Electrohydrodynamic Stability of Self-gravitating Fluid Cylinder

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Abstract: Electrohydrodynamic Stability consists of a fluid cylinder with self-gravitation A self-gravitating tenuous medium surrounds it. that is permeated by a transversely variable electric field while being affected by self-gravitating forces, Capillary, and Electrical Forces is covered across all axisymmetric and (non)axisymmetric perturbation types. The problem is solved and all individual solutions are excluded. The model stabilizes as a result of continuous Electric field stability in all perturbation modes. In a narrow area of the axisymmetric perturbation. It has been established that self-gravitating and capillary forces have a destabilizing effect. However, the present model instability is improved and modified by self-gravitating and capillary forces' stabilizing effects in all large axisymmetric fields and (non)axisymmetric domains.

1 Introduction

Many scholars, including Lin [9], Rayleigh [1], Drazin and Reid [10],were interested in the stability of a full jet fluid under surface tension, whether it is optimum or not. Rayleigh [1] pioneered the fundamental computational techniques for investigating numerous stability-related issues and also established the stability criteria. The cutoff wavenumber normalised is related to the cylinder unit's radius, according to the stability study results of an ideal liquid cylinder subjected to capillary forces, whereas the highest instability growth rate is shown when the wavenumber is about 0.0697. Simplifying solenoidal vectors into toroidal and poloidal amounts was addressed by Chandrasekhar and Fermi [4]. Chandrasekhar [5] investigated the stability of a whole liquid jet under the influence of capillary forces and self-gravitation of all sorts of axisymmetric and (non)axisymmetric disturbance, as well as the impact of the fixed complete fluid jet's axisymmetric capillary instability due to a magnetic field perturbation measured by toroidal and poloidal values, the solenoidal vectors.C. D. S. [11]. The same was carried out for a fluid cylinder under the influence of the Lorentz force, by Chandrasekhar (with uniform magnetic field) and self-gravitating forces. In several cylindrical models, Radwan [6] created hydrodynamic and hydro-magnetic instability. Additionally, since Kelly's seminal publications [9], several researchers have examined cylindrical fluids' electrodynamic stability (Mohammed and Nayyar [8], Melcher et al. (see also Mestel [12-13]). The consistency of various Radwan and Hassan have developed cylinder models that work with self-gravitating forces as well as other forces [17–18]. For all (non)axisymmetric and axisymmetric perturbation modes, Hassan [14] examined the stability of a cylinder filled with oscillating flowing fluid under the combined influence of capillary and self-gravitation. force of electrodynamics. Hassan's research used a gravity fluid cylinder with varying electric fields. [20] study of the Capillary Electrodynamic Stability of Self. Here, we study the instability of a fluid cylinder surrounded by a self-gravitating tenuous medium and permeated by a transversely variable electric field under the action of all kinds of perturbations and the interaction of the self-gravitating, electric, and capillary forces.

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2 The Problem's Formation

Take a look at a cylinder of self-gravitating, incompressible, streaming fluid that has a uniform cross-section and a radius of (radius R_O). We presume the fluid will be dielectric, self-gravitating, permeated by a uniform electric field, and both its interior and exterior have a dielectric constant.

$$\underline{E}^i_\circ = (0 , 0 , E_\circ) \tag{1}$$

Additionally, the transversely varying electric field is permeating the nearby self-gravitating tenuous medium.

