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A weakly nonlinear Ion-acoustic waves in magnetized electron-positron plasma: a fractional model

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Abstract: In this paper, we investigate the optical soliton solutions for a fractional weakly nonlinear ion-acoustic wave in a magnetized electron–positron plasma using the fractional modified Korteweg–deVries–Zakharov–Kuznetsov (f-mKdV-ZK) model. The fractional calculus framework is employed to describe the non-local effects arising from the long-range interactions and memory effects in the plasma medium. The presence of a magnetic field introduces additional complexities to the dynamics of ion-acoustic waves in electron–positron plasmas. We derive the governing equations for the f-mKdV-ZK model and employ the reductive perturbation method to obtain the corresponding optical soliton solutions. The obtained soliton solutions reveal the influence of fractional order, weak nonlinearity, and magnetic field on the characteristics of the ion-acoustic waves. The results demonstrate the formation and propagation of stable optical solitons in the magnetized electron–positron plasma and provide insights into the fundamental behavior of such systems. This study contributes to the understanding of nonlinear wave dynamics in fractional plasmas and offers potential applications in various plasma physics and astrophysical scenarios.

Keywords: Plasma physics; Optical soliton; Fractional calculus; KdV equation; Zakharov-Kuznetsov equation.

1 Introduction

Due to its importance in comprehending a broad range of physical processes in laboratory and astrophysical plasmas, the study of nonlinear waves in plasma physics has attracted considerable attention [1,2]. The interaction between ions and charged particles, which characterizes plasma systems, is defined by ion-acoustic waves, which are one of these. Due to its applications in laboratory experiments and astrophysical conditions, such as pulsar magnetospheres and compact star objects, the study of ion-acoustic waves in magnetized electron-positron plasmas has gained attention recently. Nonlinear partial differential equations (PDEs) that represent the interaction of diverse physical processes, such as dispersion, nonlinearity, and dissipation, are frequently used to describe the behavior of ion-acoustic waves in plasmas [3,4]. The Zakharov-Kuznetsov (ZK) equation, see [5,6], and the Korteweg-deVries (KdV) equation, see [7,8], are two basic models for weakly nonlinear ion-acoustic waves in plasma systems. These equations have shed important light on how solitons, or single waves, develop and move across plasmas. Traditional KdV and ZK equations, however, ignore several significant physical factors, including memory effects in plasma systems and fractional-order derivatives resulting from long-range interactions. These non-local and memory effects in various physical systems have been captured using the framework of fractional calculus, which extends the idea of differentiation and integration to non-integer orders. The dynamics of waves and solitons in fractional plasmas have been studied using fractional calculus in the context of plasma physics [9-11].

The optical soliton solutions for a fractional weakly nonlinear ion-acoustic wave in a magnetized electron-positron plasma are the main topic of this research. The fractional modified Korteweg-deVries-Zakharov-Kuznetsov (f-mKdV-ZK) equation that we suggest incorporates fractional calculus operators is a modified version of the KdV-ZK equation. The f-mKdV-ZK equation was created by taking into account a homogeneous magnetized component of an electron-positron plasma made up of cool and hot

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electrons as well as positrons with equal temperatures [12]. The density of the positrons with hot electrons and similar temperatures is given by [12]:

$$n_p = P_h Exp\left(-\frac{eg}{KE_h}\right), n_e = E_h Exp\left(\frac{eg}{KE_h}\right), \quad (1)$$

where K is the Boltzmann constant, e is the electron charge, P_h and E_h are hot positrons and hot electrons, respectively, while $g(t,x,y,z)$ is the electric field potential. The difference in the number density of hot electrons and positrons of equal temperatures can be expressed as [12]:

$$n_p - n_e = -2P_h \sinh\left(\frac{eg}{KP_h}\right) = -\frac{1}{4\pi e}\nabla^2 g, \quad (2)$$

where Poisson's equation has been utilized to get the second equality. In this paper, we deal with the following space-time f-mKdV-ZK equation [13]:

$$\begin{aligned} &\mathcal{D}_t^\eta g + mg^2 \mathcal{D}_x^\eta g + \mathcal{D}_x^\eta \mathcal{D}_x^\eta \mathcal{D}_x^\eta g \\ &\quad + \mathcal{D}_x^\eta \mathcal{D}_y^\eta \mathcal{D}_y^\eta g + \mathcal{D}_x^\eta \mathcal{D}_z^\eta \mathcal{D}_z^\eta g = 0, \end{aligned} \quad (3)$$

where $\mathcal{D}_\tau^\eta$is the local fractional derivative of order $0 < \eta < 1$, x, y and z are the scaled space variables, t is the temporal variable and m is a dispersion coefficient. The amount of work required to move an electric unit charge from a reference point in an electric field $\overrightarrow{\mathcal{E}}$ to a particular spot is known as the electric field potential. The electric potential in the electric field $\overrightarrow{\mathcal{E}}$ at a location r can be expressed as following from line integrals [14]:

$$g = -\int_C \overrightarrow{\mathcal{E}}\, dl, \quad (4)$$

where C is an arbitrary path from some fixed reference point to **r.** We utilize the gradient theorem to obtain:

$$\overrightarrow{\mathcal{E}} = -\nabla g = -\frac{\partial g}{\partial x}\overrightarrow{e_x} - \frac{\partial g}{\partial y}\overrightarrow{e_y} - \frac{\partial g}{\partial z}\overrightarrow{e_z}. \quad (5)$$

