Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/121210

Conceptual Understanding for Systems of Linear Equations: Difficulties and Challenges

M. Tashtoush[1,2], *Y. Wardat*[3], *M. Al-Shannaq*[4,5], *R. AlAli*[6,*], *S. Saleh*[6,7] *and K. AL-Saud*[8]

[1]Department of Basic Sciences, AL-Huson University College, AL-Balqa Applied University, AL-Salt, Jordan
[2]Department of Mathematics Educations, Faculty of Education and Arts, Sohar University, Sohar, Oman
[3]Department of Mathematics Educations, Higher Colleges of Technology, Al Ain, United Arab Emirates
[4]Department of Curriculum and Instruction, Faculty of Education, Sultan Qaboos University, Muscat, Oman
[5]Department of Curriculum and Instruction, Faculty of Education, Yarmouk University, Irbid, Jordan
[6]The National Research Centre for Giftedness and Creativity, King Faisal University, Al-Hasa, Saudi Arabia
[7]Department of Educational Technology, Faculty of Education, Sohag University, Sohag, Egypt
[8]Department of Art Educations, Faculty of Education, King Faisal University, Al-Hasa, Saudi Arabia

Received: 25 Aug. 2023, Revised: 18 Nov. 2023, Accepted: 26 Nov. 2023
Published online: 1 Dec. 2023

Abstract: The conceptual understanding of algebraic concepts is challenging to students, adds to the challenge of teaching them. This study aimed to reveal the level of conceptual understanding about Systems of Linear Equations (SLEs) and its difficulties that faced students while they solving problems. The study sample consisted of (68) male and female students of linear algebra course. The conceptual understanding test was prepared with two qualitative and quantitative rubrics to classify students' levels. The study results showed a decrease in the conceptual understanding of SLEs in general, and in the area of realizing the relationships between concepts related to SLEs in particular. In contrast, the level of conceptual understanding of students was medium in the context of multiple representations. The results also showed many difficulties that students face while solving SLEs. The study recommended the need to focus on the conceptual understanding of concepts related to SLEs and to use multiple representations and connect them together when teaching mathematical concepts.

Keywords: Systems of Linear Equations, Conceptual Understanding, Rubrics, Linear Algebra.

1 Introduction

Understanding is a primary goal of learning mathematics. Each student is unique in his perspective on how he understands mathematical concepts, and links to ideas and understanding differ from one student to another, so what is meant by understanding? It isn't easy to define the definition of comprehension, but its characteristics can explain it. [1] Sees that comprehension is a process that occurs quickly within the brain and depends on a long series of learning activities; Therefore, it is challenging to acquire the mathematical concept within the brain, and both [2] describe understanding as how information is represented, constructed and linked, and that the degree of understanding depends on the number of connections between ideas, representations, and actions [3].

Conceptual understanding is one of the pillars of mathematical competence that makes parts of mathematical knowledge from facts, generalizations, principles, laws, and mathematical rules linked to each other by a close network of links. It includes producing examples and non-examples of mathematical concepts, using shapes and graphics to express them, the realization of the interconnectedness and complementarity between ideas, and identifying principles, laws, and rules related to concepts. Mathematical interpretation of the relationship between them [4-6].

Therefore, it is necessary to focus on conceptual understanding and develop its idea and provide examples that clarify what is meant by conceptual awareness that can be applied and benefited from in the classroom and which works to form links between concepts and the relationships associated with them, as it plays a vital role in the performance of students when they have an understanding Conceptually for a particular mathematical concept or topic, they implement solving procedures with ease, and understand what is behind them [7].

When the student solves SLEs consisting of two equations by the algebraic method or by the graph method, this does not indicate that he has a complete conceptual understanding of solving SLEs. Still, rather he must be able to describe or explain how to solve systems of equations algebraically or geometrically, expressing this in different forms and

*Corresponding author e-mail: ralali@kfu.edu.sa

symbols, and realizing the relationships between concepts related to SLEs such as the intersection of two straight lines, their parallelism or congruence, the exclusion of variables, the understanding of the multiple representations of SLEs, and the ability to link and integrate these concepts into its conceptual map. Thus, the student has a correct conceptual understanding of solving SLEs [5-6].

[7] Indicates that students understand mathematical ideas when they can look at them from several entrances, relate them to other concepts, represent the concept in different ways, identify the connections between these representations, and their ability to provide examples of mathematical concepts and distinguish between them; [1] Indicates the necessity of using models, schemas, and different representations of ideas and linking them with each other; the student defines and applies the principles; It perceives signs and symbols, knows and uses facts and definitions, and compares related concepts Situations that require a conscious application of concepts and relationships.

Algebra is one of the most important branches of mathematics in which algorithms and arithmetic operations are applied to abstract symbols instead of numbers. Furthermore, it deals with characters and sets laws through which symbols are manipulated. The study of SLEs is one of the essential topics in mathematics, applied sciences, as it plays a significant role in modeling many physical, engineering, social, and economic phenomena.

SLEs consist of a finite set of linear equations; a single equation includes one or more Variables. These systems search for a solution for all equations; each equation is represented by a straight line. SLE is called Consistent, and if we cannot find the solution to SLEs, then it is called In-consistence. These systems also investigate different solution states; if the system has only one solution, it is called Equivalence, and if it does not have a solution, it is called Non-Equivalence. If SLEs have many solutions, it is called Multi-Equivalence. Among the concepts related to SLEs as well; Leading Variable is represented by writing a linear equation in terms of only one variable, and the concept of row reduction; Elementary Row Operations known by multiplying an equation by a fixed number and adding it to another equation, and there is also the concept of Homogeneity, which is to be one of the two sides of the equation It is equal to zero, and Non-Homogenous, which is that one side of the equation is not equal to zero [8].

