Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/131320

Stress-Strength Reliability for Monsef Distribution

M. M. E. Abd El-Monsef[1,], E. I. Soliman[2] and M. A. El-Qurashi[2]*

[1]Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt
[2]Institute of Basic & Applied Science, Arab Academy for Science and Technology, Alexandria, Egypt

Received: 6 Nov. 2023, Revised: 26 Jan. 2024, Accepted: 26 Feb. 2024.
Published online: 1 May 2024

Abstract: The investigation of the reliability of any component tested in the field of life model is a challenge. Without a doubt, one of the most critical concerns is "stress “s” strength “S” reliability". This paper aims to discuss how the stress-strength coefficient $R = P(Y < X)$ from any statistical literature Can be estimated. System reshaping is subjected to a random force X in such a way that the system fails if the stress exceeds the force. The reliability of stress resistance is examined in this study when force X has a Monsef distribution “ME” and stress (Y_1, Y_2, Y_3, Y_4, Y_5 and Y_6) has an ME distribution, Lindley distribution “Lin”, Rayleigh distribution “Ray”, Exponential distribution “Exp”, Half-Normal distribution “HN” and Rayleigh Half-Normal distribution “RHN” respectively. Explicitly obtaining the maximum probability estimator for the unknown parameter is also possible. Additionally, the maximum likelihood estimation of “MLEs” asymptotic distribution obtained which can be used to construct a confidence interval for R. The proposed model is compared with other existence models using simulations, and an illustrative data analysis was performed. Finally, we determined that the maximum product of the spacing method yielded the best results.

Keywords: Stress-strength model, Monsef distribution; Maximum likelihood estimator, Monte Carlo simulation study.

1 Introduction

There are situations in which a system is not time-dependent and might continue functioning forever if all the inputs remain within their specified ranges. Various stressors may lead to a system's demise. They may work for a long time if the stresses are below a specific value; but, if the stresses are over that value, they may fail quickly. A system's performance or failure is largely determined by the amount of stress it is subjected to. The source of stress in an operating system of this type may vary. For instance, pressure, load, velocity, resistance, temperature, humidity, vibrations, and voltage may all influence the system's operation. Consequently, the strength of a system made of components of various strengths will be a random variable, as will the stress applied to it. When the stress exceeds the system's tolerance, the system will fail. These systems are referred to as "models of stress-strength reliability.
Birnbaum (1956) offered the seed of this concept, which he and McCarty subsequently refined (1958). The official word "stress-strength" appears in Church and Harris's [1] book title (1970).
Consequently, this component's reliability is determined by $R = P(Y < X)$. Furthermore, with the complexity increasing and automation of industrial processes, the issue of improving system reliability has become increasingly important in a variety of industries, including transportation, communications, and manufacturing. Significant financial losses can result from overestimation or underestimation of reliability variables.
The stress-strength model, estimates of $P(X < Y)$ were made for the most common distributions. such as normal derivative by Woodward and Kelley[2], Pareto derivative by Beg and Singh[3], Burr explain

*Corresponding author e-mail: mmezzat@science.tanta.edu.eg

by Awad and Gharraf[4] and Surles and Padgett[5,6], Asgharzadeh et al.[7,8,9] introduced the estimation of stress-strength reliability for Weibull distribution, logistic distribution and exponential, Valiollahi et al.[10] and Raqab and Kundu[11] estimate Weibull distribution, inverse Rayleigh distribution by Rao et al.[12], exponentiated Frechet distribution Rao et al.[13], estimators of $P(X < Y)$ were obtained for the majority of common distribution families for the situations when X and Y are independent, by Saraçoğlu et al.[14], Mokhlis[15], Mirjalili et al.[16] Jia et al.[17], Nadeb et al.[18],. Alshenawy et al.[19], El- Sherpieny et al.[20], Nassr et al.[21], and Estimation of the reliability of a stress-strength system from poisson half logistic distribution by Muhammad et al.[22]

In this article, we examine the dependability of stress strength and argue that the strength variable follows finite mixture of $\boldsymbol{ME}$ distribution and stress variables follows Rayleigh , exponential, and a half-normal distribution.

The one-parameter $\boldsymbol{ME}$ distribution is a specific instance of the generalization of mixture Erlang distribution introduced by Abd El-Monsef [25] which provides a more versatile model for lifetime data. This study examines six scenarios in which $\boldsymbol{ME}$ is followed by stress and strength which is distributed differentially.

Case 1: strength follows $\boldsymbol{ME}$.

Case 2: strength follows “$\boldsymbol{Lin}$”

Case 3: strength follows “$\boldsymbol{Ray}$”.

Case 4: strength follows “$\boldsymbol{Exp}$”.

Case 5: strength follows “$\boldsymbol{HN}$”.

Case 6: strength follows “$\boldsymbol{RHN}$”.

On the other hand, the $\boldsymbol{MLE}$s of $\boldsymbol{R}$ also can be obtained in explicit form which is the most reliable method can be used. The asymptotic distribution of the $\boldsymbol{MLE}$s of $\boldsymbol{R}$ can be easily obtained and based on that, the asymptotic confidence interval of $\boldsymbol{R}$ can be found. confidence intervals of $\boldsymbol{R}$ in proposed, which are also efficient to be used in practice. Additionally, the squared error loss function, although any other loss functions also can be easily incorporated. The Bayes estimator of $\boldsymbol{R}$ which cannot be obtained in explicit form. for simulative Monte Carlo techniques is used to compute the Bayes estimate of $\boldsymbol{R}$ and the associated credible interval. Different methods are compared using Monte Carlo simulations and one data set has been analyzed for illustrative purposes.

The paper is organized as follows: we propose the dependability of stress strength and argue that the strength variable follows a finite mixture of ME distribution and stress variables follow Rayleigh, exponential, and a half-normal distribution. in Section 2, presents some characteristics for the density function of reliability. Computations were derived in Section 3. In Section 4, Estimation of Stress-Strength Reliability by using Method of Moment (MOM) Estimation of R and the maximum likelihood estimator of the distribution parameter was explored, and a simulation study was conducted to test its consistency. Finally, the paper is concluded in Section 6.

2 Statistical Model

The reliability $R = P(Y < X)$ is determined in this section, where the random variables (X) and (Y) are independent random variables, the strength X follows $\boldsymbol{ME}$ distribution and the stress (Y) consider different cases when (X) represents "strength" and (Y) represents "stress". The joint pdf $f(x.y)$, thus the component reliability is:

$$R = P(Y < X) = \int_{-\infty}^{\infty} \int_{-\infty}^{x} f(x,y)dy\,dx \tag{1}$$

In case that the $r.v$ are statistically independent, then $f(x,y) = f_X(x)g_Y(y)$ can be expressed as:

$$R = \int_{-\infty}^{\infty} \int_{-\infty}^{x} f_X(x)g_Y(y)dy\,dx \tag{2}$$

where $f_X(x)$ and $g_Y(y)$ are pdf's of X and Y respectively.

3 Reliability Computations

Let X be the strength of the probability density functions$f_X(x)$. The pdf of X which follows $\boldsymbol{ME}$ distribution distributions parameters μ is defined by the following.

$$f_X(x) = \frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}, x, \mu > 0 \tag{3}$$

The stress Y_1 follows ME

As Y_1 follows $\boldsymbol{ME}$ distribution, pdf of Y_1 is given by

$$g_{Y_1}(y) = \frac{\lambda_1^3(y+1)^2 e^{-y\lambda_1}}{2+\lambda_1(2+\lambda_1)};\ y, \lambda_1 > 0 \tag{4}$$

as X and Y_1 are independent then from (2), the reliability function R_1 can be found as:

$$R_1 = \int_0^{\infty}\int_0^{x} \left(\frac{\lambda_1^3(y+1)^2 e^{-y\lambda_1}}{2+\lambda_1(2+\lambda_1)}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

and after the simplification, one of the ways to get it is:

$$R_1 = \frac{1}{(2+2\lambda_1+\lambda_1^2)(\lambda_1+\mu)^5(2+2\mu+\mu^2)} \times \lambda_1^3\Big(\lambda_1^4(2(1+\mu)+\mu^2) + 2\lambda_1^3(2+7\mu+6\mu^2+2\mu^3) + \mu^2(40(1+\mu)+20\mu^2+6\mu^3+\mu^4) + 2\lambda_1\mu(10+30\mu+25\mu^2+10\mu^3+2\mu^4) + \lambda_1^2(4+24\mu+42\mu^2+24\mu^3+6\mu^4)\Big)$$

The stress Y_2 follows Lin .