The Maxwell-Faraday equation states that a non-conservative and spatially variable electric field is accompanied by a time-varying magnetic field. In the Maxwell-Faraday equation, the electric field $\overrightarrow{\mathcal{E}}$ can be used to determine the magnetic field $\overrightarrow{\mathcal{B}}$ [15]:

$$\nabla \times \overrightarrow{\mathcal{E}} = -\frac{\partial \overrightarrow{\mathcal{B}}}{\partial t}, \quad (6)$$

where $\nabla\times$ is the curl operator. With the help of this model, we can explore how the non-local and memory effects in the plasma medium affect the properties of ion-acoustic waves. The dynamics of ion-acoustic waves in electron-positron plasmas are further complicated by the presence of a magnetic field. The dispersion and nonlinearity of the waves are impacted by the interaction between charged particles and the magnetic field, creating novel wave shapes and occurrences [16,17]. We study the combined impacts of fractional calculus, weak nonlinearity, and magnetic field on the production and propagation of solitons by considering the fractional weakly nonlinear ion-acoustic wave in a magnetized electron-positron plasma.

The paper is organized to be an introduction as a first section. An overview of key ideas that are important to our work will be presented in Section 2. In Section 3, we present a mathematical analysis of the governing equation by utilizing a suitable transformation to translate the fractional equation into a nonlinear integer-order ordinary differential equation. Section 4 presents the main results of this work, where we will construct the optical soliton solution for the governing equation and introduce the study of some physical concepts that are related to the governing equation. Finally, some conclusions are listed in Section 5.

2 Preliminaries

By offering a potent mathematical framework to describe phenomena involving non-locality, memory effects, power-law behavior, and fractal features, fractional calculus plays a significant role in physics. Its uses are diverse and include wave propagation, power-law correlations, anomalous diffusion, and quantum mechanics. Fractional calculus can help scientists better comprehend complex processes and open new lines of inquiry by being incorporated into the modeling and analysis of physical systems [18,19]. This section is dedicated to providing an overview of key ideas that are important to our work. The definition of local fractional derivative (LFD) is given along with a few of its crucial features. For a function $g(\tau) \in \mathcal{C}_\eta(a,b)$, we have $|g(\tau) - g(\tau_0)| < \varepsilon^\eta$, where $|\tau - \tau_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in \mathbb{R}$ [19].

Definition 1 [20]. Let $g(\tau) \in \mathcal{C}_\eta(a,b)$. Then LFD of order $\eta, 0 < \eta < 1$, of the function $g(\tau)$ at the point $\tau = \tau_0$ is defined as

$$\mathcal{D}_\tau^\eta g(\tau_0) = \frac{d^\eta g(\tau_0)}{d\tau^\eta} = \lim_{\tau \to \tau_0} \frac{\Delta^\eta (g(\tau) - g(\tau_0))}{(\tau - \tau_0)^\eta}, \quad (7)$$

where

$$\Delta^\eta (g(\tau) - g(\tau_0)) \cong \Gamma(1+\eta) \Delta (g(\tau) - g(\tau_0)). \quad (8)$$

The following theorem presents the substantial properties of the LFD.

Theorem 1 [20]. Let $g_1(\tau), g_2(\tau) \in \mathcal{C}_\eta(a,b)$. Then the attached properties are attained:

$(p1)\mathscr{D}_{\tau}^{\eta}\left[g_1(\tau)\pm g_2(\tau)\right]=\mathscr{D}_{\tau}^{\eta}g_1(\tau)\pm\mathscr{D}_{\tau}^{\eta}g_2(\tau).$
$(p2)\mathscr{D}_{\tau}^{\eta}\left[g_1(\tau)g_2(\tau)\right]=\mathscr{D}_{\tau}^{\eta}\left(g_1(\tau)\right)g_2(\tau)+g_1(\tau)\mathscr{D}_{\tau}^{\eta}g_2(\tau).$
$(p3)\mathscr{D}_{\tau}^{\eta}\left[\frac{g_1(\tau)}{g_2(\tau)}\right]=\frac{\mathscr{D}_{\tau}^{\eta}(g_1(\tau))g_2(\tau)-g_1(\tau)\mathscr{D}_{\tau}^{\eta}g_2(\tau)}{(g_2(\tau))^2},g_2(\tau)\neq 0.$
$(p4)\mathscr{D}_{\tau}^{\eta}\left[g_1(\tau)\circ g_2(\tau)\right]=g_1^{(1)}\left(g_2(\tau)\right)\mathscr{D}_{\tau}^{\eta}g_2(\tau).$
$(p5)\mathscr{D}_{\tau}^{\eta}\tau^{n\eta}=\frac{\Gamma(1+n\eta)}{\Gamma(1+(n-1)\eta)}\tau^{(n-1)\eta}.$

The reader can refer to [21-24] for more details about local fractional calculus.

