

Comparative Bayesian Analysis of GARCH and Stochastic Volatility Models using R and Stan

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Abstract: This study uses modelling and model comparison to compare three widely used GARCH models with their stochastic volatility (SV) counterparts in modelling the dynamics of inflation rates using the Bayesian approach. BRICS country consumer price index (CPI) data are used to assess these models. We find that the stochastic volatility models perform better than the GARCH models most of the time. The stochastic volatility in the leverage (SV-L) model is also demonstrated to be the most effective for the BRICS nations that we took into consideration. The article also looks at which model attributes are crucial in simulating inflation rates. It turns out that when modelling inflation rates, inflation volatility feedback is an important component to take into account. For each of the five countries we took into consideration, SV-L outperforms all other models. The study was done in rstan, a programming language for statistical inference, and the simulation uses the Hamiltonian Monte Carlo (HMC) algorithm of the Markov chain Monte Carlo (MCMC) to sample from the posterior distribution.

Keywords: Bayesian Inference, GARCH models, SV models, Stan, MCMC, LOOIC

1 Introduction

Research on understanding, analysing, and forecasting the volatility of financial time series has been ongoing for more than 37 years. Because of the potential harm they could cause to the real economy, inflation and its volatility have drawn more and more attention in the economic literature. Theoretical studies show that very fluctuating inflation would result in poor resource allocation, which may impede economic expansion and increase joblessness [1,2].

The GARCH and the stochastic volatility (SV) model are two major classes of models used to model the volatility. This time-varying autocorrelated volatility process has been modelled using these two models. A class of GARCH models can be used to simulate inflation volatility. The GARCH model is a deterministic function of available data, with the parameters of the model estimated using the conventional method. The literature explores the relationship between inflation and inflation uncertainty using classical methods. [3] express that the widely used asymmetric power GARCH model examines this relationship in both developed and emerging nations. [4] used GARCH models to investigate this relationship in the G7 nations. [5] uses monthly inflation data for Turkey to analyse this relationship as well as the influence of monetary policy on this relationship. [6] uses data from the United Kingdom to analyse the impact of inflation using the predicted conditional volatility from symmetric, asymmetric, and component GARCH-M models of inflation, as well as the link between inflation uncertainty measures. Regarding inference and prediction, the Bayesian method is particularly well suited for GARCH models and offers certain benefits over traditional estimating methods, as described by [7].

Alternatively, recent research has used SV models, where inflation uncertainty is considered as a latent variable (not a directly observed variable) that follows an AR(1) process. Financial econometrics has also extensively researched SV models. The current literature has established the importance of the leverage effect, jump components, and heavy-tailed mistakes among its generalizations for financial time series [8,9]. In empirical investigations [10,11,12], different SV-jump model specifications are compared. [13] provides a comparison exercise to examine several GARCH and SV

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models. [14] propose to approximate the likelihood function of the SV model applying Hidden Markov Models (HMM). Regrettably, these two (GARCH, SV) categories of volatility models are not interconnected, and the inflationary volatilities they predict have quite distinct characteristics. Thus, it is impossible to compare these two models using traditional econometric techniques. It is important to directly assess the model fit of such two different volatility models by performing a formalized model comparison. Analyzing the relationship between inflation and its uncertainty requires careful modeling of inflation volatility. However, this kind of comparison is not often done in the literature. [15] analyzes the dynamics of inflation rates using a comparative Bayesian model method, like the Bayes factor, to evaluate the model's fit of widely used GARCH models compared to those of its SV equivalents.

Other research has looked into the connection between inflation and inflation uncertainty. [16] re-examines two hypotheses using a nonlinear flexible regression model for four east Asian economies in light of the causal connection between inflation and inflation uncertainty. [17] estimate a multivariate augmented GARCH mean model for the inflation and production growth at business in Mexico.

In order to analyze the behavior of inflation rates using a different Bayesian technique, this paper compares the fit of popular GARCH models and their SV alternatives. Specifically, three commonly used GARCH models for explaining inflation volatility are considered for the Bayesian model evaluation: GARCH in mean, asymmetric (or leverage) GARCH, and the conventional GARCH. Additionally, three stochastic volatility models that are similar to GARCH models are chosen: (1) ordinary SV, (2) SV with a leverage impact, and (3) SV in the mean. First, each model is fitted to the associated data using Rstan. Then, by conducting a comparative analysis of GARCH models and their SV counterparts, we can determine which model among the three GARCH and three SV models performs best. Furthermore, by explicitly comparing the GARCH-GJR and GARCH-M models with the basic one, we examine which model characteristics are critical for simulating the inflation dynamics, considering both the more complex SV variations and the basic SV. Model comparison is carried out using the leave-one-out cross-validation information criterion (LOOIC), and model fitting is done using Rstan using appropriate priors.

In this study, we are interested in examining how the inflation rates of the BRICS ("Brazil, Russia, India, China, and South Africa") nations change over time. The five big emerging economies known as "BRICS" are well known for having a significant influence on world affairs. [18] examines the significance of the BRICS group as developing market representatives in the global economy. It is noteworthy that the BRICS group performed significantly better economically than industrialized nations and that the financial crisis had little to no impact on them.

2 GARCH models

In this part, we present three widely used SV models for modeling inflation uncertainty.

2.1 GARCH models

We begin by considering a typical GARCH model, GARCH(1,1) model is defined as:

$$\pi_t = \alpha + \varepsilon_t \quad (1)$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$

$$\sigma_t^2 = \beta + \gamma\sigma_{t-1}^2 + \delta\varepsilon_{t-1}^2 \quad (2)$$

Where σ_0^2 is a constant, $\varepsilon_0 = 0$, $\beta > 0$, $\gamma \geq 0$, $\delta \geq 0$ and $\gamma + \delta < 1$, where π_t is the inflation rate. $\gamma + \delta < 1$ is the condition we set to ensure that the variance process is always stationary. It is evident that historical data and model parameters determine the conditional variance σ_t^2 , which serves as a proxy for inflation volatility.

