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Dynamics of Einstein-Podolski-Rosen steering for two-spin Heisenberg XYZ states: Dzyaloshinskii-Moriya and the intrinsic decoherence effects

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Abstract: The generation of two-qubit spins XYZ-Heisenberg steerability and entanglement, in the presence of the *y*-component interaction's Dzyaloshinskii-Moriya and the intrinsic decoherence, will be investigated using Einstein-Podolski-Rosen steering and negativity. We examine the effects of the spin-spin XYZ-Heisenberg, DM *y*-component-interaction, and on intrinsic decoherence couplings on the two-qubit spins XYZ-Heisenberg steerability and entanglement dynamics. Our observations of the dynamical proprieties of the steerability and entanglement can be enhanced and regulated by combining the spin-spin Heisenberg and Dzyaloshinskii-Moriya interaction effects. The increase of the spin-spin XYZ-Heisenberg, DM *y*-component-interaction, and the intrinsic decoherence couplings has a high ability to generate the two-qubit spins XYZ-Heisenberg steerability and the entanglement. The generation of the entanglement's spin-spin Heisenberg states is quicker than the generation of the steerability. The steerability exhibits sudden disappearance and sudden reappearance phenomena. The quantum steering and entanglement dynamics conforms with the hierarchy principle of nonlocality information resources. The unequal spin-spin antiferromagnetic interactions enhance the generated spin-spin steerability and entanglement dynamics. The robustness of the degradation of the spin-spin steerability and entanglement, due to the decoherence effect, depends on the antiferromagnetic Heisenberg and Dzyaloshinskii-Moriya interactions.

Keywords: spins XYZ-Heisenberg; steerability; entanglement; DM interaction

1 Introduction

Nonlocality is a key and distinctive feature of quantum information science, which has two distinct approaches for searching on quantum information resources after implementing original types of nonlocality (such as quantum discord [1] Bell nonlocality [2], Einstein–Podolsky–Rosen steering [3,4,5], and entanglement [6], ...). These types have opened an epoch of unrelenting exploration of nonlocal correlations since they were discovered. The first approach is implementing new nonlocal correlation measures and the other is generating two-qubit nonlocal correlation/nonlocality. Nonlocality resources (entanglement and steering non-locality), are used to implementing many quantum protocols, as: cryptography [7], quantum memory [8], quantum computing [9], and teleportation [10]. According to the quantum nonlocality information resources hierarchy principle [11,12,13], steering stands between entanglement [6] and Bell nonlocality [14]. There are several quantifiers were introduced to quantify the two-qubit entanglement non-locality, including: Neumann entropy, concurrence, negativity, and log-negativity [15,16,17,18,19].

Due to that the steerability is a useful quantum nonlocality information resource for secure quantum information tools, recently, many works have focused on exploring the steerability and entanglement in several real quantum systems. Hierarchy's nonlocality resources and phenomena of deaths and births for discord, entanglement, Einstein–Podolsky–Rosensteering, and Bell nonlocality have been investigated in two-qubit systems [11]. Quantum steering, nonlocality and entanglement have been explored for a two-qubit X-state in structured reservoirs [20]. Analytical results have been introduced to improving of thermal Heisenberg XY steering and nonlocality via local filtering operation [21,

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22]. Through the study of the sudden death and revival of Einstein–Podolsky–Rosen steering in noisy channels, the influence of quantum state purity and excess noise on steering was analyzed and experimentally demonstrated by distributing a squeezed state through lossy and noisy channels, respectively [23]. Recently, the steering of the cavity–magnon and qubit–qubit subsystems of a hybrid quantum system was explored under different damping rates [24].

Nowadays, exploring the generation of two-qubit nonlocality resources in real two-qubit models (including: superconducting circuits [25,26], trapped ions [27], spin-Heisenberg models [28]) become a useful problem for the advancement of quantum computing [29] and quantum estimation [30]. Therefore, the generation of nonlocality resources via two-qubit Heisenberg spin chain interactions have several attentions [31,32]. The capability of the Heisenberg XY model to generate different nonlocality resources (as Bell nonlocality, steerability, concurrence entanglement) was investigated under external magnetic field, spin couplings, anisotropy as well as decoherence [33]. For two-qubit X-matrix XYZ Heisenberg states, the relationship among the quantum steering, and entanglement was examined in quantum dot systems [34].