As Y_2 follows $\boldsymbol{Lin}$, pdf of Y_2 is given by

$$g_{Y_2}(y) = \frac{\lambda_2^2(1+y)e^{-\lambda_2 y_2}}{\lambda_2+1};\ y, \lambda_2 > 0 \tag{5}$$

As X and Y_2 are independent then from (2), the reliability function R_2 is considered as:

$$R_2 = \int_0^{\infty}\int_0^{x} \left(\frac{\lambda_2^2(1+y)e^{-\lambda_2 y}}{\lambda_2+1}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

and after the simplification, it can be obtained as:

$$R_2 = \frac{1}{\big(2+\mu(2+\mu)\big)(1+\lambda_2)(\mu+\lambda_2)^4} \times \lambda_2^2 \Bigg(\mu^2(2+\mu)\big(6+\mu(3+\mu)\big) + \lambda_2\bigg(\mu\Big(8+\mu\big(20+3\mu(4+\mu)\big)\Big) + \lambda_2\Big(2+\mu\big(10+3\mu(3+\mu)\big) + \big(2+\mu(2+\mu)\big)\lambda_2\Big)\bigg)\Bigg)$$

The stress Y_3 follows Ray .

As Y_3 follows $\boldsymbol{Ray}$, pdf of Y_3 is given by

$$g_{Y_3}(y) = \frac{y}{\lambda_3^2} e^{-\frac{y^2}{2\lambda_3^2}};\ y, \lambda_3 > 0 \tag{6}$$

As X and Y_3 are independent then from (2), the reliability function R_3 is

$$R_3 = \int_0^\infty \int_0^x \left(\frac{y}{\lambda_3^2} e^{-\frac{y^2}{2\lambda_3^2}}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

and after the simplification, it can be expressed as:

$$R_3 = \frac{4+4\mu+2\mu^2-4\mu^3\lambda_3^2+2\mu^4\lambda_3^4 - e^{\frac{1}{2}\mu^2\lambda_3^2}\sqrt{2\pi}\mu^3\lambda_3\left(-\mathrm{Erf}\left[\frac{\mu\lambda_3}{\sqrt{2}}\right] + \sqrt{\frac{1}{\lambda_3^2}}\lambda_3\right)\left(1+(1-2\mu)\lambda_3^2+\mu^2\lambda_3^4\right)}{2\big(2+\mu(2+\mu)\big)}$$

where $\mathrm{erf}(u)$ is the Gauss error function which is defined as

$$\mathrm{erfc}\,(u) = 1 - \mathrm{erf}(u) = \frac{2}{\sqrt{\pi}} \int_{\mathrm{u}}^\infty e^{-t^2}\, dt$$

The stress Y_4 follows Exp.

As Y_4 follows $\boldsymbol{Exp}$, pdf of Y_4 is given by

$$g_{Y_4}(y) = \lambda_4 e^{-y\lambda_4};\ y, \lambda_4 > 0 \tag{7}$$

as X and Y_4 are independent then from (2), the reliability function R_4 is

$$R_4 = \int_0^\infty \int_0^x \left(\lambda_4 e^{-y\lambda_4}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

and the simplification it can be found as:

$$R_4 = \frac{\lambda_4\big(\lambda_4^2(2+2\mu+\mu^2) + 2\lambda_4\mu(3+3\mu+\mu^2) + \mu^2(6+4\mu+\mu^2)\big)}{(\lambda_4+\mu)^3(2+2\mu+\mu^2)}$$

The stress Y_5 follows HN .

$$g_{Y_5}(y) = \frac{\sqrt{2}}{\lambda_5\sqrt{\pi}} e^{-\frac{y^2}{2\lambda_5^2}};\ y, \lambda_5 > 0 \tag{8}$$

As X and Y_5 are independent then from (2), the reliability function R_5 is obtained as"

$$R_5 = \int_0^\infty \int_0^x \left(\frac{\sqrt{2}}{\lambda_5\sqrt{\pi}} e^{-\frac{y^2}{2\lambda_5^2}}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

and after the simplification, it can be given as

$$R_5 = \frac{\sqrt{\frac{2}{\pi}}\mu\lambda_5(2(1+\mu)-\mu^2\lambda_5^2) + e^{\frac{1}{2}\mu^2\lambda_5^2}(5+\mu(4+\mu)-\mu^2\lambda_5^2[1+2\mu-\mu^2\lambda_5^2])\mathrm{Erfc}[\frac{\mu\lambda_5}{\sqrt{2}}]}{(2+\mu)^2+1}$$

The stress Y_6 follows RHN .

$$g_{Y_6}(y) = \frac{2\lambda_6\, e^{-y^2\lambda_6}(1+y)}{1+\sqrt{\pi\lambda_6}};\ y, \lambda_6 > 0 \tag{9}$$

As X and Y_6 are independent then from (2), the reliability function R_6 is given by:

$$R_6 = \int_0^\infty \int_0^x \left(\frac{2\lambda_6\, e^{-y^2\lambda_6}(1+y)}{1+\sqrt{\pi\lambda_6}}\right)\left(\frac{\mu^3(x+1)^2 e^{-x\mu}}{2+\mu(2+\mu)}\right) dydx$$

$$R_6 = \frac{1}{8\left(1+\sqrt{\pi}\sqrt{\lambda_6}\right)\lambda^{5/2}\left(2+\mu(2+\mu)\right)} \times 2\sqrt{\lambda}_6\left(-6\lambda_6\mu^3+\mu^4+4\lambda^2\left(2+\mu(4+3\mu)\right)\right) - e^{\frac{\mu^2}{4\lambda_6}}\sqrt{\pi}\left(2\lambda_6(1-3\mu)\mu^3+\mu^5+4\lambda_6^2\mu^2(1+3\mu)-8\lambda_6^3\left(2+\mu(2+\mu)\right)\right)\mathrm{Erfc}\left[\frac{\mu}{2\sqrt{\lambda_6}}\right]$$

4 Estimation of Stress-Strength Reliability

These specified distributions are abundant in the literature and are particularly useful for making predictions about the real world and developing models. In the past few decades, biomedical analysis and reliability engineering, economics, forecasting, astronomy, demography, and insurance have all heavily relied on classical distributions to model data. a discussion of the estimation $R = P(Y < X)$when random variables X and Y are following the lifetime distributions.

4.1 Method of Moment (MOM) Estimation of R

The estimation of reliability is very common in the statistical literature. Now to compute $\hat{R}$, the parameters λ and $\lambda_i, i = 1, 2, 3, 4, 5$ and 6 should be estimated in six cases of stress.

Since the strengths X follow $\boldsymbol{ME}$ distribution (μ), and the stress have fiive cases:

Y_1 follows $\boldsymbol{ME}$ with parameter λ_1,

Y_2 follows $\boldsymbol{Lin}$ with parameter λ_2,

Y_3 follows $\boldsymbol{Ray}$ with parameter λ_3,

Y_4 follows $\boldsymbol{Exp}$ with parameter λ_4,

Y_5 follows $\boldsymbol{HN}$ with parameter λ_5,

and Y_6 follows $\boldsymbol{RHN}$ with parameter λ_6, then their population means are given by:

$$\bar{x} = \frac{6+\mu(4+\mu)}{\mu(2+\mu(2+\mu))}, \quad \bar{y}_1 = \frac{6+\lambda_1(4+\lambda_1)}{\lambda_1(2+\lambda_1(2+\lambda_1))}, \quad \bar{y}_2 = \frac{2+\lambda_2}{\lambda_2+\lambda_2^2}, \quad \bar{y}_3 = \lambda_3\sqrt{\frac{2}{m}}, \quad \bar{y}_4 = \frac{1}{\lambda_4},$$

$$\bar{y}_5 = \frac{\lambda_5}{\sqrt{n}} \quad and \quad \bar{y}_6 = \left(\frac{2+\sqrt{\pi}\sqrt{\lambda_6}}{2n\lambda_6+2n\sqrt{\pi}\,\lambda_6{}^{3/2}}\right)^{0.5}$$

4.2 Maximum Likelihood of Stress-Strength Reliability

In this subsection, the method of $\boldsymbol{MLE}$s is derived to estimate the model parameter.