2.2 Threshold GARCH model

The threshold GARCH model or GARCH-GJR model, created by [19], is a well-known GARCH model that is frequently used to explain inflation uncertainty. The influence of news, events, incidents, etc. on the decision-making of financial investors is significant and potent. Consequently, have an asymmetrical effect on financial markets. To capture this asymmetric effect [19] introduced GARCH-GJR model, the following is the definition of the conditional variance equation:

$$\sigma_t^2 = \beta + \gamma\sigma_{t-1}^2 + [\delta + \theta\mathbb{I}(\varepsilon_{t-1} < 0)]\varepsilon_{t-1}^2, \quad (3)$$

The GARCH-GJR model considers both positive and negative disturbances, capturing asymmetric (leverage) effects on the conditional variance.

The $\mathbb{I}(\cdot)$ is an indicator function that is equal to 1 if $\varepsilon_{t-1} < 0$ and 0 otherwise.

2.3 GARCH-M model

The third model is the GARCH in mean (GARCH-M) model, and its expression is provided below:

$$y_t = \alpha + \lambda \sigma_t^2 + \varepsilon_t \tag{4}$$

$$\sigma_t^2 = \beta + \gamma (y_{t-1} - \alpha - \lambda \sigma_{t-1}^2)^2 + \delta \sigma_{t-1}^2. \tag{5}$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma_t^2)$, and λ is the volatility feedback parameter which determines whether the data series is dependent on its own volatility (risk) or not. The GARCH-mean model reduces to the standard GARCH model when $\lambda = 0$.

3 Stochastic Volatility Models

In this part, we present three widely used SV models for modeling inflation uncertainty.

3.1 Stochastic Volatility Model

The expression for the first model, which is the conventional SV model, is provided below:

$$y_t = \alpha + \varepsilon_t^y, \tag{6}$$

$$h_t = \alpha_h + \rho_h (h_{t-1} - \alpha_h) + \varepsilon_t^h, \tag{7}$$

where $\varepsilon_t^y \sim \mathcal{N}(0, e^{h_t})$, $\varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2)$, with restriction $|\rho_h| < 1$ and α_h is the unconditional mean. The log-volatility h_t follows a stationary AR(1) process. The process is initialized with $h_1 \sim \mathcal{N}(\alpha_h, \sigma_h^2 / (1 - \rho_h^2))$.

3.2 SV-L model

Secondly, we examine the SV model with a leverage impact, which is the analog of the GARCH-GJR specification explained in [?]. Specifically, we consider the possibility of a correlation between the two error terms ε_t^π and ε_t^h as follows:

$$\pi_t = \alpha + \varepsilon_t^\pi \tag{8}$$

$$h_t = \alpha_h + \rho_h (h_{t-1} - \alpha_h) + \varepsilon_t^h \tag{9}$$

$$\begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^h \end{bmatrix} \sim N \left(\mathbf{0}, \begin{bmatrix} e^{h_t} & \rho e^{\frac{1}{2}h_t} \sigma_h \\ \rho e^{\frac{1}{2}h_t} \sigma_h & \sigma_h^2 \end{bmatrix} \right) \tag{10}$$

where ρ quantifies the leverage effect. The impact of leverage on equities returns will reduce as volatility increases and, for this, corresponds to a negative correlation between the leverage effect, which is defined as the rise in volatility that occurs after a decline in equity returns, corresponds to a negative correlation between ε_t^π and ε_t^h in this model. The SV-L model becomes the standard SV when $\rho = 0$.

3.3 SV-M model

The SV-M model proposed by [21] allows for the possibility of volatility feedback, similar to the GARCH-M model. In the mean equation, the SV-M model incorporates unobserved volatility as an explanatory variable, defined as:

$$\pi_t = \alpha + \lambda e^{h_t} + \varepsilon_t^\pi, \tag{11}$$

$$h_t = \alpha_h + \rho_h (h_{t-1} - \alpha_h) + \varepsilon_t^h, \tag{12}$$

where $\varepsilon_t^\pi \sim N(0, e^{h_t})$ and $\varepsilon_t^h \sim N(0, \sigma_h^2)$ both error terms are independent. The volatility feedback parameter, denoted by the term λ . When $\lambda > 0$, the inflation rate is positively impacted by inflation volatility, and the SV-M model turns into normal SV as $\lambda = 0$.

4 Model validation

Well-known Bayesian criteria like the leave-one-out (cross-validation) information criterion (LOOIC) and the Watanabe-Akaike information criterion (WAIC) [23] are commonly used. This part provides a brief introduction to LOOIC as used in this study. The WAIC [23] is an alternative that employs a more fully Bayesian method of criterion construction. According to [24], the WAIC is considered superior to the deviance information criterion (DIC).

$$LOOIC = -2 \sum_{i=1}^n \log \left(\int_{\boldsymbol{\theta}} p(y_i | \boldsymbol{\theta}) p(\boldsymbol{\theta} | y_{-i}) d\boldsymbol{\theta} \right) \quad (13)$$

4.1 Data

This analysis utilises quarterly CPI data from the Federal Reserve Economic Data for the BRICS economies. According to the data's availability, the period runs from 1960 Q1 to 2022 Q1 for India and South Africa, 1986 Q2 to 2022 Q1 for China, 1994 Q4 to 2022 Q1 for Brazil, and 1996 Q3 to 2022 Q1 for Russia. All data series are given seasonal adjustments to make the data stationary, and inflation is calculated using the following equation:

$$\pi_t = 400 * (\log CPI_t - \log CPI_{t-1}) \quad (14)$$

We model and analyse GARCH models and stochastic volatility models using this π_t .

5 Bayesian estimation and results

Both parameter estimates for GARCH type three models and explanations for SV type three models are provided in this section. Furthermore, for BRICS nation data, each model's stan convergence is explained. Section 7 defines the stan code for all six models. Garch and SV models converge for the aforementioned stan code for all five countries.