Students face many difficulties while solving SLEs. Among these difficulties is the lack of a conceptual understanding of SLEs, their inability to connect multiple representations of them and use them while solving problems, and their inability to connect the numerous representations of SLEs; such as Algebraic Expression, Graphical Expression, Matrices Expression and Data Expression, not distinguishing among students between the intersection of lines, their parallelism and their applicability, and not distinguishing between concepts related to SLEs, in addition to the suffering of students in defining variables in practical issues [9]. Furthermore, the students' incomplete thinking about the concept of the solution leads to the difficulty of finding it and the students' lack of connection between the symbolic issue and the real issue, which calls for searching for appropriate teaching methods and strategies [10-13] based on the recommendations of the 6th international conference of modern trends in innovative curricula in mathematics education held at Sultan Qaboos University in Oman, which recommended that mathematics teachers use the latest and most appropriate teaching strategies to teach their students.

Solving the mathematical problem requires translating it from the context to the abstract level, then solving the situation that has been abstracted, and then re-translating the solution to the original context of the problem, and this requires the student to have a conceptual understanding of algebraic skills and related concepts. Furthermore, the relationship to the mathematical concept depends on the concepts contained in the context and the relationships between them. Therefore, the research began with a conceptual understanding to overcome students' difficulties when solving a mathematical problem [14].

Such difficulties reflect what students have learned, as they focus on learning procedures without understanding the reason behind their application and also focus on knowing concepts without understanding them deeply, and this, in turn, will not allow the student to apply, adapt and acquire mathematical ideas in new situations; Therefore, researchers began to pay attention to the conceptual understanding of students. Hence the importance of the current study to reveal how students understand and solve systems of linear equations and their ability to link concepts related to SLEs, in addition to their ability to solve applied problems, and understand and solve SLEs through multiple representations.

Problem Statement

Regarding SLEs among the basic and important concepts that are presented to students in the middle and high school, in addition to the university level, within several courses in the disciplines of science, engineering and economics sciences. Through field observation of researchers during their work as members of the mathematics faculty for various educational levels in several universities, some of them supervising the field of practical education in the educational field, and others participating in some international and local tests, they noticed a shortcoming in the acquisition and development of mathematical concepts in general among students, and the concepts related to SLEs and their solution in particular. This shortcoming is the result of students not linking their new experiences with previous experiences, not delving into the study of mathematical concepts correctly, and their weak ability to link between the multiple representations of mathematical concepts, in addition to a weakness in their ways of thinking about SLEs and their solutions, a large number of errors they make while solving these problems, so researchers see the need to conduct such a study to find out these difficulties and challenges faced by students, and to search for the best methods and teaching strategies to develop a conceptual understanding of SLEs and their solution, and to address those errors and overcome difficulties. And provide explanations and justifications for some mathematical situations and tasks. Based on the foregoing, the problem of conceptual understanding of SLEs emerges clearly, in addition to the emergence of the problem of students' inability to solve their problems, with the need to shed light on the difficulties that students face and the mistakes they make while solving SLEs. Hence this study comes in an attempt to answer the following two questions:

1. What is the students' level of conceptual understanding of SLEs?

2. What are the difficulties and challenges that students face while solving SLEs?

Study Importance

The importance of the study stems from the importance of the topic it deals with, and the theoretical importance of the study lies in being one of the first studies that called for studying the topic of conceptual understanding in general and conceptual understanding of SLEs in particular, which researchers hope to enrich theoretical and research literature in mathematics. The practical importance of this study is that it may benefit students, faculty members, and those interested in developing the educational process in identifying how to measure the conceptual understanding of SLEs, the resulting conceptual errors among students, and the difficulties they face while solving them, which helps to develop programs and treatment plans and develop curricula in light of the results that have been reached.

Study Objectives

The current study aimed to reveal the level of conceptual understanding of SLEs and their solution by revealing the level of students' understanding of them and through their awareness of the concepts associated with them as the intersection of two straight lines in the plane, their parallelism or their indicated, the exclusion of variables, equivalence, consistency, homogeneity, root of the equation, common unknown quantities, and their multiple representations such as algebraic, graph, matrices and data. In addition to identifying the difficulties and challenges they face while solving their problems.

Limitations

- **Human limits:** The study is limited to students of the Linear Algebra course.
- **Temporal limits:** This study was applied during the 1st semester of the academic year 2022/2023.
- **Spatial boundaries:** This study is limited to Sohar University in of Oman.
- **Objective limits:** This study is limited to determining the level of conceptual understanding of SLEs and their difficulties that students face while solving them.
- This study is determined by its tools and its psychometric properties of validity and reliability that are acceptable for the purposes of scientific research, which were prepared to achieve the study's objectives.

Procedural Definitions

- **Conceptual Understanding:** The knowledge that includes understanding mathematical concepts, terms, and symbols and realizing the various relationships and ideas that are linked to each other by a close network of links and relationships that lead to a deep understanding of the mathematical concept.
- **Systems of Linear Equations (SLEs):** A finite set of linear equations, one equation includes one or more variables, and each equation is represented by a straight line, and the points at which these straight lines intersect are searched, which is called a solution to the linear system, and the solution to the linear system is to give a numerical value for each variable of the variables of the system check all its equations simultaneously.
- **Difficulties:** A set of challenges and problems that students face while learning about systems of linear equations, represented by a set of conceptual errors resulting from not studying the mathematical concepts related to systems of linear equations in depth.

Literature Review

[15] Conducted a study aimed at revealing the level of conceptual understanding of the first derivative and the difficulties that Jordan universities students face while finding and solving their applied problems. The study sample consisted of (171) students, the researchers prepared a conceptual understanding test and a scale to correct it. The

results showed a decrease in the level of conceptual understanding of the first derivative, and the results showed many difficulties that students face while finding the derivative and solving its applied problems. The study recommended the necessity of focusing on conceptual understanding and using multiple representations of the first derivative and linking them when teaching mathematical concepts.

[16] Also conducted a study aimed at knowing the effect of the mathematical modeling strategy on the understanding of mathematical concepts and solving mathematical problems for 7th-grade students. The researcher was used the experimental method, and the study sample consisted of (76) female students. The students recommended the need to revise the content of mathematics curricula at various stages and prepare guides for teachers based on mathematical modeling.

