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Generalized Connectedness in Fuzzy Bitopological Spaces

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Abstract: The primary objective of this research is to study generalized connectedness ideas in the domain of fuzzy bitopological spaces. Additionally, it presents basic theorems to figuring out how they relate to one another, and to explore some of the primary characteristics of connectedness structures. Finally, we looked at the idea of disconnection as well as all of its fundamental theories and characteristics.

Keywords: bitopological fuzzy spaces $(fbts)$, fuzzy generalized connected $(g-conn)$, fuzzy generalized closed groups $(g-closed)$, fuzzy generalizede disconnented $(g-disconn)$.
MSC: 54A40; 03E72; 47S40; 94D05; 03B52; 28E10.

1 Introduction

Our research of fuzzy bitopology was inspired by fuzzy topological spaces, which was begun in 1965 by researcher Zadeh [1], has been given priority in this project. Following this, other academics improved the idea of fuzzy topology by applying basic concepts from general topology to fuzzy environments. For instance, Chang (1968), developed certain fuzzy ideas [2]. After that, Kandil (1989) presented fuzzy bitopological spaces [3]. Then, in 1997 Balasubramanian and Sundaram constructed generalized fuzzy closed groups in the domain of fuzzy topology [4]. In addition, some scientists presented several research papers on the generalized closed group of fuzzy space, as an example [5,6]. In 2011, scientists Zahran and Al-Maghribi presented a circular on some operations in fuzzy space [7]. As are some scholars also presented different types of studies on connectedness in fuzzy topology space, such as [8,9]. Also, the scientists Fatteh and Bassan presented an important study on stronger forms of connectedness in fuzzy space [10]. In the present research, we investigate the definition of connectedness by using some forms of generalized closed groups in bitopological fuzzy spaces and find the connection between them and study some important theories.

The research is organized as follows: Section 1 (introduction), in this section we review the history of the topic, its importance, and related research. In section 2 (preliminaries), we mention a few key antecedent concepts that are important in this study. In section 3 (Generalized Connectedness Concepts in Bitopological Fuzzy Spaces) this part presents the idea of generalized connectedness notions and describes them in relation to significant theorems and specific features. Finally, we summarize our results in section 4 (Conclusion).

2 Preliminaries

In the section that follows, we cover a few antecedent concepts that are important in this study.

Definition 1.*[11] Considering that I represent the closed interval $[0,1]$ and X is nonempty, the following is known as:*

(1)*a fuzzy set H is known as a function with an X domain and I range, $H(t) \in (0,1]$ if $t \in H$, but $H(t) = 0$ if $t \notin H$.*
(2)*R contains H as referred to $H \subseteq R$ when $H(t) \leq R(t)$, wherever $t \in X$.*
(3)*$H \vee R$ is the combination of groups that defined as $(H \vee R)(t) = upper\{H(t), R(t)\}\ \forall\, t \in X$.*

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(4)*$H \wedge R$ is the junction that defiend as $(H \wedge R)(t) = lower\{H(t), R(t)\}\ \forall\, t \in X$.*

(5)*H^c is a symbol for the completeness that defiend as $(H(t))^c = 1 - H(t)$, $\forall t \in X$.*

The meanings of fuzzy topology, also bitopological fuzzy spaces are explained in the following definitions:

Definition 2.*[11] The pair (X, δ) is consider fuzzy topology if the next three conditions holds:*

1.$0,\ 1 \in \delta$, since $0(t) = 0,\ 1(t) = 1$, as $t \in X$.
2.$H \wedge R \in \delta$, $\forall H, R \in \delta$.
3.$\vee_{i \in I} H_i \in \delta$, $\forall (H_{i \in I}) \in \delta$.

As (X, δ) is referred to as "fuzzy topology space," or "fts" shortly. Also, the elements of δ are known as fuzzy open groups. When $F \in \delta$, hence F^c is fuzzy closed, and the set of all fuzzy closed groups denoted by $\mathscr{F}_\delta$.