Case 1: Maximum Likelihood for Stress follows ME with parameter λ_1:

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample of size n from $\boldsymbol{ME}$ distribution with parameters (μ) and $Y_1 = (Y_{11}, Y_{12}, \dots, Y_{1m})$ with parameter (λ_1). The $\boldsymbol{MLE}$ of the reliability given that the sample is obtained. To compute the $\boldsymbol{MLE}$ of the reliability, it is required to obtain the $\boldsymbol{MLE}$ of (μ). and (λ_1). The joint likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_{1(\mu.\lambda_1;x.y_1)} = \left(\frac{\mu^3}{2+\mu(2+\mu)}\right)^n e^{-\mu\sum_{i=1}^n x_i} \prod_{i=1}^n (x_i+1)^2 \left(\frac{\lambda_1^3}{2+\lambda_1(2+\lambda_1)}\right)^m e^{-\lambda_1\sum_{j=1}^m y_{1j}} \prod_{j=1}^m (y_{1j}+1)^2$$

$$\begin{aligned} LogL_1 = {} & 3nLog(\mu) - nLog(2+\mu(2+\mu)) - \mu\sum_{i=1}^n x_i + 2\sum_{i=1}^n Log[x_i+1] + 3mLog[\lambda_1] \\ & - mLog(2+\lambda_1(2+\lambda_1)) - \lambda_1\sum_{j=1}^m y_{1j} + 2\sum_{j=1}^m Log(y_{1j}+1) \end{aligned} \tag{10}$$

Equating the partial derivative of Eq. (10) with respect to μ and $\lambda_1$to zero, then the following equation can be obtained:

$$\frac{\partial(LogL_1)}{\partial\mu} = \frac{3n}{\mu} - \frac{2n(1+\mu)}{2+\mu(2+\mu)} - \sum_{i=1}^n x_i \tag{11}$$

$$\frac{\partial(LogL_1)}{\partial\lambda_1} = \frac{3m}{\lambda_1} - \frac{2m(1+\lambda_1)}{2+\lambda_1(2+\lambda_1)} - \sum_{j=1}^m y_{1j} \tag{12}$$

To find the estimators of $\hat{\mu}$ and $\hat{\lambda}_1$, the following nonlinear Equations (11) and (12) need to be solved. These equations cannot be solved analytically, but it can be solved using numerical optimizations available in Mathematica, MATLAB, or R.

The $\boldsymbol{MLE}$ of μ $and\ \lambda_1$ can be obtained respectively as

$$\hat{\mu} = \frac{n^2 - n\,T + T^2 + 2T\,S_X + 8n\,S_X - 2\,S_X^2}{3T\,S_X}$$

where $S_X = \sum_{i=1}^{n} x_i$

$$T = \left(-n^3 - 2(S_X)(6n^2 + 24nS_X + 5(S_X)^2) + 3\sqrt{6}\sqrt{(S_X)^2\left(n^4 + 2(S_X)(7n^3 + 27n^2S_X + 8n(S_X)^2 + (S_X)^3)\right)}\right)^{1/3}$$

$$\hat{\lambda}_1 = \frac{m^2 - m\,Z + Z^2 + 8\,m\,S_Y + 2\,Z\,S_Y - 2\,S_Y^2}{3Z\,S_Y}$$

where $S_Y = \sum_{j=1}^{m} y_{1j}$ and

$$Z = (-m^3 - 2S_Y(6m^2 + 24mS_Y + 5S_Y^2) + 3\sqrt{6}\sqrt{S_Y^2(m^4 + 2S_Y(7m^3 + 27m^2S_Y + 8mS_Y^2 + S_Y^3))})^{1/3}$$

Case 2: Maximum Likelihood for Stress follows $\boldsymbol{Lin}$ with parameter $\boldsymbol{\lambda_2}$

Let $\boldsymbol{X} = (\boldsymbol{X_1}, \boldsymbol{X_2}, \dots, \boldsymbol{X_n})$ be random sample of size $\boldsymbol{n}$ from $\boldsymbol{ME}$ distribution with parameters $(\boldsymbol{\mu})$.and $\boldsymbol{Y_2} = (\boldsymbol{Y_{21}}, \boldsymbol{Y_{22}}, \dots, \boldsymbol{Y_{2m}})$ be a random sample of size $\boldsymbol{m}$ from $\boldsymbol{Lin}$ with parameter $(\boldsymbol{\lambda_2})$, the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_2(\mu.\lambda_2; x.y_2) = \left(\frac{\mu^3}{2 + \mu(2+\mu)}\right)^n e^{-\mu\sum_{i=1}^{n} x_i} \prod_{i=1}^{n}(x_i+1)^2 \left(\frac{\lambda_2^2}{1+\lambda_2}\right)^m e^{-\lambda_2\sum_{j=1}^{m} y_{2j}} \prod_{j=1}^{m}(y_{2j}+1)$$

$$\begin{aligned} LogL_2 = 3nLog(\mu) - nLog(2 + \mu(2+\mu)) - \mu\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} Log(x_i+1) \\ + 2mLog[\lambda_2] - mLog[1+\lambda_2] - \lambda_2\sum_{j=1}^{m} y_{2j} + \sum_{j=1}^{m} Log\left[y_{2j}+1\right] \end{aligned} \quad (13)$$

Equating the partial derivative of Eq. (13) with respect to $\boldsymbol{\lambda_2}$ to zero, so the following equations can be found as:

$$\frac{\partial(LogL_2)}{\partial\lambda_2} = \frac{2m}{\lambda_2} - \frac{m}{1+\lambda_2} - \sum_{j=1}^{m} y_j \quad (14)$$

To find the estimators of $\hat{\boldsymbol{\lambda}}_{\mathbf{2}}$, the following nonlinear Equations (14) need to be solved using Mathematica. The $\boldsymbol{MLE}$ of $\boldsymbol{\lambda}_{\mathbf{2}}$can be obtained as

$$\hat{\boldsymbol{\lambda}}_{\mathbf{2}}=\frac{\boldsymbol{m}-\sum_{j=1}^{m}\boldsymbol{y}_{j}+\sqrt{\boldsymbol{m}^{2}+6\boldsymbol{m}\sum_{j=1}^{m}\boldsymbol{y}_{j}+(\sum_{j=1}^{m}\boldsymbol{y}_{j})^{2}}}{2\sum_{j=1}^{m}\boldsymbol{y}_{j}}$$

Then the $\boldsymbol{MLE}$ of $\boldsymbol{R}$ when the strength $\boldsymbol{X}$ follows $\boldsymbol{ME}$ distribution and stress $\boldsymbol{Y}$ follows $\boldsymbol{Lin}$ with parameter ($\boldsymbol{\lambda}_{\mathbf{2}}$) is given as

$$\hat{R}_2=\frac{1}{(2+\hat{\mu}(2+\hat{\mu}))(1+\hat{\lambda}_2)(\hat{\mu}+\hat{\lambda}_2)^4}\times\left(\hat{\lambda}_2^2(\hat{\mu}^2(2+\hat{\mu})(6+\hat{\mu}(3+\hat{\mu}))+\hat{\lambda}_2(\hat{\mu}(8+\hat{\mu}(20+3\hat{\mu}(4+\hat{\mu})))+\hat{\lambda}_2(2+\hat{\mu}(10+3\hat{\mu}(3+\hat{\mu}))+(2+\hat{\mu}(2+\hat{\mu}))\hat{\lambda}_2)))\right)$$

Case 3: Maximum Likelihood for Stress follows Ray distribution with parameter $\boldsymbol{\lambda}_{\mathbf{3}}$.