The study of [17] investigated the effect of using the Generative Learning Strategy for modifying misperceptions to understand mathematical concepts among 7th-grade female students in Saudi Arabia. The researcher used the experimental method, and the study sample consisted of (66) female student the study tool was an achievement test to diagnose misperceptions for understanding mathematical concepts. The results showed that the superiority of the experimental group for modifying misconceptions to understand mathematical concepts.

A study conducted by [18] showed that there are a large number of common errors for understanding mathematical concepts among students in the 8th and 10th-grades. Where the students made a mistake when they combined a variable with a constant while solving linear equations, and in solving a group of linear equations with one variable when they mixed between the additive and multiplication counterpart, and the students repeated their mistakes in understanding the mathematical concepts when they were asked about their solution to some linear equations, which indicates that these Conceptual errors are highly persistent and have deep roots in their cognitive structure. The study recommended that more studies should be conducted to reveal the reasons behind the existence of this amount of conceptual errors while solving linear equations. The study of [19] aimed to develop mathematical concepts in pre-school children through exposure of interactive activities that promote the development of mathematical concepts. The study sample consisted of (64) schools from different regions of Australia. The study tool was represented by an interview to collect data that included a set of questions and video recordings, and the results of the study showed that children are able to learn mathematical concepts through play, but there are shortcomings in learning and understanding mathematical concepts at an early age, and mathematical concepts can be developed at much earlier stages than the age of four years.

[20] Conducted a study to determine the impact of using a teaching method based on multiple representations of functions on students' comprehension of mathematical concepts. The study used a sample of two different universities in USA, students were taught one of the two universities taught the concept of functions by using multiple representations, while the students of the second university were taught in the traditional education. The results showed the superiority of the members of the first university in achievement, and the results of qualitative interviews conducted with eight members of the first university showed their flexibility in using multiple representations to understand the concept of functions, and they became skilled in using multiple representations of functions, and that the method of teaching that depends on multiple representations works to expand the knowledge network of students.

Finally, [21] study examined the difficulties faced by 9th-grade students in understanding the mathematical concepts of solving verbal problems related to equations and inequalities. Students' mistakes to know the extent of their ability to understand the mathematical concepts related to solving mathematical equations and also to know the relationship between their ability to solve equations and their achievement. The results of the study revealed the existence of some alternative misconceptions among students that hinder the progression in the process of solving verbal problems related to equations and inequalities. It is also recommended teachers and curriculum developers provide the student with different experiences through various situations and contexts to be trained to implement the process of solving verbal problems related to equations and inequalities.

2 Methodologies

Research Design

The study followed the Mixed Methodology, which combines the quantitative and qualitative approaches with each other.

Participants

The study population consisted of students of Sohar University who enrolled in the 1st semester 2022/2023. As for the study sample, it was chosen by the Intentional Method, as the students of the Linear Algebra course were selected (68) students.

Instruments

Conceptual Understanding Test (CUT): According to [22-25] the researchers preparing the (CUT), which consisted of 10 tasks through two domains: (Understanding the SLEs in the context of multiple representations, with two subdomains: Algebraic Representation and Graphing Representation), and (Understanding the relationships between concepts related to the SLEs, with three subdomains: Consistency, Equivalence and Homogeneity). In order to verify the validity of (CUT), it was presented to arbitrators from specialized University professors; they were asked to express their opinions and comments about the (CUT) tasks and their suitability for the objectives for which they were set, as well as the integrity of the linguistic formulation and its affiliation with the fields of study. Based on the judges' opinions, the amendments were made until the test became in the final form. In order to identify the significance of the effectiveness of (CUT), the coefficients of difficulty and discrimination were calculated after applying them to an exploratory sample consisting of (20) students from outside the study sample, where the coefficients of difficulty ranged (0.52-0.82), and the coefficients of discrimination ranged (0.32-0.59). To verify the reliability of (CUT), it was applied to exploratory sample twice, two weeks apart from the two applications, Cronbach Alpha coefficient was calculated, which measures the consistency of the answers of the students, where the Cronbach Alpha coefficients ranged (0.54-0.83), these values are acceptable in this study [26].

Rubrics: According to [19, 27-29] to determine the levels of conceptual understanding of SLEs, a qualitative rubric was prepared from the researchers to analyze students' performance on the (CUT). Table (1) shows the performance indicators of the qualitative rubric.

Table 1. Qualitative Rubric.

Domain	**Subdomain**	**Performance Indicators**	
		Performance Description	**Performance Level**
Understanding the SLEs in the context of multiple representations	Algebraic Representation	• Correctly arrange all equations of SLEs according to the variables. • Correctly identifies all variables to be reduced or excluded. • Correctly performs all row reduction operations. • Verifies that the solution is correct by using back substitutions method correctly.	Perfect
		• Correctly arrange some equations of SLEs according to the variables. • Cannot correctly identify all variables to be reduced or excluded. • Correctly performs some row reduction operations. • Verifies that part of the solution is correct by using back substitutions correctly.	Balanced
		• The SLEs is arranged incorrectly according to the variables. • Incorrectly identify the variables to be reduced or excluded. • Performs row reduction operations incorrectly. • Validates the solution by using back substitutions incorrectly. • No solution.	Weak
	Graphing Representation	• Correctly draw all lines of SLEs in the xy-plane. • The x and y intercepts of all lines of SLEs are represented correctly. • Correctly determines all points of intersection between equations of SLEs. • Correctly deduce the relationship between (intersections, parallelism, identical). • Verifies that the solution is correct by using back substitutions correctly.	Perfect
		• Correctly draw some lines of SLEs in the xy-plane.	Balanced