Definition 3.*A bitopological fuzzy spaces, often known as fbts, (X, δ_1, δ_2) as δ_1, δ_2 are fuzzy topology on X that is nonempty. In this essay X take on (X, δ_1, δ_2), and Y of (Y, σ_1, σ_2), such that $i, j \in \{1, 2\}$ with $i \neq j$.*

Definition 4.*[12]. A fuzzy group ω of X is referred to a fuzzy point (singleton) iff $\omega(x) = r, (0 < r \leq 1)$ with a specific $x \in X$, $\omega(y) = 0$ with each elements y of X excluding x, and it is indicated by x_r. Sometimes we refer to x_r as a fuzzy point if $0 < r < 1$. Additionally, $S(X)$ refers to the set of each fuzzy points (singletons) included in X.*

One of the main ideas of the study is the meaning of the generalized fuzzy closed group, is as bellow:

Definition 5.*Every fuzzy class H of X is known as generalized fuzzy closed if closure H is a subgroup of R, while H is a subgroup of R, which is open.*
i.e., H is generalised fuzzy closed when $cl(H) \leq R$, as $H \leq R$, and R is fuzzy open.

3 Generalized Connectedness Concepts in Bitopological Fuzzy Spaces

This part presents the notion of generalized connectedness notions by applying some forms of generalized closed groups in bitopological fuzzy spaces with characterizes them in relation to significant theorems and certain features.
The concept of generalized closed groups is the basis for constructing the definition of connectedness and we introduced it as follows:

Definition 6.*Each fuzzy subgroup H of fbts (X, δ_1, δ_2) is named as:*

(1)*Fuzzy $(i, j)-$generalized $\psi-$closed (shortly, $(i, j) - g\psi - cld$) if $\delta_j - \psi - cl(H) \leq W$, while $H \leq W$, $W \in \delta_i$, as ψ including ((α) alpha, (s) semi, (p) pre, and (β) beta).*

(2)*The $(i, j) - g\psi -$ open of X is a supplementary of fuzzy $(i, j) - g\psi - cld$.*

Remark.(1)The overall class of each fuzzy $(i, j) - g\psi-$open also $(i, j) - g\psi-$cld groups of (X, δ_1, δ_2) is showed by $\mathscr{O}^{fg\psi}_{(i,j)}$, $\mathscr{F}^{fg\psi}_{(i,j)}$, and so forth.

(2)Additionally, the class of each $g\psi-$open, also $g\psi-$cld subgroups of X with respect to the fuzzy topology δ_i is showed by $\mathscr{O}^{fg\psi}_i$, $\mathscr{F}^{fg\psi}_i$, *as* $i = \{1, 2\}$.

The following theorem explains the relationships between all varieties of generalized closed groups in fuzzy bitopology.

Theorem 1.*When (X, δ_1, δ_2) is fbts. Consequently, the following statements are true:*

(1)*$\forall (i, j) - g - cld$ is $(i, j) - g\alpha - cld$.*
(2)*$\forall (i, j) - g\alpha - cld$ is $(i, j) - gp - cld$ and $(i, j) - gs - cld$.*
(3)*$\forall (i, j) - gp - cld$ or $(i, j) - gs - cld$ is $(i, j) - g\beta - cld$.*

Proof. It is evident from Definition 6 and the connections between various fuzzy set types, where

$$\begin{array}{ccc} open \longrightarrow (\alpha\, open) & \longrightarrow & (s\, open) \\ \downarrow & & \downarrow \\ (pre\, open) & \longrightarrow & (\beta\, open) \end{array}$$

Next, we provide an introduction to the most important concepts and theories of the term of generalized connectedness:

Definition 7.*If H, R are fuzzy $(i, j) - g\psi -$ open sets, then they are separated if they are disjoint and $H \cap ((i, j) - g\psi - cl(R)) = R \cap ((i, j) - g\psi - cl(H)) = 0$. In other words, neither contains an accumalation point of the other, called Housdorff Lennes separation condition of fuzzy $(i, j) - g\psi -$ open groups.*

Definition 8.*A fuzzy subgroup H of (X, δ_1, δ_2) is named $(i, j) - g\psi - disconnected$ (in sum, $(i, j) - g\psi - disconn$) if $\exists\, G, H$ $(i, j) - g\psi -$ open sets of (X, δ_1, δ_2) as $(H \cap G), (H \cap H)$ are disjoint nonempty whose combination is H. In this instance $G \cup H$ is known as $(i, j) - g\psi-$disconnection of H.*

So, we can infer the following from the above definition:

Corollary 1.*A fuzzy group R is fuzzy $(i, j) - g\psi - connected$ (in sum, $(i, j) - g\psi - conn$) if it is a group that invalid as being a fuzzy $(i, j) - g\psi - disconn$.*

*Example 1.*In any fuzzy bitopological space we notice that 0, and any fuzzy singleton group are always fuzzy $(i, j) - g\psi - conn$ groups.