Previous investigations of the steerability and entanglement dynamics have been explored only for the two-qubit X-matrix Heisenberg XYZ states. However, the investigation of steerability and entanglement dynamics of the spin-spin XYZ-Heisenberg non-X-states, due to the presence of the Dzyaloshinskii-Moriya y-component-interactions, is very limited under the intrinsic spin-spin XYZ-Heisenberg decoherence [35].

The manuscript is organized as follows: The physical model of the spin-spin Dzyaloshinskii-Moriya XYZ-Heisenberg model and its Milburn equation's intrinsic-decoherence are discussed in Sec. (II). Different quantifiers of the nonlocal correlation (steerability and entanglement) measures are introduced in Sec (III). The two-qubit XYZ-DM-Heisenberg steerability and entanglement dynamics are discussed in Sec (IV). Finally, in Section (V), we summarize the results of the steerability and entanglement dynamics.

2 Spin-spin Dzyaloshinskii-Moriya XYZ-Heisenberg model

Here, we consider the dynamics of the steerability and entanglement for a two-qubit system generated from a XYZ-Heisenberg model by the Milburn equation's intrinsic-decoherence [35],

$$\frac{d}{dt}\hat{D}(t) = -i[\hat{D},\hat{D}] - \frac{\gamma}{2}[\hat{\mathscr{H}},[\hat{\mathscr{H}},\hat{D}]], \quad (1)$$

$\hat{D}(t)$ is the density matrix's time-dependent XYZ-Heisenberg states. γ is the intrinsic decoherence coupling. The function $\hat{\mathscr{H}}$ represents the Hamiltonian of the spin-orbit XYZ-Heisenberg interactions with the Dzyaloshinskii-Moriya interaction, and it is given by

$$\hat{\mathscr{H}} = \sum_n (l_x \hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + l_y \hat{\sigma}_n^y \hat{\sigma}_{n+1}^y + l_z \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z) \quad (2)$$

$$+\mathbf{M}.(\sigma_n \times \sigma_{n+1}), \quad (3)$$

where the spin- spin interactions are controlled by the couplings $l_k (k = x,y,z)$. $\mathbf{M} = (m_x, m_y, m_z)$ designs the antisymmetric Dzyaloshinskii-Moriya interaction vector. $\hat{\sigma}^k_{A/B}(k = x,y,z)$ are Pauli matrices. The antiferromagnetic behavior's two spins is if $l_k > 0$ and the ferromagnetic behavior is for $l_k < 0$). In the case where the two-spin Heisenberg XYZ model describes only two qubits (say A and B) with the DM y-direction interaction coupling m_y, the Hamiltonian of Eq.2 is reduced to be

$$\hat{\mathscr{H}} = \sum_{k=x,y,z} l_k \hat{\sigma}_A^k \hat{\sigma}_B^k + m_y(\sigma_A^z \sigma_B^x - \sigma_A^x \sigma_B^z). \quad (4)$$

In the asymmetric and symmetric Bell two-spin states: $|B_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_A 1_B\rangle \pm |1_A 0_B\rangle)$ and $|B_{3,4}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle \pm |1_A 1_B\rangle)$, respectively, the eigenstates $|V_i\rangle (i = 1,2,3,4)$ and the eigenvalues V_i of the Hamiltonian of Eq. (4) are

$$\begin{aligned} |E_1\rangle &= |B_1\rangle, |E_2\rangle = |B_4\rangle, \\ |E_3\rangle &= \sin\xi_1 |B_3\rangle - \cos\xi_1 |B_2\rangle, \\ |E_4\rangle &= \sin\xi_2 |B_3\rangle - \cos\xi_2 |B_2\rangle, \end{aligned} \quad (5)$$