		• The x and y intercepts of some lines of SLEs are represented correctly. • Correctly determines some points of intersection between equations of SLEs. • Correctly deduces part of the relationship between (intersection, parallel, identical). • Verifies that part of the solution is correct by using back substitutions correctly.	
		• Wrongly draw all lines of SLEs in the xy-plane. • The x and y intercepts of all lines of SLEs are incorrectly represented. • Determines the points of intersection between the equations SLEs incorrectly. • Incorrectly deduces the relationship between (intersections, parallelism, identical). • Validates the solution by using back substitutions incorrectly. • No Solution.	Weak
Understanding the relationships between concepts related to the SLEs	Consistency	• Correctly understand the relationship between the number of equations of SLEs and the consistency of the system. • Correctly relate the relationship between the number of variables of SLEs and the consistency of the system. • Correctly deduce the relationship between system consistency and other concepts related to SLEs.	Perfect
		• Correctly understand part of the relationship between the number of equations of SLEs and the consistency of the system. • Correctly relate part of the relationship between the number of variables of SLEs and the consistency. • Correctly deduce part of the relationship between system consistency and other concepts related to SLEs.	Balanced
		• Incorrectly understand the relationship between the number of equations of SLEs and the consistency. • Incorrectly relate the relationship between the number of variables of SLEs and the consistency. • Incorrectly deduce the relationship between system consistency and other concepts related to SLEs.	Weak
	Equivalence	• Correctly understand the relationship between the number of equations of SLEs and the equivalence of the system. • Correctly relate the relationship between the number of variables of SLEs and the equivalence of the system. • Correctly deduce the relationship between system equivalence and other concepts related to SLEs.	Perfect
		• Correctly understand part of the relationship between the number of equations of SLEs and the equivalence of the system. • Correctly relate part of the relationship between the number of variables of SLEs and the equivalence. • Correctly deduce part of the relationship between system equivalence and other concepts related to SLEs.	Balanced
		• Incorrectly understand the relationship between the number of equations of SLEs and the equivalence	Weak

		of the system. • Incorrectly relate the relationship between the number of variables of SLEs and the equivalence. • Incorrectly deduce the relationship between system equivalence and other concepts related to SLEs.	
	Homogeneity	• Correctly understand the relationship between the number of equations of SLEs and the homogeneity of the system. • Correctly relate the relationship between the number of variables of SLEs and the homogeneity. • Correctly deduce the relationship between system homogeneity and other concepts related to SLEs.	Perfect
		• Correctly understand part of the relationship between the number of equations of SLEs and the homogeneity. • Correctly relate part of the relationship between the number of variables of SLEs and the homogeneity. • Correctly deduce part of the relationship between homogeneity and other concepts related to SLEs.	Balanced
		• Incorrectly understand the relationship between the number of equations of SLEs and the homogeneity. • Incorrectly relate the relationship between the number of variables of SLEs and the homogeneity. • Incorrectly deduce the relationship between homogeneity and other concepts related to SLEs.	Weak

In addition to the previous qualitative rubric, another quantitative rubric was prepared to complement the qualitative rubric, through which the qualitative and quantitative rubrics were linked with each other to classify students' conceptual understanding levels through three levels (Low, Medium, High), by giving three points to the answer that corresponds to the criteria of the level of *Perfect*, two points to the answer that corresponds to the criteria of the level of *Balanced*, and one point to the answer for the level of *Weak*, table (2) show this.

Table 2. Levels of Conceptual Understanding of SLEs.

Level	Degree
Low	1.67 – 1.00
Medium	2.35 – 1.68
High	3.00 – 2.36

To verify the validity of the rubrics, they were presented to arbitrators from specialized professors; they were asked to express their opinions and comments about the performance indicators and their suitability for the goals for which they were set, as well as the integrity of the language and the extent to which they belong to the fields of study. Based on the opinions of the arbitrators, the amendments made to the rubrics became final. To verify the reliability, two researchers analyzed the exploratory sample of students' answers independently and objectively with a two-week difference between the two evaluations and compared the results for each of them independently and the agreement coefficient between them was measured by Holsti Equation, the consistency coefficients between them was (0.93).

Data Analysis

The data was entered into the computer memory and analyzed using the SPSS by calculating the percentage and extracting the mean scores for the (CUT), the two domains of the test, subdomains and the test as a whole.

3 Results

To answer the first question, the student's written answers on the (CUT) for SLEs were analyzed according to the qualitative and quantitative rubrics, calculating the numbers of students, percentage and extracting the mean scores for the 1st and 2nd domains and as a whole and categorizing them according to the levels of conceptual understanding (high, medium, low), table (3) shows this.

Table 3. Student Numbers, Percentages, and Mean Scores for the (CUT).

Domain	Subdomain	Task	Performance Level						Mean	Level
			Perfect		Balanced		Weak			
			No	%	No	%	No	%		
Understanding the SLEs in the context of multiple representations	Algebraic Representation	**1**	13	19.11	34	50.00	21	0.88	1.71	Medium
		2	14	20.58	35	51.47	20	9.41	1.85	Medium
	Graphing Representation	**3**	11	16.17	25	36.76	32	7.05	1.57	Low
		4	12	17.64	23	33.82	33	8.52	1.57	Low
Whole 1st domain			**12**	**17.64**	**25**	**36.76**	**31**	**5.58**	**1.78**	**Medium**
Understanding the relationships between concepts related to the SLEs	Consistency	**5**	12	17.64	24	35.29	32	47.05	1.64	Low
		6	14	20.58	28	41.17	26	38.23	1.71	Medium
	Equivalence	**7**	13	19.11	18	26.47	37	54.41	1.42	Low
		8	12	17.64	22	32.35	34	50.00	1.50	Low
	Homogeneity	**9**	13	19.11	32	47.05	23	33.82	1.85	Medium
		10	13	19.11	28	41.17	27	39.70	1.71	Medium
Whole 2nd domain			**13**	**19.11**	**19**	**27.94**	**36**	**52.94**	**1.42**	**Low**
Whole Test			**13**	**19.11**	**22**	**32.35**	**33**	**48.52**	**1.57**	**Low**

Table (3) Shows that the level of conceptual understanding was generally low, with mean score (1.57). For the first domain which is the Understanding the SLEs in the context of multiple representations, it was medium with mean score (1.78). Also, in the context of algebraic representation, the level of conceptual understanding was medium with mean score for the first task was (1.71) and the second (1.85), in the context of graphic representation, the level was low with mean scores of the third and fourth tasks was (1.57). For the second domain, which is the Understanding the relationships between concepts related to the SLEs, then the level of conceptual understanding of the students was low, with mean score (1.42), the concepts of consistency and equivalence, then the level was low. The mean score was less than (1.71) for the (5-8) tasks. Finally, the level of students' conceptual understanding of the concept of homogeneity was medium with mean score for the ninth and tenth tasks were (1.85) and (1.71), respectively.
Figures (1) and (2) represent the percentages for each of the first and second domains of the conceptual understanding test for SLE, while Figures (3) represents the whole CUT percentages.