Theorem 2.*A fuzzy group R is fuzzy $(i, j) - g\psi - conn$ $\iff$ it is not a merger of two fuzzy nonempty $(i, j) - g\psi -$ open separated groups.*

Proof. We prove equivlently, that R is fuzzy $(i,j)-g\psi-disconn \iff R$ is the union of two non empty $(i,j)-g\psi-open$ separated groups. Assume R is fuzzy $(i,j)-g\psi-disconn$ with $G\cup H$ is fuzzy $(i,j)-g\psi-$disconnection of R. Then R is the union of non empty sets $(R\cap G)$, $(R\cap H)$. So, we need to show that each of $(R\cap G)$ and $(R\cap H)$ contains no accumulation point of the other. Thus, let p be an accumulation point of $R\cap G$, $p\in(R\cap H)$. Then H is $(i,j)-g\psi-open$ including p and so H contains a point of $(R\cap G)$ other than p. So, $(R\cap G)\cap H\neq 0$ but $(R\cap G)\cap H=0$, according $p\notin(R\cap H)$. Similarly, where p is an accumulation point of $(R\cap H), p\notin(R\cap G)$. Thus $(R\cap G)$ and $(R\cap H)$ are fuzzy $(i,j)-g\psi-open$ separated sets.

Proposition 1. *If $G\cup H$ is fuzzy $(i,j)-g\psi-$disconnection of R with C is fuzzy $(i,j)-g\psi-conn$ subset of R. Then $C\cap H=0$ or $C\cap G=0$ and so either $C\leq G$ or $C\leq H$.*

Proof. Since $C\leq R$, and $R\leq(G\cup H)$, then $C\leq(G\cup H)$, and $G\cap H\leq R^c$, and hence $G\cap H\leq C^c$. Thus if both $C\cap G$ and $C\cap H$ are non empty, then $G\cup H$ form $(i,j)-g\psi-$disconnection of C. But C is fuzzy $(i,j)-g\psi-conn$, hence the following condition is met $C\cap H=0$ and $C\cap G=0$.

Theorem 3. *If R and C are fuzzy $(i,j)-g\psi-conn$ which are not separated, then $R\cup C$ is fuzzy $(i,j)-g\psi-conn$ group.*

Proof. Suppose $R\cup C$ is $(i,j)-g\psi-disconn$, and $G\cup H$ is $(i,j)-g\psi-$disconnection of $R\cup C$. Since R is fuzzy $(i,j)-g\psi-conn$ subset of $R\cup C$, either $R\leq G$ or $R\leq H$ by Proposition 1. Similarly for C either $C\leq G$ or $C\leq H$. Now, if $R\leq G$ and $C\leq H$ or $(C\leq G, R\leq H)$, hence $(R\cup C)\cap G=R$, $(R\cup C)\cap H=B$, but this contradiction of the hypothesis R,C are fuzzy $(i,j)-g\psi-conn$. Hence either $R\cup C\leq G$ or $R\cup C\leq H$. So, $G\cup H$ is not fuzzy $(i,j)-g\psi-$disconnection of $R\cup C$. It means, $R\cup C$ is fuzzy $(i,j)-g\psi-conn$ group.

Corollary 2. *If $\Lambda=\{R_i\}$ is a class of fuzzy $(i,j)-g\psi-conn$ groups as no two members of Λ are separated, then $C=\bigcup_i R_i$ is fuzzy $(i,j)-g\psi-conn$ group.*

Proof. Suppose C is fuzzy $(i,j)-g\psi-disconn$ and $G\cup H$ is fuzzy $(i,j)-g\psi-disconnection$ of C. Since $\forall R_i$ of Λ is fuzzy $(i,j)-g\psi-conn$, as by Proposition 1 is contained either G or H and disjoint from the other. Furthermore, any R_{i_1}, R_{i_2} of Λ are not separated sets and so by Theorem 3, $R_{i_1}\cup R_{i_2}$ is fuzzy $(i,j)-g\psi-conn$. Thus $R_{i_1}\cup R_{i_2}$ is contained in G or H and disjoint from the other. Accordingly, all members of Λ, so $C=\bigcup_i R_i$ must be contained in either G or H and disjoint from the other. But this goes against the idea that $G\cup H$ is fuzzy $(i,j)-g\psi-disconnection$ of C. Therefore C is fuzzy $(i,j)-g\psi-conn$ group.