5.1 Bayesian Estimation of GARCH Models

Table 1 presents the expected results for the GARCH models. It is evident for all countries that the majority of estimated parameters across the three GARCH models are significantly different from zero. Let's take the results for Brazil as an example. In the typical GARCH model, the 95% credible interval for the parameter α is between (2.296, 2.671), which excludes zero and indicates that the estimate is statistically significant. The estimated value of α in this model is 2.485. The GARCH-GJR model yields a similar result to the GARCH model. However, the GARCH-M model has a substantially lower estimated value of α . This can be attributed to the presence of the parameter λ in the mean equation. There is a statistically significant difference between zero and the parameters δ and γ that describes the stability of the inflation volatility equation. The sum of the two parameters, δ and γ , falls between 0.78 and 0.92. Based on these values, we can conclude that the equation for inflation volatility is highly stable in each of the three variations. Similar results are obtained when this is applied to the remaining four nations. The nature of the inflation rates are then further investigated using model features. We start by thinking about the leverage effect. Its credible interval, which includes zero, the estimated value of θ for Brazil is -0.241 , which is not statistically distinct from zero, showing no asymmetric effect. Russia yields a similar outcome. Furthermore, the estimated value of θ for China, India, and South Africa is statistically significant, suggesting that bad news at time $t - 1$ and the coefficient of indicator function θ is negative, then inflation volatility will decrease at time t . There is mixed evidence for the leverage effect for the BRICS countries.

The volatility feedback's parameter λ is the last thing we look into. The volatility feedback's critical function in simulating inflation rates has been demonstrated. Let's use the outcomes for Brazil as an example. The statistically significant volatility parameter λ , calculated at 0.213, indicates that the inflation rate is positively impacted by inflation volatility. South Africa and China are reached the same conclusions. λ is not statistically significant for India or Russia. From Figure 1 to 15 displays the stan plot of GARCH models and the trace plot of all models shows that our model converges to the targetted distribution.

Table 1: GARCH model Bayesian estimation: estimation of the posterior means .

Countries	Models	α	β	δ	γ	θ	λ
Brazil	GARCH	2.485 (0.002)	0.225 (0.004)	0.645 (0.003)	0.133 (0.006)		
	GARCH-GJR	2.493 (0.001)	0.269 (0.002)	0.830 (0.004)	0.090 (0.002)	-0.241 (0.005)	
	GARCH-M	2.303 (0.004)	0.231 (0.002)	0.708 (0.004)	0.101 (0.003)		0.213 (0.003)
Russia	GARCH	5.653 (0.012)	7.958 (0.082)	0.821 (0.004)	0.073 (0.002)		
	GARCH-GJR	5.618 (0.006)	8.149 (0.044)	1.033 (0.018)	0.064 (0.001)	-0.187 (0.017)	
	GARCH-M	5.516 (0.010)	7.105 (0.048)	0.820 (0.003)	0.083 (0.002)		0.010 (.0001)
India	GARCH	2.807 (0.005)	1.242 (0.019)	0.330 (0.003)	0.487 (0.005)		
	GARCH-GJR	2.708 (0.003)	1.097 (0.010)	0.458 (0.003)	0.512 (0.003)	-0.263 (0.003)	
	GARCH-M	2.804 (0.027)	1.715 (0.020)	0.396 (0.006)	0.355 (0.005)		-0.014 (0.004)
China	GARCH	0.823 (0.002)	0.315 (0.003)	0.684 (0.002)	0.243 (0.003)		
	GARCH-GJR	0.845 (0.001)	0.393 (0.003)	1.122 (0.005)	0.181 (0.002)	-0.579 (0.006)	
	GARCH-M	0.632 (0.002)	0.366 (0.003)	0.694 (0.002)	0.255 (0.002)		0.114 (0.001)
South Africa	GARCH	2.454 (0.004)	0.001 (0.0001)	0.114 (0.001)	0.880 (0.001)		
	GARCH-GJR	2.324 (0.300)	0.341 (0.004)	0.300 (0.002)	0.695 (0.002)	-0.181 (0.002)	
	GARCH-M	0.901 (0.005)	0.097 (0.001)	0.106 (0.0001)	0.889 (0.001)		0.306 (0.001)