Let $\boldsymbol{X}=(\boldsymbol{X}_{\mathbf{1}},\boldsymbol{X}_{\mathbf{2}},\ldots,\boldsymbol{X}_{\boldsymbol{n}})$ be a random sample of size $\boldsymbol{n}$ from $\boldsymbol{ME}$ with parameters $(\boldsymbol{\mu})$. and $\boldsymbol{Y}_{\mathbf{3}}=(\boldsymbol{Y}_{\mathbf{31}},\boldsymbol{Y}_{\mathbf{32}},\ldots,\boldsymbol{Y}_{\mathbf{3m}})$ be a random sample of size $\boldsymbol{m}$ from $\boldsymbol{R}$ with parameter ($\boldsymbol{\lambda}_{\mathbf{3}}$), the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_3(\mu.\lambda_3;x.y_3)=\left(\frac{\mu^3}{2+\mu(2+\mu)}\right)^n e^{-\mu\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}(x_i+1)^2\left(\frac{1}{\lambda_3^2}\right)^m e^{-\frac{1}{2\lambda_3^2}\sum_{j=1}^{m}y_{3j}^2}\prod_{j=1}^{m}y_{3j}$$

$$\begin{aligned}LogL_3&=3nLog(\mu)-nLog(2+\mu(2+\mu))-\mu\sum_{i=1}^{n}x_i+2\sum_{i=1}^{n}Log(1+x_i)\\&-2mLog(\lambda_3)-\sum_{j=1}^{m}Log(y_{3j})-\frac{1}{2\lambda_3^2}\sum_{j=1}^{m}(y_{3j}^2)\end{aligned}\tag{15}$$

Equating the partial derivative of Eq. (15) with respect to $\boldsymbol{\lambda}_{\mathbf{3}}$to zero, then the following equation can be obtained

$$\frac{\partial(LogL_3)}{\partial\lambda_3}=\frac{2m}{\lambda_3}+\frac{\sum_{j=1}^{m}y_{3j}^2}{\lambda_3^3}\tag{16}$$

To find the estimators of $\hat{\boldsymbol{\lambda}}_{\mathbf{3}}$, the following nonlinear Equations (16) need to solved using Mathematica. The $\boldsymbol{MLE}$ of $\boldsymbol{\lambda_3}$can be obtained as

$$\hat{\lambda}_3 = \left(\frac{\sum_{j=1}^{m} y_j^2}{2\,m}\right)^{\frac{1}{2}}$$

Then the $\boldsymbol{MLE}$ of $\boldsymbol{R_3}$ when the strength $\boldsymbol{X}$ follows $\boldsymbol{ME}$ and stress $\boldsymbol{Y_3}$ follows $\boldsymbol{R}$ distribution with parameter $(\boldsymbol{\lambda_3})$ is given as

$$\hat{R}_3 = \frac{4 + 4\hat{\mu} + 2\hat{\mu}^2 - 4\hat{\mu}^3\lambda_3^2 + 2\hat{\mu}^4\hat{\lambda}_3^4 - e^{\frac{1}{2}\hat{\mu}^2\hat{\lambda}_3^2}\sqrt{2\pi}\hat{\mu}^3\hat{\lambda}_3\left(1 - \mathrm{Erf}\left[\frac{\hat{\mu}\hat{\lambda}_3}{\sqrt{2}}\right]\right)\left(1 + (1-2\hat{\mu})\hat{\lambda}_3^2 + \hat{\mu}^2\hat{\lambda}_3^4\right)}{2\big(2 + \hat{\mu}(2+\hat{\mu})\big)}$$

Case 4: Maximum Likelihood for Stress follows $\boldsymbol{Exp}$ with parameter $\boldsymbol{\lambda_4}$.

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample of size n from $\boldsymbol{ME}$ with parameters (μ). and $Y_4 = (Y_{41}, Y_{42}, \dots, Y_{4m})$ be a random sample of size m from $\boldsymbol{Exp}$ with parameter (λ_4), the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_4(\mu.\lambda_4; x.y_4) = \left(\frac{\mu^3}{2+\mu(2+\mu)}\right)^n e^{-\mu\sum_{i=1}^{n} x_i} \prod_{i=1}^{n} (x_i+1)^2\,(\lambda_4)^m e^{-\lambda_4\sum_{j=1}^{m} y_{4j}}$$

$$\begin{aligned} LogL_4 = {} & 3nLog[\mu] - nLog[2+\mu(2+\mu)] - \mu\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} Log[1+x_i] \\ & + mLog[\lambda_4] - \lambda_4\sum_{j=1}^{m} y_j \end{aligned} \quad (17)$$

Equating the partial derivative of Eq. (17) with respect to $\lambda_4$to zero, then the following equation can be expressed as

$$\frac{\partial(LogL_4)}{\partial\lambda_4} = \frac{m}{\lambda_4} - \sum_{j=1}^{m} y_{4j} \quad (18)$$

To determine the estimators of $\hat{\lambda}_4$, so the following nonlinear Equations (18). Need to be solved using Mathematica. The $\boldsymbol{MLE}$ of $\lambda_4$can be found as

$$\hat{\lambda}_4 = \frac{m}{\sum_{j=1}^{m} y_j}$$

Then the $\boldsymbol{MLE}$ of R_4 when the strength X follows $\boldsymbol{ME}$ and stress Y_4 follows $\boldsymbol{Exp}$ with parameter (λ_4) is given as

$$\hat{R}_4 = \frac{\hat{\lambda}_4 \left(\hat{\lambda}_4^2(2 + 2\hat{\mu} + \hat{\mu}^2) + 2\hat{\lambda}_4\hat{\mu}(3 + 3\hat{\mu} + \hat{\mu}^2) + \hat{\mu}^2(6 + 4\hat{\mu} + \hat{\mu}^2)\right)}{\left(\hat{\lambda}_4 + \hat{\mu}\right)^3 (2 + 2\hat{\mu} + \hat{\mu}^2)}$$

Case 5: Maximum Likelihood for Stress follows $HN$with parameter $\boldsymbol{\lambda_5}$.

Suppose $X = (X_1, X_2, \dots, X_n)$ is sample taken from $\boldsymbol{ME}$ with parameters (μ) and $Y_5 = (Y_{51}, Y_{52}, \dots, Y_{5m})$ be a random sample of size m from $\boldsymbol{HN}$ with parameter (λ_5), the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_5(\mu.\lambda_5; x.y_5) = \left(\frac{\mu^3}{2 + \mu(2 + \mu)}\right)^n e^{-\mu \sum_{i=1}^{n} x_i} \prod_{i=1}^{n} (x_i + 1)^2 \left(\frac{1}{\lambda_5}\right)^m \left(\frac{2}{\pi}\right)^{\frac{m}{2}} e^{-\frac{1}{2\lambda_5^2} \sum_{j=1}^{m} y_{5j}^2}$$

$$LogL_5 = -nLog[2 + \mu(2 + \mu)] - \mu \sum_{i=1}^{n} x_i + 3nLog[\mu] + 2\sum_{i=1}^{n} Log[1 + x_i] + \frac{m}{2} Log[2] - mLog[\lambda_5] - \frac{m}{2} Log[\pi] - \frac{1}{2{\lambda_5}^2} \sum_{j=1}^{m} {y_{5j}}^2 \tag{19}$$

Equating the partial derivative of Eq. (19) with respect to $\lambda_5$to zero, then we derive the following equation.

$$\frac{\partial(LogL_5)}{\partial\lambda_5} = \frac{\sum_{j=1}^{m} y_{5j}^2 - m\lambda_5^2}{\lambda_5^3} \tag{20}$$

To find the estimators of $\hat{\lambda}_5$, the following nonlinear Equations (20) should be solved. using Mathematica. The $\boldsymbol{MLE}$ of $\lambda_5$can be obtained as

$$\hat{\lambda}_5 = \left(\frac{\sum_{j=1}^{m} y_j^2}{\mathrm{m}}\right)^{\frac{1}{2}}$$

Then the $\boldsymbol{MLE}$ of R_5 when the strength X follows $\boldsymbol{ME}$ and stress Y_5 follows $\boldsymbol{HN}$ with parameter (λ_5) is given as

$$\hat{R}_5 = \frac{\sqrt{\frac{2}{\pi}}\hat{\lambda}_5\hat{\mu}\left(2+2\hat{\mu}-\hat{\lambda}_5^2\hat{\mu}^2\right)+e^{\frac{\hat{\lambda}_5^2\hat{\mu}^2}{2}}\left(2+2\hat{\mu}-\left(-1+\hat{\lambda}_5^2\right)\hat{\mu}^2-2\hat{\lambda}_5^2\hat{\mu}^3+\hat{\lambda}_5^4\hat{\mu}^4\right)Erfc\left[\frac{\hat{\lambda}_5\hat{\mu}}{\sqrt{2}}\right]}{2+2\hat{\mu}+\hat{\mu}^2}$$

Case 6: Maximum Likelihood for Stress follows $\boldsymbol{RHN}$ with parameter $\boldsymbol{\lambda_6}$.