Corollary 3. *If $\Lambda=\{R_i\}$ is the class of fuzzy $(i,j)-g\psi-conn$ groups with a non empty intersection, then $C=\bigcup_i R_i$ is fuzzy $(i,j)-g\psi-conn$ group.*

Proof. As $\bigcap_i R_i\neq 0$, thus any two members of Λ are not disjoint and so no separated, thus by Corollary 2, $C=\bigcup_i R_i$ is fuzzy $(i,j)-g\psi-conn$ set.

Theorem 4. *If R is fuzzy $(i,j)-g\psi-conn$ subgroup of X and $R\leq C\leq(i,j)-g\psi-cl(R)$. Then C is fuzzy $(i,j)-g\psi-conn$, due to this in particular $(i,j)-g\psi-cl(R)$ is $(i,j)-g\psi-conn$.*

Proof. Assume C is fuzzy $(i,j)-g\psi-disconn$, with $G\cup H$ is fuzzy $(i,j)-g\psi-$disconnection of C. While R is fuzzy $(i,j)-g\psi-conn$ subset of C, then by Proposition 1 either $R\cap H=0$ or $R\cap G=0$, if we choose $R\cap H=0$. Hence $R\leq H^c$ which is fuzzy $(i,j)-g\psi-cld$. So $R\leq C\leq(i,j)-g\psi-cl(R)\leq H^c$. Consequently, $C\cap H=0$. But this contraducts the idea that $G\cup H$ is fuzzy $(i,j)-g\psi-$disconnection of C. Thus C is fuzzy $(i,j)-g\psi-conn$.

Theorem 5. *If R is fuzzy subgroup of (X,δ_1,δ_2), after that R is fuzzy $(i,j)-g\psi-conn$ of $X \iff R$ is fuzzy $(i,j)-g\psi-conn$ in terms of the relative topology on R*

Proof. Suppose R is fuzzy $(i,j)-g\psi-disconn$ of X with $G\cup H$ is $(i,j)-g\psi-$disconnection of R. Now, G, H are fuzzy $(i,j)-g\psi-open$ groups of X with $G\cap H=0$, so $R\cap G$ and $R\cap H$are disjoint $(i,j)-g\psi-open$ groups regarding to fuzzy topology on R. Since $(R\cap G)$ and $(R\cap H)$ form a $(i,j)-g\psi-$disconnection of R by fuzzy $(i,j)-g\psi-open$ groups of R, thus R is fuzzy $(i,j)-g\psi-disconn$ with respect to the relative topology on R.
In contrast, consider R is fuzzy $(i,j)-g\psi-disconn$ regarding to fuzzy topology on R, and let G^*, H^* form $(i,j)-g\psi-$disconnection of R. Then $\exists G,H$ of X as $G^*=G\cap R$, $H^*=H\cap R$, but $R\cap G^*=R\cap(R\cap G)=R\cap G$, and $R\cap H^*=R\cap(R\cap H)=R\cap H$, hence $G\cup H$ is $(i,j)-g\psi-$disconnection of R by fuzzy $(i,j)-g\psi-open$ groups of X.This means R is fuzzy $(i,j)-g\psi-disconn$ of X.

Definition 9. *An fbts (X,δ_1,δ_2) is known as fuzzy $(i,j)-g\psi-conn$ iff X is not combination of two nonempty fuzzy $(i,j)-g\psi-open$ groups.*
In different terms, iff $\nexists$ non empty fuzzy $(i,j)-g\psi-open$ sets R, C as $R\cup C=X=1$, and $R\cap C=\phi=0$.

Theorem 6. *Consider X is fbts. Then the next instances are equivalent:*

(1) *X is fuzzy $(i,j)-g\psi-conn$.*
(2) *0, 1 are the only fuzzy subset of X which are both $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$.*

Proof. Suppose X is fuzzy $(i,j)-g\psi-conn$, R is non empty $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$ group of X as $R\cap R^c=0, R\cup R^c=1$. Now let $C_1=R$, $C_2=R^c$. Thus C_1, C_2 are complement of each other, so they are both $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$. For C_1, C_2 are nonempty $(i,j)-g\psi-open$ groups of X, we have $C_1\cap C_2=R\cap R^c=0$. Also, $C_1\cup C_2=R\cup R^c=1$. So, they form a separation of X, that contradicts the fact X is fuzzy $(i,j)-g\psi-conn$.
Inversely, suppose X is fuzzy $(i,j)-g\psi-disconn$. So, $\exists$ fuzzy $(i,j)-g\psi-open$ groups R,C as $R\cup C=1, R\cap C=0$. Here $R^c=C$, so R is both $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$ which contradicts that X has only 0 and 1 which are both $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$. So, X is fuzzy $(i,j)-g\psi-conn$.