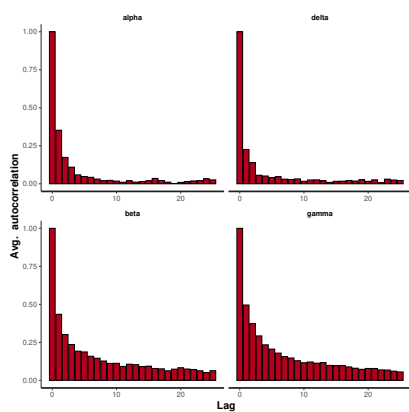


Fig. 1: Standard GARCH model for Brazil autocorrelation plot

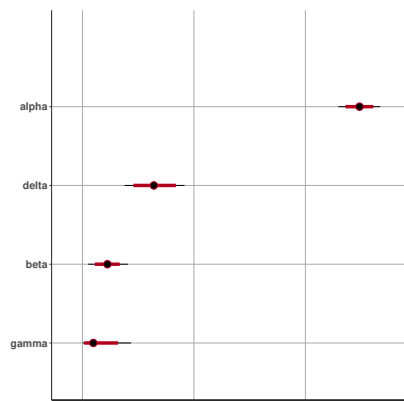


Fig. 2: Standard GARCH model for Brazil caterpillar plot

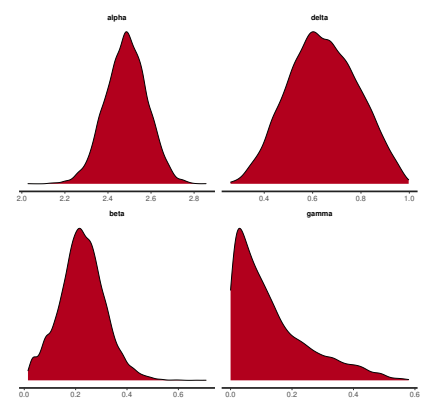


Fig. 3: Standard GARCH model for Brazil posterior density plot

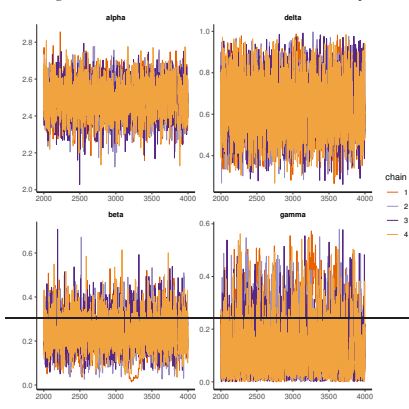


Fig. 4: Standard GARCH model for Brazil traceplot

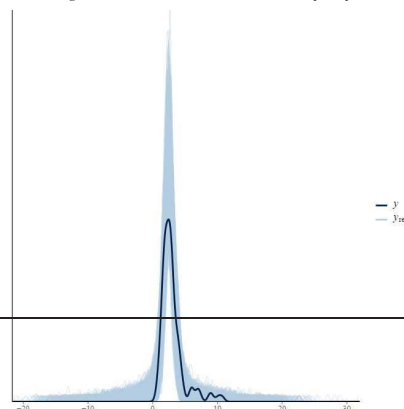


Fig. 5: Standard GARCH model for Brazil PPD plot

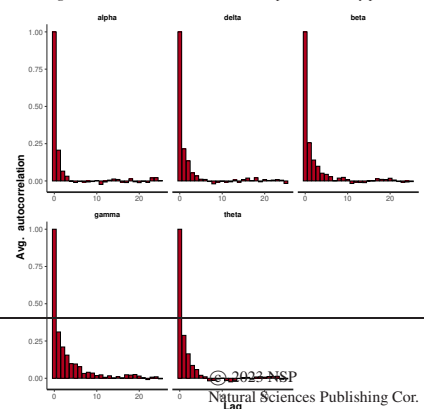


Fig. 6: GARCH-GJR model for Brazil autocorrelation plot

Table 2: SV model Bayesian estimation: estimation of the posterior means .

Countries	Models	α	α_h	ρ_h	σ_h^2	ρ	λ
Brazil	SV	2.435 (0.001)	1.306 (0.016)	0.973 (0.001)			
	SV-L	3.022 (0.006)	1.439 (0.013)	0.972 (0.001)	0.001 (0.0001)	0.496 (0.029)	
	SV-M	-4.732 (0.109)	2.108 (0.022)	0.975 (0.001)	0.109 (0.004)		0.136 (0.005)
Russia	SV	3.766 (0.004)	2.131 (0.004)	0.969 (0.0001)			
	SV-L	5.607 (0.015)	4.445 (0.14)	0.968 (.0001)	0.001 (0.0001)	0.689 (0.021)	
	SV-M	-4.920 (0.321)	1.801 (0.22)	0.963 (0.001)	0.170 (0.002)		0.208 (0.010)
India	SV	2.934 (0.002)	1.497 (0.006)	0.969 (0.001)			
	SV-L	3.061 (0.003)	2.182 (0.017)	0.972 (0.001)	0.001 (0.0001)	0.385 (0.003)	
	SV-M	-5.073 (0.124)	1.534 (0.007)	0.971 (0.001)	0.029 (0.001)		1.091 (0.022)
China	SV	0.831 (0.001)	1.250 (0.018)	0.969 (0.001)			
	SV-L	1.919 (0.006)	2.414 (0.009)	0.970 (0.0001)	0.001 (0.0001)	0.900 (0.010)	
	SV-M	-5.240 (0.869)	0.855 (0.027)	0.964 (0.0001)	0.166 (0.003)		0.344 (0.016)
South Africa	SV	2.322 (0.001)	1.103 (0.023)	0.976 (0.001)			
	SV-L	3.087 (0.007)	1.956 (0.029)	0.968 (0.001)	0.001 (0.0001)	0.820 (0.029)	
	SV-M	-6.116 (0.680)	0.897 (0.006)	0.981 (0.001)	0.019 (0.001)		2.132 (0.211)

5.2 Stochastic volatility models Bayesian estimation

From Table 2 the majority of the posterior estimates for all three SV versions are statistically significant, which is consistent with the results from the GARCH models. A posterior parameter estimate of ρ_h ranges between 0.96 to 0.98 implying the all models' ρ_h values fall within a small range .