$$\underline{E}^e_\circ = \left(0 , \frac{\beta\, E_\circ\, R_\circ}{r} , 0 \right) \tag{2}$$
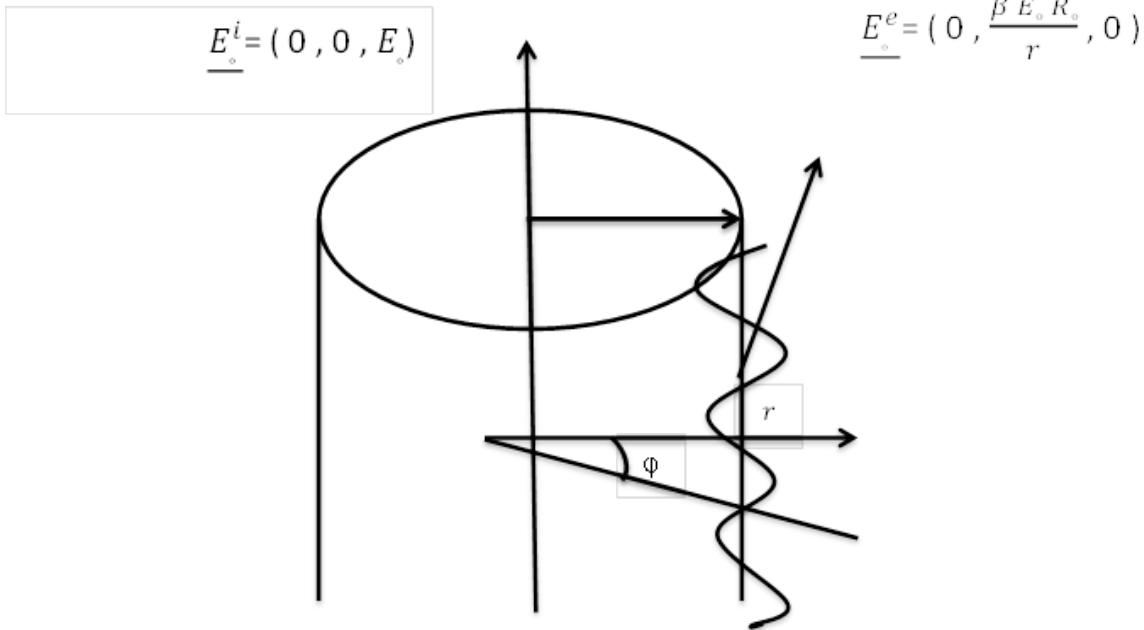

Fig. I: self-gravitation Electrohydrodynamic cylindrical Fluid sketch.

Where β is any parameter, $E_\circ$ is the electric field's strength, and The components of $\underline{E}^i$ and $\underline{E}^e$ are taken into account along the cylindrical coordinate (r, ϕ, z) system where the axis of the fluid cylinder and the Z-axis are congruent. When velocity, electric forces, self-gravitating forces, and capillary forces are coupled, they have an additive influence on the fluid. Shown in Fig (I)
conventional hydrodynamic equations and the Maxwell equations in combination, electromagnetic theory, and Newtonian self-gravitating equations make up fundamental equations (see Chandrasekhar [5] and Lin [2]). The basic equations are provided as follows.

In the fluid

$$\rho\left[\frac{\partial \underline{u}}{\partial t} + (\underline{u} . \nabla)\, \underline{u}\right] = -\nabla P + \rho\nabla v + \frac{1}{2}\nabla\left(\varepsilon^i\left(\underline{E}^i . \underline{E}^e\right)\right) \tag{3}$$

$$\nabla . \underline{u} = 0 \tag{3}$$

$$\nabla . (\varepsilon\underline{E})^i = 0 \tag{4}$$

$$\nabla \wedge \left(\varepsilon^i\underline{E}^i\right) \tag{5}$$

$$\nabla^2 v = -4\,\pi\rho G \tag{6}$$

Surface of the fluid cylinder

$$P_s = T\left(\nabla . \underline{N}\right) \tag{7}$$

In the tenuous medium around it

$$\nabla . \underline{E}^{(e)} = 0 \tag{8}$$

$$\nabla \wedge \left(\varepsilon^e \underline{E}^e\right) \tag{9}$$

$$\nabla^2 v^e = 0 \tag{10}$$

Where ρ is mass density, $\underline{u}$ is vector of velocity, p is kinetic force, v is self-gravitating potential, G stands for gravity constant , T coefficient of surface tension., $\underline{N}$ is the exterior unit vector the surface is perpendicular to it. and $\underline{E}^e$, v^e are the fluid cylinder's surrounding tenuous mediums.

