = \lambda H_1(.)$, and accelerated model in which the total lifetime under accelerated conditions is given by $Y = \frac{X}{\lambda}$, where $\lambda > 1$ denotes the accelerating factor. We adopt the accelerated model, the WED under stress conditions can be formulated by

$$F_2(y) = 1 - \frac{1}{\beta} e^{-\theta\lambda y}(\beta + 1 - e^{-\lambda\beta\theta y}),\qquad y > 0,\ \beta,\ \theta > 0, \lambda > 1, \tag{4}$$

and the corresponding PDF and hazard rate functions are given by

$$f_2(y) = \frac{\beta+1}{\beta}\lambda\theta e^{-\theta\lambda y}(1 - e^{-\beta\theta\lambda y}), \tag{5}$$

$$H_2(t) = (\beta+1)\lambda\theta\frac{1 - e^{-\beta\theta\lambda t}}{\beta + 1 - e^{-\lambda\beta\theta t}}. \tag{6}$$

Under partially constant-stress ALT model with Type-II censoring scheme, suppose a random sample of size n of a product is divided randomly into two samples of sizes n_1 and n_2, where $n = n_1 + n_2$. The sample of size n_1 is tested under normal stress conditions and the sample of size n_2 is tested under stress conditions. Prior the experiment is running, two integer m_1 and m_2 are proposed (numbers of failure which is needing under normal and stress conditions). The Type-II censoring sample under partially constant-stress ALT is denoted by $\mathbf{X} = \left(X_{j1;n},\ X_{j2;n},\ \ldots,\ X_{jm_j;n_j}\right),\ j = 1,\ 2$. If m_j-th failure times of the n_j units is originally have a continuous distribution with CDF given by $F_j(x)$ and PDF given by $f_j(x)$, then the joint likelihood function of observed Type-II censoring samples $\mathbf{x} = \left(x_{j1;n},\ x_{j2;n},\ \ldots,\ x_{jm_j;n_j}\right)$, $j = 1, 2$, is given by

$$f(\mathbf{x}|\Theta) = \prod_{j=1}^{2} C_j R_j^{n_j - m_j}(x_{jm_j;n_j}) \left(\prod_{i=1}^{m_j} f_j(x_{ji;n_j})\right), \tag{7}$$

where $C_j = \frac{n_j}{n_j - m_j}$, $R_j(.) = 1 - F_j(.)$ and $\Theta = \{\beta, \theta, \lambda\}$, λ is called the accelerated factor.

3 Point Estimation

The model is formulated for given Type-II censoring sample $\mathbf{x} = (x_{j1;n},\ x_{j2;n},\ \ldots,\ x_{jm_j;n_j})$, $j = 1,\ 2$, under partially constant-stress ALT. Also, the point estimate of the model parameters are discussed by maximum likelihood and Bayes approaches.

3.1 Maximum Likelihood Estimation (MLE)

Under observed Type-II censoring sample $\mathbf{x} = \left(x_{j1;n}, x_{j2;n},\ \ldots,\ x_{jm_j;n_j}\right)$, $j = 1,\ 2$, the likelihood function (7) with distribution given by (2) and (4) is formulated by

$$L(\Theta|x) \propto \left(\frac{1}{\beta}\right)^{n_1+n_2} (\beta+1)^{m_1+m_2} (\theta)^{m_1+m_2} \lambda^{m_2}$$
$$\exp\left\{-\theta\sum_{i=1}^{m_1} x_{1i} - \theta\lambda\sum_{i=1}^{m_2} x_{2i} + \sum_{i=1}^{m_1}\log\left[1-e^{-\beta\theta x_{1i}}\right]\right.$$
$$+\sum_{i=1}^{m_2}\log\left[1-e^{-\beta\theta\lambda x_{2i}}\right] - (n_1-m_1)\,\theta x_{1m_1}$$
$$-(n_2-m_2)\,\theta\lambda x_{2m_2} + (n_1-m_1)\log\left[\beta+1-e^{-\beta\theta x_{1m_1}}\right]$$
$$\left.+(n_2-m_2)\log\left[\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right]\right\}. \quad (8)$$

The joint likelihood function (8), by taking the natural logarithm is reduced to

$$\ell(\Theta|x) = -(n_1+n_2)\log\beta + (m_1+m_2)\log[\beta+1]$$
$$+(m_1+m_2)\log\theta + m_2\log\lambda - \theta\sum_{i=1}^{m_1} x_{1i} - \theta\lambda\sum_{i=1}^{m_2} x_{2i}$$
$$+\sum_{i=1}^{m_1}\log\left[1-e^{-\beta\theta x_{1i}}\right] + \sum_{i=1}^{m_2}\log\left[1-e^{-\beta\theta\lambda x_{2i}}\right] - \theta x_{1m_1}$$
$$-\theta\lambda x_{1m_2} + (n_1-m_1)\log\left[\beta+1-e^{-\beta\theta x_{1m_2}}\right]$$
$$+(n_2-m_2)\log\left[\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right]. \quad (9)$$

