Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/170614

A Soft Ideal Topological Space: New Closed Set

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Received: 2 Sep. 2023, Revised: 2 Oct.. 2023, Accepted: 22 Oct. 2023
Published online: 1 Nov. 2023

Abstract: To use the topological structure to solve specific problems in many fields, it has become important to generalize it. So, in this work, we begin with an introduction to some of the ideas of ideal topology and soft topology that will be combined to show and explain the generalization of soft ideal generalized closed sets ($\tilde{\mathscr{I}}\mathscr{G}C_s$). A soft topological structure's consideration of soft ideals, we explore the generalization of sets and look into the novel idea of α-generalized closed sets ($\alpha\mathscr{G}C_s$). Various properties and ideas of $\alpha\mathscr{G}C_s$ are also presented in a soft ideal topological framework. Last but not least, we provide various applications that can be used in conjunction with the novel sets.

Keywords: Spaces with soft topologies; soft ideal; soft ideal generalized closed sets; α-soft ideal generalized closed sets.

1 Introduction

Molodtsov [25] first proposed the concept of soft sets as a generic mathematical tool in order to deal with ambiguous things. In [25, 26], Molodtsov used soft theory to a variety of topics, such as soft function, game theory, operations research, probability, and more. The features and applications of soft set theories are rapidly being investigated [19, 20, 31]. Soft set operations have been redefined, and a decision-making framework based on these new operations has been established [2, 7]. It was introduced to generalize relevant ideas of soft separability [1, 4, 19, 30]. The ideal concept is based on two primary variables [11, 13, 14, 17], the first of which is a local function and the second of which is a closure operator. In several fascinating applications, such as biochemistry and approximation spaces in [6, 15], the concept of ideal was applied. Ideal is used to generalize the topological vacuum and generate finer topology to solve some problems for open and closed sets [5, 12, 29]. The soft ideal has been coupled to soft topology to offer novel relationships as a topological soft ideal structure [18]. We will go over the fundamental definitions and outcomes of soft topology, ideal topology, and generalized closed set ($\mathscr{G}C_s$) theory that we will need in this study in this section.

Firstly, a local function of the ideal $\mathscr{I}$ concept is $()^*:\wp(\mathscr{N})\rightarrow\wp(\mathscr{N})$, where $\mathscr{N}$ is the space carries topology σ, and the secondly Kuratowski closure operator for $\sigma^*(\mathscr{I})$ as introduced by Kuratowski [21] and Vaidyanathasamy [37], which are defined by $\aleph^*(\mathscr{I},\sigma)=\{n\in\mathscr{N}:\mho\Cap\aleph\notin\mathscr{I}$ for every $\mho\in\sigma(n)\}$, $\aleph\sqsubseteq\mathscr{N}$, where $\sigma(n)=\{\mho\in\sigma:n\in\mho\}$, and $Cl^*(\aleph)=\aleph\Cup\aleph^*(\mathscr{I},\sigma)$, that is an ideal $\mathscr{I}$ on a space $\mathscr{N}$ which carries topology σ is the family of $\mathscr{N}$, which satisfy:

1-If $\aleph\in\mathscr{I},C\sqsubseteq\aleph$, then $C\in\mathscr{I}$.

2-If $\aleph\in\mathscr{I},C\in\mathscr{I}$, implies $\aleph\Cup C\in\mathscr{I}$[37]. The topology induced by ideal $\sigma^*(\mathscr{I})$ is finer than σ, induced from the base $\beta(\mathscr{I},\sigma)=\{\mho\text{-}L:\mho\in\sigma$ and $L\in\mathscr{I}\}$.

Definition 1.*A subset B of a topological space $(\mathscr{N},\sigma)$ is known as*
1-$\mathscr{G}C_s$ [22] if $Cl(B)\sqsubseteq\mho$ at $B\sqsubseteq\mho$ with $\mho$ is open,
2-$\hat{\mathscr{G}}C_s$ [38] if $Cl(B)\sqsubseteq\mho$ at $B\sqsubseteq\mho$ with $\mho$ is semi-open,
3-$\mathscr{G}^*C_s$ [16] if $Cl(B)\sqsubseteq\mho$ at $B\sqsubseteq\mho$ with $\mho$ is $\hat{\mathscr{G}}$-open,
4-pre C_s [27] if $Cl(Int(B))\sqsubseteq B$,
5-αC_s [28] if $Cl(Int(Cl(B)))\sqsubseteq B$,

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6-Semi C_s [8] if $Int(Cl(B)) \sqsubseteq B$, and
7-bC_s [3] if $Cl(Int(B)) \sqcap Int(Cl(B)) \sqsubseteq B$.

Definition 2.[25] *Let $\mathscr{R}$ be a set of parameters and $\mathscr{N}$ be an initial universe, A be a non-empty subset of $\mathscr{R}$ and $\wp(\mathscr{N})$ indicate the power set of $\mathscr{N}$. A soft set(S_s)over $\mathscr{N}$ is a pair (Φ, A) denoted by Φ_A, where Φ is maps provided by $\Phi : A \to \wp(\mathscr{N})$. That is to say, a parameterized subset family of the universe $\mathscr{N}$ is a S_s over $\mathscr{N}$. For a specific reason, $r \in A$, $\Phi(r)$ may be considered the set of r-approximate elements of the set (Φ, A) and if $r \notin A$, then $\Phi(r) = \phi$ that is $\Phi_A = \{\Phi(r) : r \in A \sqsubseteq \mathscr{R}, \Phi : A \to \wp(\mathscr{N})\}$. $SS(\mathscr{N})_A$ denotes the family of all these soft sets SS_s over $\mathscr{N}$.*

Definition 3.*If σ is a set of SS_s distributed across a universe $\mathscr{N}$ using a set of parameters that can't be changed $\mathscr{R}$, then $\sigma \sqsubseteq SS(\mathscr{N})_{\mathscr{R}}$ is named a soft topology on $\mathscr{N}$ [35] if*
(a) $\tilde{\mathscr{N}}, \tilde{\phi} \in \sigma$, that $\tilde{\phi}(r) = \phi$ and $\tilde{\mathscr{N}}(r) = \mathscr{N}$, for all $r \in \mathscr{R}$,
(b)$\Cup_i \tilde{A}_i \in \sigma$ for all $\tilde{A}_i \in \sigma$,
(c)$\tilde{A}_1 \Cap \tilde{A}_2 \in \sigma$ for all $\tilde{A}_1, \tilde{A}_2 \in \sigma$.
A sts over $\mathscr{N}$ is defined as the triplet $(\mathscr{N}, \sigma, \mathscr{R})$.