3 State of equilibrium

We examine the equilibrium state and show how the variables may be calculated in a situation where $\underline{u} = 0$, $\frac{\partial}{\partial \varphi} = 0$ and $\frac{\partial}{\partial z} = 0$ using equation (3) we get

$$v_{\circ} = -\pi\rho Gr^2 \tag{11}$$

$$v_{\circ}^{(e)} = 2\pi\rho GR_{\circ}^2 \left[\ln\frac{R_{\circ}}{r} - \frac{1}{2}\right] \tag{12}$$

$$P_{os} = \frac{T}{R_{\circ}} \tag{13}$$

Additionally, after exerting the remaining pressure across the boundary surface at $r = R_{\circ}$ the The formula for the fluid pressure distribution in the equilibrium state is

$$P_{\circ} = \frac{T}{R_{\circ}} + \pi G\rho^2\left[R_{\circ}^2 - r^2\right] + \frac{E_{\circ}^2}{2}\left[\varepsilon^i - \beta^2\varepsilon^e\right] \tag{14}$$

4 Perturbed State

Every variable quantity $Q(r, \varphi, z, t)$ may be represented as follows for little variations from the equilibrium condition.
$$Q(r, \varphi, z, t) = Q_{\circ}(r) + \eta_{\circ}(t)Q_1(r, \varphi, z) + \cdots \tag{15}$$

Here Q stands for $\rho, \underline{u}, \underline{E}^{(e)}, \underline{E}^{(i)}, v^e, v^i, \underline{N}_S$ and P_S . $\eta(t)$ is a parameter with dimensions that measures the magnitude of the disturbance. $\eta(t)$ may be expressed as
$$\eta(t) = \eta_{\circ}e^{(\sigma t)} \tag{16}$$

Where $\varepsilon_{\circ}(= \varepsilon$ at $t = 0)$ is the initial amplitude, and σ is the growth rate, $\sigma = i\omega$
A normal mode may be expressed as
$$r = R_{\circ} + R_1 , R_1 \ll R_{\circ} \tag{17}$$

With
$$R_1 = \eta(t) e^{i(kz+m\varphi)} \tag{18}$$

Where R_1 The longitudinal wave number, It is k an actual number. and the transverse wave number, which is an integer, is the elevation surface wave recorded from the equilibrium site. By using the expansion (16), the pertinent perturbation equations in the fundamental equations (3) to (11), are provided by

$$\frac{\partial u_1}{\partial t} = -\nabla\pi_1^i \tag{19}$$

$$\pi_1^i = \frac{P_1}{\rho} - v_1^i - \frac{1}{\rho}\left(\varepsilon^i\left(\underline{E}_\circ^i \cdot \underline{E}_1^i\right)\right) \tag{20}$$

$$\nabla \cdot \underline{u}_1 = 0 \tag{21}$$

$$\nabla \cdot \left(\varepsilon^i \underline{E}_1^i\right) = 0 \tag{22}$$

$$\nabla \wedge \varepsilon^i \underline{E}_1^i = 0 \tag{23}$$

$$\nabla^2 v_1^i = 0 \tag{24}$$

Near the fluid-tenuous media contact

$$P_{1s} = -\frac{T}{R_\circ}\left(\eta + \frac{\partial^2 \eta}{\partial \varphi^2} + R_\circ^2 \frac{\partial^2 \eta}{\partial z^2}\right) \tag{25}$$