The estimate value of the parameters vector which maximize the log-likelihood function (9) can obtain by the first partially derivatives (likelihood equations) as follows:

$$\frac{\partial\ell(\Theta|\mathbf{x})}{\partial\beta} = -\frac{(n_1+n_2)}{\beta} + \frac{(m_1+m_2)}{\beta+1} + \theta\sum_{i=1}^{m_1}\frac{x_{1i}e^{-\beta\theta x_{1i}}}{1-e^{-\beta\theta x_{1i}}}$$
$$+\theta\lambda\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta x_{2i}}} + \frac{(n_1-m_1)(1+\theta x_{1m_2}e^{-\beta\theta x_{1m_2}})}{\beta+1-e^{-\beta\theta x_{1m_2}}}$$
$$+\frac{(n_2-m_2)(1+\lambda\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}})}{\beta+1-e^{-\beta\lambda\theta x_{2m_2}}} = 0, \quad (10)$$

$$\frac{\partial\ell(\Theta|\mathbf{x})}{\partial\theta} = \frac{(m_1+m_2)}{\theta} - \sum_{i=1}^{m_1} x_{1i} - \lambda\sum_{i=1}^{m_2} x_{2i} + \beta\sum_{i=1}^{m_1}\frac{x_{1i}e^{-\beta\theta x_{1i}}}{1-e^{-\beta\theta x_{1i}}}$$
$$+\beta\lambda\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta x_{2i}}} - x_{1m_1} - \lambda x_{1m_2} + \frac{(n_1-m_1)\,\beta x_{1m_2}e^{-\beta\theta x_{1m_2}}}{\beta+1-e^{-\beta\theta x_{1m_2}}}$$
$$+\frac{(n_2-m_2)\,\beta\lambda x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}}{\beta+1-e^{-\beta\lambda\theta x_{2m_2}}} = 0, \quad (11)$$

and

$$\frac{\partial\ell(\Theta|\mathbf{x})}{\partial\lambda} = \frac{m_2}{\lambda} + \theta\sum_{i=1}^{m_2} x_{2i} + \beta\theta\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta\lambda x_{2i}}}$$
$$-\theta\lambda x_{1m_2} + \frac{(n_2-m_2)\,\beta\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}}{\beta+1-e^{-\beta\lambda\theta x_{2m_2}}} = 0. \quad (12)$$

Equations (10) to (12) show MLE of the model, the parameters are obtained by solving three non-linear equations. Newton Raphson method can be employed as iteration method to solve this problem. Also, MLE of the reliability and failure rate function are given by

$$\hat{R}(t) = \frac{1}{\hat{\beta}}e^{-\hat{\theta}x}(\beta+1-e^{-\hat{\beta}\hat{\theta}x}),$$

and

$$\hat{H}(t) = (\hat{\beta}+1)\hat{\theta}\frac{1-e^{-\hat{\beta}\hat{\theta}x}}{\hat{\beta}+1-e^{-\hat{\beta}\hat{\theta}x}}. \quad (13)$$

3.2 Bayesian Estimation

In this section, we adopt Bayesian approach for obtaining the Bayes estimators of the model, parameters and the corresponding credible intervals. The parameters β and θ of WED have independent gamma prior density and non-informative prior distribution of accelerated parameter, the posterior distribution is formulated. Hence, the Bayes estimators of unknown model parameters under squared error loss (SEL) function are formulated for point and symmetric credible intervals as follows:

Suppose the prior information is formulated as

$$P^*(\Theta) = P_1^*(\beta)P_2^*(\theta)P_3^*(\lambda), \quad (14)$$

where

$$P_1^*(\beta) \propto \beta^{a-1}\exp(-b\beta),\ \beta>0;\ a,b>0, \quad (15)$$

$$P_2^*(\theta) \propto \theta^{c-1}\exp(-d\theta),\ \theta>0;\ c,d>0, \quad (16)$$

and

$$P_3^*(\lambda) \propto \lambda^{-1}. \quad (17)$$

The joint posterior distribution can be formulated by

$$P(\Theta) = \frac{P_1^*(\beta)P_2^*(\theta)P_3^*(\lambda)L(\Theta|\mathbf{x})}{\iiint_{\Theta} P_1^*(\beta)P_2^*(\theta)P_3^*(\lambda)L(\Theta|\mathbf{x})\,d\beta\, d\theta\, d\lambda}. \quad (18)$$

From the likelihood function (8) and joint prior distribution (14), the joint posterior distribution can be formulated by

$$P(\Theta|\mathbf{x}) \propto \beta^{a-n_1-n_2-1}(\beta+1)^{m_1+m_2}\theta^{c+m_1+m_2-1}\lambda^{m_2-1}$$
$$\times\exp\left\{-b\beta - d\theta + \theta\sum_{i=1}^{m_1} x_{1i} + \theta\lambda\sum_{i=1}^{m_2} x_{2i}\right.$$
$$+\sum_{i=1}^{m_1}\log\left[1-e^{-\beta\theta x_{1i}}\right] + \sum_{i=1}^{m_2}\log\left[1-e^{-\beta\theta\lambda x_{2i}}\right]$$
$$-\theta x_{1m_1} - \theta\lambda x_{2m_2} + (n_1-m_1)\log\left[\beta+1-e^{-\beta\theta x_{1m_1}}\right]$$
$$\left.+(n_2-m_2)\log\left[\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right]\right\}. \quad (19)$$

Based on posterior distribution (19), the Bayes estimate of the model parameters depends on selection of loss function. In this section, we adopt squared error loss function and hence, the Bayes estimate is taken to be posterior mean. Also, the posterior distribution (19) has shown that, the analytical solution can not be obtained. Therefore, different methods can be employed for the problem, such as numerical integration, Lindelly approximation and Markov Chen Monte Carlo method (MCMC). The empirical posterior distribution can be obtained by using the MCMC method as follows:

Baysian approach under MCMC

From the posterior distribution (19), we formulate the full conditional posterior distribution as follows:

$$P(\beta|\theta,\lambda,\mathbf{x}) \propto \beta^{a-n_1-n_2-1}(\beta+1)^{m_1+m_2}\exp\{-b\beta\}, \tag{20}$$

$$P_1(\theta|\beta,\lambda,\mathbf{x}) \propto \theta^{c+m_1+m_2-1}\exp\left\{-\theta\left(d+\sum_{i=1}^{m_1}x_{1i}+x_{1m_1}\right)\right\} \tag{21}$$

$$P(\lambda|\beta,\theta,\mathbf{x}) \propto \lambda^{m_2-1}\exp\left\{-\lambda\left(\theta\sum_{i=1}^{m_2}x_{2i}+\theta x_{2m_2}\right)\right\}, \tag{22}$$

and

$$h(\beta,\theta,\lambda,\mathbf{x}) \propto \left(d-\sum_{i=1}^{m_1}x_{1i}+x_{1m_1}\right)^{-(c+m_1+m_2 0}\left(-\theta\sum_{i=1}^{m_2}x_{2i}+\theta x_{2m_2}\right)^{-m_2}$$
$$\times\exp\left\{\sum_{i=1}^{m_1}\log\left[1-e^{-\beta\theta x_{1i}}\right]+\sum_{i=1}^{m_2}\log\left[1-e^{-\beta\theta\lambda x_{2i}}\right]\right.$$
$$+(n_1-m_1)\log\left[\beta+1-e^{-\beta\theta x_{1m_1}}\right])$$
$$\left.+(n_2-m_2)\log\left[\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right]\right\}. \tag{23}$$

The full conditional distribution reduce to two gamma function, proper function of β and general function of the model parameters. The function $h(\beta,\theta,\lambda,\mathbf{x})$ is called the weight function. Hence, for generating a sample from distributions, we apply Metropolis–Hastings (MH) method, [11] with Gaussian proposal distribution. The description of the algorithm is used to generate MCMC sampling described as follows:

Algorithm 2 (Importance sample algorithms)

1.Begin with intial values $\Theta^{(0)}=\{\beta^{(0)},\,\theta^{(0)},\,\lambda^{(0)}\}$=$\{\hat{\beta},\,\hat{\theta},\,\hat{\lambda}\}$ and put $\kappa=1$
2.Generate $\theta^{(\kappa)}$ from gamma density (37).
3.Generate $\lambda^{(\kappa)}$ from gamma density (38).
4.Generate $\beta^{(\kappa)}$ for (36) with MH algorithms with normal proposal distribution.
5.Compute the value $h(\beta^{(\kappa)},\theta^{(\kappa)},\lambda^{(\kappa)}|\mathbf{x})$.
6.Set $\kappa=\kappa+1$.
7.Steps from (2) to (6) are repeted N times.
8.The Bayes estimate of any function $g(\beta,\,\theta,\,\lambda)$ under a SEL function is defined by

$$\tilde{g}_B(\beta,\,\theta,\,\lambda)=\frac{\frac{1}{N-M}\sum_{i=M+1}^{N}g\left(\beta^{(i)},\,\theta^{(i)},\,\lambda^{(i)}\right)h(\beta^{(i)},\theta^{(i)},\lambda^{(i)}|\mathbf{x})}{\frac{1}{N-M}\sum_{i=M+1}^{N}h(\beta^{(i)},\theta^{(i)},\lambda^{(i)}|\mathbf{x})}, \tag{24}$$

where M is the number of iteration needing to reach stationary distribution.
9.Also the posterior variance of $g(\beta_1,\,\beta_2,\,\theta)$ is calculated by

$$V(g(\beta,\,\theta,\,\lambda))=\frac{\frac{1}{N-M}\sum_{i=M+1}^{N}(g\left(\beta^{(i)},\,\theta^{(i)},\,\lambda^{(i)}\right)-\tilde{g}_B)^2h(\beta^{(i)},\,\theta^{(i)},\,\lambda^{(i)}|\mathbf{X})}{\frac{1}{N-M}\sum_{i=M+1}^{N}h(\beta^{(i)},\,\theta^{(i)},\,\lambda^{(i)}|\mathbf{X})}. \tag{25}$$

4 Interval Estimation

In this section, we adopt asymptotic confidence interval, bootstrap confidence interval and Bayes credible intervals.