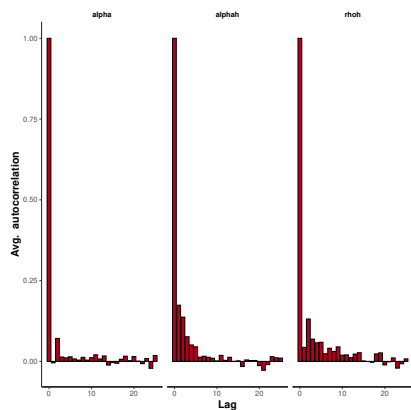


Fig. 16: standard SV model for Brazil autocorrelation plot

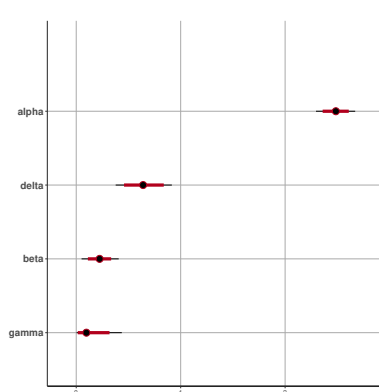


Fig. 17: standard SV model for Brazil caterpillar plot

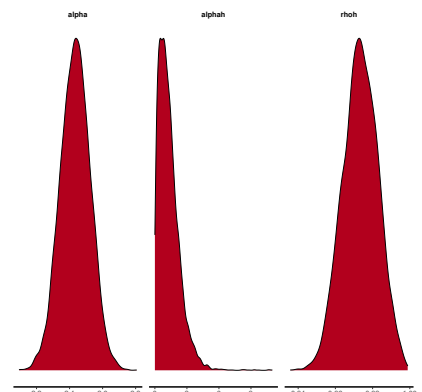


Fig. 18: standard SV model for Brazil posterior density plot

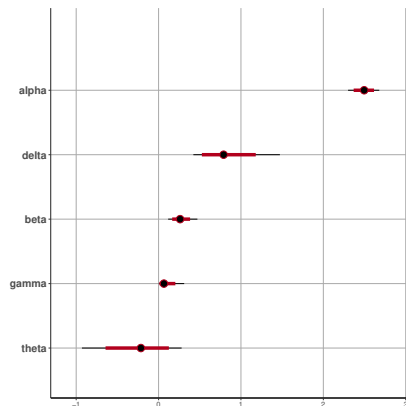


Fig. 7: GARCH-GJR model for Brazil caterpillar plot

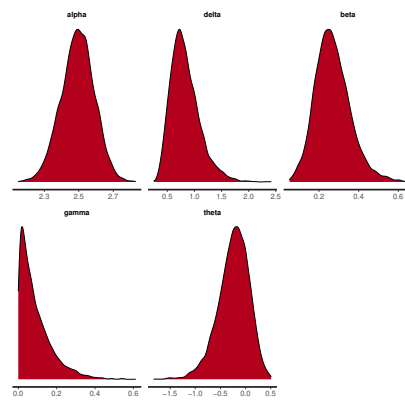


Fig. 8: GARCH-GJR model for Brazil posterior density plot

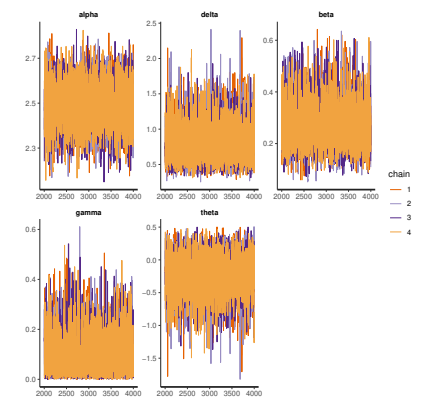


Fig. 9: GARCH-GJR model for Brazil traceplot

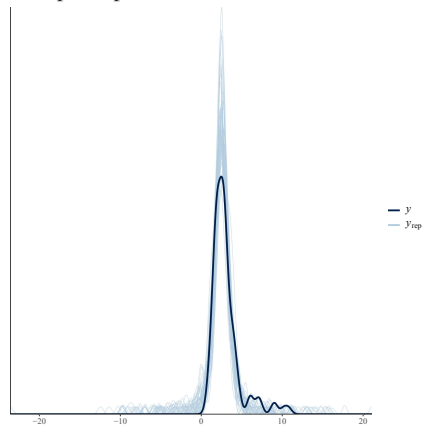


Fig. 10: GARCH-GJR model for Brazil PPD plot

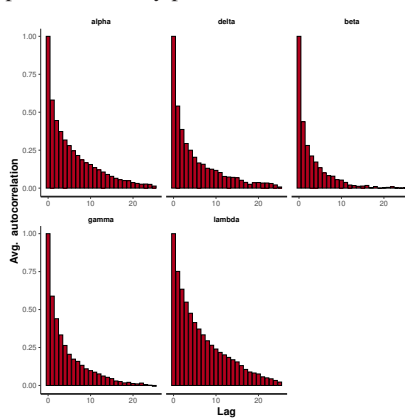


Fig. 11: GARCH M model for Brazil autocorrelation plot

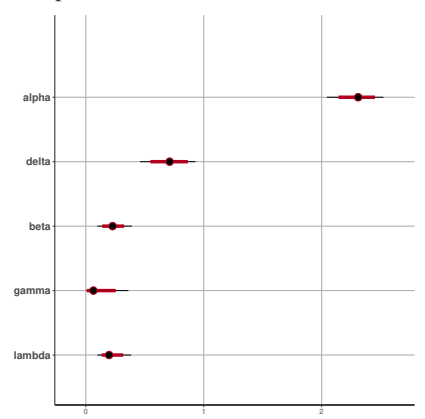


Fig. 12: GARCH M model for Brazil caterpillar plot

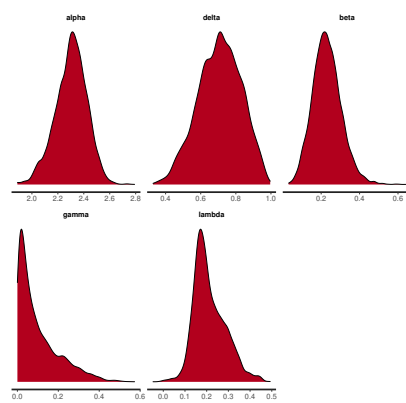


Fig. 13: GARCH M model for Brazil posterior density plot

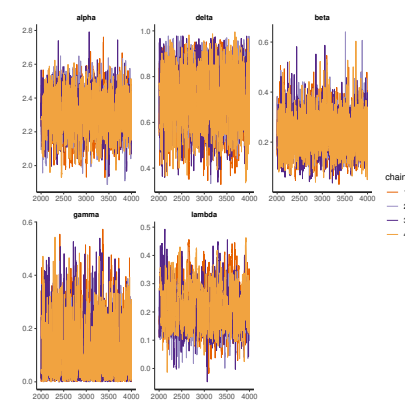


Fig. 14: GARCH M model for Brazil traceplot

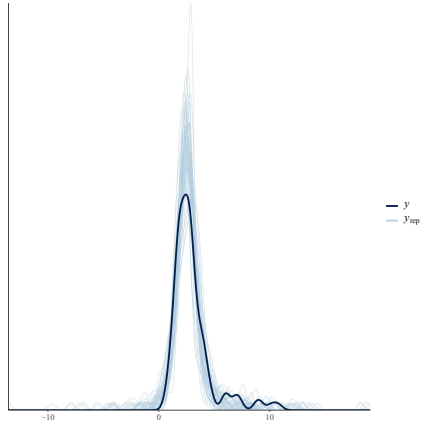


Fig. 15: GARCH M model for Brazil PPD plot

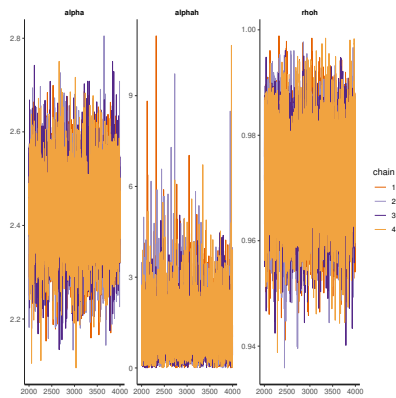


Fig. 19: standard SV model for Brazil traceplot

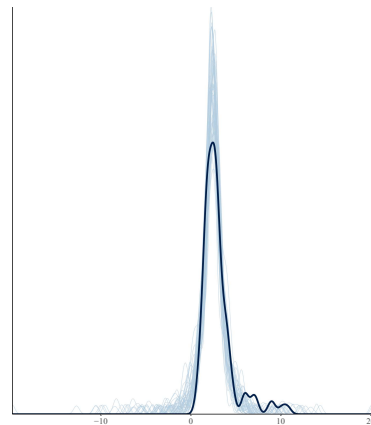


Fig. 20: standard SV model for Brazil PPD plot

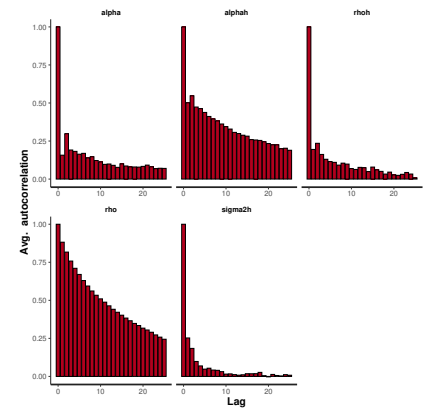


Fig. 21: SV-L model for Brazil autocorrelation plot

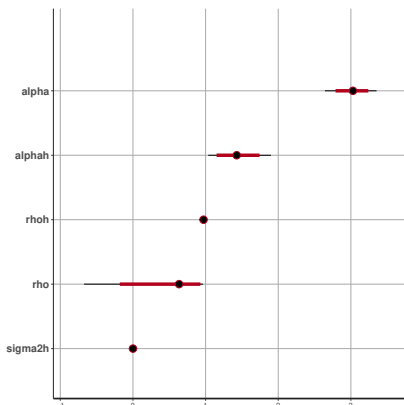


Fig. 22: SV-L model for Brazil caterpillar plot

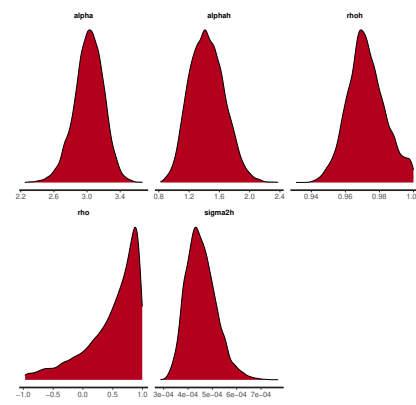


Fig. 23: SV-L model for Brazil posterior density plot

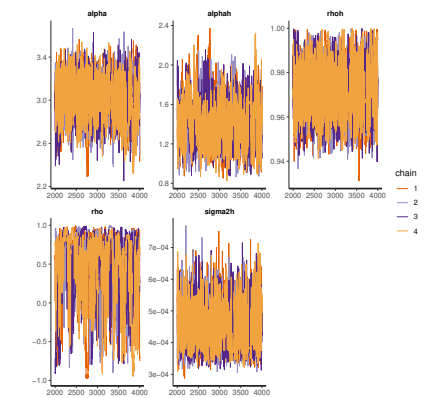


Fig. 24: SV-L model for Brazil trace plot

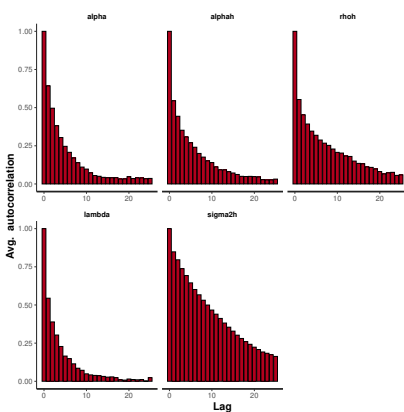


Fig. 25: SV-M model for Brazil autocorrelation plot

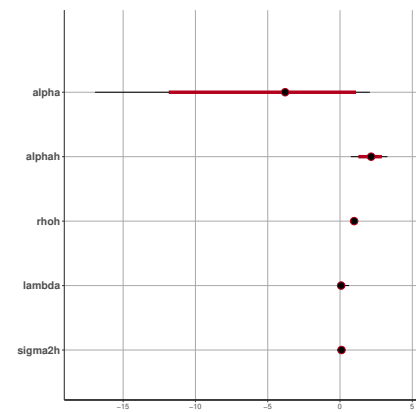


Fig. 26: SV-M model for Brazil caterpillar plot

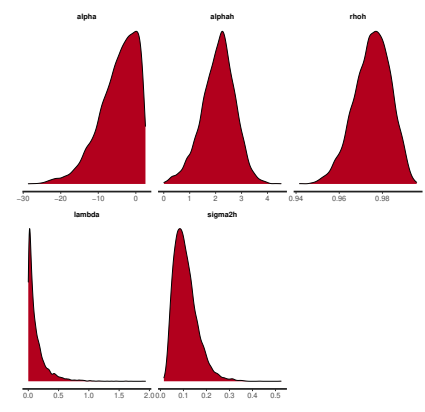


Fig. 27: SV-M model for Brazil posterior density plot

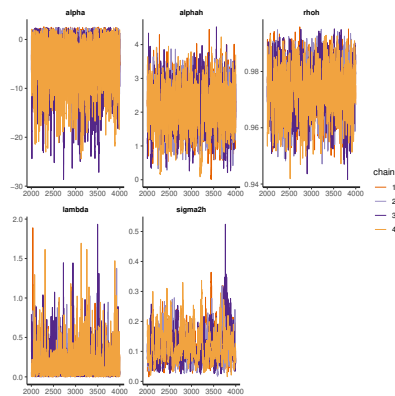


Fig. 28: SV-M model for Brazil traceplot

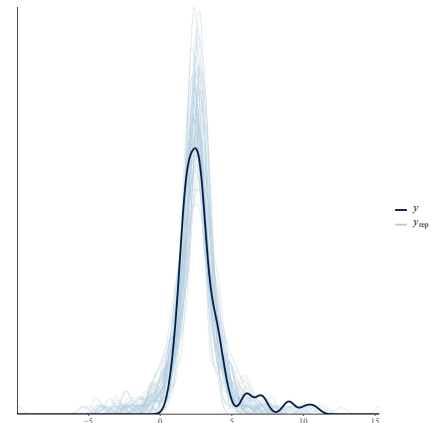


Fig. 29: SV-M model for Brazil PPD plot