Around the fluid, in the tenuous midrange

$$\nabla \cdot \left(\varepsilon^e \underline{E}_1^e\right) = 0 \tag{26}$$

$$\nabla \wedge \underline{E}_1^e = 0 \tag{27}$$

$$\nabla^2 v_1^e = 0 \tag{28}$$

5 Fourier Analysis

a method for tackling cylindrical stability issues that uses linear perturbation and the space-time dependence (18) and (19) of the relevant perturbed quantity $Q_1(r, \varphi, z, t)$ May be expressed by

$$Q_1(r, \varphi, z, t) = Q_1(r)e^{(i(kz+m\varphi)+\sigma t)} \tag{29}$$

Take the divergence of equation (20) and using equations (22) and (23), we get

$$\nabla^2 \pi_1^i = 0 \tag{30}$$

Equations (23) and (28) state that the corresponding magnetic field perturbation, $\underline{E}_1^{i,e}$ can be derived from the ascolor function say,

$$\underline{E}_1^{i,e} = -\nabla \psi_1^{i,e} \tag{31}$$

If the equations (32),(24),and (27), we obtain

$$\nabla^2 \psi_1^{i,e} = 0 \tag{32}$$

We obtain the solution to equations (29), (31), (32), and (33) by using the expansion (30).

$$r^{-1}\frac{d}{dr}\left(r\frac{dQ_1(r)}{dr}\right) - \left(\frac{m^2}{r^2} + k^2\right)Q_1(r) = 0 \tag{33}$$

Where, $Q_1(r)$ stands for $\pi_1^i(r)$, $v_1^i(r)$, $v_1^e(r)$, and $\psi_1^i(r)$. The solution of the ordinary second-order differential equation (34) is expressed using standard Bessel functions of order m. Other than a single resolution, we obtain

$$v_1^i = A^i I_m(kr)e^{(i(kz+m\varphi)+\sigma t)} \tag{34}$$

$$v_1^e = A^e K_m(kr)e^{(i(kz+m\varphi)+\sigma t)} \tag{35}$$

$$\pi_1^i = B^i I_m(kr)e^{(i(kz+m\varphi)+\sigma t)} \tag{36}$$

$$\psi_1^i = C^i I_m(kr)e^{(i(kz+m\varphi)+\sigma t)} \tag{37}$$

$$\psi_1^e = C^E K_m(kr)e^{(i(kz+m\varphi)+\sigma t)} \tag{38}$$

6 Stability Criterion

6-1 Self-gravitating condition

1- A steady self-gravitating potential over the equilibrium surface is required at $r = R_\circ$

$$v_1^i + R_1 \frac{\partial v_\circ^i}{\partial r} = v_1^e + R_1 \frac{\partial v_\circ^e}{\partial r} \tag{39}$$

2- The self-gravitating potential's derivative needs to X`.LOUAYJT be continuous throughout the equilibrium surface at $r = R_\circ$

$$\frac{\partial v_1^i}{\partial r} + R_1 \frac{\partial^2 v_\circ^i}{\partial r^2} = \frac{\partial v_1^i}{\partial r} + R_1 \frac{\partial^2 v_\circ^e}{\partial r^2} \tag{40}$$

From which, we get

$$A^i = 4\pi\rho R_\circ^2 G K_m(x) \tag{41}$$

$$A^e = 4\pi\rho R_\circ^2 G I_m(x) \tag{42}$$

6-2 Kinematic condition

The fluid tenuous medium interface velocity and there must be consistency between the fluid velocity vector's normal component at $r = R_\circ$ i.e.

$$\underline{N}.\underline{u} = \frac{\partial r}{\partial t} \tag{43}$$

Where

$$\underline{N} = (1,0,0) + \eta_\circ\left(0, -\frac{im}{R_\circ}, -ik\right) e^{(i(kz+m\varphi)+\sigma t)} \tag{44}$$

We get

$$B^i = -\frac{\sigma^2 R_\circ^2}{x I_m'(x)} \tag{45}$$

6-3 Electric condition

1- The electric field potential's normal component must also be constant over the equilibrium surface. at $r = R_\circ$

$$\underline{N}.\left(\varepsilon^i \underline{E}^i - \varepsilon^e \underline{E}^e\right) = 0 \tag{46}$$

$$\underline{E} = \underline{E}_\circ + R_1 \frac{\partial \underline{E}_\circ}{\partial r} + \underline{E}_1 \tag{47}$$