4.1 Approximate confidence intervals

The estimators of confidence intervals of the model parameters need to compute the Fisher information matrix. For this we require to compute the minus expectation of the second derivative of the log-likelihood function. But, in several cases this expectation is more serious to compute under high vector of parameters. The natural alternative approximate information matrix is defined by $i(\Theta)$ as follows:

$$i(\Theta)=-\left(\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\Theta_i\partial\Theta_j}\right),\ i.j=1,2,3. \tag{26}$$

The approximate information matrix at the MLE values of the parameters is denoted by $i^0(\hat{\Theta})$ and defined by

$$i^0(\hat{\Theta})=-\left(\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\Theta_i\partial\Theta_j}\right)|_{\hat{\Theta}},\ i.j=1,2,3. \tag{27}$$

The second derivatives of the log-likelihood function are given by

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\beta^2}=\frac{(n_1+n_2)}{\beta^2}-\frac{(m_1+m_2)}{(\beta+1)^2}-\theta^2\sum_{i=1}^{m_1}\frac{x_{1i}^2e^{-\beta\theta x_{1i}}}{\left(1-e^{-\beta\theta x_{1i}}\right)^2}-\theta^2\lambda^2\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta x_{2i}}\right)^2}-\frac{(n_1-m_1)\left(1+(2+(\beta+1)\theta x_{1m_2})\theta x_{1m_1}e^{-\beta\theta x_{1m_1}}\right)}{\left(\beta+1-e^{-\beta\theta x_{1m_1}}\right)^2}-\frac{(n_2-m_2)\left(1+(2+(\beta+1)\lambda\theta x_{1m_2})\lambda\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}\right)}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}, \tag{28}$$

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\theta^2}=-\frac{(m_1+m_2)}{\theta^2}-\beta^2\sum_{i=1}^{m_1}\frac{x_{1i}^2e^{-\beta\theta x_{1i}}}{\left(1-e^{-\beta\theta x_{1i}}\right)^2}-\beta^2\lambda^2\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta x_{2i}}\right)^2}-\frac{(n_1-m_1)\beta^2x_{1m_2}^2e^{-\beta\theta x_{1m_2}}}{\left(\beta+1-e^{-\beta\theta x_{1m_2}}\right)^2}-\frac{(n_2-m_2)\beta^2\lambda^2x_{1m_2}^2e^{-\beta\lambda\theta x_{2m_2}}}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}, \tag{29}$$

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\lambda^2}=-\frac{m_2}{\lambda^2}-\beta^2\theta^2\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta\lambda x_{2i}}\right)^2}-\theta x_{2m_2}-\frac{(n_2-m_2)\beta^2\theta^2x_{2m_2}^2e^{-\beta\lambda\theta x_{2m_2}}}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}, \tag{30}$$

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\theta\partial\beta}=\sum_{i=1}^{m_1}\frac{x_{1i}e^{-\beta\theta x_{1i}}}{1-e^{-\beta\theta x_{1i}}}-\beta\theta\sum_{i=1}^{m_1}\frac{x_{1i}^2e^{-\beta\theta x_{1i}}}{\left(1-e^{-\beta\theta x_{1i}}\right)^2}+\lambda\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta x_{2i}}}-\beta\theta\lambda\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta x_{2i}}\right)^2}+\frac{(n_1-m_1)\beta x_{1m_2}e^{-\beta\theta x_{1m_2}}}{\beta+1-e^{-\beta\theta x_{1m_2}}}+\frac{(n_2-m_2)\beta\lambda x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}}{\beta+1-e^{-\beta\lambda\theta x_{2m_2}}}, \tag{31}$$

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\theta\partial\lambda}=\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\lambda\partial\theta}=-\sum_{i=1}^{m_2}x_{2i}+\beta\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta x_{2i}}}+(\beta\lambda)^2\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta x_{2i}}\right)^2}-x_{1m_2}+\frac{(n_2-m_2)\beta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}}{\beta+1-e^{-\beta\lambda\theta x_{2m_2}}}+\frac{(n_2-m_2)(\beta\lambda x_{1m_2})^2e^{-\beta\lambda\theta x_{2m_2}}}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}, \tag{32}$$

$$\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\beta\partial\lambda}=\frac{\partial^2\ell(\Theta|\mathbf{x})}{\partial\lambda\partial\beta}=\theta\sum_{i=1}^{m_2}\frac{x_{2i}e^{-\beta\theta\lambda x_{2i}}}{1-e^{-\beta\theta x_{2i}}}+(\theta\lambda)^2\sum_{i=1}^{m_2}\frac{x_{2i}^2e^{-\beta\theta\lambda x_{2i}}}{\left(1-e^{-\beta\theta x_{2i}}\right)^2}+\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)\times\frac{(n_2-m_2)\left(\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}-(\lambda\theta x_{1m_2})^2e^{-\beta\lambda\theta x_{2m_2}}\right)}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}-\frac{(n_2-m_2)\left(1+\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}\right)\left(\theta x_{1m_2}e^{-\beta\lambda\theta x_{2m_2}}\right)}{\left(\beta+1-e^{-\beta\lambda\theta x_{2m_2}}\right)^2}. \tag{33}$$

Under the properties of MLE model parameters are distributed with bibariate normal distribution. The $(1-2\gamma)100\%$ approximate confidence intervals are formulated by

$$\begin{cases}\hat{\beta}\mp Z_\gamma\varepsilon_{11}\\ \hat{\theta}\mp Z_\gamma\varepsilon_{22}\\ \hat{\lambda}\mp Z_\gamma\varepsilon_{33},\end{cases} \tag{34}$$

where, Z_γ is standard normal probability with right tailed γ. Also, ε_{11}, ε_{22} and ε_{33} are the elements of diagonal approximate information matrix.

In different cases the lower bound of the confidence intervals may be less than 0, which contradicts with the prerequisite $\beta,\ \theta,\ \lambda>0$. Log-transformation and delta method are used in order to avoid this situation.