with

$$\xi_{1,2} = \tan^{-1}\{\frac{2m_y}{l_x + l_z \mp \sqrt{4m_y^2 + l_s^2}}\},$$

where $l_s = l_x + l_y$. The Hamiltonian eigenvalues E_k are,

$$E_{1,2} = \pm l_x + ly \mp l_z,$$

$$E_{3,4} = -l_y \pm \sqrt{4m_y^2 + l_s^2}.$$

By using the eigenstates'Hamiltonian $|E_i\rangle (i = 1,2,3,4)$ of Eq. (5) and the eigenvalues' Hamiltonian E_i of Eq. (6), the matrix density's spin-spin DM-XYZ-Heisenberg states D_{AB} [35] of Eq. (1) is given by

$$\hat{D}(t) = \sum_{m,n=1}^{4} D_{deco}(t) U_{unit}(t) \langle E_m|\hat{M}(0)|E_n\rangle |E_m\rangle\langle E_n|. \quad (6)$$

The decoherence $D_{deco}(t)$ and the unitary evolution $U_{unit}(t)$ have respectively the following expressions:

$$D_{deco}(t) = e^{-\frac{\gamma}{2}(E_m - E_n)^2 t}, U_{unit}(t) = e^{-i(V_m - V_n)t}. \quad (7)$$

To exporting the generation of the Einstein-Podolski-Rosen steering and entanglement's XYZ-DM-Heisenberg states due to spin-spin and Dzyaloshinskii-Moriya y-component interactions, we assume that the two spins start their initial pure up states $|11\rangle$.

3 Nonlocal steerability and entanglement measures

The considered two-qubit XYZ-DM-Heisenberg nonlocal steerability and entanglement quantifiers are

–Quantum steering:
Here, the maximal violation of Einstein-Podolsky-Rosen steering inequalities will used to quantify the two-qubit XYZ-DM-Heisenberg nonlocal steerability [3]. The Einstein-Podolsky-Rosen steering measure is given by

$$S(t) = \max\{0, \frac{\sqrt{\Delta^2 - \Delta^2_{min}} - 1}{\sqrt{2} - 1}\}, \quad (8)$$

where $\Delta = \sqrt{\mathbf{c}}$ with the diagonal matrix $\mathbf{r} = \{c_1, c_2, c_3\}$ of the correlation matrix [36]. $\Lambda_{min} = \min\{|c_1|, |c_2|, |c_3|\}$. For a non-steerable two-qubit state, $S(t) = 0$, while $S(t) = 1$ for a maximum steerable two-qubit state.

–Negativity
Here, we investigate also the generated entanglement's two-spin DM-Heisenberg states by using the negativity [16]. For the time-dependent spin-spin XYZ-DM-Heisenberg states $M(t)$, the negativity is defined the absolute sum of the negative eigenvalues λ_i of the matrix $(M(t))^T$ of the partial transpose of the two-qubit XYZ-DM-Heisenberg state $M(t)$ of Eq.1, with respect to A/B-spin. The negativity function [16] is given by

$$N(t) = -2\sum_i \lambda_i, \quad (9)$$

For separable two-spin DM-Heisenberg states, $N(t) = 0$ whereas for maximally two-spin DM-Heisenberg states $N(t) = 1$. In general, $0 \leq N(t) \leq 1$.

In the following, we use the Einstein-Podolsky-Rosen steering and negativity functions to explore the ability of the increase of the effects of the spin-spin and the DM interaction couplings to creation of the two-qubit state steerability and the negativity entanglement under the intrinsic decoherence effects.