If

$$\psi_\circ^i = -E_\circ r \tag{48}$$

$$\psi_\circ^e = -R_\circ E_\circ \left[1 + \log\frac{r}{R_\circ}\right] \tag{49}$$

$$\psi_1^i + R_1 \frac{\partial \psi_\circ^i}{\partial r} = \psi_1^e + R_1 \frac{\partial \psi_\circ^e}{\partial r} \tag{50}$$

$$c^i = \frac{i E_\circ [\beta\varepsilon^e - \varepsilon^i] K_m(x)}{\varepsilon^i I_m'(x) K_m(x) - \varepsilon^e K_m'(x) I_m(x)} \tag{51}$$

$$c^e = \frac{i E_\circ [\beta\varepsilon^e - \varepsilon^i] I_m(x)}{\varepsilon^i I_m'(x) K_m(x) - \varepsilon^e K_m'(x) I_m(x)} \tag{52}$$

Where $x = kR_\circ$ is dimensionless, the longitudinal wave number.
The condition can be written as follows for the issue at hand:

$$P_{1s} = P_1 + R_1 \frac{\partial P_\circ}{\partial r} + \frac{1}{2}\varepsilon^i\left(2E_\circ^i . E_1^i\right) - \frac{1}{2}\varepsilon^e\left(2E_\circ^e . E_1^e\right) \tag{53}$$

BY substituting from equations (15), (21), and(26) we get

$$\sigma^2 = \frac{T}{\rho R_\circ^3}\left[1 - m^2 - x^2\right]\frac{x I_m'(x)}{I_m(x)} + 4\pi G\rho\left[I_m(x)K_m(x) - \frac{1}{2}\right]\frac{x I_m'(x)}{I_m(x)} + \frac{-E_\circ^2}{\rho R_\circ^2}\left[\frac{[\varepsilon^i - \beta\varepsilon^e]^2 x^2 I_m'(x) K_m'(x)}{\varepsilon^i I_m'(x)K_m(x) - \varepsilon^e K_m'(x)I_m(x)}\right] \tag{54}$$

7 Case Limitations

The relationship (55) is a combined stability of a linear equation requirements for a cylinder for liquids working just under its own gravitational pull, a fluid cylinder acting under the influence of one acting as a result of electric force, and one acting under the influence of capillary force (55) are a connection between the wavenumber x and m, modifications to Bessel functions,and the oscillation frequency, or temporal amplification $I_m(x), K_m(x)$ and its derivation, the parameter β for the parameters, the transverse electric field $\rho, \underline{E}, G, T, and R_\circ$ of the problem and with the fundamental quantity $\frac{T}{\rho R_\circ^3}$ as well as $\frac{1}{\sqrt{4\pi G\rho}}$, and $\sqrt{\frac{E_\circ^2}{\rho R_\circ^2}}$ time unit. Given that the current problem is a little more generic than the previous analysed situations, stability requirements for many problems with various characteristics can be obtained as limiting examples with sufficient alternatives from the general dispersion relation (55). Several simplifications, such as

1- $G = 0$, $\underline{E} = 0$, $and\ m = 0$, are necessary to obtain the following dispersion relation from (55)

$$\sigma^2 = \frac{T}{\rho R^3}[\ 1 - x^2]\left(\frac{xI'_\circ(x)}{I_\circ(x)}\right) \tag{55}$$

2- $G = 0$, $\underline{E} = 0$, and $\beta = 0$, while $m \geq 1$ are necessary to obtain the following dispersion relation from (55)

$$\sigma^2 = \frac{T}{\rho R^3}[\ 1 - m^2 - x^2]\left(\frac{xI'_m(x)}{I_m(x)}\right) \tag{56}$$

3- $T = 0$, $\underline{E} = 0$, $\beta = 0$, and $m = 0$ are necessary to obtain the following dispersion relation from (55)

$$\sigma^2 = 4\pi G\rho\left[I_\circ(x)K_\circ(x) - \frac{1}{2}\right]\left(\frac{xI'_\circ(x)}{I_\circ(x)}\right) \tag{57}$$