Suppose $X = (X_1, X_2, \ldots, X_n)$ be a random sample of size n from $\boldsymbol{ME}$ distribution with parameters (μ) and $Y_6 = (Y_{61}, Y_{62}, \ldots, Y_{6m})$ be a random sample of size m from $\boldsymbol{RHN}$ with parameter (λ_6) then, the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L_6(\mu.\lambda_6; x.y_6) = \left(\frac{\mu^3}{2+\mu(2+\mu)}\right)^n e^{-\mu\sum_{i=1}^n x_i}\prod_{i=1}^n (x_i+1)^2 \left(\frac{2\lambda_6}{1+\sqrt{\pi}\sqrt{\lambda_6}}\right)^m e^{-\lambda_6\sum_{j=1}^m y_{6j}^2}\prod_{j=1}^m (1+y_{6j})$$

$$\begin{aligned} LogL_6 = {} & 3nLog[\mu] - nLog[2+\mu(2+\mu)] - \mu\sum_{i=1}^n x_i + 2\sum_{i=1}^n Log[1+x_i] - mLog\left[1+\sqrt{\pi}\sqrt{\lambda_6}\right] \\ & + mLog[2\lambda_6] - \lambda_6\sum_{j=1}^m y_{6j}^2 + \sum_{j=1}^m Log\left[1+y_{6j}\right] \end{aligned} \tag{21}$$

Equating the partial derivative of Eq. (21) with respect to λ_6 to zero, following equation can be achieved

$$\frac{\partial(LogL_6)}{\partial\lambda_6} = \frac{m}{\lambda_6} - \frac{m\sqrt{\pi}}{2(\sqrt{\lambda_6}+\sqrt{\pi}\lambda_6)} - \sum_{j=1}^m y_{6j}^2 \tag{22}$$

To obtain the estimators of $\hat{\lambda}_6$, we need to solve the following nonlinear Equations (22). using Mathematica. The $\boldsymbol{MLE}$ of $\lambda_6$can be obtained as

$$\hat{\lambda}_6 = \frac{Q^2 + 2QW(m\pi + W) + W^2(m^2\pi^2 - 16m\pi W + 4W^2)}{6\pi QW^2}$$

where $W = \sum_{j=1}^m y_{6j}^2$ and

$$\begin{aligned} Q = {} & (-m^3\pi^3W^3 + 51m^2\pi^2W^4 - 48m\pi W^5 + 8W^6 \\ & + 3\sqrt{3}\pi^{3/2}\sqrt{m^3W^7(-2m^2\pi^2 + 71m\pi W - 16W^2)})^{1/3} \end{aligned}$$

Then the $\boldsymbol{MLE}$ of R_6 when the strength X follows $\boldsymbol{ME}$ and stress Y_6 follows $\boldsymbol{RHN}$ with parameter (λ_6) is given as

$$\hat{R}_6 = \frac{2\sqrt{\hat{\lambda}_6}(\hat{\mu}^4 - 6\hat{\mu}^3\hat{\lambda}_6 + 4(2+\hat{\mu}(4+3\hat{\mu}))\hat{\lambda}_6^2) - e^{\frac{\hat{\mu}^2}{4\hat{\lambda}_6}}\sqrt{\pi}\mathrm{Erfc}[\frac{\hat{\mu}}{2\sqrt{\hat{\lambda}_6}}](\hat{\mu}^5 + 2\hat{\lambda}_6(\hat{\mu}^3 - 3\hat{\mu}^4 + 2\hat{\mu}^2(1+3\hat{\mu})\hat{\lambda}_6 - 4(2+\hat{\mu}(2+\hat{\mu}))\hat{\lambda}_6^2))}{8(2+\hat{\mu}(2+\hat{\mu}))(1+\sqrt{\pi}\sqrt{\hat{\lambda}_6})\hat{\lambda}_6^{5/2}}$$

4.3 Numerical Evaluation

In different cases, the system reliability $\boldsymbol{R}$ has evaluated for some specific values of the parameters involved in the expression of $\boldsymbol{R}$. A system with two components is analyzed using a graphical method for stated values of the reliability expressions in section 3. If the parameters for stress, strength, or probability is changed, the reliability of a two-component system will change in the same way. Since the strengths X follow $\boldsymbol{ME}$ distribution (μ), and the stress have in six cases of stress.:

Case 1: Stress follows $\boldsymbol{ME}$ with parameter $\boldsymbol{\lambda_1}$:

Table 1 and Fig 1 and 2 indicate that the reliability value decreases as the strength parameter values increase. these results show that the Increased reliability is correlated with the increased stress.

Table 1: Iteration in R_1 when strength and stress follow $\boldsymbol{ME}$

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.5000	0.2100	0.1038	0.0582	0.0359	0.0237	0.0165	0.0120	0.0090	0.0069
0.2	0.7900	0.5000	0.3178	0.2106	0.1458	0.1049	0.0780	0.0597	0.0468	0.0374
0.3	0.8962	0.6822	0.5000	0.3685	0.2766	0.2120	0.1658	0.1321	0.1071	0.0882
0.4	0.9418	0.7894	0.6315	0.5000	0.3976	0.3195	0.2598	0.2139	0.1783	0.1503
0.5	0.9641	0.8542	0.7234	0.6024	0.5000	0.4165	0.3493	0.2952	0.2517	0.2163
0.6	0.9763	0.8951	0.7880	0.6805	0.5835	0.5000	0.4297	0.3711	0.3223	0.2817
0.7	0.9835	0.9220	0.8342	0.7402	0.6507	0.5703	0.5000	0.4395	0.3878	0.3436
0.8	0.9880	0.9403	0.8679	0.7861	0.7048	0.6289	0.5605	0.5000	0.4470	0.4009
0.9	0.9910	0.9532	0.8929	0.8217	0.7483	0.6777	0.6122	0.5530	0.5000	0.4530
1	0.9931	0.9626	0.9118	0.8497	0.7837	0.7183	0.6564	0.5991	0.5470	0.5000

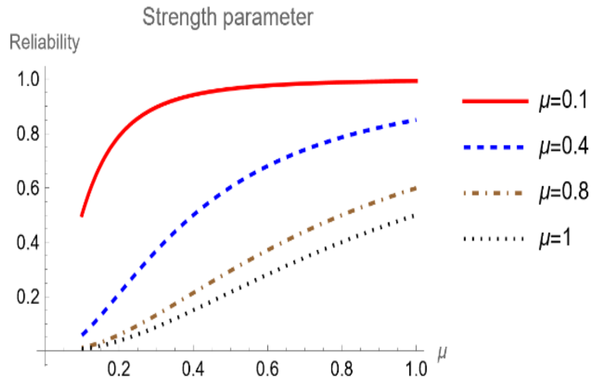

Fig 1: Alteration in R_1 for fixed strength

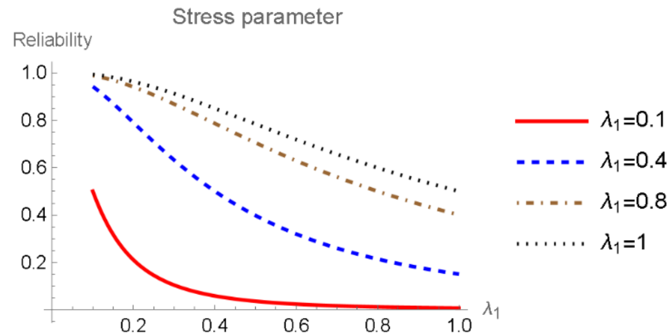

Fig 2: Alteration in R_1 for fixed stress

Case 2: Stress follows $\boldsymbol{Lin}$ with parameter $\boldsymbol{\lambda_2}$

Table 2 and Fig 3 and 4 indicate that the reliability value decreases as the strength parameter values increase. Increased reliability is correlated with increased stress.

