

# An Optimization Model for Cargo Container Loading Problems

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Received: 26 Aug. 2023, Revised: 28 Sept. 2023, Accepted: 30 Oct. 2023.

Published online: 1 Nov. 2023.

**Abstract:** Recently there has been a significant increase in demand for services that provide quick and dependable delivery and door-to-door pickup. As a result, networks for international air express carriers' services have quickly expanded, creating increasingly competitive markets for airlines. To design a stochastic-demand cargo container loading planning model that reduces overall operating expenses while upholding operational limitations is the main objective of this research work. To achieve this goal, we must consider all of the relevant operational requirements. The model is presented in the form of an NP-hard nonlinear mixed integer stochastic program, which is the classification for the problem. To solve the problem, we devise a direct search strategy.

**Keywords:** Supply Chain Management, Air Express Carriers, Optimization, Nonlinear Mixed Integer Programs, Container Loading Plannings.

## 1 Introduction

Supply chain Management and logistics have been identified as major areas of concern for the business analyst in current global and competitive business scenario. In past few decades, much effort has been taken to technically advanced the different aspects of the supply chain processes like warehouse management, inventory control, information sharing among partners and others. As the competition increases, the business organization needs to improve the different logistics processes to make it more cost efficient. Global uneven geographical distribution of resources and customers across the world always put much extra pressure on the logistics services to be more efficient. Any disruption can cause delay of the products and raw materials, which can further result in poor customer service. It has been realized that most of the consumer goods stays long hours in transportation or on shelves as compared to their own production time. Extra unnecessary inventory is also a consequence of the poor and weak logistics network [1].

In the extremely competitive business of overnight package delivery, finding ways to save costs is very necessary. If a business has no ability to come up with methods of lowering its expenditures and pass those reductions on to its consumers, its competitors will find other ways to do it. The cost of loading aircraft in a timely and effective manner is one of the individual costs that is the most significant for businesses that ship products by airfreight. It only takes a short period of time for items to be delivered to their destinations when they are flown by airplane. Following the loading of these goods into containers, the containers are next loaded into aircraft. Therefore, if air transport can find a way to maximize the application of these containers, they will be able to lower the amount of space needed for containers, utilize the fewest possible containers, and spend less money on aircraft [2].

Pure containers and mixed containers are both often used in the operations of air freight carrier businesses [3]. The pure containers each hold a shipment sent to the same location as the original package. Once these containers have been delivered to the hub, they will be able to be immediately moved (a process known as trans-loading) from one airplane to another. There are packages destined for a variety of locations that are contained inside the mixed containers. These need a sorting operation to be carried out at the hub in order to distinguish between the encased goods and put them into other containers. These containers are then carried and transported by different aircraft to various locations. These two distinct methods of loading containers each have an effect that is distinct both at the point of origin and at the hub. When loading pure containers at the origins of the logistics system and managing mixed containers at the hub, sorting operations are required by law. It is an extremely difficult challenge of identifying the loading plan of containers at the

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origins and at the hub, which includes the quantity to be packed in mixed or pure type container and required number of containers. This task is connected to the origin and the hub's operational expenses and limitations, which can be attributed by many variables, including container formations, cargo OD demands, and containers arrangement according to its configuration. Furthermore, stochastic disruptions of regular cargo need regularly operations, which inevitably have an impact on the choice of the container's loading. As a consequence of this, it is very difficult, if not impossible, to determine the best option without first conducting a methodical investigation.

In this paper, we construct a Stochastic-Demand Cargo Container Loading Plan Model (SDCCLPM) from the point of view of an air express carrier. Our goal is to minimize the total container operating expenses while adhering to the connected functional limitations. We do this by taking into account the stochastic disruption of daily cargo demands that arise during real-world operations. In terms of optimization, the model is defined as an NP-hard nonlinear mixed integer program, which describes its difficulty level. For the purpose of solving the large-scale model, we suggest using a direct search approach.

### Problem Formulation

Even though it is common practice for cargo aircraft to have many stops on their flight itineraries, the time-definite limitation of express carriers' guarantees of delivering packages the next day makes it impractical for them to do large number of stops. The majority of carriers' flights are nonstop, meaning that they go to their final destination directly without making any further stops. Operations at airports may be broken down into three categories: origins activities, destinations activities, and those at the hub.