The pivotal $\Phi=\frac{\log\hat{\Theta}_i-\log\Theta_i}{\mathrm{Var}(\log\hat{\Theta}_i)}$ has standard normal distribution. The 100(1-2γ)% approximate confidence interval of $\Theta=\{\beta,\theta,\lambda\}$ is given by

$$\left(\frac{\hat{\Theta}_i}{\exp\left(\gamma_\alpha\sqrt{\mathrm{Var}(\log\hat{\Theta}_i)}\right)},\hat{\Theta}_i\exp\left(\gamma_\alpha\sqrt{\mathrm{Var}(\log\hat{\Theta}_i)}\right)\right),\quad i=1,2,3, \tag{35}$$

where $\mathrm{Var}(\log\hat{\Theta}_i)=\frac{\mathrm{Var}(\hat{\Theta}_i)}{\hat{\Theta}_i}$ and $i=1,2,3$. For more details, see [12,13].

4.2 Bootstrap confidence intervals

Bootstrap techniques are not only used in parameter ostentation problems but also to estimate bias and variance of estimator or calibrate hypothesis tests. Bootstrap techniques are described as resembling methods. The bootstrap techniques are defined in parametric and non-parametric methods, see [14,15]. Here we adopted parametric bootstrap technique to built two different confidence intervals. In literature the parametric bootstrap technique, percentile bootstrap technique and bootstrap-t technique can be found in [16, 18]. The following algorithm is employed to present percentile bootstrap technique for formulation bootstrap confidence intervals:

Algorithm 1 (Bootstrap confidence interval)

1.From the original Type-II GHCS sample $\mathbf{x} = \left(x_{j1;n},\, x_{j2;n},\, \ldots,\, x_{jm_j;n_j}\right),\ j = 1,\ 2,$ the MLEs $\Theta = \{\hat{\beta}, \hat{\theta}, \hat{\lambda}\}$ are obtained.

2.Generate Type-II samples of sizes m_1 and m_2 from WED$(\hat{\beta}, \hat{\theta})$ and accelerated WED$(\hat{\beta}, \hat{\theta}, \hat{\lambda})$ respectively and denote by $\mathbf{x}^* = \left(x^*_{j1;n},\, x^*_{j2;n},\, \ldots,\, x^*_{jm_j;n_j}\right),\ j = 1, 2.$

1.For given bootstrap Type-II sample X^* the MLEs $\Theta^* = \{\hat{\beta}^*, \hat{\theta}^*, \hat{\lambda}^*\}$ are obtained.

4.Steps (2) and (3) are repeated **N** times and each time computed bootstrap estimate $\Theta^* = \{\hat{\beta}^*, \hat{\theta}^*, \hat{\lambda}^*\}$.

5.The bootstrap sample estimate $\Theta^{*(i)} = \{\hat{\beta}^{*(i)}, \hat{\theta}^{*(i)}, \hat{\lambda}^{*(i)}\},\ i = 1,\ 2,\ \ldots,\ \mathbf{N}$ are arranged in ascending order $\Theta^*_{(i)} = \{\hat{\beta}^*_{(i)}, \hat{\theta}^*_{(i)}, \hat{\lambda}^*_{(i)}\}$.

Percentile Bootstrap Confidence Interval (PBCI)

Suppose that, the ordered sample is described by $F(x) = P(\hat{\Theta}^*_{(i)} \leqslant x),\ i = 1,\ 2,\ 3,$ the cumulative distribution function of Θ^*, where Θ^*_1 mean $\hat{\beta}^*$ and others. So, the point bootstrap estimate is defined by

$$\hat{\Theta}^*_i = \frac{1}{\mathbf{N}} \sum_{j=1}^{\mathbf{N}} \Theta^*_{(j)}. \tag{36}$$

Also, the $100(1-2\gamma)\%$ PBCIs are given by

$$(\hat{\Theta}^*_{i\text{boot}(\gamma)},\ \hat{\Theta}^*_{i\text{boot(1-}\gamma)}), \tag{37}$$

where of $\hat{\Theta}^*_{i\text{boot}} = F^{-1}(x)$.

4.3 Bayesian credible interval

1.The posterior variance of $g\,(\beta_1,\ \beta_2,\ \theta)$ is calculated by

$$V(g(\beta,\ \theta,\ \lambda)) = \frac{\frac{1}{N-M}\sum\limits_{i=M+1}^{N}\left(g\left(\beta^{(i)},\ \theta^{(i)},\ \lambda^{(i)}\right) - \tilde{g}_B\right)^2 h(\beta^{(i)},\ \theta^{(i)},\ \lambda^{(i)}|\mathbf{X})}{\frac{1}{N-M}\sum\limits_{i=M+1}^{N} h(\beta^{(i)},\ \theta^{(i)},\ \lambda^{(i)}|\mathbf{X})} \tag{38}$$

2.As given in [17] the credible interval or HPD credible intervals of any function $\varphi(\theta_1, \theta_2, \beta)$ can be construct as follows:

I.We put the posterior sample $\varphi^{(i)}(\theta_1^{(i)}, \theta_2^{(i)}, \beta^{(i)})$ and the corresponding weighted function $w^{(i)} = \frac{\Pi(\theta_1^{(i)}, \theta_2^{(i)}, \beta^{(i)})}{\sum_{i=S^*+1}^{S} \Pi(\theta_1^{(i)}, \theta_2^{(i)}, \beta^{(i)})},\ i = 1, 2, \ldots S - S^*,$ in ascending order $(w_{(i)},\ \varphi_{(i)}),\ i = 1, 2, \ldots S - S^*$.