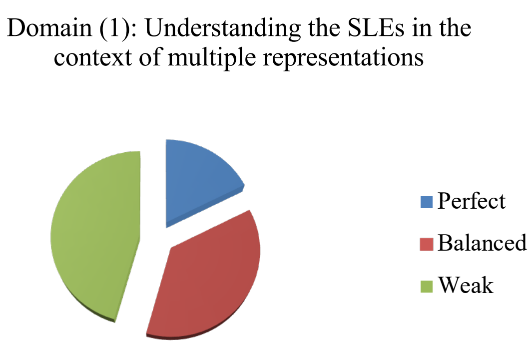

Fig 1. Percentages for the (CUT) for the 1st domain.

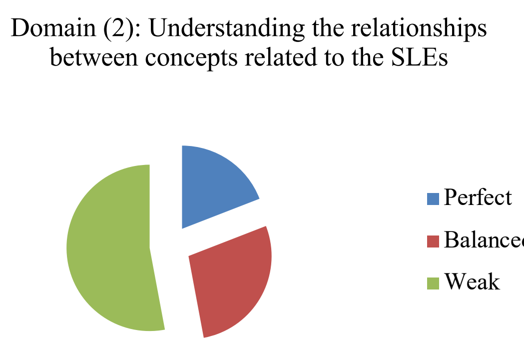

Fig 2. Percentages for the (CUT) for the 2nd domain.

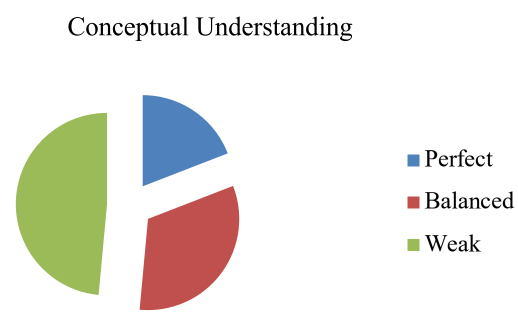

Fig 3. Percentages for the (CUT).

To answer the second question, the conceptual understanding test tasks that exceeded the percentage of students' responses and which did not fully achieve the performance indicator required of each task by more than (70%) were listed and arranged in descending order according to the percentage, see table (4).

Table 4. Percentage of students' responses to the (CUT)

Performance Indicators	Number	Ratio
Represent the x and y intercepts for a linear equation in the xy-plane	63	%92.6
Draw a linear equation in the xy-plane	62	%91.2
It relates the relationship between the concept of consistency and the concept of Equivalence for SLEs	60	%88.2
Doing row reduction; Elementary Row Operations	58	%85.2
Draw equations of SLEs and determine the relationship between them (intersection, parallel, identical)	56	%82.3
Write SLEs describing applied problems	53	%77.9
It relates the relationship between the concept of homogeneity and the concept of equivalence for SLEs	51	%75.0
Finds the points of intersection between the equations of SLEs	49	%72.0
Validates the solution using the back-substitution process	48	%70.5

Tables (4) shows that the most prominent difficulties faced by students, which exceeded the percentage of responses that did not achieve the performance indicators required of the tasks on them, were represented in: Represent the x and y intercepts for a linear equation in the xy-plane and Draw a linear equation in the xy-plane, they obtained a percentage of (92.6%), and Draw a linear equation in the xy-plane obtained a percentage of (91.2%). The relationship between the concept of consistency and the concept of Equivalence for SLEs and Doing row reduction; Elementary Row Operations they obtained a percentage of (88.2%) and (85.2%) respectively. This indicates a low level of students' understanding of SLEs not only in the context of multiple representations but also in their awareness of the relationship between them and related concepts; The difficulties that students face lie in the difficulty of understanding SLEs and solving them conceptually.

4 Discussions and Conclusions

It is clear that from the results above the level of conceptual understanding of SLEs and solving their problems was generally low. In the context of multiple representations it was medium, this may be due to the presence of qualitative differences in the students' understanding of the concept of SLEs in the context of multiple representations; or perhaps because the majority of students prefer algebraic representation to graphing; perhaps because the algebraic representation is more familiar than the graph during their previous school years, and their teachers are more accustomed to the algebraic representation than the graph, or because the students are more accustomed to using symbols and algebraic structures than to using lines, curves and points of intersection, even though the students use incorrect properties of the representation algebraic; such as applying elementary row operations, distributing multiplication to addition, incorrectly decoding parentheses, or using wrong algebraic generalizations. The reason may also be attributed; the familiarity of using simple class operations more than drawing lines and curves at the xy-plane, which requires finding the x and y intercepts, determining the points of intersection between the equations of SLEs and determining the cases of intersection, parallelism, matching the equations of the system, as the results indicate that more than half of the students could not those who drew the lines of SLEs correctly, and they could not distinguish between the three cases (intersection, parallelism, identical). Figure (4) shows the answer of one of the students to the third task, which was classified within the level (weak) according to the qualitative rubric and within the level (low) according to the quantitative rubric, the student shows multiple errors; such as representing the x and y intercepts, not drawing the SLEs correctly, defining the relationship between the slopes of the two straight lines incorrectly, and not determining the intersection relationship between the system lines.