Primary tasks of a gateway, especially at an airport, include gathering products that are being sent out (exports) and distributing items that are being brought in (imports). The first step in the export procedure is the package retrieval at the origin location, performed by on-road carrier. Every day at end, total packages destined for export are brought back to station, where they are stored until a shuttle takes them to gateway. Thereby items are sorted and allocated to either type containers, afterwards the containers are stacked into airplanes.

Pure containers take a longer time to be put together at an origin due to the prolonged process of package categorization, whereas mixed containers require less time to be transloaded from aircraft to others at the hub. This difference in time requirements is due to the fact that pure containers are more popular than mixed containers. Mixed containers, consequently, take less work at gateway, whereby fewer assets available, but involve additional people and an extensive amount of time to sort packages at the hub. If infinite space at the hub is available for contains, marginal operational expense is stable while being less than that at each origin, then it is self-evident that parcels are placed into mixed containers at every point to achieve the lowest possible total operational expense. Nevertheless, in most cases, the capacity for processing at hub is restricted, and the peripheral operational expense does not remain same. In fact, the available operational time window has a negative correlation with the cost of package handling operations. This correlation is established by the flight schedule. To put it another way, the lower the expenses for handling are, the larger operational time window in the flight schedule has to be. For example, in the event that an aircraft from its origins to its hub is scheduled to arrive late, it is required to construct pure containers in order to ensure that related parcels reach in time to make their immediate connecting flights. The strategy of loading for the cargo containers will undoubtedly be altered as a direct result of the flight schedule. In this particular piece of work, the difficulty primarily centers on the strategy for loading cargo containers in accordance with a specified flight schedule. In the future, studies into the possibility of integrating a plan for loading cargo containers and a schedule for flight operations are possible.

Currently, the air carrier designs its container loading strategy based on staff expertise with a set and anticipated demand. The load up plan of containers, in particular, is planned individually and separately at each gateway, ignoring the interactions between each gateway and hub from a system viewpoint. Presently, every OD volume is categorized as split or not for convenience. An inefficient and ineffective approach like this would produce subpar results, especially for big service networks. Furthermore, in actual operations, stochastic perturbations of daily cargo demand frequently happen. The issue is that the container loading plan in the existing system is created using a fixed and forecasted cargo demand. Thus, the resulting solution might not satisfy the actual demand and perhaps be subpar.

Numerous studies have been conducted on topics related to cargo, including air carriers features [4], international air carriers network planning [5]–[8], air cargo flight arrangement [9]–[11], competition analysis and configuration analysis for air cargo carriers [12]–[14], air freight approaches [15], hub location selection [16]–[27], vessel loading problem [28], and cargo loading. However, all of them lack the research's specific emphasis and do not provide air carriers effective container loading options. To the best of the authors' knowledge, no research on planning issues including stochastic disturbances in operations has been successful in addressing loading plan issues for air cargo. This includes studies in other disciplines [29]–[33].

In the present paper, SDCCLPM is developed and taking air carrier viewpoint solution method are defined along with at

every origin and hub the handling capacities of cargo containers, the stochastic cargo demands, transportation cost by third party overflow volume, the loading capacity of aircraft, cost function of handling the container at each gateway and at hub, and other parameters are discussed and analyzed.

## 2 Methodologies

The research method used is a literature study, namely studying literature books by collecting information from reference books and journals about similar research that has been done before.

### Assumptions

1. All system-wide cargo OD requests have been fulfilled.
2. A third party may transfer overflows.
3. In the Asia Pacific operations, a single aircraft fleet is anticipated.
4. There is no general air freight service provided.
5. Direct flights are available between the gateways and the hub.
6. In terms of operational efficiency, a mixed container may be comparable to a ratio of a pure container.
7. The one-hub operation's flight schedule is provided.
8. Each gateway's or the hub's container handling capacity and cost function are listed.