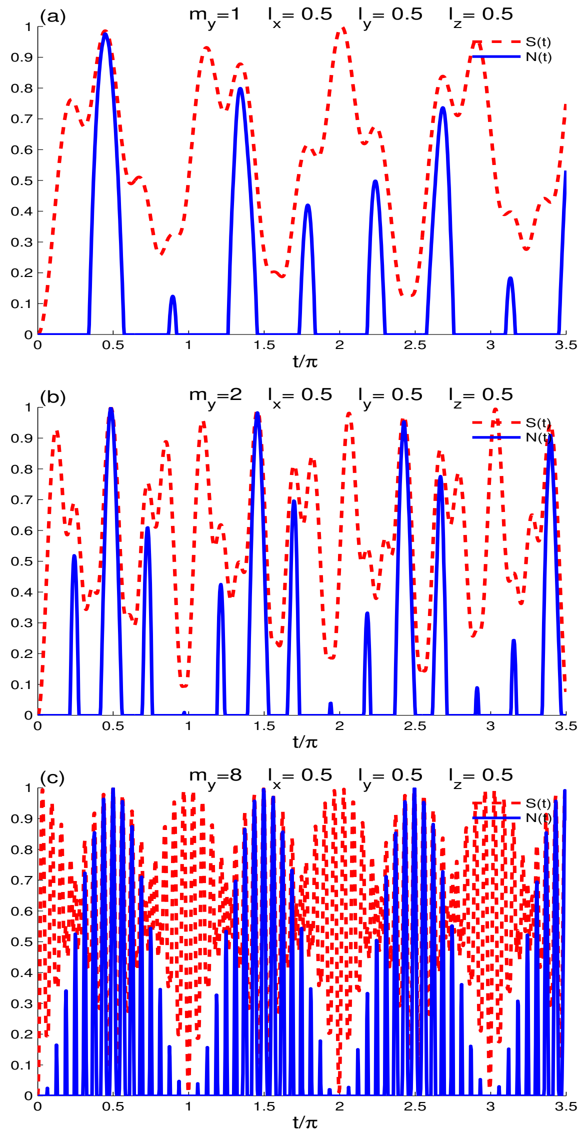

Fig. 1: The dynamics of the Einstein-Podolsky-Rosen steering $S(t)$ and negativity $N(t)$ functions are shown without decoherence effect for weak equal spin-spin couplings $l_k = 0.5$, and for different DM y-component- couplings: $m_y = 1$ in (a), $m_y = 2$ in (b), and $m_y = 8$ in (c).

4 The XYZ-DM-Heisenberg steerability and entanglement

Under increasing DM y-component-interaction m_y and intrinsic decoherence γ for various cases of the spin-spin Heisenberg antiferromagnetic interactions l_k, the creation of the two-qubit state steerability and the negativity entanglement will be investigated.

In Fig.1, the dynamics of the Einstein-Podolsky-Rosen steering and the negativity are shown without decoherence effect ($\gamma = 0$) for weak equal spin-spin couplings $l_k = 0.5$, and for different DM y-component- couplings: $m_y = 1$ in (a), $m_y = 2$ in (b), and $m_y = 8$ in (c). From Fig.1a, we find that the weak equal spin-spin antiferromagnetic couplings $l_k = 0.5$ in presence of the DM y-component- couplings $m_y = 1$ lead to generate partial and maximal Einstein-Podolsky-Rosen steering and the negativity as nonclassical two-qubit information resources having different ir-regular oscillatory dynamics. In this case, the generation of the entanglement's spin-spin Heisenberg states is quicker

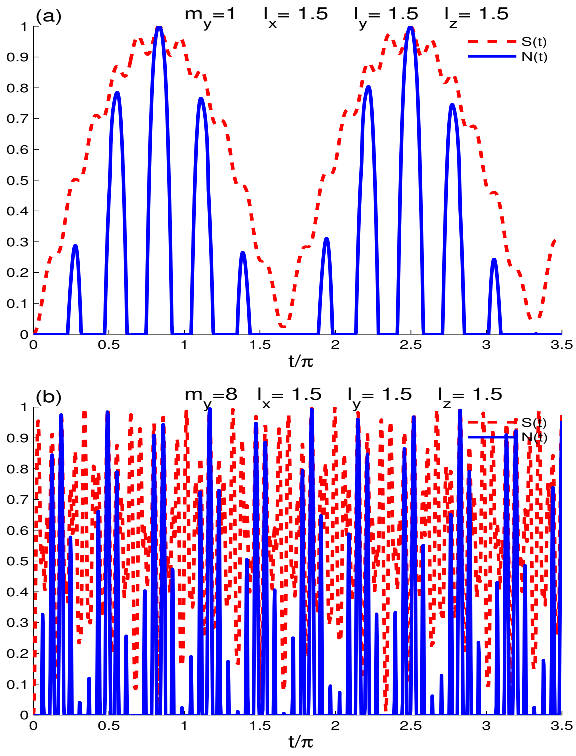

Fig. 2: The Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,c are shown but for strong spin-spin Heisenberg couplings $l_k = 1.5$.