3 Mathematical analysis

Here, we convert the model (3) into a nonlinear ordinary differential equation (ODE) of integer order by use of an appropriate traveling wave transformation. In order to achieve this, we take into account the following complex fractional traveling wave transformation:

$$g(t,x,y,z)=G(\xi),$$

$$\xi=\frac{\beta_1}{\Gamma(1+\eta)}x^{\eta}+\frac{\beta_2}{\Gamma(1+\eta)}y^{\eta}+\frac{\beta_3}{\Gamma(1+\eta)}z^{\eta}-\frac{\alpha}{\Gamma(1+\eta)}t^{\eta}, \tag{9}$$

where β_1,β_2,β_3 and α are constants. The constant α refer to the speed of the traveling wave. The function $G(\xi)$ is the wave shape. Using the chain rule of the LFD, p4 in Theorem 1, and transformation in (9), the following relations are obtained:

$$-\alpha\frac{dG}{d\xi}+\beta_1 mG^2\frac{dG}{d\xi}+\left(\beta_1^3+\beta_1\beta_2^2+\beta_1\beta_3^2\right)\frac{d^3G}{d\xi^3}=0. \tag{10}$$

By integrating (10) with respect to ξ and considering the integrating constant be zero, we have:

$$-\alpha G+\frac{\beta_1 m}{3}G^3+\left(\beta_1^3+\beta_1\beta_2^2+\beta_1\beta_3^2\right)\frac{d^2G}{d\xi^2}=0. \tag{11}$$

We are using this nonlinear ODE to construct the desired optical soliton solutions for the governing model (3).

4 Optical soliton solutions

Self-sustaining wave packets known as optical solitons keep their shape and speed while traveling through a medium. These extraordinary things are very important in physics because of their special characteristics. First of all, optical solitons have exceptional stability, which enables them to travel over great distances without dispersion or distortion. Due to their durability and ability to transmit information without deterioration, they are essential in high-capacity optical communication systems. Second, optical solitons have a balance between nonlinearity and dispersion, which enables them to self-focus and self-trap. Due to this characteristic, they are able to combat light's inclination to diffuse naturally and create powerful, locally focused pulses. Additionally, the study of nonlinear phenomena, including nonlinear dynamics, nonlinear wave propagation, and nonlinear optics, has been aided by the use of optical solitons. They give researchers a base for analyzing complicated wave interactions, nonlinear scattering, and pattern generation. Also, optical solitons are useful in many different disciplines. They have been used to limit signal degradation and enable effective signal processing in fiber optics for long-distance data transfer. Ultrafast lasers, all-optical switches, and optical data storage systems have all used soliton-based devices [25,26]. In order to solve the space-time fractional model precisely, we aim to create bright and kink soliton solutions for the model (3) using a suitable assumption of the solutions for ODE (11).

4.1 Bright soliton solutions

We assume that the nonlinear ODE (11) has a solution in the form:

$$G(\xi)=\frac{A\,\mathrm{sech}(k\xi)}{\sqrt{1+B\,\mathrm{sech}^2(k\xi)}}, \tag{12}$$

where A,B and k are constants to be determined by substitution the solution (12) into (11) and solve the obtained algebraic system after some mathematical simplifications. We construct it as:

$$\alpha=k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right), \tag{13}$$

$$A=\pm\sqrt{\frac{-6k^2\left(\beta_1^2+\beta_2^2+\beta_3^2\right)}{m}}, \tag{14}$$

$$B=1, \tag{15}$$

where k is arbitrary and provided that $m<0$. Using this results in (13-15), the wave profile can be given as:

$$G_{1,2}(\xi)=\frac{\pm\sqrt{\frac{-6k^2(\beta_1^2+\beta_2^2+\beta_3^2)}{m}}\,\mathrm{sech}(k\xi)}{\sqrt{1+\mathrm{sech}^2(k\xi)}}. \tag{16}$$

For researching and forecasting the behavior of waves in plasma systems, it is essential to comprehend how the dispersion coefficient affects the governing equation. Researchers can affect wave propagation, soliton dynamics, and wave interactions by regulating or changing the dispersion coefficient, which advances knowledge and the creation of plasma physics applications. The wave profile (16) is plotted in Figure 1. We consider the constants $\beta_i=1$ for $i=1,2,3$, while the parameter k and the dispersion coefficient m have considered in different values to show their impact on the behavior of the obtained bright wave. We clearly notice that the waveform is affected by the change in both parameters, whether by expanding or narrowing the wave packets during their propagation. The electric field potential can be given using the obtained wave profile (16):

$$g_{1,2}(t,x,y,z)=\pm\sqrt{\frac{-6k^2(\beta_1^2+\beta_2^2+\beta_3^2)}{m}}\times\frac{\operatorname{sech}\left(k\left(\frac{\beta_1}{\Gamma(1+\eta)}x^\eta+\frac{\beta_2}{\Gamma(1+\eta)}y^\eta+\frac{\beta_3}{\Gamma(1+\eta)}z^\eta-\frac{\left(k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right)\right)}{\Gamma(1+\eta)}t^\eta\right)\right)}{\sqrt{1+\operatorname{sech}^2\left(k\left(\frac{\beta_1}{\Gamma(1+\eta)}x^\eta+\frac{\beta_2}{\Gamma(1+\eta)}y^\eta+\frac{\beta_3}{\Gamma(1+\eta)}z^\eta-\frac{\left(k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right)\right)}{\Gamma(1+\eta)}t^\eta\right)\right)}}. \tag{17}$$