To formulate the SDCCLPM, we employ approaches from integer programming. The decision variables are as below,  $\rho_{ij}^b$ ,  $m_{ij}^b(w)$ ,  $\gamma_{ij}$  and  $n_{ij}(w)$ :

- $\rho_{ij}^b$  The percentage of the demand from  $i$  to  $j$  to be loaded into the  $b^{\text{th}}$  kind (AMJ or AKE)'s pure containers;
- $\gamma_{ij}$  how much of the demand from points  $i$  to  $j$  will be placed into mixed containers;
- $m_{ij}^b(w)$  The number of  $b^{\text{th}}$  kind (AMJ or AKE)'s pure containers (in integers) transported from  $i$  to  $j$  in the  $w^{\text{th}}$  stochastic scenario;
- $n_{ij}(w)$  the container volume that, in the stochastic scenario, a third-party transports from point  $i$  to point  $j$  (in AMJ equivalents, which can be a real number);

Following are the others symbol explanation,

- $w$  stochastic  $w^{\text{th}}$  state
- $\Omega$  Total stochastic states
- $\square_w$   $w$  scenario probability
- OD Total OD pairs
- $d_{ij}(w)$  stochastic state  $w^{\text{th}}$  demand volume from  $i$  to  $j$
- $\square_i$  the handling of a mixed container at origin  $i$  equivalent to a wholly pure container;
- $\square_i$  the hub's equivalent of a wholly pure container for managing a mixed container being transported from the origin  $i$ ;
- $b$   $b = 1$  for AMJ and  $b = 2$  for AKE; in  $b^{\text{th}}$  container.
- $s$  The hub's handling capability (in totally pure AMJ counterparts);
- $u_i$  The origin  $i$  handling capability (completely pure AMJ counterparts);
- $f_i()$  At origin  $i$  cost function for mixed and pure containers
- $P_b$  The ability of the  $b^{\text{th}}$  kind of containers to be loaded onto aircraft;
- $v_b$  The  $b^{\text{th}}$  type of container's AMJ equivalent;
- $g_i()$  At hub cost function shipment from gateway  $i$  for handling mixed container

$c_b$	At hub handling $b$ th container for fully or partially
$c_{ij}$	Transporting AMJ from $i$ to $j$ by third party.
AC	Containers total set.
SP	Origins total set.
TP	Destination set.

According to this formulation [32], the SDCCLPM is a nonlinear mixed integer programming:

$$\text{Minimize } \sum_{w \in \Omega} \delta_w \left[ \sum_i f_i(\rho_{ij}^b d_{ij}(w), \gamma_{ij} d_{ij}(w) |_{\forall j}) + \sum_i \sum_j \sum_b c_b m_{ij}^b \right. \\ \left. + \sum_i g_i(\gamma_{ij} d_{ij}(w) |_{\forall j}) + \sum_{ij \in OD} c_{ij} n_{ij}(w) \right] \quad (1)$$

Subject to

$$\sum_b \rho_{ij}^b d_{ij}(w) + \gamma_{ij} d_{ij}(w) + n_{ij}(w) = d_{ij}(w), \quad \forall ij \in OD, \forall w \in \Omega; \quad (2)$$

$$\sum_j \sum_b \rho_{ij}^b d_{ij}(w) + \alpha_i \sum_j \gamma_{ij} d_{ij}(w) \leq u_i, \quad \forall i \in SP, \quad \forall w \in \Omega; \quad (3)$$

$$\sum_i \beta_i \sum_j \gamma_{ij} d_{ij}(w) \leq s, \quad \forall w \in \Omega; \quad (4)$$

$$(\rho_{ij}^b / v_b) d_{ij}(w) \leq m_{ij}^b, \quad \forall ij \in OD, \quad \forall b \in AC, \quad \forall w \in \Omega; \quad (5)$$

$$\sum_j m_{ij}^b(w) \leq p_b, \quad \forall i \in SP, \quad \forall b \in AC, \quad \forall w \in \Omega; \quad (6)$$

$$\sum_j \sum_b v_b m_{ij}^b(w) + \sum_j \gamma_{ij} d_{ij}(w) \leq \sum_b v_b p_b, \quad \forall i \in SP, \quad \forall w \in \Omega; \quad (7)$$

