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A successful and ideal randomized orthogonal additive response model

Manzoor A. Khanday[1,], Tanveer A. Tarray [2], S. Alkhazaleh [3], A. Ibrahim [1], Pratyay Das[1], Akansha Rawat [1], and Shiv Shanker Pandey[1]*

[1]Department of Statistics, Faculty of Chemical Engineering and Physical Sciences, Lovely Professional University Punjab, India
[2]Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir, India
[3]Department of Mathematics, Faculty of Science and Information Technology, Jadara University, Irbid, Jordan

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Abstract: An innovative additive randomized response technique (RRT) has been put forth in this study. The proposed model's characteristics have been researched. Theoretically, it has been demonstrated that under highly realistic circumstances, the recommended additive model is superior to the existing one. RRTs are used to lessen response biases in sensitive research question self-report surveys (e.g., on socially undesirable characteristics). Evidence suggests that they cannot entirely eradicate self-protective response mechanisms. There are RRTs made especially to assess the effectiveness of such tactics in order to solve this issue. Also provided in support of the current study are numerical examples.

Keywords: Randomized response sampling, orthogonality, Mean Squared Error, Estimation of proportion, sensitive variable.

1 Introduction

One issue with studies on high-risk behaviour is that participants may knowingly or unknowingly give false information. A social desirability bias has been noted as a significant factor in the distortion of standardized personality tests in psychological studies. Concerns concerning the veracity of survey findings/results about issues including drunk driving, marijuana usage, tax evasion, illicit drug use, induced abortion, shoplifting, child abuse, family issues, test cheating, HIV/AIDS, and sexual conduct are shared by survey researchers. Lack of an accurate indicator of their incidence or prevalence is the biggest obstacle to investigating some sensitive social issues (such as drug use, induced abortion, tax fraud, etc.). To gather reliable information on such sensitive and private issues, instead there is a need for alternate processes other than open surveys. Warner was the first to present such an alternate method as "randomized response technique" (RRT). It offers the chance to lessen response biases brought on by dishonest responses to delicate queries. In the context of sensitive and private issues, the availability of reliable information is crucial. However, relying solely on open surveys may not always be the most suitable approach but Fuzzy theory can be the solution [1-7].

As a result, the method guarantees a high level of privacy protection in many situations. Many improvements are suggested in the literature as a result of Warner's ground breaking study. A nice overview of randomized response method advancements might be found in [15-23]. Due to their delicate nature, many social and psychological phenomena of considerable societal importance are challenging to experimentally explore. For instance, the ongoing corona virus disease 2019 (COVID-19) pandemic [24-30] has resulted in an alarming rise in the prevalence of intimate partner violence (IPV), according to recent reports from the German news outlet [21] The model's description from [8] is provided [8] additive model:

Allow k scrambling variables, represented by Sj , j = 1,2,…,k whose cruel θj (i.e. E(Sj) =θj) and variance γ_j^2 (i.e. V(Sj) = γ_j^2) are known. To determine the proportion of the k darkened areas, (see Ref. [15]) proposed optimum new orthogonal additive model known as (POONAM), each responder chosen from the sample is asked to rotate a spinner. P1, P2, … Pkare, let's say, orthogonal to the k scrambling variables' meanings $\theta_1, \theta_2, \ldots, \theta_k$ such that:

$$\sum_{j=1}^{k} P_j \theta_j = 0 \tag{1.1}$$

and

*Corresponding author e-mail: manzoor.28711@lpu.co.in or manzoorstat@gmail.com

$$\sum_{j=1}^{k} P_j = 1 \tag{1.2}$$

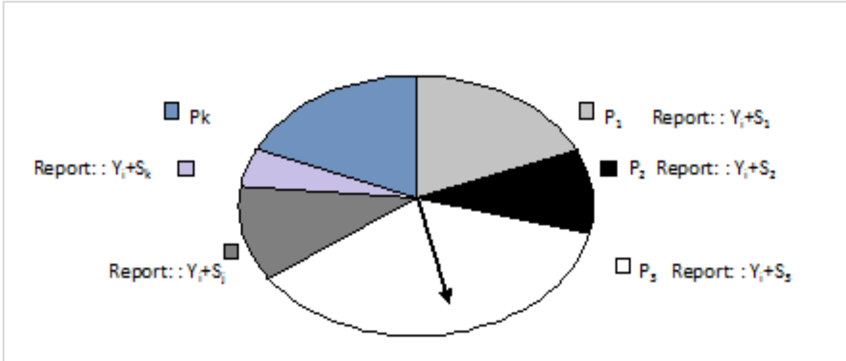

Fig. 1 Spinner for POONAM

If the cursor now pauses in the j^{th} shadowed region, the i^{th} responder having the sensitive variable's actual value, let's say Yi, is asked to submit the response's scrambled form, Z_i, as follows:

$$Z_i = Y_i + S_j \tag{1.3}$$

Assuming that simple random sampling with replacement is used to choose the sample of size n from the population (SRSWR). Singh (2010) proposed an objective method of estimating the population mean μ_Y as

$$\hat{\mu}_Y = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{1.4}$$

The variance of $\hat{\mu}_Y$ is given by

$$V(\hat{\mu}_Y) = \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j(\theta_j^2 + \gamma_j^2)\right] \tag{1.5}$$

2 The proposed procedure

Let S_j, j = 1,2,…,k be k the process of mixing together variables such that their distribution is understood. In brief, let E(Sj) =θj and V(Sj) = γ_j^2 get noticed. Subsequently, in the suggested additive model, each participant chosen from the sample is asked to spin a spinner, as shown in Fig. 2, to determine the percentage of the k shaded regions, say P_1, P_2, … P_k are orthogonal to the means of the k scrambling variables, say $\theta_1, \theta_2, \ldots, \theta_k$ such that:

$$\sum_{j=1}^{k} P_j \theta_j = 0 \tag{2.1}$$

and

$$\sum_{j=1}^{k} P_j = 1 . \tag{2.2}$$

The i^{th} respondent holding the genuine value of the sensitive variable, let's say Y_i, is asked to report the scrambled response if the pointer stops in the j^{th} darkened region. Z_i^* as:

$$Z_i^* = Y_i + S_j^*, \tag{2.3}$$

where $S_j^* = \dfrac{(a_j S_j + b_j \theta_j)}{(a_j + b_j)}$; and ($a_j$,b_j) being appropriately selected constants that have real-world values and are functions of established parameters for the scrambling variable S_j such as γ_j, θj , Cj (= γj / θj) , $\beta_2(S_j) = \dfrac{\mu_4(S_j)}{\gamma_j^4}$ (coefficient of kurtosis) , $G_1(S_j) = \dfrac{\mu_3(S_j)}{\gamma_j^3}$ is the Fisher's measure of skewness, $\mu_3(S_j)$ and $\mu_4(S_j)$ are third and fourth central moments of the scrambling variable Sj etc. Let's use basic random sampling with replacement to select a sample of size n from the population (SRSWR). The following theorems are then proven.

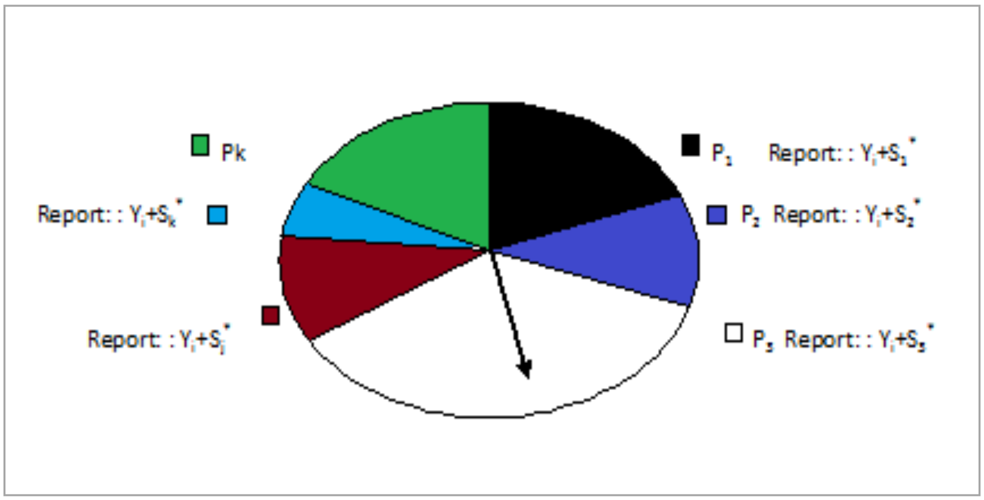

Fig. 2 Spinner for proposed procedure.