II.For the ordered pairs $(w_{(i)},\ \varphi_{(i)})$, define the $\alpha-$th quantile of the marginal posterior of φ by

$$\hat{\varphi}^{(\alpha)} = \begin{cases} \varphi_{(1)}, \text{ if } \alpha = 0 \\ \varphi_{(k)}, \text{ if } \sum\limits_{i=1}^{k-1} w_{(i)} < \alpha < \sum\limits_{i=1}^{k} w_{(i)} \end{cases}. \tag{39}$$

III.The $100(1-2\alpha)\%$ credible intervals of φ is given by

$$(\varphi^{(\alpha)}, \varphi^{(1-\alpha)}) \tag{40}$$

V.The $100(1-2\alpha)\%$ HPD credible intervals of φ is given by

$$(\varphi^{(L/(S-S^*))}, \varphi^{(\{L+[(1-2\alpha)(S-S^*)]\}/(S-S^*))})$$

where $L = 1, 2, \ldots, (S-S^*) - [(1-2\alpha)(S-S^*)]$. Then, 100 (1-2$\alpha$)% HPD credible intervals is the smallest interval width among all credible intervals.

5 Simulation Studies

In this section, we discuss the problem of statistical inferences of WED under partially constant-stress ALTs model with Type-II censoring scheme. We measure the estimate values of the parameters based on the proposed model under different methods of estimations. This problem is assessed under formulation of Monte Carlo simulation study. Also, we compare the estimation methods for different choices of the parameters values and censoring scheme. Throughout the study, we discuss the effect of parameter changing and for different sample sizes and effected sample sizes. Therefore, we generate 1000 size sample and through sample compute average estimate (AE) and the corresponding mean squared error (MSE) Table 1 and 3. For interval estimation, we compute average interval length (AIL) and the corresponding coverage percentile (CP) Tables 2 and 4. The prior information are selected to satisfy that, the true parameter value are equal to mean of prior density and denoted by P^1. If the prior information is weaker then we use non-informative prior information which is denoted by P^0 and the parameter of prior distribution is selected equal to the value 0.0001. The true parameters values are selected to be $(\beta, \theta) = \{(0.2, 1.0), (2.5, 2)\}$ and $\lambda = \{1.5, 2.5\}$, respectively. The numerical computations are adopted with respect to the following algorithms.

Algorithm 3 (Monte Carlo simulation algorithms)

1:Generate samples of size m_1 and m_2 from WED under normal and stress conditions, respectively.

2:For the joint sample compute MLEs and Bayes estimate of model parameters.

3:For the joint sample compute the approximate confidence interval, bootstrap confidence interval and credible intervals.

4:The Steps from (1) to (3) are repeated 1000 times.

Table 1: The MSE of MLE and Bayes estimte when (β,θ,λ)={0.2, 2.5, 1.5}.

	MLE			Bayes P^0			Bayes P^1		
(n_1,n_2,m_1,m_2)	α	β	λ	α	β	λ	α	β	λ
(30,30,15,15)	0.142	0.455	0.325	0.135	0.418	0.314	0.098	0.385	0.287
(30,30,15,25)	0.127	0.415	0.288	0.118	0.418	0.271	0.071	0.351	0.266
(30,30,25,15)	0.112	0.400	0.272	0.105	0.411	0.265	0.066	0.342	0.253
(30,30,25,25)	0.101	0.388	0.259	0.091	0.402	0.254	0.057	0.331	0.240
(50,30,25,25)	0.105	0.388	0.259	0.091	0.402	0.254	0.057	0.331	0.240
(30,50,25,25)	0.111	0.379	0.245	0.096	0.407	0.248	0.061	0.324	0.233
(50,50,30,30)	0.098	0.363	0.232	0.090	0.392	0.240	0.053	0.311	0.219
(50,50,40,30)	0.092	0.358	0.233	0.087	0.388	0.241	0.046	0.304	0.214
(50,50,30,40)	0.095	0.362	0.231	0.091	0.384	0.240	0.039	0.308	0.209
(50,50,40,40)	0.084	0.341	0.218	0.083	0.357	0.218	0.033	0.285	0.192
(50,50,50,40)	0.077	0.337	0.208	0.072	0.344	0.209	0.031	0.269	0.184
(50,50,40,50)	0.079	0.334	0.208	0.077	0.340	0.207	0.033	0.264	0.180
(50,50,50,50)	0.057	0.302	0.181	0.062	0.319	0.179	0.024	0.225	0.141

Table 2: The AIL and the corresponding CP when (β,θ,λ)={0.2, 2.5, 1.5}.