$$\sum_i m_{ij}^b(w) \leq p_b, \quad \forall j \in TP, \quad \forall b \in AC, \quad \forall w \in \Omega; \quad (8)$$

$$\sum_i \sum_b v_b m_{ij}^b(w) + \sum_i \gamma_{ij} d_{ij}(w) \leq \sum_b v_b p_b, \quad \forall j \in TP, \quad \forall w \in \Omega; \quad (9)$$

$$\sum_b \rho_{ij}^b + \gamma_{ij} \leq 1, \quad \forall ij \in OD; \quad (10)$$

$$0 \leq \rho_{ij}^b \leq 1, \quad \forall ij \in OD, \quad \forall b \in AC; \quad (11)$$

$$0 \leq \gamma_{ij} \leq 1, \quad \forall ij \in OD; \quad (12)$$

$$n_{ij}(w) \geq 0, \quad \forall ij \in OD, \quad \forall w \in \Omega; \quad (13)$$

$$m_{ij}^b(w) \geq 0, \quad m_{ij}^b(w) \in I, \quad \forall ij \in OB, \quad \forall b \in AC, \quad \forall w \in \Omega; \quad (14)$$

Minimizing the total cost predicted value of container handling the main objective of the system, as expressed by Eq. (1). For each OD pair, the objective function includes four terms: the first denotes overall cost of mixed/pure containers handled at origins; next represents pure/mixed containers overall cost handled at the hub; the third denotes pure/mixed containers total cost handled at the hub; and the fourth represents the total transportation cost by a third party. It is important to keep in mind that at the destination gateways the handling cost of containers is not included in the objective function since it is fixed and is not influenced by the loading strategy. Overflows occur when there are more passengers than the system can handle at a given origin or hub, or when there are more passengers than seats on a certain aircraft.

For each case, Eq. (2) guarantees that all of the OD demand is placed into pure/mixed AMJ/AKE containers or is carried by a third-party overflow volume. For the sake of simplicity, all the overflow capacity and mixed containers are modelled as AMJ counterparts. Once the total number of entirely and partly pure containers and gateway overflow

volume has been established, the resulting quantity of mixed containers may be separated into specific AMJ and AKE containers, as per aircraft capability of loading. For each gateway and scenario, the container handling capacity limitation is given by Eq. (3) in entirely pure AMJ equivalents. Take note that the converted containers now contain just pure AMJ. Equal quantities of pure AMJ have been added to the pure AKE containers. For each case, the hub's ability to handle mixed containers is denoted in totally pure AMJ equivalents by Eq. (4). It is important to remember that this limitation does not apply to the processing of pure containers, since this requires far less work than the handling of mixed containers does. However, if clean containers are used, modification in constraints can be possible as  $\sum_i \sum_j \sum_b v_b m_{ij}^b + \sum_i \beta_i \sum_j y_{ij} d_{ij} \leq s$ , here the handling capacity is represented by  $s$  which is for both mixed and pure containers.

In each case, the quantity of pure containers required to store the container's volume is calculated using Eq (5). For any scenario, pure AMJ and AKE containers quantity that need to be transported to hub from every origin should not surpass the airplane loading capacity for pure AKE and AMJ containers, as shown by Eq. (6). Given the aeroplane loading capacity, Eq. (7) shows that containers (AMJ equivalents) quantity that may be transported from origin to hub must not be more than the containers maximum quantity. When travelling from the hub to gateway, the aeroplane size limitation for pure AMJ and AKE containers is described by Eqs. (8), which are analogous to Eqs. It's important to remember that AMJ and AKE containers may be mixed and matched in whatever spare space remains on the plane's upper or lower deck. In the same manner as Eq. (7), (9) shows that containers (in AMJ equivalents) quantity being transferred to destination from every hub must not surpass airplane capacity. Equation (10) guarantees less than or equal to one percent of every OD requirement will be put in mixed containers. Equation (11) guarantees that there is no negative value for the proportion of every OD demand loaded into pure AKE A/MJ containers. Non-negative fractions of OD for loading into mixed containers are denoted by Eq. (12). The non-negativeness of the AMJ equivalent container volume transferred by the third party for each OD pair is guaranteed by Eq. (13). Since pure AKE/AMJ containers quantities transferred for every OD pair is guaranteed to be non-negative by Eq. (14), this is a safe assumption to make for all scenarios.