We then consider the significance of the leverage effect in modeling inflation rates. The estimated parameter values of the SV model are comparable to those of its GARCH counterpart. In the SV-L model, the parameter ρ can be compared with its corresponding parameter in the GARCH-GJR model results. Notably, adding the leverage effect to Brazil leads to an increase in the marginal likelihood, with the 95% credible interval of the leverage effect parameter ρ excluding zero. Furthermore, a negative shock at time $t - 1$ reduces the volatility at time t , indicating a positive correlation ($\rho = 0.50$), which is consistent with the results of the GARCH-GJR model. The ρ value for the other four countries is also statistically significant.

Next, we examine the significance of inflation volatility feedback in illuminating inflation rate dynamics. For all five countries, it is calculated that the SV-M volatility feedback parameter λ is positive and significantly different from zero, similar to the results of GARCH-M. This highlights the importance of the parameter λ in modeling inflation rates. Additionally, compared to GARCH-M, the estimate of the volatility parameter λ under SV-M is substantially larger, indicating stronger volatility feedback. These results apply to all other countries considered.

Figures 16 to 29 display the stan plot of SV models, and the trace plot of all models shows convergence to the target distribution. The trace plot presents the evolution of each parameter over time for five distinct Markov chains, represented by different colors and combined in the same graph. This indicates that the Markov chains have converged to a common and consistent state. The PPD plot demonstrates that the model's predictions, based on the posterior distribution, are consistent with the data, indicating a good fit.

6 Analysis of models and recommendations

In this study, we employed a Bayesian estimator to assess three popular GARCH specifications and the SV models that they correspond to when modelling inflation rates for the BRICS nations. From Table 3 we discover that stochastic volatility models' looic value is lower than that of their corresponding GARCH models. The model with the lowest looic value is considered to be the best model. Therefore, when compared to the corresponding GARCH models, the three stochastic volatility models perform the best. Additionally, it is demonstrated that the SV-L model is the best one for each of the five nations because the SV-L model for all five nations has the lowest looic value among the stochastic volatility models. Finally, Nearly all GARCH models for the provided BRICS data show negligible variation among themselves, making it impossible to determine which model is best based on the LOOIC or WAIC value from this data.