Definition 4.[18] *If across a universe $\mathscr{N}$, $\tilde{I}$ is a non-null collection of SS_s with a predetermined set of parameters $\mathscr{R}$, then $\tilde{I} \sqsubseteq SS(\mathscr{N})_R$ is referred to as a soft ideal on $\mathscr{N}$ with a predetermined point set $\mathscr{R}$ if*
(a) $(\Phi, \mathscr{R}) \in \tilde{I}$ and $(\Psi, \mathscr{R}) \in \tilde{I} \Rightarrow (\Phi, \mathscr{R}) \tilde{\Cup} (\Psi, \mathscr{R}) \in \tilde{I}$,
(b) $(\Phi, \mathscr{R}) \in \tilde{I}$ and $(\Psi, \mathscr{R}) \tilde{\sqsubseteq} (\Phi, \mathscr{R}) \Rightarrow (\Psi, \mathscr{R}) \in \tilde{I}$.

Definition 5.[36] *If $\alpha Cl(\aleph) \sqsubseteq \mho$, whenever $\aleph \sqsubseteq \mho$, and $\mho$ is open in a topological space $(\mathscr{N}, \sigma)$, a subset $\aleph$ of a $(\mathscr{N}, \sigma)$ is named an $\alpha\mathscr{G}C_s$.*

Definition 6.[33] *If I is an ideal on a topological space $(\mathscr{N}, \sigma)$. A subset $\aleph$ of $\mathscr{N}$ is alleged to be $\mathscr{I}\mathscr{G}C_s$ if $\aleph^* \sqsubseteq \mho$, at $\aleph \sqsubseteq \mho$, $\mho$ is open.*

Definition 7.[9] *An ideal I is named*
1-codense if $\sigma \Cap I = \phi$,
2-completely codense if $P_o(\mathscr{N}) \Cap I = \phi$, where $P_o(\mathscr{N})$ indicates the $(\mathscr{N}, \sigma)$ family of all preopen sets.

Definition 8.[23] *If the space $\mathscr{N}$ carries topology σ with an ideal on $\mathscr{N}$. A subset $\aleph$ of $\mathscr{N}$ is named $\alpha I\mathscr{G}C$ if $\aleph^* \sqsubseteq \mho$, whenever $\aleph \sqsubseteq \mho$ and $\mho$ is α-open.*

Definition 9.[19] *In a sts $(\mathscr{N}, \sigma, \mathscr{R})$, a soft set (Φ, A) is named $\mathscr{G}C$ if $Cl(\Phi, A) \tilde{\sqsubseteq} \mho$ at $(\Phi, A) \tilde{\sqsubseteq} \mho$ and $\mho$ is soft open in $\mathscr{N}$.*

At successive times we have noticed that the concept of soft, ideal and $\mathscr{G}C_s$ along with topology has been discussed by some researchers and we in this work are moving in the same direction for generalization. In the context of the soft ideal in the soft topological space, we introduce the notion of $\alpha\mathscr{G}C_s$ related to soft ideal in a soft topological space (sts). Also, in a topological space via soft ideal, various features of $\alpha\mathscr{G}C_s$ are introduced. Finally, we offer some applications that can used with a novel set.

2 A new closed set via a soft ideal topological space

In the second Part, we present $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ for a sts with soft ideal, along with several examples to demonstrate their properties.

Definition 10.*Let $(\mathscr{N}, \sigma, \mathscr{R})$ denote a sts and $\tilde{\mathscr{I}}$ denote a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ as the set of parameters. Then $\Phi^*_{\mathscr{R}}$=$(\Phi, \mathscr{R})^*$=$\Cup\{n_r \in (\mathscr{N}, \mathscr{R}) : \mathscr{O}_{n_r} \Cap \Phi_{\mathscr{R}} \notin \tilde{\mathscr{I}}$ for every $\mathscr{O}_{n_r} \in \sigma\}$ is referred to as a soft ideal local function of $\Phi_{\mathscr{R}}$=$(\Phi, \mathscr{R})$ concerning to σ, where $\mathscr{O}_{n_r}$ is a σ-soft open set containing n_r.*

Definition 11.*Let $\tilde{\mathscr{I}}$ denote a soft ideal over a sts $(\mathscr{N}, \sigma, \mathscr{R})$ with $\mathscr{R}$ as the set of parameters. A subset $\Phi_{\mathscr{R}}$ of $\mathscr{N}$ is referred to as $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ if $\Phi^*_{\mathscr{R}} \sqsubseteq \mho$ at $\Phi_{\mathscr{R}} \sqsubseteq \mho$ and $\mho$ is α-soft open.*

*Example 1.*Let $\mathscr{N} = \{h_1, h_2, h_3\}$ with topologies $\tilde{\sigma} = \{\tilde{\phi}, \tilde{\mathscr{N}}, (\Phi_1, \mathscr{R}), (\Phi_2, \mathscr{R}), (\Phi_4, \mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi}, (\Phi_2, \mathscr{R})\}$, such that $(\Phi_1, \mathscr{R}), (\Phi_2, \mathscr{R}), ..., (\Phi_8, \mathscr{R})$ are soft sets over $\mathscr{N}$ as well as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_2, h_3\}$,
$\Phi_4(r) = \{h_1, h_2\}$,
$\Phi_5(r) = \{h_1, h_3\}$,
$\Phi_6(r) = \{h_3\}$,
$\Phi_7(r) = \mathscr{N}$,
$\Phi_8(r) = \phi$.
It goes without saying that, $(\Phi_6, \mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Definition 12.*Let $\tilde{\mathscr{I}}$ denote a soft ideal over a sts $(\mathscr{N}, \sigma, \mathscr{R})$ with $\mathscr{R}$ as the set of parameters. Then if $(\mathscr{N}, \mathscr{R})$-$(\Phi, \mathscr{R})$ is $\alpha\tilde{\mathscr{I}}C_s$, a subset $\Phi_{\mathscr{R}}$ of $\mathscr{N}$ is designated $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open set.*