This electric field potential is depicted in Figure 2. We consider the parameters $\beta_1=0.2,\beta_2=0.1,\beta_3=1,m=1,k=1$ at integer derivative order $\eta=1$. The fractional order directly affects the shape of the implicit bright soliton (16). To show this impact, we present Figure 3 at fractional derivative order $\eta=0.99$ and $\eta=0.9$. The bright soliton is not clearly shown in Figure 3 due to the effect of the fractional derivative on it. It should also be noted here that the definition used in this work, the LFD definition, its effect differs from the effect of other definitions of the fractional derivative such as the truncated M-fractional derivative definition in [27], and this leads us to the fact that the physical interpretation of the fractional derivative may differ according to the definition used. For more illustration, we depict the electric field potential $g(t,x,y,z)$ in the 2D plot at different fractional derivative orders in Figure 4, while the dispersion coefficient m is considered at $m=1$ and $m=2$.
The electric field $\overrightarrow{\mathscr{E}}$ can be obtained using the electric field potential $g(t,x,y,z)$ in (17) with the aid of (5). It is crucial to notice that the specific effects of the fractional derivative on the electric field rely on the specific plasma system being considered, including its unique features and governing equations. With the addition of the fractional derivative, the description of the electric field dynamics becomes more flexible and complicated, improving the ability to accurately depict non-local and memory effects in some plasma systems. The electric field $\overrightarrow{\mathscr{E}}$ can be obtained using the electric field potential $g(t,x,y,z)$ with the aid of (5). It is crucial to notice that the specific effects of the fractional derivative on the electric field rely on the specific plasma system being considered, including its unique features and governing equations. With the addition of the fractional derivative, the description of the electric field dynamics becomes more flexible and complicated, improving the ability to accurately depict non-local and memory effects in some plasma systems. Figure 5 show the electric field $\overrightarrow{\mathscr{E}}$ that that related to (17) at $\beta_1=1,\beta_2=1,\beta_3=1,m=0.1,k=0.1$ such that the fractional derivative order is consider $\eta=1$ and $\eta=0.5$.
Within the context of the mKdV-ZK equation in plasma physics, the difference in the number density of hot electrons and positrons of equal temperatures has a significant bearing. This difference in number density denotes a fundamental asymmetry in the plasma system, which can result from a variety of physical causes, such as different energy sources or different electron and positron transport parameters. Understanding and quantifying this asymmetry are essential steps in understanding the plasma's general behavior. Hot electron and positron interactions are nonlinear because there is a variation in number density between the two species. These interactions might take the form of particle entrapment, wave-particle interactions, or the creation of brand-new waves and structures. The strength and form of these interactions are directly influenced by the difference in number density, which has an impact on the plasma's overall dynamics. The mKdV-ZK equation's main focus, the excitation and propagation of waves inside the plasma system, is also influenced by the number density differential. The difference in number density has an impact on system-specific instabilities and wave excitations. Mode conversion, parametric instabilities, or the activation of electromagnetic or electrostatic waves with specific properties are a few examples. Therefore, understanding how the number density difference affects these wave occurrences is essential for forecasting and studying them. Additionally, the transit of hot electrons and positrons inside the plasma is impacted by the number density imbalance. This difference drives particle fluxes, diffusion, and heat conduction, which in turn affects plasma transport characteristics and overall energy transfer. The system's complex interplay between particle dynamics, wave propagation, and energy redistribution is influenced by the discrepancy in number density. The difference in the number density of hot electrons and positrons of equal temperatures can be obtained using (2). We depict this difference in Figure 6 at $\beta_1=1,\beta_2=1,\beta_3=1,k=1$ and the fractional derivative order considered at different values to show its impact on the constructed results. In Figure 6, we notice that the

difference in the number of hot electrons and positrons with equal temperatures is affected by the fractional derivative when changing the order of the derivative. Also, the dispersion coefficient m has a prominent role in this change in general.

4.2 Kink soliton solutions

To construct a kink soliton solution for the f-mKdV-ZK model (3), we assume that the ODE (11) has solution in the form:

$$G(\xi) = \frac{A\sinh(k\xi)}{\sqrt{B+\sinh^2(k\xi)}}, \tag{18}$$

Substituting (18) into ODE (11) and some mathematical simplification led us to obtain an algebraic system. Solving the obtained algebraic system gives us the following result:

$$\alpha = -2k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right), \tag{19}$$

$$A = \pm\sqrt{\frac{-6k^2\left(\beta_1^2+\beta_2^2+\beta_3^2\right)}{m}}, \tag{20}$$

$$B = 1, \tag{21}$$

where k is arbitrary and provided that the dispersion coefficient $m<0$. The presence of anomalous dispersion in the medium where the wave is propagating is indicated when the dispersion coefficient is negative. Shorter wavelength waves propagate more quickly than longer wavelength waves, a phenomenon known as abnormal dispersion. The dispersion coefficient describes the relationship between the wave's frequency and wavenumber in terms of wave propagation. A negative dispersion coefficient indicates that higher frequency (shorter wavelength) waves travel farther through the medium than lower frequency (longer wavelength) waves. The effects of this behavior on wave phenomena are significant. The negative dispersion coefficient in the case of pulses or wave packets leads the higher frequency wave components to advance before the lower frequency wave components. As a result, over time, the pulse or wave packet may spread or stretch.