7 Appendix A

Stan codes

stan code for the standard GARCH model defined in section (2.1) is given below:

```
Garch="
data{
int N;
vector[N]y;
```

Table 3: GARCH and SV model validation using Looic and Waic.

Countries	Models	LOOIC	WAIC	Models	LOOIC	WAIC
Brazil	SV	313.7	313.1	GARCH	314.7	314.3
	SV-L	53.6	40.1	GARCH-GJR	314.8	314.9
	SV-M	181.9	320	GARCH-M	359.6	360.3
Russia	SV	502.4	499.2	GARCH	625.5	631.1
	SV-L	340.9	318.9	GARCH-GJR	627.2	629.8
	SV-M	501.9	498	GARCH-M	626.3	629.5
India	SV	1107.6	1105.3	GARCH	1126	1125.9
	SV-L	148.8	134.6	GARCH-GJR	1124.9	1124.5
	SV-M	1112.9	1111.8	GARCH-M	1129.5	1130.4
China	SV	519.1	516.9	GARCH	534.0	533.9
	SV-L	108.8	86.2	GARCH-GJR	532.7	532.7
	SV-M	523.9	521.7	GARCH-M	523.2	522.7
South Africa	SV	1026.1	1024.5	GARCH	1050.9	1050
	SV-L	521.8	503.6	GARCH-GJR	981.4	1044
	SV-M	1031.3	1027.9	GARCH-M	1044.2	1045.5

```

real<lower=0>sigma1;
}
parameters{
real alpha;
real<lower=0> gamma;
real<lower=0,upper=1-gamma> delta;
real<lower=0> beta;
}
transformed parameters{
real<lower=0> sigma[N];
sigma[1]=sigma1;
for(n in 2:N){
sigma[n]=sqrt(beta+gamma*pow(sigma[n-1],2)+delta*pow(y[n-1]-alpha,2));
}
}
model{
alpha~normal(0,10);
gamma~normal(0,10)T[0,];
delta~normal(0,10)T[0,];
beta~normal(0,10)T[0,];
y~normal(alpha,sigma);
}
generated quantities{
vector[N] yrep;
real log_lik[N];
for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|alpha,sigma[n]);
}
for(n in 1:N){
yrep[n]=normal_rng(alpha,sigma[n]);
}

```

```
}}
"
```

stan code for GARCH-GJR model defined in section (2.2) is given below:

```
Garch_gjr="
data{
int N;
vector[N]y;
real<lower=0>sigmal;
}
parameters{
real alpha;
real theta;
real<lower=0> gamma;
real<lower=0, upper=1-gamma-theta> delta;
real<lower=0> beta;
}
transformed parameters{
real<lower=0> sigma[N];
sigma[1]=sigmal;
for(n in 2:N){
if((y[n-1]-alpha)<0){
sigma[n]=sqrt(beta+gamma*pow(sigma[n-1],2)+(delta+theta)*pow(y[n-1]-alpha,2));
}
else{
sigma[n]=sqrt(beta+gamma*pow(sigma[n-1],2)+delta*pow(y[n-1]-alpha,2));
}
}}
model{
alpha~normal(0,10);
theta~uniform(-delta,1-delta-gamma);
gamma~normal(0,10)T[0,];
delta~normal(0,10)T[0,];
beta~normal(0,10)T[0,];
y~normal(alpha,sigma);
}
generated quantities{
vector[N] yrepgjr;
real log_lik[N];
for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|alpha,sigma[n]);
}
for(n in 1:N)
yrepgjr[n]=normal_rng(alpha,sigma[n]);
}
"
```