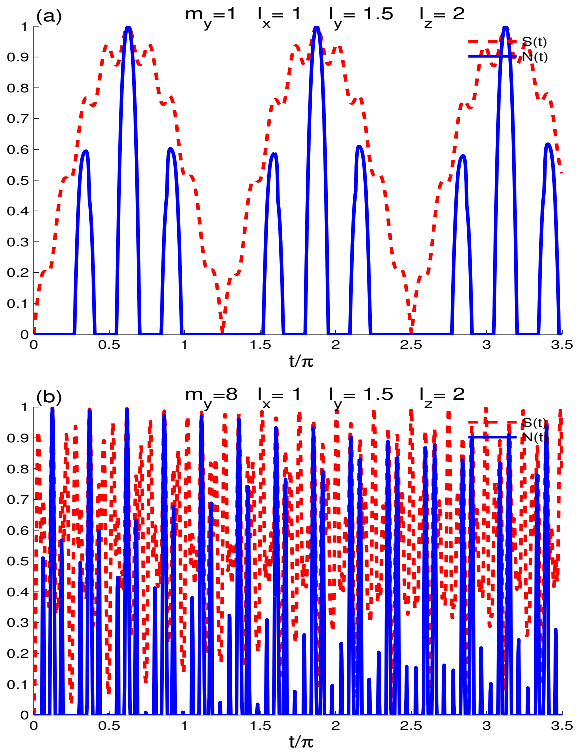

Fig. 3: The Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,c are shown but for strong spin-spin antiferromagnetic couplings $(J_x, J_y, J_z) = (1, 1.5, 2)$.

than the the generation of the steerability. After a particular large time point t_{rev} (this time is called "revival time point of the steerability RTPS"), the Einstein-Podolsky-Rosen steering revivals suddenly and grows to its partial and maximal steerability. The steerability may exhibit a sudden disappearance and sudden reappearance phenomenon. The phenomena of the sudden death and revival of Einstein–Podolsky–Rosen steering happens, severally, at the ends of the time intervals of the steerability disappearance. This phenomenon is confirmed, experimentally [23]. From the irregular oscillatory entanglement dynamics of the dashed curve, we find that the spin-spin Heisenberg states does not return to its initial disentangled state. This means that the generated spin-spin Heisenberg states can have partial entanglement during the time intervals of the steerability disappearance. The results confirm that the dynamics of the quantum steering and entanglement's spin-spin Heisenberg states agree with the hierarchy principle of two-qubit nonlocality information resources [11, 12, 13].

Figs.1b,c illustrate the dynamics of the generated quantum steering and entanglement's spin-spin Heisenberg states, due to weak equal spin-spin antiferromagnetic interactions $l_k = 0.5$, for different cases where the DM y-component-coupling are: $m_y = 1$ in (b) and $m_y = 8$ in (c). In the cases of the Figs.1b,c, we find that the weak equal spin-spin antiferromagnetic interactions $l_k = 0.5$ have a strong ability to generate the spin-qubit nonlocality information resources (steerability, negative entanglement) when the coupling of the DM y-component-interaction is increased. The frequencies, regularity, and amplitudes of the steerability and entanglement quantifiers are improved. The time intervals of the steerability disappearance are reduced by increasing the DM y-component-interaction coupling. For a strong DM y-component-interaction coupling $m_y = 8$, the generated Einstein-Podolsky-Rosen steering and negativity information resources have different π-regular oscillatory dynamics with high frequencies and large amplitudes. During each time period, the DM-Heisenberg qubits can have maximal steerability and entanglement at the same time.

Fig.2 visualizes the Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,c for strong spin-spin Heisenberg antiferromagnetic couplings $l_k = 1.5$. By comparing Fig.1a and Fig.2a, we note that the strong Heisenberg antiferromagnetic couplings enhances and regulates spin-spin steerability and entanglement dynamics with a large period. The negativity appears as cover for the upper bounds of the maxim's