Using result (19-21), the wave profile $G(\xi)$ can be given as:

$$G_{1,2}(\xi) = \frac{\pm\sqrt{\frac{-6k^2\left(\beta_1^2+\beta_2^2+\beta_3^2\right)}{m}}\sinh(k\xi)}{\sqrt{1+\sinh^2(k\xi)}}. \tag{22}$$

When using the traveling wave transformation, the wave profile is crucial. It fulfills numerous essential functions during the investigation, enabling a thorough comprehension of the wave behavior and its ramifications. The significance of the wave profile is highlighted as: by providing a thorough description of the wave's shape, amplitude, and temporal and spatial properties, the wave profile makes it possible to characterize wave behavior. This thorough comprehension of the wave's characteristics assists in comprehending the underlying physical events. Additionally, the wave profile is crucial in determining the various types of waves. Since each of these wave types exhibits unique characteristics in their profiles, it is possible to discern between solitons, kink waves, periodic waves, and other wave types by looking at the profile. For linking wave patterns with the corresponding physical processes, this identification is essential. The investigation of wave stability and propagation is made easier by the wave profile. Determining whether the wave maintains its form and amplitude over time is part of the stability assessment process. One can learn more about the factors that contribute to wave stability or cause variations in its behavior by keeping an eye on how the profile evolves. Moreover, the wave profile can be used to check the precision of analytical conclusions made using the traveling wave transformation. Figure 7 shows the inferred kink waves at different values of the dispersion coefficient m and the parameter k. We can notice that the amplitude of the lattice wave is affected directly with the change in the values of parameter k, while the amplitude of the wave is affected inversely with the change in the value of the dispersion coefficient m. The electric field potential $g(t,x,y,z)$ can be given using the obtained wave profile (22):

$$g_{1,2}(t,x,y,z) = \pm\sqrt{\frac{-6k^2(\beta_1^2+\beta_2^2+\beta_3^2)}{m}} \times \frac{\sinh\left(k\left(\frac{\beta_1}{\Gamma(1+\eta)}x^\eta + \frac{\beta_2}{\Gamma(1+\eta)}y^\eta + \frac{\beta_3}{\Gamma(1+\eta)}z^\eta - \frac{\left(-2k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right)\right)}{\Gamma(1+\eta)}t^\eta\right)\right)}{\sqrt{1+\sinh^2\left(k\left(\frac{\beta_1}{\Gamma(1+\eta)}x^\eta + \frac{\beta_2}{\Gamma(1+\eta)}y^\eta + \frac{\beta_3}{\Gamma(1+\eta)}z^\eta - \frac{\left(-2k^2\beta_1\left(\beta_1^2+\beta_2^2+\beta_3^2\right)\right)}{\Gamma(1+\eta)}t^\eta\right)\right)}}. \tag{23}$$

To illustrate the kink soliton in (23), we present Figure 8 which shows the electric field potential

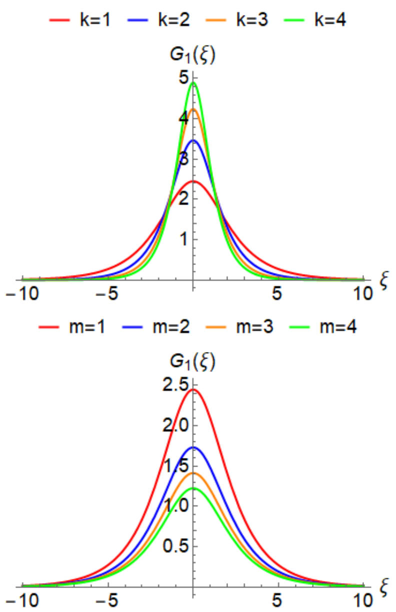

Fig. 1: The wave profile $G_1(\xi)$in (16) at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1$: (a) $k = 1,2,3,4$; (b) $m = 1,2,3,4$.

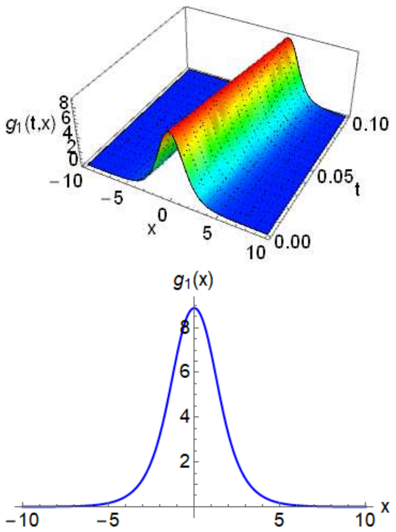

Fig. 2: The electric field potential $g(t,x,y,z)$in (17) at $\beta_1 = 0.2, \beta_2 = 0.1, \beta_3 = 1, m = 1, k = 1$and derivative order $\eta = 1$such that: (a) 3D plot; (b) 2D plot.