Theorem 1.*The following are equivalent if $(\mathscr{N}, \sigma, \mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$, $\mathscr{R}$ as the set of parameters , and $\Phi_{\mathscr{R}} \sqsubseteq \mathscr{N}$.*
1-$\Phi_{\mathscr{R}}$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed set.
2-$Cl^*(\Phi_{\mathscr{R}}) \sqsubseteq \mho$ whenever $\Phi_{\mathscr{R}} \sqsubseteq \mho$ and $\mho$ is α-soft open in $(\mathscr{N}, \sigma, \mathscr{R})$.
3-For all $n_r \in Cl^*(\Phi_{\mathscr{R}})$, $\alpha Cl(\{n_r\}) \Cap \Phi_{\mathscr{R}} \neq \phi$.
4- There is no nonempty α-soft closed set in $Cl^*(\Phi_{\mathscr{R}})$-$\Phi_{\mathscr{R}}$.
5-There is no nonempty α-soft closed set in $\Phi^*_{\mathscr{R}}$-$\Phi_{\mathscr{R}}$.

*Proof.*1 $\Rightarrow$ 2: If $\Phi_{\mathscr{R}}$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed, then $\Phi^*_{\mathscr{R}} \sqsubseteq \mho$ whenever $\Phi_{\mathscr{R}} \sqsubseteq \mho$ and $\mho$ is α-soft open. So, $Cl^*(\Phi_{\mathscr{R}}) \sqsubseteq \mho$ whenever $\Phi_{\mathscr{R}} \sqsubseteq \mho$ and $\mho$ is α-soft open. Then, 2 is proved.
2 $\Rightarrow$ 3: Let $n_r \in Cl^*(\Phi_{\mathscr{R}})$ and $\alpha Cl(\{n_r\}) \Cap \Phi_{\mathscr{R}}$=$\phi$, then $\Phi_{\mathscr{R}} \sqsubseteq \mathscr{N}$-$\alpha Cl(\{n_r\})$. By (2), $Cl^*(\Phi_{\mathscr{R}}) \sqsubseteq \mathscr{N}$-$\alpha Cl(\{n_r\})$, which is contradiction to $n_r \in Cl^*(\Phi_{\mathscr{R}})$. Then, 3 is proved.
3 $\Rightarrow$ 4: Let $\Phi'_{\mathscr{R}} \sqsubseteq Cl^*(\Phi_{\mathscr{R}})$-$\Phi_{\mathscr{R}}$, $\Phi'_{\mathscr{R}} \sqsubseteq \mathscr{N}$-$\Phi_{\mathscr{R}}$ is α soft

closed and $n_r \in \Phi'_{\mathscr{R}}$. Then $\Phi_{\mathscr{R}} \sqsubseteq \mathcal{N}\text{-}\Phi'_{\mathscr{R}}$ and hence $\alpha Cl(\{n_r\}) \Cap \Phi_{\mathscr{R}}=\phi$. Since $n_r \in Cl^*(\Phi_{\mathscr{R}})$ by (3), $\alpha Cl(\{n_r\}) \Cap \Phi_{\mathscr{R}} \neq \phi$. Therefore, there is no nonempty α-soft closed set in $Cl^*(\Phi_{\mathscr{R}})\text{-}\Phi_{\mathscr{R}}$.
4 $\Rightarrow$ 5: Since $Cl^*(\Phi_{\mathscr{R}})\text{-}\Phi_{\mathscr{R}}=(\Phi_{\mathscr{R}} \Cup \Phi^*_{\mathscr{R}})\text{-}\Phi_{\mathscr{R}} = (\Phi_{\mathscr{R}} \Cup \Phi^*_{\mathscr{R}}) \Cap \Phi^c_{\mathscr{R}}=(\Phi_{\mathscr{R}} \Cap \Phi^c_{\mathscr{R}}) \Cup (\Phi^*_{\mathscr{R}} \Cap \Phi^c_{\mathscr{R}}) = \Phi^*_{\mathscr{R}}\text{-}\Phi_{\mathscr{R}}$ contains no nonempty α-soft closed.
5 $\Rightarrow$ 1: Assume that $\Phi_{\mathscr{R}} \sqsubseteq \mho, \mho$ is α-soft open set. Then $\mathcal{N}\text{-}\mho \sqsubseteq \mathcal{N}\text{-}\Phi_{\mathscr{R}}$ and so $\Phi^*_{\mathscr{R}} \Cap (\mathcal{N}\text{-}\mho) \sqsubseteq \Phi^*_{\mathscr{R}} \Cap (\mathcal{N}\text{-}\Phi_{\mathscr{R}})=\Phi^*_{\mathscr{R}}\text{-}\Phi_{\mathscr{R}}$. Therefor, $\Phi^*_{\mathscr{R}} \Cap (\mathcal{N}\text{-}\mho) \sqsubseteq \Phi^*_{\mathscr{R}}\text{-}\Phi_{\mathscr{R}}$. Since $\Phi^*_{\mathscr{R}}$ is always soft closed set, $\Phi^*_{\mathscr{R}} \Cap (\mathcal{N}\text{-}\mho)$ is a α-soft closed set contained in $\Phi^*_{\mathscr{R}}\text{-}\Phi_{\mathscr{R}}$. Therefore, $\Phi^*_{\mathscr{R}} \Cap (\mathcal{N}\text{-}\mho)=\phi$ and hence $\Phi^*_{\mathscr{R}} \sqsubseteq \mho$. Therefore, $\Phi_{\mathscr{R}}$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Definition 13.*A subset $\Phi_{\mathscr{R}}$ of a sts $(\mathcal{N},\sigma,\mathscr{R})$, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathcal{N}$ with $\mathscr{R}$ as the set of parameters is called $\tilde{*}$-closed if $\Phi^*_{\mathscr{R}} \sqsubseteq \Phi_{\mathscr{R}}$.*

Theorem 2.*Each $\tilde{*}$-closed set is $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ but not the other way around.*

*Proof.*Suppose that $\Phi_{\mathscr{R}}$ is $\tilde{*}$-closed, then $\Phi^*_{\mathscr{R}} \sqsubseteq \Phi_{\mathscr{R}}$. Let $\Phi_{\mathscr{R}} \sqsubseteq \mho$ such that $\mho$ is α-soft open. Then $\Phi^*_{\mathscr{R}} \sqsubseteq \mho$. Hence $\Phi_{\mathscr{R}}$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Contrary to popular belief, this is not always the case, as we will demonstrate in the subsequent instance.