Theorem 2.1 An unbiased estimator of the population mean μ_Y is given by

$$\hat{\mu}_{ST} = \frac{1}{n}\sum_{i=1}^{n} Z_i^* \tag{2.4}$$

Proof. Let E1 and E2 denote the expectation over the sampling design and the randomization device respectively, we have

$$E(\hat{\mu}_{ST}) = E_1 E_2\left[\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right]$$

$$= E_1\left[\frac{1}{n}\sum_{i=1}^{n} E_2(Z_i^*)\right]$$

$$= E_1\left[\frac{1}{n}\sum_{i=1}^{n}\left\{Y_i \sum_{j=1}^{k} P_j + \sum_{j=1}^{k} P_j E_2(S_j^*)\right\}\right]$$

$$= E_1\left[\frac{1}{n}\sum_{i=1}^{n}\left\{Y_i \sum_{j=1}^{k} P_j + \sum_{j=1}^{k} P_j \theta_j\right\}\right]$$

$$= E_1\left[\frac{1}{n}\sum_{j=1}^{n} Y_i\right] = \mu_Y$$

, since $\sum_{j=1}^{k} P_j = 1 \; and \; \sum_{j=1}^{k} P_j\theta_j = 0,$

which completes the theorem.

The variance of the proposed estimator $\hat{\mu}_{ST}$ is given in the following theorem.

Theorem 2.2 The variance of the proposed estimator $\hat{\mu}_{ST}$ is given by

$$V(\hat{\mu}_{ST}) = \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\theta_j^2(1+\eta_j^2 C_j^2)\right], \tag{2.5}$$

where

$\eta_j = a_j / (a_j + b_j)$ and Cj = γj / θj ; j =1,2,…,k. (2.6)

Proof. Let V1 and V2 denote the variance over the sampling design and over the proposed randomization device, respectively, then we have

$$V(\hat{\mu}_Y) = E_1V_2(\hat{\mu}_Y) + V_1E_2(\hat{\mu}_Y)$$

$$= E_1\left[V_2\left(\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right)\right] + V_1\left[E_2\left(\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right)\right]$$

$$= E_1\left[\frac{1}{n^2}\sum_{i=1}^{n} V_2(Z_i^*)\right] + V_1\left[\frac{1}{n}\sum_{i=1}^{n} E_2(Z_i^*)\right]$$

$$= E_1\left[\frac{1}{n^2}\sum_{i=1}^{n} V_2(Z_i^*)\right] + V_1\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right] \quad , \quad \text{since} \;\; E_2(Z_i^*) = Y_i$$

$$= \frac{\sigma_y^2}{n} + E_1\left[\frac{1}{n^2}\sum_{i=1}^{n} V_2(Z_i^*)\right] \tag{2.7}$$

Note that:

$$V_2(Z_i^*) = \sum_{j=1}^{k} P_j E_2(Y_i + S_j^*)^2 - Y_i^2$$

$$= \sum_{j=1}^{k} P_j E_2(Y_i^2 + S_j^{*2} + 2Y_iS_j^*) - Y_i^2$$

$$= Y_i^2\sum_{j=1}^{k} P_j + 2Y_i\sum_{j=1}^{k} P_j\theta_j + \sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2C_j^2\right\} - Y_i^2$$

$$= Y_i^2 + \sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2C_j^2\right\} - Y_i^2; \; \text{since} \;\; \sum_{j=1}^{k} P_j = 1 \text{ and } \sum_{j=1}^{k} P_j\theta_j = 0,$$

$$= \sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2C_j^2\right\} \tag{2.8}$$

where $\eta_j = a_j / (a_j + b_j)$ and Cj = γj / θj ; j =1,2,…,k.