It should be noted that in stochastic optimization problem researchers in [34] suggested theoretical ideas such as the expected value of perfect information (EVPI), which can be utilized in stochastic solution performance, and the value of the stochastic solution (VSS). Assuming that there are  $S$  stochastic possibilities, let  $\bar{x}(s)$  and denote the best solution and associated best objective value for the given scenario. The wait-and-see solution (WS) can then be used to get ideal solution expected value, where  $WS = ES(z(\bar{x}(s), s))$ . The here-and-now solution or the stochastic programming model's optimal value, can be represented as  $HN = \min_x ESz(x, S)$ . Hence the variation in here-and-now and WS solution is EVPI which can be written as  $EVPI = HN - WS$ . Furthermore, the  $S$  scenarios expectation  $\bar{s}$  (or  $E(S)$ ) be assumed than the mean value problem is expressed as  $EV = \min_x z(x; \bar{s})$  with the relevant best possible solution can be expressed as  $\bar{x}(\bar{s})$ . Henceforth, EV can be represented as  $EEV = E_s(z(\bar{x}(\bar{s}), S))$ . Hence, VSS stochastic solution can be expressed as HN and EEN difference such as  $VSS = EEV - HN$ . Theoretically, both VSS and EVPI are nonnegative quantities.

Referring to [4], three different minimization problems properties are depicted below:

$$WS \leq HN \leq EEV, \tag{15}$$

$$0 \leq EVPI \leq HN - EV \leq EEV - EV, \tag{16}$$

$$0 \leq VSS \leq EEV - EV, \tag{17}$$

If  $EV = EEV$ , as shown in Eqs. (16) and (17), then the stochastic components cannot affect the optimum solution since VSS and EVPI equivalent to zero. That is to say, the same optimum solution may be found in the face of any stochastic requirement. Therefore, a stochastic programming model of the issue is unnecessary. In current study, we will additionally assess the VSS and EVPI for the SDCCLPM to better comprehend the stochastic solution's efficiency.

### 3 Results

#### 3.1 The Basic Approach

The model that will be discussed and developed in this research is the rumor spreading model introduced [4] which is commonly known as the DK model In this model the closed population and homogeneous mixture are divided into three groups, those who are not aware of the rumor, those who have heard about it and are actively spreading it, and those who have heard the rumor but no longer spread it These groups are called fools, spreaders and stiflers,

respectively It is assumed that rumors spread through the population through direct contact between the spreader and the fools Every time a spreader interacts with a fool, the fool becomes a spreader When a spreader contacts a stifter, the spreader turns into a stifter and when a spreader meets another spreader, the spreader that started it becomes a stifter Another basic assumption is that someone who knows the rumor will continue to tell it until he or she decides that the rumor is no longer worthy of being called "news" In the [13] rumors spread through direct contact of the disseminator with other individuals Therefore, when a spreader contacts another spreader, only the initiating spreader becomes the stifter [14] have worked on the process of disseminating various ideas [15] studied the process of spreading rumors with denial and skepticism, two models were established to accommodate skepticism Many models in the literature represent the dynamics of spreading rumors by a system of ordinary differential without any time delay To reflect the real dynamic behavior of the model that depends on the system's past history, it makes sense to incorporate a time delay into this system In fact, the inclusion of a delay in the rumor-spreading model makes it more realistic by allowing a description of the effect of the situation when a fool contacts the spreader, the fool may experience a latent period before becoming a propagator.