*Example 2.*Suppose $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_2,\mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi}\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ defined as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_2,h_3\}$,
$\Phi_4(r) = \{h_1,h_2\}$,
$\Phi_5(r) = \{h_1,h_3\}$,
$\Phi_6(r) = \{h_3\}$,
$\Phi_7(r) = \mathcal{N}$,
$\Phi_8(r) = \phi$.
It goes without saying that, $(\Phi_1,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. But $(\Phi_1,\mathscr{R})$ is not a $\tilde{*}$-closed set.

*Remark.*As seen in the instances below, the $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ and $\alpha\tilde{\mathscr{I}}$-closed set are independent of one another.

*Example 3.*Let $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_1,\mathscr{R}),(\Phi_6,\mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi},(\Phi_5,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ as well as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_3\}$,
$\Phi_4(r) = \{h_1,h_2\}$,
$\Phi_5(r) = \{h_1,h_3\}$,
$\Phi_6(r) = \{h_2,h_3\}$,
$\Phi_7(r) = \mathcal{N}$,
$\Phi_8(r) = \phi$.
It is self-evident that $(\Phi_2,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, and it is not an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ because of $Cl^*(Int(Cl^*\Phi_2(r))) \not\sqsubseteq \Phi_2(r)$.

*Example 4.*Let $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_1,\mathscr{R}),(\Phi_5,\mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi},(\Phi_2,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ defined as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_2,h_3\}$,
$\Phi_4(r) = \{h_1,h_2\}$,
$\Phi_5(r) = \{h_1,h_3\}$,
$\Phi_6(r) = \{h_3\}$,
$\Phi_7(r) = \mathcal{N}$,
$\Phi_8(r) = \phi$.
It goes without saying that, $(\Phi_6,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}C_s$, and it is not an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ because of $(\Phi_6,\mathscr{R})^* \not\sqsubseteq \mho$.

*Remark.*As seen in the instances below, the $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ set and semi $\tilde{\mathscr{I}}$-closed set are unrelated of one another.

*Example 5.*Let $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_1,\mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi},(\Phi_3,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ defined as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_2,h_3\}$,
$\Phi_4(r) = \{h_1,h_2\}$,
$\Phi_5(r) = \{h_1,h_3\}$,
$\Phi_6(r) = \{h_3\}$,
$\Phi_7(r) = \mathcal{N}$,
$\Phi_8(r) = \phi$.
It goes without saying that, $(\Phi_4,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, and it is not semi soft ideal closed because of $Int(Cl^*(\Phi_4(r))) \not\sqsubseteq \Phi_4(r)$.

*Example 6.*Let $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),(\Phi_4,\mathscr{R})\}$, and $\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi},(\Phi_1,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,$ $(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ as well as:
$\Phi_1(r) = \{h_1\}$,
$\Phi_2(r) = \{h_2\}$,
$\Phi_3(r) = \{h_3\}$,
$\Phi_4(r) = \{h_1,h_2\}$,
$\Phi_5(r) = \{h_1,h_3\}$,
$\Phi_6(r) = \{h_2,h_3\}$,
$\Phi_7(r) = \mathcal{N}$,
$\Phi_8(r) = \phi$.
It goes without saying that, $(\Phi_2,\mathscr{R})$ is semi soft ideal closed set but is not an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ since $(\Phi_2,\mathscr{R})^* \not\sqsubseteq \mho$.

*Remark.*Every pre-soft ideal closed set need not be an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

*Example 7.*Let $\mathcal{N} = \{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma} = \{\tilde{\phi},\tilde{\mathcal{N}},(\Phi_1,\mathscr{R}),(\Phi_5,\mathscr{R})\}$, and
$\mathscr{R} = \{r\}$, $\tilde{\mathscr{I}} = \{\tilde{\phi},(\Phi_2,\mathscr{R})\}$,
such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),\ldots,(\Phi_8,\mathscr{R})$ are SS_s over $\mathcal{N}$ defined as:
$\Phi_1(r) = \{h_1\}$,

$\Phi_2(r)=\{h_2\}$,
$\Phi_3(r)=\{h_3\}$,
$\Phi_4(r)=\{h_1,h_2\}$,
$\Phi_5(r)=\{h_1,h_3\}$,
$\Phi_6(r)=\{h_2,h_3\}$,
$\Phi_7(r)=\mathscr{N}$,
$\Phi_8(r)=\phi$.

It goes without saying that, $(\Phi_3,\mathscr{R})$ is pre-soft ideal closed set. But $(\Phi_3,\mathscr{R})$ is not $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ because of $(\Phi_3,\mathscr{R})^* \not\sqsubseteq \mho$ whenever $(\Phi_3,\mathscr{R})\sqsubseteq\mho$.

*Remark.*In the following examples, the $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ and the $b\tilde{\mathscr{I}}$-closed sets are unrelated of one another.