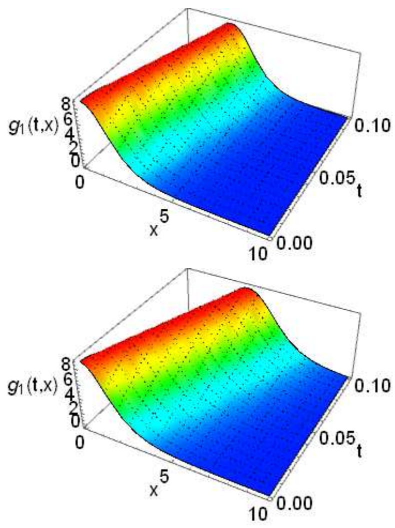

Fig. 3: The electric field potential $g(t,x,y,z)$in (17) at $\beta_1 = 0.2, \beta_2 = 0.1, \beta_3 = 1, m = 1, k = 1$such that: (a) fractional derivative order $\eta = 0.99$; (b) fractional derivative order $\eta = 0.9$.

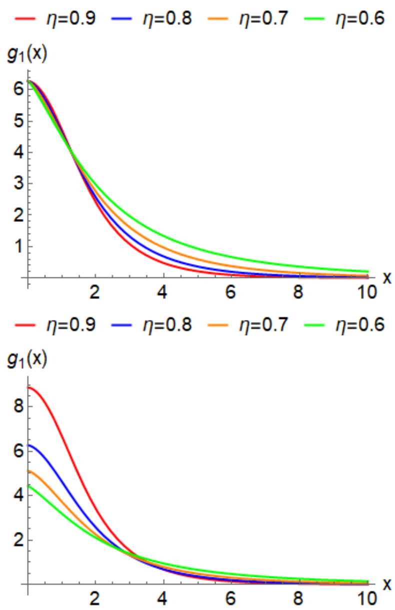

Fig. 4: The electric field potential $g(t,x,y,z)$in (17) at $\beta_1 = 0.2, \beta_2 = 0.1, \beta_3 = 1, k = 1$such that: (a) dispersion coefficient $m = 1$; (b) dispersion coefficient $m = 2$.

$g_2(t,x,y,z)$ at derivative order $\eta = 1$ in 2D and 3D. It is certain that the fractional derivative affects the behavior of the inferred soliton, so we show in Figure 9 the electric field potential $g(t,x,y,z)$ in (23) by considering the fractional derivative at different orders. Figure 10 presents the behavior of the kink soliton in (23) in a 2D plot, where different orders of the fractional derivative and different values of the dispersion coefficient m were considered. We notice that the soliton is affected by the change in the order of the fractional derivative in Figure 10, as the amplitude of the wave is less the lower the order of the fractional derivative, with the need to note that its shape has not changed. The dispersion coefficient m also has a role in the amplitude of the inferred wave. As we mentioned earlier, the fractional derivative affects

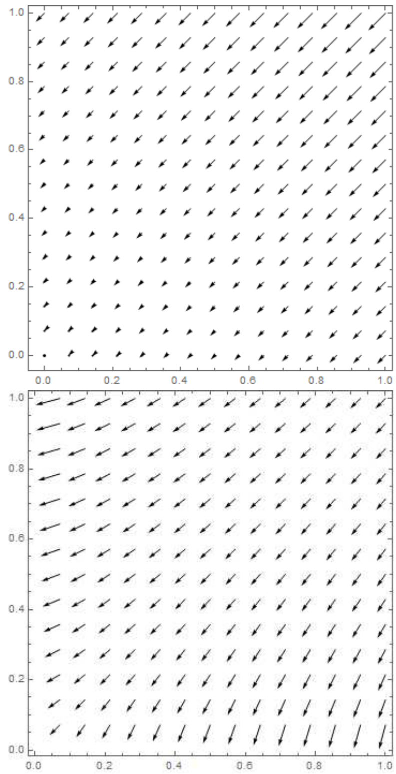

Fig. 5: The electric field *overrightarrow*$\mathcal{E}_1$at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1, m = 0.1$such that: (a) fractional derivative order $\eta = 0.9$, $k = 0.1$; (b) fractional derivative order $\eta = 0.5$, $k = 0.1$.

the behavior of the solution in a certain way, but this effect is linked to several factors, the most important of which is the definition used for the fractional derivative, as we notice in [27] the ruling model has been studied considering another definition of the fractional derivative, and the effect of the fractional derivative of the solutions they derived was in [27] somewhat different. With the help of (5), the electric field $\overrightarrow{\mathcal{E}}$ may be constructed using the electric field potential $g(t,x,y,z)$.

We show the electric field in (23) in Figure 11 in different fractional derivative orders. Using (2) and (23), we can obtain the difference in the number density of hot electrons and positrons of equal temperature. It is depicted in Figure 12. Figure 12(a) shows the difference in the number density of hot electrons and isothermal positrons $n_p - n_e$ when the dispersion coefficient $m = -0.1$ by which this difference is somewhat larger when compared to Fig. 12(b) when the scattering coefficient is considered $m = -0.5$.