Table 2: Alteration in R_2 when strength has $\boldsymbol{ME}$ and stress has $\boldsymbol{Lin}$

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.6861	0.4051	0.2592	0.1784	0.1298	0.0985	0.0772	0.0622	0.0512	0.0429
0.2	0.8870	0.6827	0.5182	0.4001	0.3158	0.2546	0.2092	0.1748	0.1482	0.1273
0.3	0.9474	0.8149	0.6780	0.5617	0.4683	0.3942	0.3353	0.2882	0.2502	0.2191
0.4	0.9712	0.8826	0.7752	0.6725	0.5824	0.5059	0.4418	0.3882	0.3434	0.3056
0.5	0.9823	0.9205	0.8365	0.7489	0.6667	0.5930	0.5285	0.4726	0.4244	0.3827
0.6	0.9883	0.9433	0.8768	0.8027	0.7293	0.6606	0.5983	0.5427	0.4934	0.4499
0.7	0.9918	0.9578	0.9044	0.8415	0.7765	0.7135	0.6546	0.6006	0.5518	0.5078
0.8	0.9939	0.9676	0.9238	0.8702	0.8127	0.7553	0.7003	0.6487	0.6011	0.5576
0.9	0.9954	0.9744	0.9380	0.8919	0.8409	0.7888	0.7376	0.6888	0.6430	0.6005
1	0.9964	0.9792	0.9486	0.9086	0.8632	0.8158	0.7684	0.7224	0.6786	0.6375

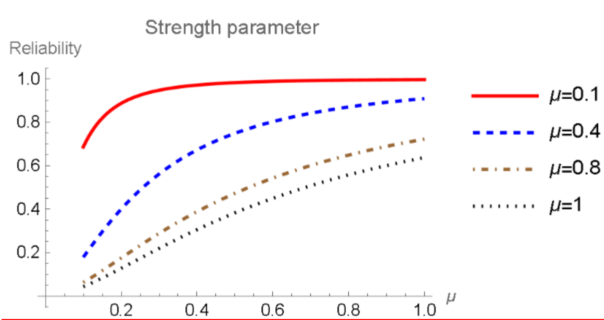

Fig 3: Alteration in R_2 for fixed strength

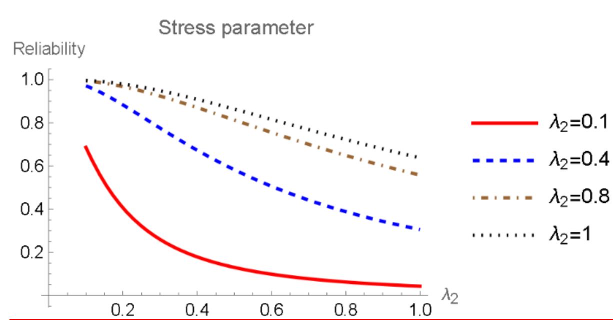

Fig 4: alteration in R_2 for fixed stress

Case 3: Stress follows Ray distribution with parameter λ_3.

Table 3 and Fig 5 and 6 indicate that the reliability value decreases as the strength parameter values increase. Increased reliability is correlated with increased stress.

Table 3: Alteration in R_3 when strength has $\boldsymbol{ME}$ and stress have $\boldsymbol{Ray}$

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.9999	0.9995	0.9986	0.9969	0.9946	0.9916	0.9878	0.9835	0.9785	0.9731
0.2	0.9998	0.9989	0.9968	0.9932	0.9880	0.9815	0.9735	0.9643	0.9541	0.9429
0.3	0.9997	0.9982	0.9946	0.9886	0.9803	0.9698	0.9573	0.9430	0.9273	0.9104
0.4	0.9996	0.9973	0.9920	0.9834	0.9715	0.9567	0.9394	0.9199	0.8988	0.8763
0.5	0.9994	0.9962	0.9890	0.9774	0.9617	0.9424	0.9201	0.8954	0.8689	0.8413
0.6	0.9993	0.9950	0.9856	0.9707	0.9509	0.9269	0.8996	0.8698	0.8384	0.8059
0.7	0.9990	0.9936	0.9817	0.9634	0.9393	0.9105	0.8782	0.8435	0.8074	0.7706
0.8	0.9988	0.9920	0.9775	0.9555	0.9269	0.8933	0.8562	0.8169	0.7765	0.7359
0.9	0.9985	0.9902	0.9729	0.9469	0.9138	0.8755	0.8337	0.7900	0.7458	0.7019
1	0.9982	0.9883	0.9680	0.9379	0.9002	0.8571	0.8109	0.7633	0.7156	0.6689

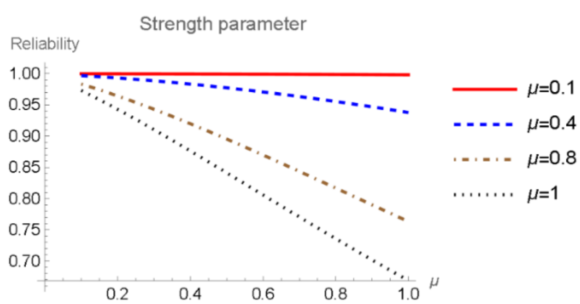

Fig 5: Alteration in R_3 for fixed strength

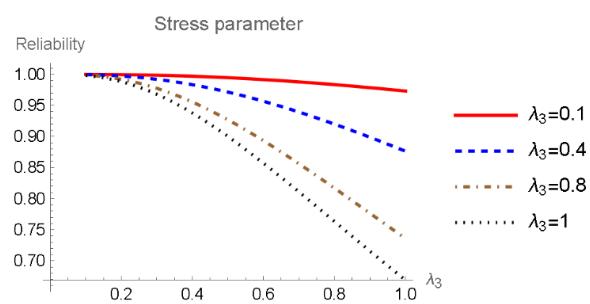

Fig 6: Alteration in R_3 for fixed stress

Case 4: Stress follows Exp with parameter λ_4.

Table 4 and Fig 7 and 8 indicate that the reliability value decreases as the strength parameter values increase. Increased reliability is correlated with increased stress.

Table 4: Alteration in R_4 when strength has $\boldsymbol{ME}$ and stress have $\boldsymbol{Exp}$.