*Example 8.*Let $\mathscr{N}=\{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma}=\{\tilde{\phi},\tilde{\mathscr{N}},(\Phi_1,\mathscr{R}),(\Phi_5,\mathscr{R})\}$, and $\mathscr{R}=\{r\}$, $\tilde{\mathscr{I}}=\{\tilde{\phi},(\Phi_3,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),...,(\Phi_8,\mathscr{R})$ are SS_s over $\mathscr{N}$ as well as:
$\Phi_1(r)=\{h_1\}$,
$\Phi_2(r)=\{h_2\}$,
$\Phi_3(r)=\{h_3\}$,
$\Phi_4(r)=\{h_1,h_2\}$,
$\Phi_5(r)=\{h_1,h_3\}$,
$\Phi_6(r)=\{h_2,h_3\}$,
$\Phi_7(r)=\mathscr{N}$,
$\Phi_8(r)=\phi$.

It goes without saying that, $(\Phi_4,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. But not an $b\tilde{\mathscr{I}}$-closed set since $Cl^*(Int(\Phi_4(r)))\sqcup Int(Cl^*(\Phi_4(r))\not\sqsubseteq\Phi_4(r)$.

*Example 9.*Let $\mathscr{N}=\{h_1,h_2,h_3\}$ with soft topology $\tilde{\sigma}=\{\tilde{\phi},\tilde{\mathscr{N}},(\Phi_1,\mathscr{R}),(\Phi_5,\mathscr{R})\}$, and $\mathscr{R}=\{r\}$, $\tilde{\mathscr{I}}=\{\tilde{\phi},(\Phi_2,\mathscr{R})\}$, such that $(\Phi_1,\mathscr{R}),(\Phi_2,\mathscr{R}),...,(\Phi_8,\mathscr{R})$ are SS_s over $\mathscr{N}$ as well as:
$\Phi_1(r)=\{h_1\}$,
$\Phi_2(r)=\{h_2\}$,
$\Phi_3(r)=\{h_3\}$,
$\Phi_4(r)=\{h_1,h_2\}$
$\Phi_5(r)=\{h_1,h_3\}$
$\Phi_6(r)=\{h_2,h_3\}$
$\Phi_7(r)=\mathscr{N}$,
$\Phi_8(r)=\phi$.

It goes without saying that, $(\Phi_3,\mathscr{R})$ is not an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ since, $\Phi_3^*(\mathscr{R})\not\sqsubseteq\Phi_5(\mathscr{R})$ whenever $\Phi_3(\mathscr{R})\sqsubseteq\Phi_5(\mathscr{R})$. But it is $b\tilde{\mathscr{I}}$-closed set.

Theorem 3.*Each $\tilde{\mathscr{I}}\hat{\mathscr{G}}C_s$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Let $(\Phi,\mathscr{R})\sqsubseteq\mho$ such that $\mho$ is α-soft open. Every α-soft open clearly is α-soft semi-open. Because $(\Phi,\mathscr{R})$ is $\tilde{\mathscr{I}}\hat{\mathscr{G}}C_s$t, $(\Phi,\mathscr{R})^*\sqsubseteq\mho$, implying that $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 4.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters. Then for every $(\Phi,\mathscr{R})\in\tilde{\mathscr{I}}$, $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Let $(\Phi,\mathscr{R})\sqsubseteq\mho$ such that $\mho$ is α-soft open set. Because of $(\Phi,\mathscr{R})^*=\phi$ for all $(\Phi,\mathscr{R})\in\tilde{\mathscr{I}}$, then $(\Phi,\mathscr{R})^*\sqsubseteq(\Phi,\mathscr{R})$. That is $(\Phi,\mathscr{R})^*\sqsubseteq\mho$. Hence, for all $(\Phi,\mathscr{R})\in\tilde{\mathscr{I}}$, $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 5.*If $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ in the sts $(\mathscr{N},\sigma,\mathscr{R})$ and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters, and $(\Phi,\mathscr{R})\sqsubseteq(\Psi,\mathscr{R})\sqsubseteq Cl^*(\Phi,\mathscr{R})$, then $(\Psi,\mathscr{R})$ is also an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Let $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, $(\Phi,\mathscr{R})\sqsubseteq(\Psi,\mathscr{R})\sqsubseteq Cl^*(\Phi,\mathscr{R})$ and $(\Psi,\mathscr{R})\sqsubseteq\mho$ and $\mho$ is α-soft open set, we have $(\Phi,\mathscr{R})\sqsubseteq\mho$. Hence, $(\Phi,\mathscr{R})^*\sqsubseteq\mho$ because $(\Phi,\mathscr{R})$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. Since $(\Psi,\mathscr{R})\sqsubseteq Cl^*(\Phi,\mathscr{R})$, we have $Cl^*(\Psi,\mathscr{R})\sqsubseteq Cl^*(\Phi,\mathscr{R})\sqsubseteq\mho$. Therefore $(\Psi,\mathscr{R})^*\sqsubseteq\mho$ whenever $(\Psi,\mathscr{R})\sqsubseteq\mho$, that is $(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 6.*If $(\Phi,\mathscr{R})$, $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ in the sts $(\mathscr{N},\sigma,\mathscr{R})$ and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters, then $(\Phi,\mathscr{R})\Cup(\Psi,\mathscr{R})$ is also an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Let $(\Phi,\mathscr{R})\Cup(\Psi,\mathscr{R})\sqsubseteq\mho$ where $\mho$ is α-soft open in $\mathscr{N}$. Then $(\Phi,\mathscr{R})\sqsubseteq\mho$ and $(\Psi,\mathscr{R})\sqsubseteq\mho$. Since $(\Phi,\mathscr{R})$, $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, then $(\Phi,\mathscr{R})^*\sqsubseteq\mho$ and $(\Psi,\mathscr{R})^*\sqsubseteq\mho$. From $((\Phi,\mathscr{R})\Cup(\Psi,\mathscr{R}))^*$=$(\Phi,\mathscr{R})^*\Cup(\Psi,\mathscr{R})^*\sqsubseteq\mho$. Hence $(\Phi,\mathscr{R})\Cup(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 7.*If $(\Phi,\mathscr{R})$, $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ in the sts $(\mathscr{N},\sigma,\mathscr{R})$ and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters, then $(\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R})$ is also an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Let $(\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R})\sqsubseteq\mho$ where $\mho$ is α-soft open in $\mathscr{N}$. Then $(\Phi,\mathscr{R})\sqsubseteq\mho$ and $(\Psi,\mathscr{R})\sqsubseteq\mho$. Since $(\Phi,\mathscr{R})$, $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, then $(\Phi,\mathscr{R})^*\sqsubseteq\mho$ and $(\Psi,\mathscr{R})^*\sqsubseteq\mho$. From $((\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R}))^*\sqsubseteq\mho$. Hence $(\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 8.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters, then $(\Phi,\mathscr{R})^*$ is always $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ for every subset $(\Phi,\mathscr{R})$ of $(\mathscr{N},\sigma,\mathscr{R})$.*