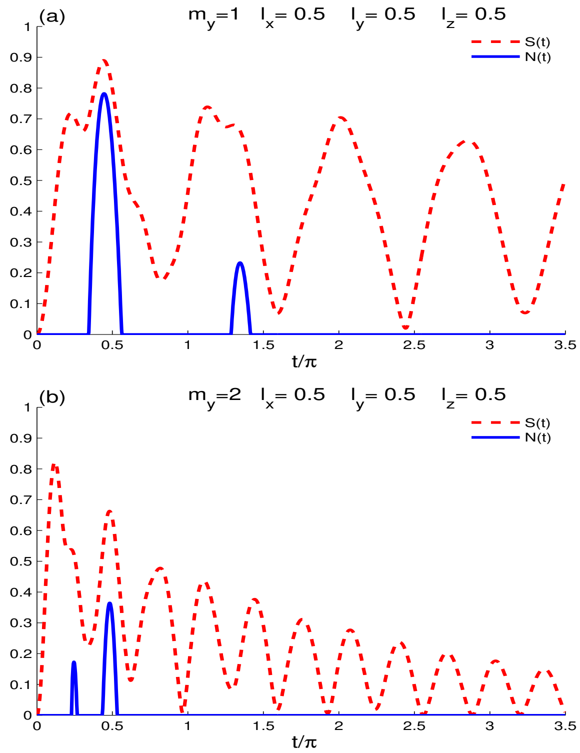

Fig. 4: The Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,b are shown but for decoherence coupling, $\gamma = 0.01$.

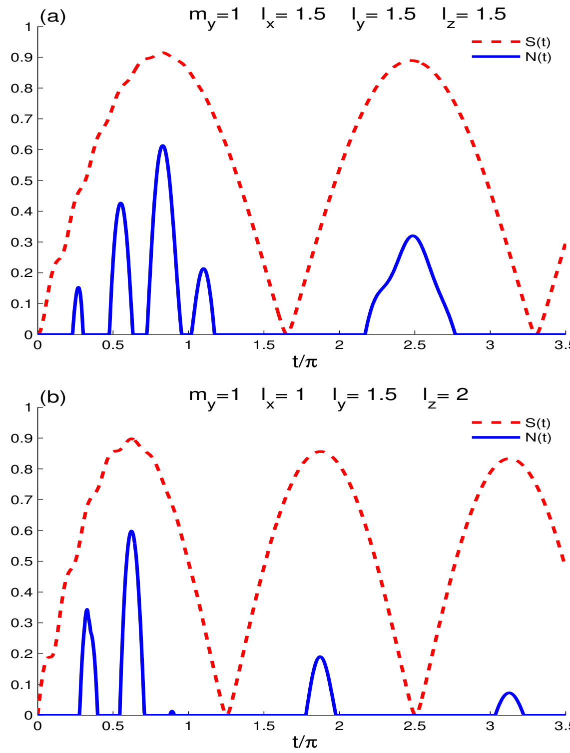

Fig. 5: The Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.2a and 3a are shown in the presence of the decoherence $\gamma = 0.01$.

Einstein-Podolsky-Rosen steering. The generated two-spin states reach maximum steerability and entanglement at the same time. Figs.1c and Fig.2b confirm the effect of the increase of the DM y-component-interaction $m_y = 8$. This increase enhances the generated spin-spin steerability and entanglement dynamics. They have semi-regular oscillatory dynamics with high frequencies and large amplitudes. The revival time point of steerability and the time intervals of the steerability disappearance are reduced by increasing the DM y-component-interaction coupling.

Fig.3a displays the Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,c but for strong spin-spin antiferromagnetic couplings $(l_x, l_y, l_z) = (1, 1.5, 2)$. From the results of Figs.1a, 2a, and Fig.3a, we find that the strong unequal spin-spin antiferromagnetic couplings lead to rapid oscillation of the generated spin-spin steerability and entanglement dynamics. The frequencies, regularity, and amplitudes of the steerability and entanglement are affected. The revival time point of steerability and the time intervals of the steerability disappearance are enhanced. The rapid growth of the Einstein-Podolsky-Rosen steering and negativity dynamics in Fig.3b confirms the observed effect of the DM y-component-interaction $m_y = 8$. The regularity, frequency and amplitudes of the spin-spin Einstein-Podolsky-Rosen steering and negativity are enhanced with irregular dynamics. The time intervals of the steerability disappearance are reduced by increasing the DM y-component-interaction coupling. As the evolves, the amplitudes of the spin-spin Einstein-Podolsky-Rosen steering decrese while of the negativity enhance. The amount of the Einstein-Podolsky-Rosen steering is a good indicator to the generated two spin-spin negativity entanglement. This accords with the hierarchy principle between the steerability and the entanglement [11,12,13].