		MLE			Boot			Bayes P^0			Bayes P^1		
(n_1,n_2,m_1,m_2)		α	β	λ	α	β	λ	α	β	λ	α	β	λ
(30,30,15,15)	AIL	5.254	0.624	3.852	5.741	0.751	4.125	5.224	0.600	3.804	4.421	0.541	3.330
	CP	0.89	0.88	0.90	0.90	0.88	0.89	0.90	0.91	0.90	0.92	0.91	0.90
(30,30,15,25)	AIL	5.115	0.527	3.771	5.587	0.674	4.002	5.040	0.554	3.340	4.250	0.503	3.240
	CP	0.91	0.89	0.90	0.90	0.91	0.92	0.93	0.92	0.92	0.94	0.93	0.93
(30,30,25,15)	AIL	5.085	0.501	3.745	5.544	0.639	3.970	5.005	0.519	3.315	4.221	0.479	3.218
	CP	0.91	0.90	0.91	0.90	0.91	0.89	0.93	0.92	0.91	0.94	0.92	0.92
(30,30,25,25)	AIL	5.019	0.441	3.700	5.495	0.601	3.914	4.971	0.491	3.280	4.200	0.424	3.185
	CP	0.93	0.90	0.92	0.90	0.93	0.91	0.93	0.94	0.92	0.94	0.95	0.93
(50,30,25,25)	AIL	5.012	0.435	3.703	5.488	0.598	3.907	4.970	0.484	3.283	4.207	0.415	3.181
	CP	0.92	0.91	0.90	0.90	0.89	0.91	0.91	0.94	0.91	0.94	0.91	0.92
(30,50,25,25)	AIL	5.017	0.438	3.711	5.482	0.589	3.911	4.972	0.480	3.281	4.194	0.418	3.192
	CP	0.92	0.92	0.90	0.92	0.90	0.91	0.92	0.94	0.92	0.94	0.93	0.91
(50,50,30,30)	AIL	4.992	0.401	3.680	5.424	0.531	3.865	4.914	0.442	3.239	4.152	0.389	3.157
	CP	0.94	0.93	0.92	0.92	0.95	0.93	0.93	0.95	0.92	0.93	0.93	0.95
(50,50,40,30)	AIL	4.962	0.370	3.654	5.401	0.502	3.845	4.875	0.414	3.212	4.109	0.341	3.121
	CP	0.93	0.93	0.94	0.93	0.95	0.94	0.94	0.92	0.93	0.92	0.97	0.96
(50,50,30,40)	AIL	4.967	0.373	3.658	5.398	0.501	3.839	4.869	0.417	3.215	4.112	0.343	3.118
	CP	0.94	0.92	0.93	0.93	0.94	0.91	0.94	0.95	0.93	0.93	0.94	0.92
(50,50,40,40)	AIL	4.925	0.339	3.631	5.367	0.472	3.814	4.838	0.400	3.182	4.081	0.312	3.087
	CP	0.93	0.94	0.93	0.92	0.94	0.93	0.94	0.94	0.92	0.96	0.94	0.95
(50,50,50,40)	AIL	4.875	0.300	3.573	5.314	0.427	3.771	4.802	0.352	3.131	4.041	0.289	3.044
	CP	0.95	0.94	0.94	0.92	0.94	0.96	0.94	0.94	0.97	0.93	0.94	0.94
(50,50,40,50)	AIL	4.870	0.307	3.582	5.317	0.414	3.775	4.812	0.345	3.125	4.039	0.281	3.047
	CP	0.94	0.92	0.94	0.93	0.94	0.93	0.94	0.92	0.93	0.93	0.95	0.97
(50,50,50,50)	AIL	4.819	0.254	3.514	5.282	0.379	3.744	4.752	0.301	3.082	4.002	0.231	3.014
	CP	0.93	0.93	0.94	0.94	0.94	0.94	0.94	0.97	0.92	0.94	0.95	0.94

5:Compute the estimate values of MSE, AIL and CP and reported in Tables from 1–4.

6 Conclusions

Life testing experiment or reliability analysis of products require more information about the life of product. Hence, for the optimal censoring scheme which can serve this problem, we applied the Type-II censoring scheme. The mechanism of ALTs that saves the reliability results more quickly for a high reliable product is applied. In our paper, we consider the products have WE lifetime distribution. The unknown model parameters are

Table 3: The MSE of MLE and Bayes estimte when (β, θ, λ)={1.2, 0.5, 2.0}.

	MLE			Bayes P^0			Bayes P^1		
(n_1,n_2,m_1,m_2)	α	β	λ	α	β	λ	α	β	λ
(30,30,15,15)	0.354	0.245	0.543	0.315	0.211	0.514	0.245	0.154	0.411
(30,30,15,25)	0.318	0.217	0.512	0.291	0.187	0.489	0.214	0.122	0.387
(30,30,25,15)	0.308	0.211	0.504	0.287	0.181	0.478	0.205	0.120	0.369
(30,30,25,25)	0.287	0.189	0.472	0.257	0.149	0.436	0.179	0.089	0.315
(50,30,25,25)	0.281	0.182	0.477	0.251	0.144	0.425	0.173	0.091	0.318
(30,50,25,25)	0.272	0.191	0.477	0.265	0.141	0.434	0.172	0.093	0.310
(50,50,30,30)	0.233	0.142	0.438	0.224	0.115	0.401	0.122	0.051	0.284
(50,50,40,30)	0.200	0.111	0.405	0.187	0.089	0.379	0.088	0.032	0.232
(50,50,30,40)	0.203	0.115	0.402	0.191	0.084	0.373	0.084	0.028	0.235
(50,50,40,40)	0.173	0.091	0.378	0.145	0.059	0.345	0.044	0.021	0.214
(50,50,50,40)	0.142	0.088	0.370	0.138	0.051	0.320	0.032	0.014	0.192
(50,50,40,50)	0.135	0.082	0.369	0.131	0.051	0.323	0.035	0.011	0.194
(50,50,50,50)	0.1.1	0.052	0.301	0.075	0.032	0.284	0.024	0.009	0.165

Table 2: The AIL and the corresponding CP when (β, θ, λ)={1.2, 0.5, 2.0}.