*Proof.*Let $(\Phi,\mathscr{R})^*\sqsubseteq\mho$, $\mho$ be α-soft open. Because $((\Phi,\mathscr{R})^*)^*\sqsubseteq(\Phi,\mathscr{R})^*$, we have $((\Phi,\mathscr{R})^*)^*\sqsubseteq\mho$ whenever $(\Phi,\mathscr{R})^*\sqsubseteq\mho$ and $\mho$ is α-soft open. Then, $(\Phi,\mathscr{R})^*$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 9.*If $(\Phi,\mathscr{R})$ and $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open sets in the sts $(\mathscr{N},\sigma,\mathscr{R})$ and $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters, then $(\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R})$ is also an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open set.*

*Proof.*If $(\Phi,\mathscr{R})$ and $(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open sets then, $(\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})$ and $(\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R})$ are $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed sets. From Theorem 2.22, $(\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})$ $\Cup$ $(\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. that $(\mathscr{N},\mathscr{R})$-$((\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R}))$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. Hence, $(\Phi,\mathscr{R})\Cap(\Psi,\mathscr{R})$ is also an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open set.

Theorem 10.*Let $(\mathscr{N},\sigma,\mathscr{R})$ be a sts and $\tilde{\mathscr{I}}$ be a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the same set of parameters, then every $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, which is α-open is $\tilde{*}$-closed set.*

*Proof.*Let $(\Phi,\mathscr{R})$ be an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. Then $(\Phi,\mathscr{R}) \sqsubseteq (\Phi,\mathscr{R})$ that $(\Phi,\mathscr{R})^* \sqsubseteq (\Phi,\mathscr{R})$ since $(\Phi,\mathscr{R})$ is α-soft open. Therefore, $(\Phi,\mathscr{R})$ is $\tilde{*}$-closed set.

Theorem 11.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters and $(\Phi,\mathscr{R})$ be an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed. Then the following are interchangeable.*
(1) $(\Phi,\mathscr{R})$ is a $\tilde{*}$-closed,
(2) $Cl^*(\Phi,\mathscr{R})$-$(\Phi,\mathscr{R})$ is α-soft closed set,
(3) $(\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R})$ is α-soft closed set.

Proof.(1) $\Rightarrow$ (2): If $(\Phi,\mathscr{R})$ is a $\tilde{*}$-closed, then $(\Phi,\mathscr{R})^* \sqsubseteq (\Phi,\mathscr{R})$ and so
$Cl^*(\Phi,\mathscr{R})$-$(\Phi,\mathscr{R})$=$((\Phi,\mathscr{R}) \Cup (\Phi,\mathscr{R})^*)$-$(\Phi,\mathscr{R})$=$\phi$.
Hence $Cl^*(\Phi,\mathscr{R})$-$(\Phi,\mathscr{R})$ is α-soft closed set.
(2) $\Rightarrow$ (3): Since $Cl^*(\Phi,\mathscr{R})$-$(\Phi,\mathscr{R}) = (\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R})$ and so $(\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R})$ is α-soft closed set.
(3) $\Rightarrow$ (1): If $(\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R})$ is α-soft closed set, then by Theorem 2.5, $(\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R}) = \phi$ and so $(\Phi,\mathscr{R})$ is $\tilde{*}$-soft closed.

Theorem 12.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters and $(\Phi,\mathscr{R}) \sqsubseteq (\mathscr{N},\mathscr{R})$. Then $(\Phi,\mathscr{R})$ is $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed set if and only if $(\Phi,\mathscr{R})$=$(H,\mathscr{R})$-$\mathscr{M}$, where $(H,\mathscr{R})$ is $\tilde{*}$-soft closed and $\mathscr{M}$ contains no nonempty α-soft closed.*

*Proof.*If $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed, then by Theorem 2.5(5), $\mathscr{M}$=$(\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R})$ contains no nonempty α-soft closed. If $(H,\mathscr{R})$=$Cl^*(\Phi,\mathscr{R})$, then $(H,\mathscr{R})$ is $\tilde{*}$-soft closed such that $(H,\mathscr{R})$-$\mathscr{M}$=$((\Phi,\mathscr{R}) \Cup (\Phi,\mathscr{R})^*)$-$((\Phi,\mathscr{R})^*$-$(\Phi,\mathscr{R}))$=$((\Phi,\mathscr{R}) \Cup (\Phi,\mathscr{R})^*) \Cap ((\Phi,\mathscr{R})^* \Cap (\Phi^c,\mathscr{R}))^c$=$((\Phi,\mathscr{R}) \Cup (\Phi,\mathscr{R})^*) \Cap (((\Phi,\mathscr{R})^*)^c \Cup (\Phi,\mathscr{R}))$ =$((\Phi,\mathscr{R}) \Cup (\Phi,\mathscr{R})^*) \Cap ((\Phi,\mathscr{R}) \Cup ((\Phi,\mathscr{R})^*)^c$=$(\Phi,\mathscr{R}) \Cup ((\Phi,\mathscr{R})^* \Cap ((\Phi,\mathscr{R})^*)^c$=$(\Phi,\mathscr{R})$.
Conversely, let $(\Phi,\mathscr{R})$=$(H,\mathscr{R})$-$\mathscr{M}$ where $(H,\mathscr{R})$ is $\tilde{*}$-closed and $\mathscr{M}$ contains no non empty α-soft closed set. Suppose $\mho$ be a α-soft open set such that $(\Phi,\mathscr{R}) \sqsubseteq \mho$. Then $(H,\mathscr{R})$-$\mathscr{M} \sqsubseteq \mho$ that is $(H,\mathscr{R}) \Cap (\mathscr{N}$-$\mho) \sqsubseteq \mathscr{M}$. Now $(\Phi,\mathscr{R}) \sqsubseteq (H,\mathscr{R})$ and $(H,\mathscr{R})^* \sqsubseteq (H,\mathscr{R})$, then $(\Phi,\mathscr{R})^* \sqsubseteq (H,\mathscr{R})^*$ and so, $(\Phi,\mathscr{R})^* \Cap (\mathscr{N}$-$\mho) \sqsubseteq (H,\mathscr{R})^* \Cap (\mathscr{N}$-$\mho) \sqsubseteq (H,\mathscr{R}) \Cap (\mathscr{N}$-$\mho) \sqsubseteq \mathscr{M}$. By hypothesis, since $(\Phi,\mathscr{R})^* \Cap (\mathscr{N}$-$\mho)$ is α-soft closed, $(\Phi,\mathscr{R})^* \Cap (\mathscr{N}$-$\mho) = \phi$ and so $(\Phi,\mathscr{R})^* \sqsubseteq \mho$. Hence $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 13.*Let $(\mathscr{N},\sigma,\mathscr{R})$ be a sts and $\tilde{\mathscr{I}}$ be a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters. If $(\Phi,\mathscr{R})$ and $(\Psi,\mathscr{R})$ are subsets of $(\mathscr{N},\mathscr{R})$ such that $(\Phi,\mathscr{R}) \sqsubseteq (\Psi,\mathscr{R}) \sqsubseteq Cl^*(\Phi,\mathscr{R})$ and $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed set, then $(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.*