stan code for GARCH-M model defined in section (2.3) is given below:

```
Garch_m="
data{
int N;
vector[N]y;
real<lower=0>sigmal;
}
parameters{
real alpha;
real<lower=0>gamma;
real<lower=0, upper=1-gamma>delta;
real<lower=0>beta;
```

```

real lambda;
}
transformed parameters{
real<lower=0>sigma[N];
vector[N]mu;
sigma[1]=sigma1;
mu[1]=(sigma1)^2;
for(n in 2:N){
sigma[n]=sqrt(beta+gamma*pow(sigma[n-1],2)+
delta*pow(y[n-1]-alpha-lambda*pow(sigma[n-1],2),2));
}
for(n in 2:N){
mu[n]=beta+gamma*mu[n-1]+delta*pow(sigma[n-1],2);
}
}
model{
alpha~normal(0,10);
lambda~normal(0,100);
gamma~normal(0,10)T[0,];
delta~normal(0,10)T[0,];
beta~normal(0,5)T[0,];
y~normal(alpha+lambda*mu, sigma);
}
generated quantities{
vector[N] yrepgm;
real log_lik[N];
for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|alpha+lambda*mu[n], sigma[n]);
}
for(n in 1:N)
yrepgm[n]=normal_rng(alpha+lambda*mu[n], sigma[n]);
}
"

```

stan code for standard SV model defined in section (3.1) is given below:

```

SV="
data{
int N;
vector[N]y;
}
parameters{
real<lower=0>h1;
real alpha;
real<lower=-1,upper=1> rhoh;
vector[N]eh;
real<lower=0> sigma2h;
real<lower=0> alphah;
}
transformed parameters{
real<lower=0> sigma[N];
real h[N];
h[1]=h1;
for(n in 2:N){
h[n]=alphah+rhoh*(h[n-1]-alphah)+eh[n];
}
for(n in 1:N){
sigma[n]=sqrt(exp(h[n]));
}
}

```

```

model{
  alpha~normal(0,10);
  alphah~normal(0,10);
  h1~normal(alphah,sqrt(sigma2h/(1-rhoh^2)));
  rhoh~normal(0.97,0.01)T[-1,1];
  eh~normal(0,sqrt(sigma2h));
  sigma2h~inv_gamma(5,0.16)T[0,];
  y~normal(alpha,sigma);
}
generated quantities{
  vector[N] yrepgsv;
  real log_lik[N];
  for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|alpha,sigma[n]);}
  for(n in 1:N)yrepgsv[n]=normal_rng(alpha,sigma[n]);}
"

```

stan code for SV-L model defined in section (3.2) is given below:

```

SV_L="
data{
  int N;
  row_vector[N]y;
}
parameters{
  real alpha;
  real<lower=0>h1;
  real<lower=0> alphah;
  real<lower=-1,upper=1>rhoh;
  real<lower=-1,upper=1>rho;
  vector[N]eh;
  real<lower=0>sigma2h;
}
transformed parameters{
  matrix[2,2] cov[N];
  real<lower=0> sigma[N];
  real h[N];
  vector[N] ep;
  vector[2] Y[N];
  vector[2]mu[N];
  for(n in 1:N){
  mu[n][1]=0;
  mu[n][2]=0;
  }
  for(n in 1:N){
  ep[n]=y[n]-alpha;
  }
  for(n in 1:N){
  Y[n][1]=ep[n];
  Y[n][2]=eh[n];
  }
  h[1]=h1;
  for(n in 2:N){
  h[n]=alphah+rhoh*(h[n-1]-alphah)+eh[n];
  }
  for(n in 1:N){
  sigma[n]=sqrt(exp(h[n]));
  }
}

```

```

for(n in 1:N){
cov[n][1,1]=(sigma[n])^2;
cov[n][1,2]=rho*exp((1.0/2)*h[n])*sqrt(sigma2h);
cov[n][2,1]=rho*exp((1.0/2)*h[n])*sqrt(sigma2h);
cov[n][2,2]=sigma2h;
}
}
model{
alpha~normal(0,10);
alphah~normal(0,10);
h1~normal(alphah,sqrt(sigma2h^2/(1-rhoh^2)));
rhoh~normal(0.97,0.01)T[-1,1];
rho~uniform(-1,1);
eh~normal(0,sqrt(sigma2h));
sigma2h~inv_gamma(5,0.16)T[0,];
for(n in 1:N){
target+=multi_normal_lpdf(Y[n]|mu[n],cov[n]);
}
generated quantities{
real log_lik[N];
vector[2] yrep[N];
for(n in 1:N){log_lik[n]=multi_normal_lpdf(Y[n]|mu[n],cov[n]);}
for(n in 1:N){yrep[n]=multi_normal_rng(mu[n],cov[n]);}
}
"

```

stan code for SV-M model defined in section (3.3) is given below:

```

SV_M="
data{
int N;
vector[N]y;
}
parameters{
real alpha;
real<lower=0>h1;
real<lower=0>alphah;
real<lower=0>lambda;
real<lower=-1,upper=1>rhoh;
vector[N]eh;
real<lower=0>sigma2h;
}
transformed parameters{
real mu;
real<lower=0>sigma[N];
real h[N];
h[1]=h1;
for(n in 2:N){
h[n]=alphah+rhoh*(h[n-1]-alphah)+eh[n];
}
for(n in 1:N){
sigma[n]=sqrt(exp(h[n]));
}
mu=alpha+lambda*exp((alphah+(sigma2h/2)-rhoh*alphah)/(1-rhoh));
}
model{
alpha~normal(0,10);
alphah~normal(0,10);

```

```

h1~normal(alpha, sqrt(sigma2h^2/(1-rhoh^2)));
lambda~normal(0,100)T[0,];
rhoh~normal(0.97,.01)T[-1,1];
eh~normal(0,sqrt(sigma2h));
sigma2h~inv_gamma(5,0.016)T[0,];
y~normal(mu,sigma);
}
generated quantities{
vector[N] yrepgmsv;
real log_lik[N];
for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|mu,sigma[n]);}
for(n in 1:N)yrepgmsv[n]=normal_rng(mu,sigma[n]);}
}

```

Garch and SV models are converging for the aforementioned stan code for all five countries

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