Putting (2.8) in (2.7) we get

$$V(\hat{\mu}_Y) = \frac{\sigma_y^2}{n} + E_1\left[\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2C_j^2\right\}\right]$$

$$= \frac{\sigma_y^2}{n} + \frac{1}{n}\sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2 C_j^2\right\}$$

$$= \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2 C_j^2\right\}\right] \qquad (2.9)$$

This completes the proof of the Theorem.

3 Efficiency Comparison

The proposed estimator $\hat{\mu}_{ST}$ will be more efficient than the estimator $\hat{\mu}_Y$ if

$V(\hat{\mu}_{ST}) < V(\hat{\mu}_Y)$ if

i.e. if $\frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2 C_j^2\right\}\right] < \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\theta_j^2(1+C_j^2)\right]$

i.e if $\sum_{j=1}^{k} P_j\theta_j^2\left\{1+\eta_j^2 C_j^2\right\} < \sum_{j=1}^{k} P_j\theta_j^2(1+C_j^2)$

i.e if $\sum_{j=1}^{k} P_j\theta_j^2\left\{\eta_j^2 - 1\right\}C_j^2 < 0$

i.e. if $\sum_{j=1}^{k} P_j\gamma_j^2\left\{\eta_j^2 - 1\right\} < 0 \quad \forall\ j=1,2,\ldots,k$,

i.e. if $\left\{\eta_j^2 - 1\right\} < 0\ \forall\ j=1,2,\ldots,k$,

i.e. if $\left|\eta_j^2\right| < 1\ \forall\ j=1,2,\ldots,k$. (3.1)

It follows from (3.1) that we should choose the value of (aj ,bj) in such way that

$$\left|\frac{a_j}{a_j+b_j}\right| < 1. \qquad (3.2)$$

We have calculated the suggested estimator's percent relative efficiency (PRE) of the estimator $\hat{\mu}_{ST}$ with respect to Singh's (2010) estimator $\hat{\mu}_Y$ by using the formula:

$$PRE(\hat{\mu}_{ST}, \hat{\mu}_Y) = \frac{\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\left\{(\theta_j^2 + \gamma_j^2)\right\}\right]}{\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\left\{(\theta_j^2 + \eta_j^2\gamma_j^2)\right\}\right]} \times 100 \qquad (3.3)$$

where η_j is given in (2.6).

Considering the respondents' participation, we made the following decision: γ=40, γ_1= 30, γ_2= 40, γ_3 = 20, γ_4= 10, P_1=0.01, P_2=0.02, P_3=0.03, P_4=0.04 with k = 4. In addition we choose different values $\sigma_y^2, \theta_1, \theta_2, \theta_3$ *and* θ_4 as listed in Table 1.

Table 1. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$.

σ_Y^2	θ_1	θ_2	θ_3	θ_4	PREs						
					$a_j \to b_j$	$\frac{b_j}{2}$	$\frac{3b_j}{4}$	b_j	$\frac{5b_j}{4}$	$\frac{5b_j}{2}$	$\frac{7b_j}{4}$
					$\eta_j \to 1/5$	1/3	3/7	1/2	5/9	3/5	7/11
	1	5	10	-10.25	538.02	407.78	328.31	274.76	241.06	218.70	203.60
	5	10	15	-17.50	310.22	268.99	237.43	212.50	194.95	182.40	173.47
	10	15	20	-25.00	215.74	198.95	184.58	172.15	162.76	155.68	150.44
25	15	20	25	-32.50	171.97	163.40	155.64	148.58	143.02	138.70	135.42
	1	5	10	-10.25	343.28	291.56	253.38	224.05	203.82	189.56	179.51
	5	10	15	-17.50	251.87	226.76	206.20	189.06	176.49	167.22	160.48
	10	15	20	-25.00	195.53	182.80	171.62	161.73	154.12	148.31	143.96
125	15	20	25	-32.50	163.61	156.35	149.72	143.62	138.78	134.99	132.10
	1	5	10	-10.25	268.41	239.05	215.49	196.15	182.14	171.91	164.51
	5	10	15	-17.50	218.88	201.42	186.54	173.71	164.03	156.76	151.39
	10	15	20	-25.00	181.33	171.18	162.10	153.94	147.57	142.66	138.95
225	15	20	25	-32.50	156.98	150.72	144.94	139.58	135.30	131.92	129.34
	1	5	10	-10.25	228.78	209.14	192.60	178.49	167.95	160.07	154.27
	5	10	15	-17.50	197.66	184.52	173.02	162.87	155.07	149.12	144.68
	10	15	20	-25.00	170.81	162.42	154.82	147.90	142.44	138.19	134.97
325	15	20	25	-32.50	151.61	146.11	141.00	136.23	132.39	129.36	127.02