*Proof.*Since $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-closed set, then by Theorem 2.5(4), $Cl^*(\Phi,\mathscr{R})$-$(\Phi,\mathscr{R})$ contains no nonempty α-soft closed. Since $(Cl^*(\Psi,\mathscr{R})$-$(\Psi,\mathscr{R})) \sqsubseteq (Cl^*(\Phi,\mathscr{R}) - (\Phi,\mathscr{R}))$ and so $Cl^*(\Psi,\mathscr{R})$-$(\Psi,\mathscr{R})$ contains no nonempty α-soft closed. Hence $(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 14.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters and $(\Phi,\mathscr{R}) \sqsubseteq (\mathscr{N},\mathscr{R})$. Then, $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open if and only if $(H,\mathscr{R}) \sqsubseteq Int^*(\Phi,\mathscr{R})$ at $(H,\mathscr{R})$ is α-soft closed and $(H,\mathscr{R}) \sqsubseteq (\Phi,\mathscr{R})$.*

*Proof.*Let $(\Phi,\mathscr{R})$ be an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open, $(H,\mathscr{R}) \sqsubseteq (\Phi,\mathscr{R})$ and $(H,\mathscr{R})$ be α-soft closed set. Then $((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})) \sqsubseteq ((\mathscr{N},\mathscr{R})$-$(H,\mathscr{R}))$ and $((\mathscr{N},\mathscr{R})$-$(H,\mathscr{R}))$ is α-soft open. Since $(\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$, then $((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))^* \sqsubseteq ((\mathscr{N},\mathscr{R}) - (H,\mathscr{R}))$ and $((\mathscr{N},\mathscr{R})$-$Int^*(\Phi,\mathscr{R})) = Cl^*((\mathscr{N},\mathscr{R}) - (\Phi,\mathscr{R})) \sqsubseteq ((\mathscr{N},\mathscr{R})$-$(H,\mathscr{R}))$. Hence, $(H,\mathscr{R}) \sqsubseteq Int^*(\Phi,\mathscr{R})$.
Conversly, let $((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})) \sqsubseteq \mho$ where $\mho$ is α-soft open. Then $((\mathscr{N},\mathscr{R})$-$\mho) \sqsubseteq (\Phi,\mathscr{R})$ and $((\mathscr{N},\mathscr{R})$-$\mho)$ is α-soft closed. By hypothesis, we have $((\mathscr{N},\mathscr{R})$-$\mho) \sqsubseteq int^*(\Phi,\mathscr{R})$ and hence $((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))^* \sqsubseteq Cl^*((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})) = ((\mathscr{N},\mathscr{R})$-$Int^*(\Phi,\mathscr{R}))$ therefore $((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ and $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open set.

Theorem 15.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters and $(\Phi,\mathscr{R}) \sqsubseteq (\mathscr{N},\mathscr{R})$. If $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open and $Int^*(\Phi,\mathscr{R}) \sqsubseteq (\Psi,\mathscr{R}) \sqsubseteq (\Phi,\mathscr{R})$, then $(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open.*

*Proof.*Since $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open, $(\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. By Theorem 2.5(4),
$Cl^*((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})) - ((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))$ contains no nonempty α-soft closed set. Since $Int^*(\Phi,\mathscr{R}) \sqsubseteq Int^*(\Psi,\mathscr{R})$, $(\mathscr{N},\mathscr{R})$-$Cl^*((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R})) \sqsubseteq (\mathscr{N},\mathscr{R})$-$Cl^*((\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R}))$ which implies that $Cl^*((\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R})) \sqsubseteq Cl^*((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))$ and so $Cl^*((\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R}))$-$((\mathscr{N},\mathscr{R})$-$(\Psi,\mathscr{R})) \sqsubseteq Cl^*((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))$-$((\mathscr{N},\mathscr{R})$-$(\Phi,\mathscr{R}))$. That is $(\Psi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}$-open.