Corollary 4. *Suppose X is fbts. Hence the coming instances are equivalent:*

(1) *X is fuzzy $(i,j)-g\psi-disconn$.*
(2) *$\exists$ fuzzy subset of X which both $(i,j)-g\psi-open$ also $(i,j)-g\psi-cld$.*

Corollary 5. *X is fuzzy $(i,j)-g\psi-conn \iff \nexists$ fuzzy point sets of X which their union equal 1 and their intersection equal 0 except 0 and 1.*

Theorem 7. *When (X,δ_1,δ_2) is fbts. Hence the coming instances are equivalent:*

(i) *X is fuzzy $(i,j)-g\psi-conn$.*
(ii) *When R,H are nonempty fuzzy $(i,j)-g\psi-open$ groups of X with $R\cup H=1$ and $R\cap H=0$, then R, H are complementary to each other.*
(iii) *If R,H are fuzzy $(i,j)-g\psi-cld$ groups of X with $R\cup H=1$ and $R\cap H=0$, then R, H are complementary to each other.*

Proof. $(i)\Rightarrow(ii)$, Asumme that (ii) is not true and $R\neq H^c, R\cup H=1, R\cap H=0$. Then by De Morgan's rule $(R\cup H)^c=1^c=0\Rightarrow R^c\cap H^c=0$ and $(R\cap H)^c=0^c=1\Rightarrow R^c\cup H^c=1$. Now $(i,j)-g\psi-cl(R^c)\cap H^c=0$, also $R^c\cap(i,j)-g\psi-cl(H^c)=0$. Since $R,H\neq 0$ and their union equal 1 and their intersection equal 0, hence they form a separation of X, thus X is not fuzzy $(i,j)-g\psi-conn$, which is a contradiction. So, $R=H^c$.
$(ii)\iff(iii)$, It is clear by applying De Morgan's low.
$(iii)\iff(i)$, Suppose that (i) is not true. Then $\exists$ two nonzero $(i,j)-g\psi-open$ grous R,H as $(i,j)-g\psi-cl(R)\cap H=0=R\cap(i,j)-g\psi-cl(H)$. This is implies that (iii) is not true as they are complementary to each other.

Theorem 8. *Suppose (X,δ_1,δ_2) is fbts. Thus, the following claims are true:*

(1) *$\forall$ δ_j-conn, is $(i,j)-g-conn$.*
(2) *$\forall$ $(i,j)-g-conn$ is $(i,j)-g\alpha-conn$.*
(3) *$\forall$ $(i,j)-g\alpha-conn$, is $(i,j)-gs-conn$ and $(i,j)-gp-conn$.*
(4) *$\forall$ $(i,j)-gs-conn$, or $(i,j)-gp-conn$ is $(i,j)-g\beta-conn$.*

Proof. It is clear from the concept of connectedness in Definition 9, Theorem 2, and the connections between different forms of $(i,j)-g\psi-cld$ sets in Theorem 1. Also, based on the reference [13] which discussed the relationships between these groups in detail and explained the inverse relationships as mentioned in remark 3 in the reference.

Remark. The next diagram showing the connections between different forms of fuzzy $(i,j)-g\psi-conn$.

$$\begin{array}{ccc}
\delta_j-conn & & \\
\downarrow & & \\
(i,j)-g-conn & & \\
\downarrow & & \\
(i,j)-g\alpha-conn & \longrightarrow & (i,j)-gp-conn \\
\downarrow & & \downarrow \\
(i,j)-gs-conn & \longrightarrow & (i,j)-g\beta-conn
\end{array}$$

Fig.1: Presents the relationship through all varieties of fuzzy $(i,j)-g\psi-conn$.

(1) The reverse of the previous graphic is not typically true, and this is clear from Definition 9, Theorem 2, from the relationships between (i,j) generalized ψ groups previously illustrated in Theory 1, and remak 3 and examples 2 to 7 in [13].
(2) The concept of fuzzy $(i,j)-gp-$conn with $(i,j)-gs-$conn are separate.