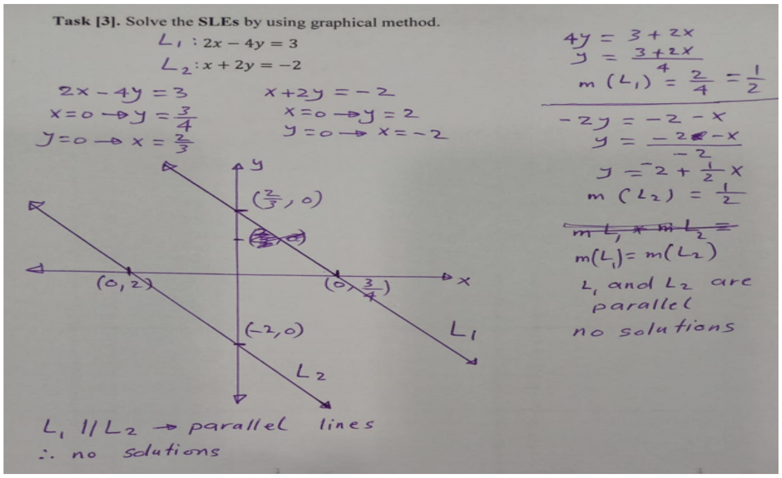

Fig 4. Student's answer to the third task.

It was also noted that there is a missing solution, where the student begins to answer the task, then the solution is not completed, but rather stops at some point because he believes that the solution is finished, this may be due to the lack of appropriate teaching strategies [30-31] would enhance the multiple representations of SLEs in their multiple contexts, especially algebraic representation, this is consistent with the National Strategic Vision for Education 2040 in Oman, which calls for quality education and high-level educational outcomes, by raising the efficiency of teachers, preparing them and training them on the latest and most important teaching strategies [33].

For the second domain, the level of conceptual understanding was low in the realization of the relationships between the concepts of consistency and equivalence, and medium in the concept of homogeneity; This may be due to the fact that learning algebra in general, and learning SLEs in particular, is not done in the right way through the use of appropriate teaching strategies [30-32], instead of being taught as a language with its logical structure and related to other mathematical themes and concepts, it is taught as a dead language that contains a huge amount of laws and theories.

The inability of the students to realize the relationship between the consistency of the SLEs through the number of equations of the system or the number of its variables, and the realization of the relationship between the consistency of SLEs through its equivalence or homogeneity, indicates that there is a general weakness of the students represented in the lack of understanding of many theories and algebraic generalizations in a true way, or it may be due to the lack of a conceptual basis in the cognitive structure of students, which may refer to traditional teaching methods based on memorization and the failure to use teaching methods and strategies that would enhance the conceptual understanding of students. Figure (5) shows the answer of one of the students to the fifth task, which was classified within the level (balanced) according to the qualitative rubric, and within the level (medium) according to the quantitative rubric, the student shows his inability to link the relationship between the concepts of SLEs that the homogeneous system must be equivalent through the presence of the zero solution.

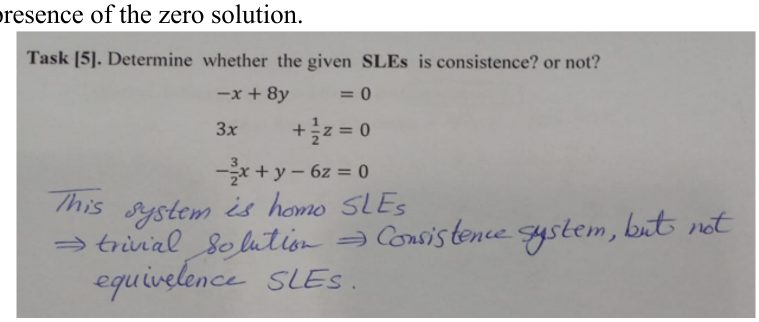

Fig 5. Student's answer to the fifth task.

As for understanding the relationships between equivalence concepts and other concepts related to SLEs; It is noticed that about half of students do not have a conceptual basis about the cognitive structure of basic algebraic theories and laws, and they cannot comprehend the relationship between the graphic representation of SLEs and the cases of drawing the lines of the system in the xy-plane (intersection, parallelism, identical). Therefore, the students cannot be relating the relationships between the consistence of SLEs and intersection of its lines, in-consistence of SLEs and parallelism of its lines and multi-consistence of SLEs and identical of its lines. Perhaps the reason for this may be because the conceptual understanding of SLEs and solving their problems constitute an abstract algebraic ability and represents a higher level of performing algebraic operations. Thus, a student who is not proficient in drawing the straight lines of SLEs, he will not be able to realize the relationships between the concepts of equivalence and other related concepts.

The students' inability to distinguish between homogeneous systems, and not realizing the relationship between the consistency or equivalence of SLEs through its homogeneity, as the students did not realize that the homogeneous SLEs must be consistent through the existence of the zero solution, and the homogeneous SLEs whose number of variables is more than the number of its equations, it is multi-consistence, and the non-homogeneous SLEs can be consistence, in-consistence or multi-consistence, all of which indicates that there is a general weakness among students represented in the lack of a correct algebraic conceptual structure and a good conceptual inventory to link the most important basic concepts related to SLEs as indicated some studies [2, 27, 31, 34], despite the students' exposure to a wide range of algebraic mathematical concepts, but it seems that students' weakness it is related to the lack of interest in the environment rich in algebraic concepts and the use of sensory and semi-sensory representations such as drawings and shapes to illustrate basic algebraic ideas.

The results show the diversity of common errors in the basic algebraic concepts, and it was noted that the highest percentage of difficulties and common errors in the basic algebraic concepts related to SLEs is represented in the graphic representation of SLEs and determining the relationship between them and the representation of the x and y intercepts of the lines of SLEs and the points of intersection between them. The reason for this can be attributed to the fact that the sources of difficulties faced by students lie in the difficulty of conceptually understanding SLEs through the context of graphing due to the students not acquiring the skills of drawing curves for linear equations during their years of study, and perhaps this is due to the student's preference for algebraic representation over graphing as we explained previously. Among the most prominent difficulties and challenges that the students also faced; writing SLEs that describes applied problems and their solutions. Students face difficulties in how to convert the verbal problem into SLEs, and the reason for this can be attributed to the great linguistic weakness that students suffer when converting or translating the written problem into SLEs, which causes them difficulties in understanding and learning the language of mathematics that indicated some studies [29-32].