5 Discussion and Conclusions

The investigation of the fractional modified Korteweg-deVries-Zakharov-Kuznetsov (f-mKdV-ZK) model utilizing the local fractional derivative has been the main subject of this paper. We built bright and kink soliton solutions within this framework by using the traveling wave transformation. Our research covered a wide range of topics, including wave profile analysis, electric field potential investigation, electric field examination, and comparison of the number density of hot electrons and positrons at equal temperatures. The

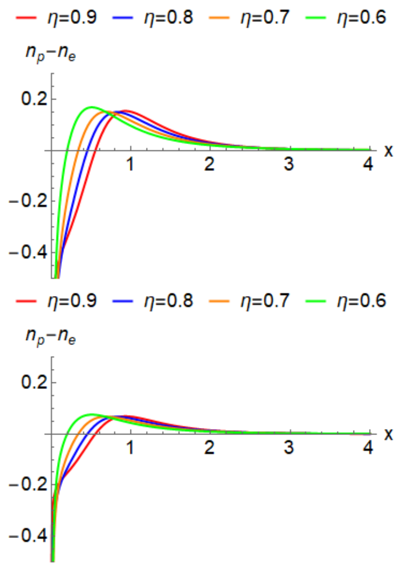

Fig. 6: The difference in the number density of hot electrons and positrons of equal temperatures $n_p - n_e$at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1, k = 1$such that: (a) dispersion coefficient $m = 1$; (b) dispersion coefficient $m = 5$.

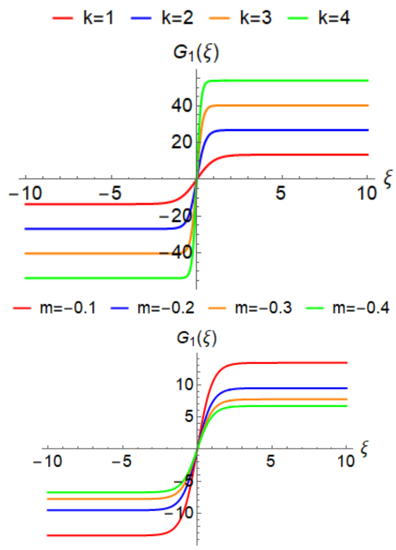

Fig. 7: The wave profile $G_1(\xi)$in (22) at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1$: (a) $k = 1,2,3,4$; (b) $m = -0.1,-0.2,-0.3,-0.4$.

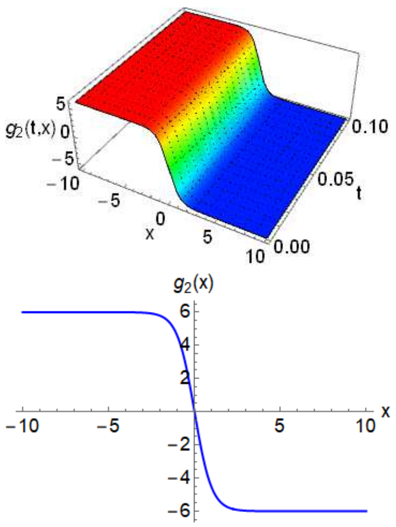

Fig. 8: The electric field potential $g_2(t,x,y,z)$in (22) at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1, m = -0.5, k = 1$and derivative order $\eta = 1$such that: (a) 3D plot; (b) 2D plot.

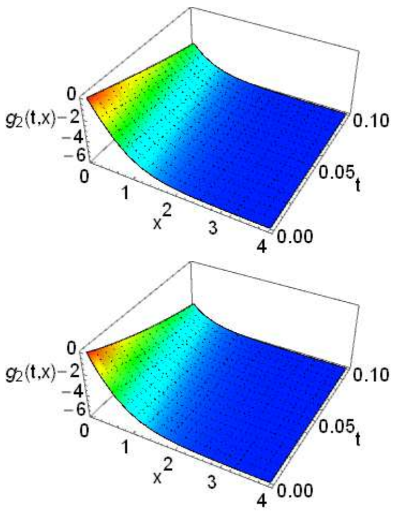

Fig. 9: The electric field potential $g_2(t,x,y,z)$in (23) at $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1, m = -0.5, k = 1$such that: (a) fractional derivative order $\eta = 0.99$; (b) fractional derivative order $\eta = 0.98$.