Theorem 9. *When (X,δ_1,δ_2) is fbts and R,H form a separation of X. Then for any fuzzy set G in X, $G\cap R$ and $G\cap H$ are fuzzy $(i,j)-g\psi-disconn$.*

Proof. Since R,H are separated. Then R,H are non empty $(i,j)-g\psi-open$ sets, as $R\cap((i,j)-g\psi-cl(H))=H\cap((i,j)-g\psi-cl(R))=0$. Now $G\cap R\leq R\Rightarrow(i,j)-g\psi-cl(G\cap R)\leq(i,j)-g\psi-cl(R)$. Also, $G\cap H\leq H\Rightarrow(i,j)-g\psi-cl(G\cap H)\leq(i,j)-g\psi-cl(H)$. Now,$((i,j)-g\psi-cl(G\cap R))\cap(G\cap H)\leq(i,j)-g\psi-cl(R)\cap H=0$. Hence $((i,j)-g\psi-cl(G\cap R))\cap(G\cap H)=0$.
In a similar way, we get $(G\cap R)\cap((i,j)-g\psi-cl(G\cap H))=0$. Hence $G\cap R$, $G\cap H$ form a separation. So $G\cap R$, $G\cap H$ are fuzzy $(i,j)-g\psi-disconn$.

Definition 10. *A mapping $f:(X,\delta_1,\delta_2)\rightarrow(Y,\sigma_1,\sigma_2)$ termed a fuzzy $(i,j)-generalized\ \psi-continuous$ (briefly, $(i,j)-g\psi-conts$) when each corresponding image of all fuzzy open of (Y,σ_j) is fuzzy $(i,j)-g\psi-open$ of X.*

Theorem 10. *The image of fuzzy $(i,j)-g\psi-conn$ under $(i,j)-g\psi-$continuous mapping is fuzzy σ_j-conn.*

Proof. Let $f:(X,\delta_1,\delta_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be fuzzy $(i,j)-g\psi-conts$, onto function, and (X,δ_1,δ_2) be

$(i,j)-g\psi-conn$ space. Also, assume R, H are fuzzy σ_j-open groups of Y, and Y is fuzzy $\sigma_j-disconn$. Then $Y = R\cup H$. Since all fuzzy σ_j-open set is $(i,j)-g\psi-open$, and f is fuzzy $(i,j)-g\psi-conts$, then $X = f^{-1}(R)\cup f^{-1}(H)$, which is contradiction that X is fuzzy $(i,j)-g\psi-conn$ space. Therefore Y is fuzzy σ_j-conn.

Definition 11. *A mapping $f:(X,\delta_1,\delta_2)\to(Y,\sigma_1,\sigma_2)$ termed a fuzzy $(i,j)-generalized\,\psi-irresolute$ mapping $(briefly,(i,j)-g\psi-irres)$ when each corresponding image of all fuzzy $(i,j)-g\psi-open$ group of X is fuzzy $(i,j)-g\psi-open$ of Y.*

Theorem 11. *If $f:(X,\delta_1,\delta_2)\to(Y,\sigma_1,\sigma_2)$ is fuzzy $(i,j)-g\psi-irresolute$ with surjective mapping, then Y is fuzzy $(i,j)-g\psi-conn$, where X is fuzzy $(i,j)-g\psi-conn$ space.*

Proof. Suppose $f:(X,\delta_1,\delta_2)\to(Y,\sigma_1,\sigma_2)$ is fuzzy $(i,j)-g\psi-irresolute$, surjective function, with X is $(i,j)-g\psi-conn$ space. In addition, assume R, H are fuzzy $(i,j)-g\psi-open$ groups of Y, also Y is fuzzy $(i,j)-g\psi-disconn$. After that $Y=R\cup H$. As f is fuzzy $(i,j)-g\psi-irresolute$, thus $X=f^{-1}(R)\cup f^{-1}(H)$, this runs counter to X is fuzzy $(i,j)-g\psi-conn$ space. Therefore Y is $(i,j)-g\psi-conn$.

4 Conclusion

In the above research, we defined also examined the terms of different varieties of generalized connectedness ideas in fuzzy bitopology space, as well as some relationships between them. Then we examined some fundamental theorems and characteristics of these concepts.

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