		MLE			Boot			Bayes P^0			Bayes P^1		
(n_1,n_2,m_1,m_2)		α	β	λ	α	β	λ	α	β	λ	α	β	λ
(30,30,15,15)	AIL	3.458	2.147	5.421	3.879	2.456	5.774	3.411	2.101	5.381	3.114	1.854	4.547
	CP	0.90	0.88	0.90	0.89	0.88	0.87	0.91	0.90	0.90	0.93	0.91	0.89
(30,30,15,25)	AIL	3.411	2.103	5.382	3.832	2.414	5.725	3.275	2.069	5.341	3.074	1.812	4.500
	CP	0.90	0.90	0.91	0.91	0.91	0.91	0.93	0.92	0.92	0.91	0.93	0.92
(30,30,25,15)	AIL	3.414	2.101	5.377	3.835	2.417	5.721	3.270	2.063	5.347	3.071	1.815	4.503
	CP	0.92	0.90	0.90	0.91	0.91	0.92	0.93	0.92	0.93	0.94	0.93	0.95
(30,30,25,25)	AIL	3.374	2.069	5.332	3.802	2.379	5.682	3.225	2.024	5.312	3.024	1.777	4.466
	CP	0.92	0.91	0.92	0.92	0.93	0.91	0.94	0.94	0.92	0.92	0.93	0.91
(50,30,25,25)	AIL	3.371	2.064	5.335	3.807	2.382	5.680	3.219	2.027	5.304	3.018	1.781	4.462
	CP	0.92	0.90	0.93	0.90	0.91	0.91	0.92	0.94	0.93	0.94	0.93	0.92
(30,50,25,25)	AIL	3.366	2.061	5.325	3.803	2.377	5.682	3.214	2.018	5.301	3.012	1.775	4.458
	CP	0.94	0.92	0.93	0.92	0.92	0.91	0.94	0.94	0.95	0.94	0.93	0.94
(50,50,30,30)	AIL	3.311	2.007	5.274	3.761	2.319	5.631	3.169	1.890	5.251	2.974	1.714	4.401
	CP	0.92	0.93	0.94	0.92	0.93	0.93	0.93	0.92	0.92	0.93	0.93	0.96
(50,50,40,30)	AIL	3.287	1.891	5.238	3.725	2.289	5.600	3.114	1.856	5.221	2.942	1.678	4.365
	CP	0.94	0.92	0.91	0.94	0.95	0.90	0.94	0.93	0.93	0.93	0.92	0.94
(50,50,30,40)	AIL	3.279	1.894	5.242	3.727	2.282	5.603	3.119	1.851	5.219	2.914	1.666	4.361
	CP	0.92	0.93	0.94	0.92	0.92	0.91	0.94	0.93	0.93	0.93	0.95	0.96
(50,50,40,40)	AIL	3.242	1.865	5.215	3.692	2.249	5.570	3.049	1.800	5.179	2.841	1.614	4.312
	CP	0.94	0.94	0.92	0.92	0.90	0.93	0.93	0.94	0.93	0.96	0.93	0.95
(50,50,50,40)	AIL	3.211	1.832	5.181	3.662	2.214	5.535	3.014	1.761	5.142	2.809	1.571	4.276
	CP	0.93	0.92	0.94	0.90	0.94	0.92	0.94	0.93	0.94	0.93	0.92	0.95
(50,50,40,50)	AIL	3.214	1.837	5.178	3.649	2.212	5.541	3.007	1.756	5.133	2.801	1.569	4.272
	CP	0.93	0.92	0.93	0.93	0.93	0.93	0.92	0.92	0.94	0.94	0.93	0.92
(50,50,50,50)	AIL	3.120	1.761	5.100	3.691	2.125	5.472	2.874	1.700	5.025	2.706	1.503	4.214
	CP	0.92	0.93	0.92	0.94	0.92	0.94	0.95	0.93	0.93	0.93	0.92	0.94

estimated by ML and Bayes methods for point estimate. Also, asymptotic confidence, bootstrap confidence and Bayes credible intervals are computed. The numerical results obtained from Monte Carlo simulation study have shown that;

1 The Type-II censoring scheme under proposed model serve well for all choices of censoring schemes and parameter values.
2 The results under maximum likelihood estimation and Bayes non-informative prior P^0 are more closed.
3 The informative priors P_1 serve better than non-informative prior and maximum likelihood estimations.
4 The higher values of affected sample size m_1 and m_2 have minimum MSE and AIL.

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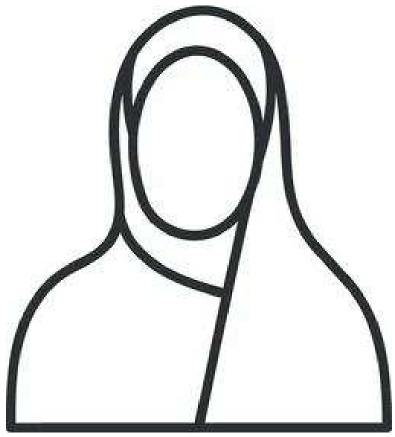

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