Fig.4 visualizes of the Einstein-Podolsky-Rosen steering and negativity dynamics of Figs.1a,b in the presence of the intrinsic spin-spin-Heisenberg model decoherence coupling. For a small intrinsic decoherence coupling $\gamma = 0.01$, in the case where the weak equal spin-spin antiferromagnetic couplings $l_k = 0.5$, the regularity, frequency and amplitudes of the Einstein-Podolsky-Rosen steering and negativity are degraded (see Fig.4a). The time intervals of the steerability disappearance are increased by increasing the decoherence coupling. The degradation of the spin-spin entanglement dynamics is more robust against the intrinsic decoherence effect. For a strong the intrinsic decoherence, the spin-spin steerability disappears, completely. By increasing the DM coupling, the frequencies and the amplitudes of the spin-spin Einstein-Podolsky-Rosen steering and negativity dynamics reduce (see Fig.4b). The time intervals of the steerability disappearance are increased by increasing the DM y-component-interaction coupling. We can find that the DM m_y-component coupling reduce the resistance ability of the generated spin-spin steerability and entanglement dynamics against the intrinsic decoherence. The DM m_y-component coupling enhance the intrinsic decoherence effects. After a particular time, the spin-spin steerability disappears, completely. While the generated spin-spin negativity amplitudes reduces to reaching its partial entanglement.

Fig.5a shows robustness of the generation of the spin-spin steerability and entanglement against the decoherence in the presence of the strong spin antiferromagnetic interaction. In this case, the robustness of the generated steerability and entanglement dynamics against the intrinsic decoherence is enhanced Fig.5b shows that the strong DM y-component-interaction coupling leads to rapid disappearance of the spin-spin steerability. The time intervals of the steerability disappearance are increased. The Einstein-Podolsky-Rosen steering is fragile against the intrinsic decoherence. The results of the Figs.4 and 5 shows that the degradation of the spin-spin steerability and entanglement, due to the decoherence effect, depends on the spin-spin antiferromagnetic Heisenberg and Dzyaloshinskii-Moriya interactions.

5 Conclusion

Here, we have investigated the generation of two-qubit spins XYZ-Heisenberg steerability and entanglement, in the presence of the y-component interaction's Dzyaloshinskii-Moriya (DM) and the intrinsic decoherence, will be investigated using Einstein-Podolski-Rosen steering and negativity. We examine the effects of the spin-spin XYZ-Heisenberg, DM y-component-interaction, and on intrinsic decoherence couplings on the two-qubit spins XYZ-Heisenberg steerability and entanglement dynamics. The increase of the spin-spin XYZ-Heisenberg, DM y-component-interaction, and the intrinsic decoherence couplings has a high ability to generate the two-qubit spins XYZ-Heisenberg steerability and entanglement. The generation of the entanglement's spin-spin Heisenberg states is quicker than the the generation of the steerability. The phenomena of the sudden death and revival of Einstein–Podolsky–Rosen steering happens, severally, at the ends of the time steerability disappearance intervals. The generated spin-spin Heisenberg states can have entanglement during the time steerability disappearance intervals. The steerability and entanglement dynamicsagree with the hierarchy principle of nonclassical two-qubit information resources. The spin-spin antiferromagnetic interactions have a strong ability to generate and regulated the spin-qubit nonlocality information resources (steerability, negative entanglement) when the coupling of the DM y-component-interaction is increased. The strong unequal spin-spin antiferromagnetic couplings lead to rapid oscillation of the generated spin-spin steerability and entanglement dynamics. The degradation of the spin-spin entanglement dynamics is more robust against the intrinsic decoherence effect. While the generated steerability is fragile against the intrinsic decoherence. The results are shown that the degradation of the spin-spin steerability and entanglement, due to the decoherence effect, depends on the antiferromagnetic Heisenberg and Dzyaloshinskii-Moriya interactions.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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