Theorem 16.*If $(\mathscr{N},\sigma,\mathscr{R})$ is a sts, $\tilde{\mathscr{I}}$ is a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the set of parameters and $(\Phi,\mathscr{R}) \sqsubseteq (\mathscr{N},\mathscr{R})$. Then, every subset of $(\mathscr{N},\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ if and only if α-soft open set is $\tilde{*}$-closed.*

*Proof.*Let every subset of $(\mathscr{N},\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$. If $\mho \sqsubseteq (\mathscr{N},\mathscr{R})$ is α-soft open, then $\mho$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$ and so $\mho^* \sqsubseteq \mho$. Hence $\mho$ is $\tilde{*}$-closed. Conversely, suppose that every α-soft open set is $\tilde{*}$-closed. If $\mho$ is α-soft open set such that $(\Phi,\mathscr{R}) \sqsubseteq \mho \sqsubseteq (\mathscr{N},\mathscr{R})$, then $(\Phi,\mathscr{R})^* \sqsubseteq \mho \sqsubseteq (\mathscr{N},\mathscr{R})$ and $(\Phi,\mathscr{R})$ is an $\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$.

Theorem 17.*For a sts $(\mathscr{N},\sigma,\mathscr{R})$ and $\tilde{\mathscr{I}}$ is a completely codense a soft ideal over $\mathscr{N}$ with $\mathscr{R}$ the same set of parameters. The following are then equivalent.*
1-$\mathscr{N}$ is soft normal,
2-For any soft closed sets that are disjointed $\Phi_{\mathscr{R}}$ and $\Psi_{\mathscr{R}}$, There are disjoint $\tilde{\mathscr{I}}g$-open sets $\mho_1$ and $\mho_2$ such that $\Phi_{\mathscr{R}} \sqsubseteq \mho_1$ and $\Psi_{\mathscr{R}} \sqsubseteq \mho_2$,
3-For any soft closed $\Phi_{\mathscr{R}}$ and soft open set $\mho_2$ containing

$\Phi_{\mathscr{R}}$, there exist an $\tilde{\mathscr{I}}\mathscr{G}$-open set $\mho_1$ such that $\Phi_{\mathscr{R}} \sqsubseteq \mho_1 \sqsubseteq Cl^*(\mho_1) \sqsubseteq \mho_2$.

*Proof.*1 $\Rightarrow$ 2 The evidence is provided by the fact that each open set is $\tilde{\mathscr{I}}\mathscr{G}$-open.
2 $\Rightarrow$ 3 Let $\Phi_{\mathscr{R}}$ be soft closed and $\mho_2$ is an soft open set containing $\Phi_{\mathscr{R}}$. Since $\Phi_{\mathscr{R}}$ and $\mathscr{N}$-$\mho_2$ are Separate soft closed sets, there are varying $\tilde{\mathscr{I}}\mathscr{G}$-open $\mho_1$ and $\mho_3$ such that $\Phi_{\mathscr{R}} \sqsubseteq \mho_1$ and $(\mathscr{N}\text{-}\mho_2) \sqsubseteq \mho_3$. Since $\mathscr{N}$-$\mho_2$ is $\hat{\mathscr{G}}$-closed and $\mho_3$ is $\tilde{\mathscr{I}}\mathscr{G}$-open, $\mathscr{N}$-$\mho_2 \sqsubseteq Int^*(\mho_3)$ and so $\mathscr{N}$-$Int^*(\mho_3) \sqsubseteq \mho_2$. Again $\mho_1 \Cap \mho_3 = \phi$ which implies that $\mho_1 \Cap Int^*(\mho_3) = \phi$ and so $\mho_1 \sqsubseteq \mathscr{N}$-$Int^*(\mho_3)$ which implies that $Cl^*(\mho_1) \sqsubseteq \mathscr{N}$-$Int^*(\mho_3) \sqsubseteq \mho_2$. Then there is an $\tilde{\mathscr{I}}\mathscr{G}$-open set $\mho_1$ such that $\Phi_{\mathscr{R}} \sqsubseteq \mho_1 \sqsubseteq Cl^*(\mho_1) \sqsubseteq \mho_2$.
3 $\Rightarrow$ 1 If $\Phi_{\mathscr{R}}$ and $\Psi_{\mathscr{R}}$ are two disjoint soft closed subsets of $\mathscr{N}$. By hypothesis, there is an $\tilde{\mathscr{I}}\mathscr{G}$-open set $\mho_1$ that $\Phi_{\mathscr{R}} \sqsubseteq \mho_1 \sqsubseteq Cl^*(\mho_1) \sqsubseteq \mathscr{N} - \Psi_{\mathscr{R}}$. Since $\mho_1$ is $\tilde{\mathscr{I}}\mathscr{G}$-open, $\Phi_{\mathscr{R}} \sqsubseteq Int^*(\mho_1)$. Since $\tilde{\mathscr{I}}$ is completely codense, by Theorem 6 [33], $\sigma^* \sqsubseteq \sigma^{\alpha}$ and so $Int^*(\mho_1)$ and $\mathscr{N}$-$Cl^*(\mho_1) \in \sigma^{\alpha}$. Hence $\Phi_{\mathscr{R}} \sqsubseteq Int^*(\mho_1) \sqsubseteq Int(Cl(Int(Int^*(\mho_1)))) = G$ and $\Psi_{\mathscr{R}} \sqsubseteq \mathscr{N}$-$Cl^*(\mho_1) \sqsubseteq Int(Cl(Int(\mathscr{N}\text{-}Cl^*(\mho_1)))) = H$, G and H are the required disjoint soft open sets containing $\Phi_{\mathscr{R}}$ and $\Psi_{\mathscr{R}}$ respectively, which proves 1.

3 Conclusion

We merged ideal topology and soft topology to provide generalized new sets, such as α-generalized closed sets pertaining to soft ideal in sts. In a soft ideal topological space, numerous features of $\alpha\mathscr{G}C_s$ are also discussed. Finally, using the various examples provided, we present several applications on α-generalized soft closed sets. We shall investigate neighborhoods ideal with soft topology in the future to explain the generalization of soft neighborhoods ideal generalized closed sets.

4 List of abbreviations

sts: soft topological space
$\mathscr{G}C_s$: generalized closed set.
$I\mathscr{G}C_s$: ideal generalized closed set.
$\alpha\tilde{\mathscr{I}}\mathscr{G}C_s$: α-soft ideal generalized closed set.
$\alpha\mathscr{G}C_s$: α-generalized closed set.
S_s: soft set.
SS_s: soft sets

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Acknowledgement

This study is supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2023/R/1444).

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