Firstly, we take MILP with the following form:

$$\text{Minimize } P = c^T x \quad (18)$$

$$\text{Subject to } Ax \leq b \quad (19)$$

$$x \geq 0 \quad (20)$$

$$x_j \text{ integer for some } j \in J \quad (21)$$

The  $(x_B)_k$  optimal basic feasible vector component solved as continuous MILP is represented as:

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \dots - \alpha_{kj}(x_N)_j - \dots - \alpha_{kn} - m(x_N)n - m \quad (22)$$

Keep in mind that the Simplex procedure's final tableau contains this statement. The division of  $k$  into the integer and fractional components is that provided if  $(x_B)_k$  is an integer variable and we suppose that  $k$  is not an integer.

$$\beta_k = [\beta_k] + f_k, 0 \leq f_k \leq 1 \quad (23)$$

Let's say we want to raise  $(x_B)_k$  to the next integer over  $([\beta]+1)$ . We may raise a specific non-basic variable, such as  $(x_N)_{j^*}$ , over its limit of zero based on the concept of suboptimal solutions, such that  $\alpha_{kj^*}$ , is elements of vector  $\alpha_{j^*}$ , is negative. Let  $\Delta_{j^*}$  be the change in the non-variable  $(x_N)_{j^*}$ , such that the scalar  $(x_B)_k$  numerical value is an integer. In light of Eqn.(25),  $\Delta_{j^*}$  may be written as

$$\Delta_{j^*} = \frac{1 - f_k}{-\alpha_{kj^*}} \quad (24)$$

The remainder of the nonbasic remain at zero. As can be observed, using the partitioning of  $k$  provided in (23) and putting (24) into (22) for  $(x_N)_{j^*}$  we get:

$$(x_B)_k = [\beta] + 1$$

Thus,  $(x_B)_k$  is now an integer.

It is now clear that a nonbasic variable plays an important role to integerize the relevant fundamental variable. To prove that in integerizing problem non integer variable must present, the following result is required.

**Theorem.** Suppose an optimum solution exists for the MILP problem (18)–(21), in which case some of the nonbasic variables will be used. The variables  $(x_N)_j$ ,  $j = 1, \dots, n$ , and  $n$  must not be non-integers.

**Proof.** Using slack variables to solve the issue continuously (which are non-integer, except in equality constraint case). The nonbasic vector  $x_N$  would include all integer variables and have integer values if basic variables  $x_B$  is considered to be made of slack variables.

It is obvious that when the scalar  $(x_N)_{j^*}$  value rises to  $\Delta_{j^*}$ , the other components of the vector  $x_B$ ,  $(x_B)_{i \neq k}$ , will likewise be impacted. The element of  $x_B$  corresponding to a positive vector  $\alpha_{j^*}$ , i.e.,  $\alpha_{ij^*}$  for  $i \neq k$  would therefore diminish and finally could reach zero. However, the non-negativity constraint prevents vector  $x$  components from falling below zero. In order to determine the nonbasic  $(x_N)_{j^*}$  movement in a way that  $x$  components remain viable, a formula known as the minimum ratio test is required. Two examples would be included in this ratio test.

1.  $(x_B)_{i \neq k}$  a fundamental variable, drops to zero (lower bound) first.

2. The fundamental variable  $(x_B)_k$  becomes an integer.

For each of the two aforementioned scenarios, one would specifically compute

$$\theta_1 = \min_{i \neq k | \alpha_{j^*} > 0} \left\{ \frac{\beta_i}{\alpha_{j^*}} \right\} \tag{25}$$

$$\theta_2 = \Delta_{j^*} \tag{26}$$

nonbasic  $(x_N)_{j^*}$  can be release for zero bound in ways that  $x$  continue to be feasible, and it dependence  $s_i$  on the  $\theta^*$  as shown below.

$$\theta^* = \min(\theta_1, \theta_2) \tag{27}$$

The variable  $(x_B)_{i \neq k}$  will be at lower bound prior to  $(x_B)_k$  become integer when the  $\theta^* = \theta_1$  condition is met. The feasibility will be kept if  $\theta^* = \theta_2$ , along with the variable  $(x_B)_k$  will be an integer. Similarly, the variable  $(x_B)_k$  numerical value can be reduced to  $[\beta_k]$ . for that scenario the nonbasic variable  $(x_N)_{j^*}$  movement will relate to positive element  $\alpha_{j^*}$ .