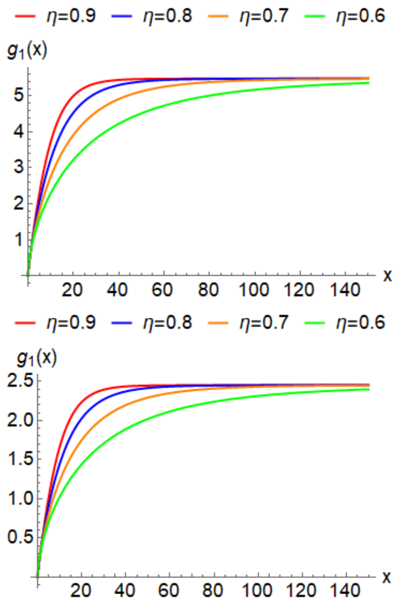

Fig. 10: The electric field potential $g_1(t,x,y,z)$in (23) at $\beta_1 = 0.1, \beta_2 = 2, \beta_3 = 1, k = 1$such that: (a) dispersion coefficient $m = -1$; (b) dispersion coefficient $m = -5$.

data were represented graphically using instructive figures, which facilitated comprehension of our conclusions. The use of the fractional derivative, which added non-local and memory effects to the model, is one of the study's standout features. We were able to better capture the nuances of plasma behavior by integrating the fractional derivative, which also allowed us to go beyond the conventional integer-order models in terms of our comprehension of plasma dynamics. It was thoroughly investigated how the fractional derivative affected the outcomes, shedding light on the importance of using fractional calculus in plasma physics research. By carrying out this study, we have improved our comprehension of the complex dynamics present in the f-mKdV-ZK model as well as our knowledge of applications of fractional calculus in plasma physics. These findings serve as a starting point for additional research and serve as a model for future studies looking at the effects of fractional derivatives in different plasma systems. In the end, this research advances plasma modeling, wave propagation analysis, and our general understanding of complicated plasma phenomena. It also contributes to the subject of plasma physics.

Supplementary Materials: The following supporting information can be downloaded at: www.mdpi.com/xxx/s1, Figure S1: title; Table S1: title; Video S1: title.

Author Contributions: Conceptualization, Mohammed Alquran ; methodology, Asad Freihat; software, Asad Freihat; validation, Mohammed Alabedalhadi; formal analysis, Mohammed Alquran; investigation, Mohammed Alquran; resources, Asad Freihat; data curation, Mohammed Alquran; writing original draft preparation, Mohammed Alabedalhadi; writing review and editing, Mohammed Alquran and Asad Freihat; visualization, Mohammed Alquran. All authors have read and agreed to the published version of the manuscript.

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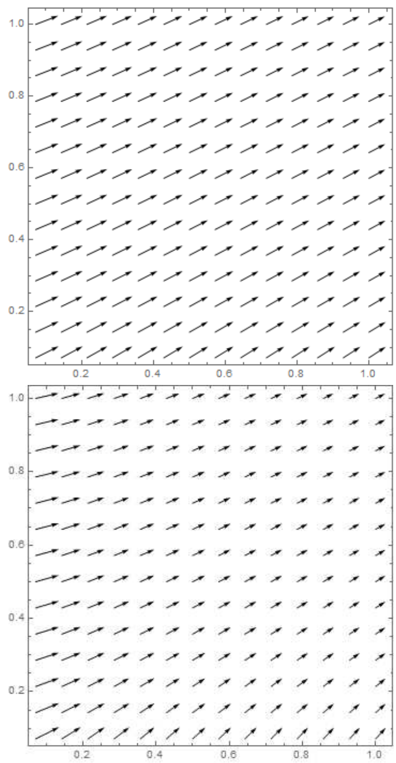

Fig. 11: The electric field $overrightarrow\mathcal{E}_1$at $\beta_1 = 2, \beta_2 = 1, \beta_3 = 1, m = -0.1$such that: (a) fractional derivative order $\eta = 0.9$, $k = 0.1$; (b) fractional derivative order $\eta = 0.7$, $k = 0.1$.

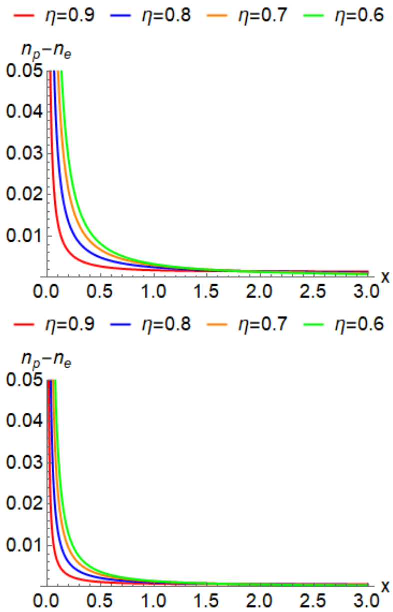

Fig. 12: The difference in the number density of hot electrons and positrons of equal temperatures $n_p - n_e$ at $\beta_1 = 2, \beta_2 = 1, \beta_3 = 1, k = 0.1$ such that: (a) dispersion coefficient$m = -0.1$; (b) dispersion coefficient $m = -0.5$.

Data Availability Statement: Data supporting reported results can be obtained on request from the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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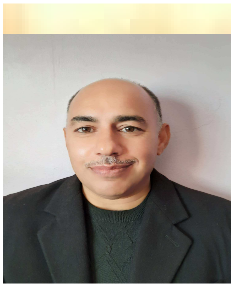

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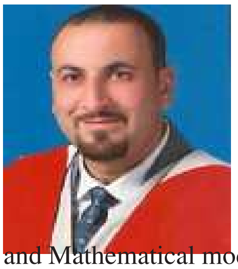

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