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.8620	0.6733	0.5358	0.4378	0.3661	0.3119	0.2698	0.2364	0.2093	0.1871
0.2	0.9549	0.8484	0.7390	0.6436	0.5638	0.4975	0.4424	0.3962	0.3572	0.3241
0.3	0.9791	0.9148	0.8346	0.7548	0.6815	0.6163	0.5591	0.5092	0.4656	0.4274
0.4	0.9882	0.9460	0.8862	0.8209	0.7568	0.6966	0.6416	0.5919	0.5473	0.5073
0.5	0.9925	0.9628	0.9169	0.8633	0.8077	0.7534	0.7020	0.6543	0.6104	0.5704

0.6	0.9949	0.9728	0.9365	0.8919	0.8437	0.7949	0.7476	0.7025	0.6603	0.6211
0.7	0.9963	0.9793	0.9498	0.9121	0.8700	0.8263	0.7828	0.7406	0.7004	0.6625
0.8	0.9971	0.9836	0.9592	0.9269	0.8899	0.8505	0.8106	0.7712	0.7332	0.6968
0.9	0.9977	0.9867	0.9661	0.9381	0.9052	0.8697	0.8330	0.7962	0.7603	0.7256
1	0.9982	0.9889	0.9713	0.9467	0.9174	0.8851	0.8512	0.8170	0.7831	0.7500

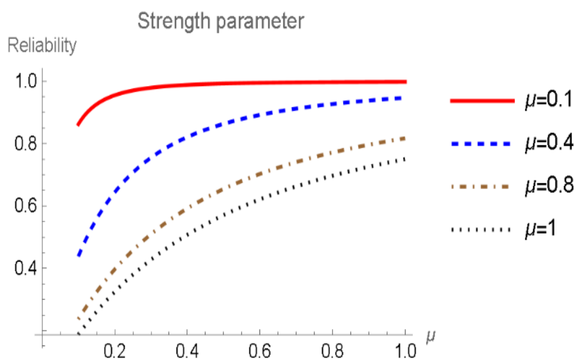

Fig 7: Alteration in R_4 for fixed strength

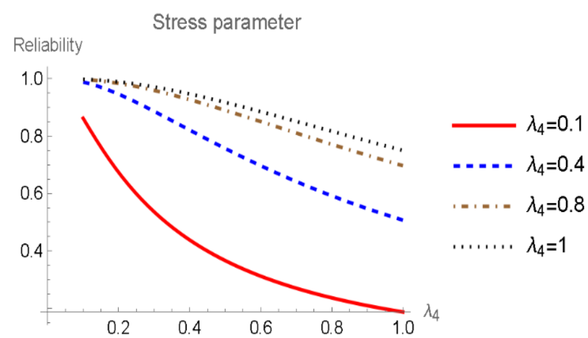

Fig 8: Alteration in R_4 for fixed stress

Case 5: Stress follows $\boldsymbol{HN}$with parameter $\boldsymbol{\lambda_5}$.

Table 5 and Fig 9 and 10 indicate that the reliability value decreases as the strength parameter values increase. Increased reliability is correlated with increased stress.

Table 5: Alteration in R_5 when strength has $\boldsymbol{ME}$ and stress have $\boldsymbol{HN}$

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	1.0000	0.9997	0.9991	0.9981	0.9966	0.9947	0.9924	0.9897	0.9865	0.9831
0.2	0.9999	0.9994	0.9980	0.9958	0.9927	0.9886	0.9837	0.9780	0.9716	0.9645
0.3	0.9998	0.9989	0.9968	0.9932	0.9882	0.9818	0.9740	0.9652	0.9553	0.9447
0.4	0.9998	0.9984	0.9953	0.9902	0.9831	0.9741	0.9635	0.9514	0.9381	0.9238
0.5	0.9997	0.9978	0.9937	0.9869	0.9775	0.9658	0.9521	0.9368	0.9200	0.9023
0.6	0.9996	0.9972	0.9918	0.9831	0.9714	0.9569	0.9401	0.9215	0.9014	0.8804
0.7	0.9995	0.9964	0.9897	0.9791	0.9648	0.9474	0.9275	0.9057	0.8824	0.8583
0.8	0.9993	0.9956	0.9874	0.9747	0.9578	0.9375	0.9145	0.8895	0.8632	0.8362
0.9	0.9992	0.9947	0.9850	0.9700	0.9504	0.9271	0.9010	0.8731	0.8439	0.8143
1	0.9990	0.9937	0.9823	0.9650	0.9427	0.9164	0.8873	0.8565	0.8247	0.7926

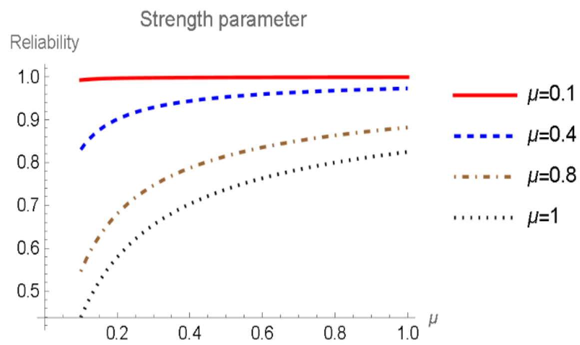

Fig 9: Alteration in R_5 for fixed strength

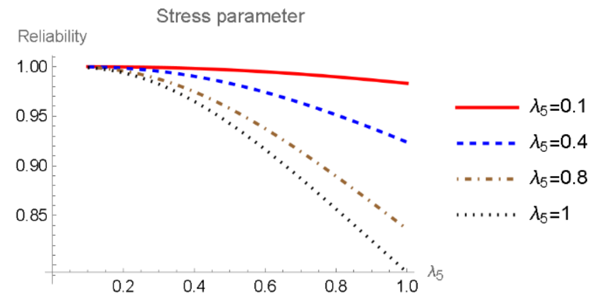

Fig 10: Alteration in R_5 for fixed stress

Case 6: Stress follows RHN with parameter λ_6.

Table 6 and Fig 11 and 12 indicate that the reliability value decreases as the strength parameter values increase. Increased reliability is correlated with increased stress.

Table 6: Alteration in R_6 when strength has ME and stress have RHN

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.9926	0.9589	0.9013	0.8301	0.7542	0.6796	0.6097	0.5460	0.4890	0.4387
0.2	0.9965	0.9791	0.9461	0.9009	0.8483	0.7922	0.7355	0.6804	0.6280	0.5791
0.3	0.9978	0.9859	0.9623	0.9287	0.8877	0.8423	0.7947	0.7468	0.6999	0.6548
0.4	0.9983	0.9893	0.9708	0.9436	0.9097	0.8711	0.8299	0.7874	0.7450	0.7034
0.5	0.9987	0.9913	0.9759	0.9530	0.9238	0.8901	0.8534	0.8151	0.7763	0.7377
0.6	0.9989	0.9926	0.9794	0.9595	0.9337	0.9036	0.8704	0.8354	0.7995	0.7635
0.7	0.9990	0.9936	0.9820	0.9642	0.9411	0.9137	0.8833	0.8509	0.8175	0.7836
0.8	0.9992	0.9943	0.9839	0.9678	0.9467	0.9216	0.8935	0.8633	0.8319	0.7999
0.9	0.9992	0.9949	0.9854	0.9707	0.9513	0.9280	0.9018	0.8734	0.8438	0.8134
1	0.9993	0.9953	0.9866	0.9730	0.9550	0.9332	0.9086	0.8818	0.8537	0.8248

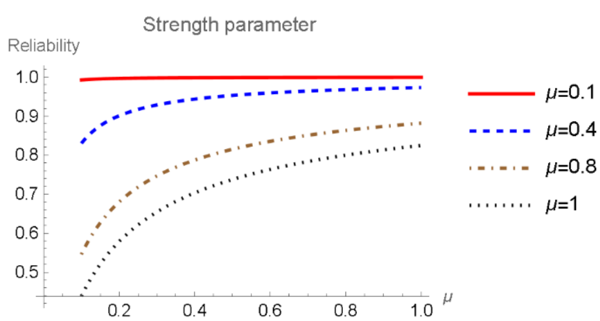

Fig 11: Alteration in R_6 for fixed strength

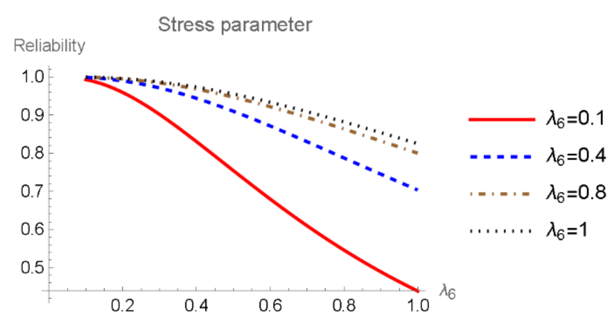

Fig 12: Alteration in R_6 for fixed stress

5. Simulation Results

In this section, the performance of the estimators is evaluated using Monte Carlo simulations. To generate simulated samples, several parameter values from independent sources are employed. $\boldsymbol{ME}(\mu)$ and $\boldsymbol{ME}(\lambda_1)$ are of sizes n and m respectively, (5,5), (10, 10), (20, 20), (30, 30), (40, 40) and (100, 100), with two different sets of parameter values, namely and (1, 0.5)

The following sample sizes are considered; $(n, m) = (5, 5)$, $(10, 10)$, $(20,20)$, $(30,30)$, $(40,40)$, $(50,50)$ and $(100,100)$.