$$\Delta_{j^*} = \frac{f_k}{\alpha_{kj}} \tag{28}$$

The ratio test  $\theta^*$  is still required to maintain feasibility. Take into account the movement of a certain non-basic variable,  $\Delta$  (, as shown in Eqns (22). The matching component of the vector  $\alpha$  (is the only factor that needs to be calculated. Using a vector  $\alpha_j$  as an example,

$$\alpha_j = B^{-1}a_j, j = 1, \dots, n - m \tag{29}$$

Determining the relevant column of matrix  $[B]^{-1}$  is thus necessary to get a specific vector  $\alpha_j$  element. Consider the case when the value of element  $\alpha_{kj^*}$  are required, where  $v_k^T$  is the k-th column vector of  $[B]^{-1}$ .

$$v_k^T = e_k^T B^{-1} \tag{30}$$

Consequently, it is possible to get  $\alpha_{kj^*}$  value from

$$\alpha_{kj^*} = v_k^T a_{j^*} \tag{31}$$

The process represented by Eqns. (30) and (31) is known as the pricing operation in the language of Linear Programming (LP). The decrease in OB function due to freeing a nonbasic variable from its constraint may be quantified using the vector of decreased costs  $d_j$ . It follows that vector  $d_j$  should considered when selecting whether nonbasic should be freed throughout the integerizing procedure in order to avoid degradation.

Integer-feasible solutions have a lower limit that is given by the least continuous solution. However, the magnitude of change of a specific no basic variables, as in Eqns. (24) or (28), is dependent on subsequent member of vector  $\alpha_j$ . In order to integerize variable  $(x_B)_k$ , one must first release a nonbasic variable  $(x_N)_{j^*}$  and this release reduces the value of the objective function.

$$\left| \frac{d_k}{\alpha_{kj^*}} \right| \tag{32}$$

where  $|a|$  denotes scalar  $a$ 's absolute value.

Then, we use the following method to determine nonbasic variable can be raised from its limit of zero to minimize continuous solution deterioration,

$$\min_j \left\{ \left| \frac{d_k}{\alpha_{kj^*}} \right| \right\}, j = 1, \dots, n - m \tag{33}$$

Partitioning the constraints into those for basic (B), nonbasic (N), and superbasic (S), variables allow us to derive a formula for writing down "active constraints."

$$\begin{bmatrix} B & S & N \\ & & I \end{bmatrix} \begin{bmatrix} x_b \\ x_N \\ x_S \end{bmatrix} = \begin{bmatrix} b \\ b_N \end{bmatrix} \quad (34)$$

or

$$Bx_b + Sx_S + Nx_N = b \quad (35)$$

$$x_N = b_N \quad (36)$$

Assuming that the B basis matrix is nonsingular and square, we get:

$$x_B = \beta - Wx_S - \alpha x_N \quad (37)$$

Where,

$$\beta = B^{-1}b \quad (38)$$

$$W = B^{-1}S \quad (39)$$

$$\alpha = B^{-1}N \quad (40)$$

To show that the nonbasic variables constrained to bounds, we use the expression (36). The "nearly" elementary form of Eqn (37) demonstrates this., one may use the integerizing approach developed for the MILP issue that we addressed before. In particular, while the solution would be degenerate, the integerizing procedure would allow us to free a nonbasic variable from its constraint, Eqn (36), and replace it with a comparable basic variable.

### 3.2 Pivoting

At now, a certain basic variable,  $(x_B)_k$ , is being integerized, which results in subsequent nonbasic variable,  $(x_N)_{j^*}$ , to be unconstrained by the value zero. Let's assume the maximum allowed deviation of  $(x_N)_{j^*}$  is:

$$\theta^* = \Delta_{j^*}$$

By moving  $(x_N)_{j^*}$  into B (in lieu of  $(x_B)_k$ ) and integer-valued  $(x_B)_k$  into S, we may preserve the integer solution while taking use of the method of shifting the basis. Since a fundamental variable has reached its minimum or maximum value, we have a degenerate solution. The new set [B,S] will be used to continue the integerizing procedure. All the integer variables can become superbasic in this instance.

### 3.3 Algorithm

At now, a certain basic variable,  $(x_B)_k$ , is being integerized, which results in subsequent nonbasic variable,  $(x_N)_{j^*}$ , to be unconstrained by the value zero. Let's assume the maximum allowed deviation of  $(x_N)_{j^*}$  is:

Stage 1.