For the difficulties that students face in perceiving the relationships between the concepts of SLEs, the results indicate that there is a noticeable weakness in the student's understanding of the concepts of consistency, equivalence, homogeneity and other related concepts. Figure (6) shows the answer of one of the students to the eighth task, where the answer of this student is classified according to the qualitative rubric (weak) and according to the quantitative rubric (low), this student made some mistake in the process of excluding the variable y (leading variable), errors to determining the slope of the two straight lines, errors to determining the type of relationship between them and errors to determining the type of equivalence, thus getting a wrong answer.

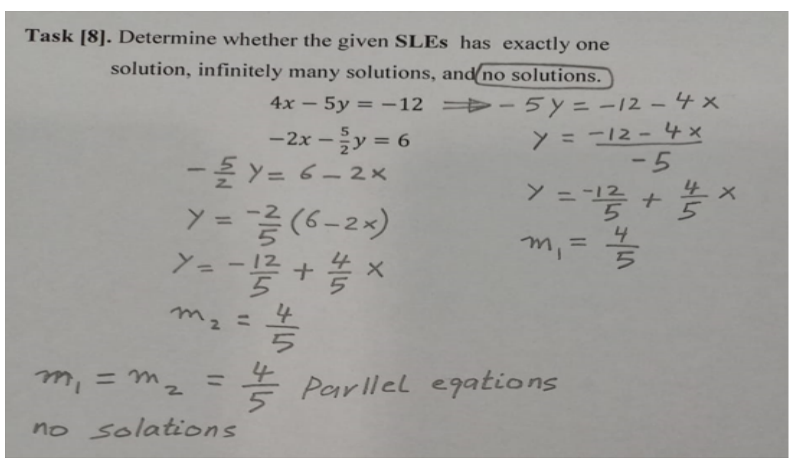

Fig 6. Student's answer to the eighth task.

In order to learn it based on understanding; it is advisable to start teaching it by linking it to the students' stock of knowledge and gradual learning by offering examples and models that link the tangible and the abstract, especially since these concepts are the basic building block of SLEs, and student's failure to learn them leads to difficulties in learning generalizations, theories and algebraic concepts. In order to overcome these difficulties, it is necessary to provide students with the necessary skills and the rule of generalization after making sure that they understand and comprehend the concepts related to SLEs.

5 Recommendations

Focusing on the conceptual understanding of mathematical concepts in general, and concepts related to SLEs in particular, the importance of recognizing and deducing relationships between the concept and the concepts that are related to it, and using multiple representations of concepts and linking them together when teaching mathematical concepts.

- Knowing the conceptual errors found in the current study while solving SLEs helps in predicting the difficulties and obstacles facing students and taking them into consideration in the lesson planning process, which helps prevent their occurrence among students.
- Organizing training courses and workshops to develop the performance of faculty members and to familiarize them with the importance of conceptual understanding as a major goal in teaching and learning mathematics.
- Conduct more studies and research that examine the conceptual understanding of SLEs and the difficulties students face while solving problems on them, including algebra and other mathematics topics and different study stages and their relationship to other variables.

Acknowledgments

The authors thank the Deanship of Scientific Research at King Faisal University, KSA. This work was financially supported by the Deanship of Scientific Research at King Faisal University, [grant number GRANT3647].

Conflicts of Interest Statement

The authors declare that there is no conflict regarding the publication of this paper.

References

[1] English, L. Data Modeling with First-Grade Students, Educational Studies in Mathematics., **81(1)**, 15-30, 2012. https://doi.org/1007/s10649-011-93773

[2] Tashtoush, M., Wardat, Y., Aloufi, F., Taani, O. The Effect of a Training Program Based on (TIMSS) to Developing the Levels of Habits of Mind and Mathematical Reasoning Skills among Pre-service Mathematics Teachers, EURASIA Journal of Mathematics, Science and Technology Education., **18(11)**, Article No em2182, (2022) https://doi.org/10.29333/ejmste/12557

[3] Hiebert, J. & Carpenter, T. Learning and teaching with understanding. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning, (pp.85–97). New York: MPC, (1992).

[4] Donlan, C. The Development of Mathematical Skills, Psychology Press, University College London, UK, (2002).

[5] Wardat, Y., Tashtoush, M., Alali, R., Jarrah, A. ChatGPT: A Revolutionary Tool for Teaching and Learning Mathematics. EURASIA Journal of Mathematics, Science and Technology Education, 19(7), Article No em2286, (2023) https://doi.org/10.29333/ejmste/13272

[6] Tashtoush, M., Wardat, Y., Elsayed, A. Mathematics Distance Learning and Learning Loss During COVID-19 Pandemic: Teachers' Perspectives. Journal of Higher Education Theory and Practice., **23(5)**, 162-174, (2023) https://doi.org/10.33423/jhetp.v23i5.5933

[7] National Council of Teachers of Mathematics (NCTM). Principles and standards for school mathematics, Reston, VA: Author, (2000).

[8] Kolman, B. Elementary Linear Algebra, 4th edition, John Wiley & Sons, Inc., (2000) uaar.edu.pk/fs/books/8.pdf

[9] Al-Shirawia, N. & Tashtoush, M. Differential Item Functioning Analysis of an Emotional Intelligence Scale for Human Resources Management at Sohar University. Information Sciences Letters., **12(11),** 2937-2952, (2023) https://doi.org/10.18576/isl/121109

[10] Fannakhosrow, M., Nourabadi, S., Huy, D., Trung, N., Tashtoush, M. A Comparative Study of (ICT)-Based and Conventional Methods of Instruction on Learners' Academic Enthusiasm for L2 Learning, Education Research International, 2022, Article ID 5478088, (2022) https://doi.org/10.1155/2022/5478088

[11] Tashtoush, M., Wardat, Y., Aloufi, F., Taani, O. The Effectiveness of Teaching Method Based on the Components of Concept-Rich Teaching in Achievement for Students of Linear Algebra Course and Their Attitudes Towards, Journal of Higher Education Theory and Practice., **22(7)**, 41-57, (2022) https://doi.org/10.33423/jhetp.v22i7.5269

[12] Rasheed, N. & Tashtoush, M. The Fertility and its Relation with Some Demographic, Economic and Social Variables in Jordan, Turkish Journal of Computer and Mathematics Education., **12(11)**, 5088-5095, 2021. https://www.turcomat.org/index.php/turkbilmat/article/view/6710

[13] Tashtoush, M., Al-Shannaq, M. & Barakat, A. The Effect of Using Self-Regulated Learning Learning Strategy to Reduce the Level of Mathematics Anxiety among Students of Al-Huson University College, Jordanian Journal of Education, Jordanian Association for Educational Sciences., **5(3)**, 306-329, (2020).