Table7: Simulation studt

(n, m) (μ, λ_1)	(5,5)	(10,10)	(20,20)	(30,30)	(40,40)	(50,50)	(100,100)
(1, 0.5)	0.0162	0.0071	0.0038	0.0025	0.0018	0.0016	0.0008
	0.0194	0.0118	0.0053	0.0031	0.0072	0.0002	0.0004
(1, 1)	0.0251	0.0128	0.0069	0.0048	0.0037	0.0029	0.0015
	0.0043	0.0002	0.0022	0.0011	0.0040	0.0042	0.0012
(1, 1.5)	0.0209	0.0114	0.0059	0.0034	0.0028	0.0023	0.0011
	0.0117	0.0092	0.0049	0.0029	0.0009	0.0007	0.0000
(1, 2)	0.0154	0.0077	0.0040	0.0025	0.0022	0.0016	0.0007
	0.0198	0.0089	0.0060	0.0021	0.0022	0.0034	0.0020
(0.5, 1)	0.0162	0.0075	0.0038	0.0026	0.0018	0.0013	0.0008
	0.0224	0.0096	0.0061	0.0020	0.0026	0.0015	0.0016
(1.5, 1)	0.0204	0.0117	0.0056	0.0038	0.0027	0.0023	0.0011
	0.0158	0.0090	0.0026	0.0023	0.0016	0.0010	0.0009
(2, 1)	0.0154	0.0082	0.0042	0.0026	0.0018	0.0015	0.0008
	0.0161	0.0157	0.0042	0.0037	0.0016	0.0031	0.0005

6. Conclusion

In this article, the stress-strength reliability for the Monsef distribution has been explored when the strength follows the Monsef distribution and the stress follows the Monsef, Lindley, Rayleigh, exponential, Half-Normal, and Rayleigh Half-Normal distributions. The maximum likelihood estimation approach is used to provide parameter estimates based on calculations and graphs. The numerical evaluation reveals that when the stress has a Monsef, Lindley, Rayleigh, exponential, half-normal, or Rayleigh half-normal distribution, the dependability increases as the stress level rises and vice versa. We derived the maximal likelihood estimation (MLE) and the method of moments (MOM) for estimating the unknown parameters that are employed in reliability estimation (R). The simulation study is carried out to assess the performance of the various offered estimation.

Conflict of Interest

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest, or non-financial interest in the subject matter or materials discussed in this manuscript.

References

[1] Church, J. D., & Harris, B. (1970). The estimation of reliability from stress-strength relationships. Technometrics, 12(1), 49-54.

[2] Woodward, W. A., & Kelley, G. D. (1977). Minimum variance unbiased estimation of P [Y< X] in the normal case. Technometrics, 19(1), 95-98.

[3] Beg, M. A., & Singh, N. (1979). Estimation of Pr (Y< X) for the Pareto distribution. IEEE Transactions on reliability, 28(5), 411-414.

[4] Awad, A. M., & Gharraf, M. K. (1986). Estimation of P (Y< X) in the Burr case: A comparative study. Communications in Statistics-Simulation and Computation, 15(2), 389-403.

[5] Surles, J. G., & Padgett, W. J. (1998). Inference for P (Y< X) in the Burr type X model. Journal of Applied Statistical Sciences, 7(4), 225-238.

[6] Surles, J. G., & Padgett, W. J. (2001). Inference for reliability and stress-strength for a scaled Burr type X distribution. Lifetime data analysis, 7, 187-200.

[7] Asgharzadeh, A., Valiollahi, R., & Raqab, M. Z. (2011). Stress-strength reliability of Weibull distribution based on progressively censored samples. SORT-Statistics and Operations Research Transactions, 103-124.

[8] Asgharzadeh, A., Valiollahi, R., & Raqab, M. Z. (2013). Estimation of the stress–strength reliability for the generalized logistic distribution. Statistical Methodology, 15, 73-94.

[9] Asgharzadeh, A., Valiollahi, R., & Raqab, M. Z. (2017). Estimation of Pr (Y< X) for the two-parameter generalized exponential records. Communications in Statistics-Simulation and Computation, 46(1), 379-394.

[10] Valiollahi, R., Asgharzadeh, A., & Raqab, M. Z. (2013). Estimation of P (Y< X) for Weibull distribution under progressive Type-II censoring. Communications in Statistics-Theory and Methods, 42(24), 4476-4498.

[11] Kundu, D., & Raqab, M. Z. (2009). Estimation of R= P (Y< X) for three-parameter Weibull distribution. Statistics & Probability Letters, 79(17), 1839-1846.

[12] Srinivasa Rao, G., Kantam, R. R. L., Rosaiah, K., & Pratapa Reddy, J. (2013). Estimation of stress–strength reliability from inverse Rayleigh distribution. Journal of Industrial and production Engineering, 30(4), 256-263.

[13] Rao, G. S., Rosaiah, K., & Babu, M. S. (2016). Estimation of stress-strength reliability from exponentiated Fréchet distribution. The International Journal of Advanced Manufacturing Technology, 86, 3041-3049.

[14] Saraçoğlu, B., Kinaci, I., & Kundu, D. (2012). On estimation of R= P (Y< X) for exponential distribution under progressive type-II censoring. Journal of Statistical Computation and Simulation, 82(5), 729-744.

[15] Mokhlis, N. A. (2005). Reliability of a stress-strength model with Burr type III distributions. Communications in Statistics-Theory and Methods, 34(7), 1643-1657.

[16] Mirjalili, M., Torabi, H., & Nadeb, H. (2016). Stress-strength reliability of exponential distribution based on Type-I progressively hybrid censored samples. Journal of Statistical Research of Iran JSRI, 13(1), 89-105.

[17] Jia, X., Nadarajah, S., & Guo, B. (2017). Bayes estimation of P (Y< X) for the Weibull distribution with arbitrary parameters. Applied Mathematical Modelling, 47, 249-259.

[18] Nadeb, H., Torabi, H., & Zhao, Y. (2019). Stress-strength reliability of exponentiated Fréchet distributions based on Type-II censored data. Journal of Statistical Computation and Simulation, 89(10), 1863-1876.

[19] Alshenawy, R., Sabry, M. A., Almetwally, E. M., & Almongy, H. M. (2021). Product spacing of stress–strength under progressive hybrid censored for exponentiated-gumbel distribution. Computers, Materials & Continua, 66(3), 2973-2995.

[20] El-Sherpieny, E. S. A., Almetwally, E. M., & Muhammed, H. Z. (2021). Bayesian and Non-Bayesian Estimation for the Parameter of Bivariate Generalized Rayleigh Distribution Based on Clayton Copula under Progressive Type-II Censoring with Random Removal. Sankhya A, 1-38.

[21] Nassr, S. G., Almetwally, E. M., & El Azm, W. S. A. (2021). Statistical inference for the extended weibull distribution based on adaptive type-II progressive hybrid censored competing risks data. Thailand Statistician, 19(3), 547-564.

[22] Muhammad, I., Wang, X., Li, C., Yan, M., & Chang, M. (2020). Estimation of the reliability of a stress–strength system from Poisson half logistic distribution. Entropy, 22(11), 1307–1334.

[23] Kotz, S., Lumelskii, Y., & Pensky, M. (2003). The stress–strength model and its generalizations: theory and applications. World Scientific Publishing, Singapore.

[24] Almetwally, E. M., & Almongy, H. M. (2020). Parameter estimation and stress-strength model of power Lomax distribution: classical methods and Bayesian estimation. Journal of Data Science, 18(4), 718-738.

[25] Abd El-Monsef, M. (2021). Erlang mixture distribution with application on COVID-19 cases in Egypt. International Journal of Biomathematics, 14(03), 2150015.