Step 1. Get row  $i^*$  the smallest integer infeasibility, such that  $\delta_{i^*} = \min\{f_i, 1 - f_i\}$  The OB function minimum deterioration can be found by using this step.

Step 2. Do a pricing operation.

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate  $\sigma_{ij} = v_{i^*}^T a_j$

With  $j$  corresponds to

$$\min_j \left\{ \left| \frac{a_j}{\sigma_{ij}} \right| \right\}$$

I. For nonbasic  $j$  at lower bound

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{(1 - \delta_{i^*})}{-\sigma_{ij}}$$

$$\text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{(1 - \delta_{i^*})}{\sigma_{ij}}$$

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$$



If  $\sigma_{ij} > 0$  and  $\delta_{i^*} = f_i$  calculate  $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

II. For nonbasic  $j$  at upper bound

If  $\sigma_{ij} < 0$  and  $\delta_{i^*} = 1 - f_i$  calculate  $\Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}}$

If  $\sigma_{ij} > 0$  and  $\delta_{i^*} = f_i$  calculate  $\Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}}$

If  $\sigma_{ij} > 0$  and  $\delta_{i^*} = 1 - f_i$  calculate  $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

If  $\sigma_{ij} < 0$  and  $\delta_{i^*} = f_i$  calculate  $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

Instead, if  $j$  is not an integer, go on to the next superbasic  $j$  (if available). The,  $j^*$  column in LB needs to be enlarged, while the,  $j^*$  column in UB needs to be lowered. If it doesn't work, try,  $i^*$ .

Step 4. Calculate

$$\alpha_{j^*} = B^{-1}\alpha_{j^*}$$

i.e. solve  $B\alpha_{j^*} = \alpha_{j^*}$  for  $\alpha_{j^*}$

Step 5. Ratio test; Due to the liberation of nonbasic,  $j^*$  from its constraints, three options for the basic variables are required to maintain feasibility.

If  $j^*$  lower bound

Let

$$A = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$$

$$B = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$$

$$C = \Delta$$

the maximum movement of  $j^*$  depends on:  $\theta^* = \min(A, B, C)$

If  $j^*$  upper bound

Let

$$A' = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$$

$$B' = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$$

$$C' = \Delta$$

The  $j^*$  movement is dependent on:  $\theta^* = \min(A', B', C')$

Step 6. There are three possibilities which can be used to change basis.

1. If  $A$  or  $A'$ 
  - $x_{B_{i'}}$  at lower bound  $l_{i'}$  becomes nonbasic
  - $x_{i^*}$  remains basic (non-integer)
  - $x_{j^*}$  (replaces  $x_{B_{i'}}$ ) and becomes basic
2. If  $B$  or  $B'$ 
  - $x_{B_{i'}}$  at upper bound  $u_{i'}$  turn into nonbasic
  - $x_{i^*}$  remains basic (non-integer)
  - $x_{j^*}$  (replaces  $x_{B_{i'}}$ ) and develop into basic
3. If  $C$  or  $C'$ 
  - $x_{i^*}$  at integer-valued develops into superbasic
  - $x_{j^*}$  (replaces  $x_{i^*}$ ) turn into basic

Step 7. If row  $i^* = \{\emptyset\}$  go to Stage 2, otherwise;

Repeat from step 1.

Stage 2. Perform a line search using integers to enhance the integer feasibility solution.

## 4 Conclusions

A good cargo container is required by air carriers' companies to compete globally with other companies. A carrier must consider in actual operations the market demand uncertainties in addition to the airport operational costs when creating an effective cargo container loading plan. We create a stochastic-demand cargo container loading plan model in this research that can address everyday stochastic needs that actually occur in practice. Subject to the associated operating limits, the purpose is to minimize the overall handling cost. The model is expressed as a nonlinear mixed integer programming that, from the perspective of optimization, is NP-hard. In order to resolve the issue, we employ a workable direct search strategy. To reduce their operational costs, increase profits, and be more competitive in the market, air express carriers should find the model and the proposed solution approach to be helpful planning aids when determining their container loading strategies.

## Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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