Table 2. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$ for $\theta_j = 0, j=1,2,\ldots,k$.

σ_Y^2	PREs						
	$a_j \to \frac{b_j}{4}$	$\frac{b_j}{2}$	$\frac{3b_j}{4}$	b_j	$\frac{5b_j}{4}$	$\frac{5b_j}{2}$	$\frac{7b_j}{4}$
	$\eta_j \to \frac{1}{5}$	1/3	3/7	1/2	5/9	3/5	7/11
25	1244.77	678.45	466.30	355.22	294.99	258.47	235.18
125	470.23	370.27	305.36	259.81	230.36	210.48	196.88
225	320.82	276.33	242.67	216.33	197.91	184.80	175.50
325	257.33	230.85	209.31	191.44	178.39	168.80	161.84
425	222.20	204.02	188.59	175.33	165.36	157.89	152.37
525	199.89	186.32	174.47	164.04	156.05	149.96	145.42
625	184.47	173.77	164.24	155.70	149.06	143.94	140.09
725	173.17	164.40	156.48	149.28	143.62	139.22	135.89
825	164.54	157.15	150.39	144.19	139.26	135.41	132.48

Table 3. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$.

σ_Y^2	θ_1	θ_2	θ_3	θ_4	PREs	
					$a_j=\theta_j, b_j=\gamma_j$	$a_j=\theta_j^2, b_j=\gamma_j^2$
25	1	1	2	-2.25	728.03	2001.40
	1	2	2	-2.75	505.45	1818.41
	2	1	3	-3.25	340.55	1584.98
	2	2	3	-3.75	239.00	1390.36
125	1	1	2	-2.25	383.36	537.72
	1	2	2	-2.75	320.09	526.40
	2	1	3	-3.25	253.68	509.02
	2	2	3	-3.75	199.62	491.03
225	1	1	2	-2.25	282.95	347.33
	1	2	2	-2.75	251.03	343.40
	2	1	3	-3.25	212.91	337.17
	2	2	3	-3.75	177.62	330.43
325	1	1	2	-2.25	235.09	272.36
	1	2	2	-2.75	214.97	270.31
	2	1	3	-3.25	189.24	267.01
	2	2	3	-3.75	163.58	263.34

Table 4. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$ for $\theta_j=0, j=1,2,\ldots,k$.

σ_Y^2	25	125	225	325
PRE's	2380.00	556.00	353.33	275.38

Table 5. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$ with k=2.