[14] Shirawia, N., Alali, R., Wardat, Y., Tashtoush, M., Saleh, S., Helali, M. Logical Mathematical Intelligence and its Impact on the Academic Achievement for Pre-Service Math Teachers. Journal of Educational and Social Research., **13(6)**, 242-257, (2023) https://doi.org/10.36941/jesr-2023-0161

[15] Atta, R. & Al-Zoubi, A. Southern Jordanian Undergraduates' Level of Conceptual Understanding of Derivative and Difficulties faced them during Solving Derivative Problems, Journal of Al-Quds Open University for Educational & Psychological Research & Studies., **8(24)**, 139-151, 2018 https://doi.org/10.1016/j.sbspro.2014.07.495

[16] Toba, R. The effect of using the mathematical modeling strategy on understanding mathematical concepts and solving the mathematical problem of the seventh-grade students in the unit of measurement (Unpublished master thesis), An-Najah University, (2014).

[17] AL-Omari, N. The effect of using the generative learning strategy in modifying the misperceptions of some mathematical concepts among the seventh graders in Al-Makhwah, (Unpublished Mater Thesis), Umm Al Qura University, KSA, (2013).

[18] Tarhe, N. Common errors in basic algebraic concepts and thinking strategies associated with these errors among eighth and tenth graders in Jerusalem (Master Thesis), Birzeit University, (2010).

[19] Abo-Obaid, A. The method of evaluating performance and its impact on the achievement and follow-up of the first grade of the secondary school towards mathematics, Psychological and Educational Studies., **7**, 25-57, 2011https://doi.org/10.1007/s10763-010-9224-5

[20] Tashtoush, M. Weakly c–Normal and cs–Normal Subgroups of Finite Groups. Jordan Journal of Mathematics and Statistics., **1(2)**, 123–132, (2008).

[21] Abo Laban, W. The difficulties faced by ninth-grade students in solving word problems related to equations and inequalities (Unpublished master thesis), Birzeit University, (1999).

[22] Balka, A., Hull, J. & Miles, H. What is conceptual understanding? Mathematics Teaching., **32(5)**, 128-139, 2015 https://doi.org/10.25896/mtch/128561

[23] Tashtoush, M., Alali, R., Wardat, Y., AL-Shraifin, N., Toubat, H. The Impact of Information and Communication Technologies (ICT)-Based Education on the Mathematics Academic Enthusiasm. Jounal of Educational and Social Research., **13(3)**, 287-296, (2023) https://doi.org/10.36941/jesr-2023-0077

[24] Tashtoush, M., Aloufi, F., Rasheed, N., Abo Al Aish, A., Az- Zo'bi, E. The Impact of Teaching Limits and Differentiation Using Blended Learning on Achievement and Motivation to Learn. Res Militaris., **13(3)**, 107-120, (2023) https://resmilitaris.net/menu-script/index.php/resmilitaris/article/view/3251

[25] Zureigat, H., Tashtoush, M., Al Jassar, A., Az- Zo'bi, E., Alomare, M. A solution of the complex fuzzy heat equation in terms of complex Dirichlet conditions using a modified Crank-Nicolson method. Advances in Mathematical Physics, Vol. 2023, Article ID 6505227., 1-8, (2023) https://doi.org/10.1155/2023/6505227

[26] Odeh, A. Measurement and evaluation in the teaching process, India, Dar Al-Amal Publisher, (2010).

[27] Tashtoush, M., Al-Shannaq, M., & Albarakat, A. The Effectiveness of Self-Regulated Learning (SRL) in Creative Thinking for CALCULUS Students, PalArch's Journal of Archaeology of Egypt/ Egyptology., **17(7)**, 6630-6652, (2020).

[28] Tashtoush, M., Wardat, Y., AlAli, R., Al-Saud, K. Cyberbullying's Impact on Student Motivation to Learn: In-Sights from Abu Dhabi Emirate Schools, Humanities and Social Science Letters., (2023).

[29] Rahim, M. The Use of Blended Learning Approach in EFL Education, International Journal of Engineering and Advanced Technology., **8(5)**, 1165-1168, (2017) https://doi.org/10.35940/ijeat.E1163.0585C19

[30] Rasheed, N. & Tashtoush, M. The Impact of Cognitive Training Program for Children (CTPC) to Development the Mathematical Conceptual and Achievement. Journal of Higher Education Theory and Practice., **23(10)**, 218-234, (2023) https://doi.org/10.33423/jhetp.v23i10.6196

[31] Sahin, S. & Baki, A. A new model for assessment of mathematical power. Procedia- Social and Behavioral Sciences., **9**, 1368–1372, 2010. https://doi.org/10.1016/j.sbspro.2010.12.336

[32] Tashtoush, M. & Rasheed, N. The Assessment of the Performance of Calculus Students in Composition Function and Finding an Inverse Function. 6th Sohar University Teaching and Learning Conference (Innovations and Applications in Teaching and Learning), 2 March, 2023, Sohar University, OMAN, (2023).

[33] Council of Education. The National Education Strategy in Sultanate of Oman 2040, (2019).

[34] Rider, L. The Effect of Multi-Representational Methods on Students' Knowledge of Function Concepts in Developmental College Mathematics. (Unpublished Ph. D. Dissertation), North Carolina State University, (2004).