σ_Y^2	θ_1	θ_2	θ_3	θ_4	PREs						
					$a_j \rightarrow \frac{b_j}{4}$	$\frac{b_j}{2}$	$\frac{3b_j}{4}$	b_j	$\frac{5b_j}{4}$	$\frac{5b_j}{2}$	$\frac{7b_j}{4}$
					$\eta_j \rightarrow \frac{1}{5}$	1/3	3/7	1/2	5/9	3/5	7/11
25	1	5	10	-10.25	946.45	585.24	423.58	331.90	279.96	247.66	226.74
	5	10	15	-17.50	715.96	494.06	377.16	305.00	262.03	234.50	216.31
	10	15	20	-25.00	508.30	391.69	318.60	268.49	236.60	215.29	200.82
	15	20	25	-32.50	373.52	311.41	267.01	233.70	211.12	195.38	184.39
125	1	5	10	-10.25	368.67	308.28	264.88	232.20	209.99	194.49	183.65
	5	10	15	-17.50	340.15	289.46	251.92	223.00	203.02	188.92	178.97
	10	15	20	-25.00	300.41	262.11	232.47	208.85	192.12	180.10	171.51
	15	20	25	-32.50	261.38	233.86	211.58	193.18	179.78	169.95	162.84
225	1	5	10	-10.25	259.68	232.59	210.63	192.45	179.20	169.47	162.42
	5	10	15	-17.50	249.15	224.71	204.64	187.86	175.52	166.41	159.78
	10	15	20	-25.00	232.79	212.24	195.03	180.39	169.49	161.37	155.41
	15	20	25	-32.50	214.45	197.93	183.78	171.51	162.23	155.23	150.05
325	1	5	10	-10.25	213.59	197.25	183.24	171.08	161.88	154.93	149.79
	5	10	15	-17.50	208.16	192.94	179.80	168.33	159.61	153.00	148.09
	10	15	20	-25.00	199.29	185.84	174.09	163.73	155.79	149.73	145.22
	15	20	25	-32.50	188.67	177.21	167.07	158.02	151.01	145.63	141.59

Table 6. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the existing estimator $\hat{\mu}_Y$ with k=2

σ_Y^2	θ_1	θ_2	θ_3	θ_4	PREs	
					$a_j = \theta_j, b_j = \gamma_j$	$a_j = \theta_j^2, b_j = \gamma_j^2$
	1	1	2	-2.25	1676.13	1720.51
	1	2	2	-2.75	1599.26	1682.81
	2	1	3	-3.25	1587.15	1701.02
25	2	2	3	-3.75	1518.21	1664.21
	1	1	2	-2.25	424.94	427.21
	1	2	2	-2.75	421.12	425.64
	2	1	3	-3.25	420.26	426.41
125	2	2	3	-3.75	416.53	424.85
	1	1	2	-2.25	281.14	281.98
	1	2	2	-2.75	279.82	281.49
	2	1	3	-3.25	279.45	281.73
225	2	2	3	-3.75	278.14	281.24
	1	1	2	-2.25	225.57	226.04
	1	2	2	-2.75	224.87	225.80
	2	1	3	-3.25	224.65	225.92
325	2	2	3	-3.75	223.95	225.68

Tables 1 and 3 demonstrate that the values of $PRE(\hat{\mu}_{ST}, \hat{\mu}_Y)$ are greater than 100. It follows that the proposed estimator $\hat{\mu}_{ST}$ is more efficient than the estimator $\hat{\mu}_Y$ due to Singh (2010) with substantial gain in efficiency. Thus, based on the outcomes of our simulation, applying the suggested estimator $\hat{\mu}_{ST}$ over Singh (2010) estimator $\hat{\mu}_Y$ is recommended for all situations close to Tables 1 and 3 respectively.

We also consider a situation where θj = 0 for j= 1,2,3,4, and rest of the parameters are kept same as in Table 1 and 3 respectively. The percent relative efficiency of the proposed estimator $\hat{\mu}_{ST}$ over Singh (2010) estimators $\hat{\mu}_Y$ has been shown in Tables 2 and 4 respectively.

Tables 2 and 4 clearly show that the percent relative efficiencies remain higher if the value of σ_y^2 is small.

We have further considered the case k = 2 and computed the $PRE(\hat{\mu}_{ST}, \hat{\mu}_Y)$ for different choices of parameters. Results are shown in Tables 5 and 6.

In light of these numerical findings, the suggested estimates $\hat{\mu}_{ST}$ is to be preferred over Singh (2010) estimator $\hat{\mu}_Y$ is recommended for all situations in actual use, the Tables 1 to 6 are comparable. It should be mentioned that while selecting a randomization method to be employed in practise, real-world survey expertise is a need.

4 Conclusions

This paper has proposed a novel additive randomised response method (RRT). Research has been done on the qualities of the suggested model. Theoretically, it has been proven that the suggested additive model outperforms the current one under extremely realistic conditions. In sensitive research question self-report surveys, RRTs are employed to reduce response biases (e.g., on socially undesirable characteristics). There are RRTs designed specifically to evaluate the efficacy of such strategies in order to address this problem.

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