4- $T = 0$, $\underline{E} = 0$, $\beta = 0$, while $m \geq 1$ are necessary to obtain the following dispersion relation from (55)

$$\sigma^2 = 4\pi G\rho\left[I_m(x)K_m(x) - \frac{1}{2}\right]\left(\frac{xI'_m(x)}{I_m(x)}\right) \tag{58}$$

5- $T = 0$, $G = 0$, while $m \geq 0$ are necessary to obtain the following dispersion relation from (55)

$$\sigma^2 = \frac{-E_\circ^2}{\rho R_\circ^2}\left[\frac{\left[\varepsilon^i - \beta\varepsilon^e\right]^2 x^2 I'_m(x)K'_m(x)}{\varepsilon^i I'_m(x)K_m(x) - \varepsilon^e K'_m(x)I_m(x)}\right] \tag{59}$$

8 Stability Discussions

The modified Bessel functions that exist in the criterion must be studied for some of their behaviour (54). The modified Bessel functions' recurrence relations are provided in the form for the nonzero real value of x (see Abramowitz and Stegun [17]).

$$2I'_m(x) = I_{m-1}(x) + I_{m+1}(x) \tag{60}$$

$$2K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \tag{61}$$

Due to the fact that for each actual value X that $I_m(x)$ is constantly favourable and steadily growing while $K_m(x)$ is monotonically declining but never negative, as we can see

$$I'_m(x)\ and\ K'_m(x) < 0 \tag{62}$$

Consequently, For each nonzero real value, we obtain x

$$\left(\frac{xI'_m(x)}{I_m(x)}\right) > 0\ \text{while}\ \left(\frac{xk'_m(x)}{k_m(x)}\right) < 0 \tag{63}$$

$m = 0$ and (non)axisymmetric modes $m \geq 1$ are both true for all axisymmetric modes.

9 Mathematical Discussions

This model's stable and unstable regions, as well as their properties, must be statistically identified using σ^* the relation of dispersion (55). Additionally, research will be done to determine how the capillary force in the most important mode of disturbance is affected by the self-gravitating and electrodynamic forces. For $m = 0$ equation (55) reads

$$\sigma^{*2} = (1 - x^2)\frac{xI'_\circ(x)}{I_\circ(x)} + N\left[I_\circ(x)K_\circ(x) - \frac{1}{2}\right]\frac{xI'_\circ(x)}{I_\circ(x)} - M^2 x^2\left[\frac{[1 - \beta\varepsilon]^2 I'_\circ(x)K'_\circ(x)}{I'_\circ(x)K_\circ(x) - \varepsilon K'_\circ(x)I_\circ(x)}\right] \tag{64}$$

Where

$$\sigma^* = \frac{\sigma}{\sqrt{\frac{T}{\rho R^3}}}\ ,N = \frac{4\pi G\rho^2 R_\circ^3}{T}\ ,\text{and}\ M^2 = \frac{E_\circ^2 \varepsilon^i R_\circ}{T}$$

are dimensionless amounts. for all wavelengths, including short and long $0 \leq X \leq 3$, the dispersion relation (65) has been calculated using several computer tools. The σ^{*2} values related to the unstable

domains and those of $\omega^* = \frac{\omega}{\sqrt{\frac{T}{\rho R_\circ^3}}}$ These data are gathered, collated, and visually shown in relation to the stable

domains. Typical values of x in the range $0 \leq X \leq 3$, such computations have been developed for various values of β, M and N.

i) For $M = 0$, $N = 0.1$, 0.4 , 0.7 , 0.9 and 1.2 it is discovered that:-

There exist unstable domains

$$0 < x < 1.1331 \qquad\qquad 0 < x < 1.3257 \qquad\qquad \text{and}\ 0 < x < 1.7496$$

while domains that are stable are

$$1.1331 < x < \infty \qquad\qquad 1.3257 < x < \infty \qquad\qquad \text{and } 1.7496 < x < \infty$$

Thus, as seen in Figure 1 , The equalities line up with the states of marginal stability.

Fig. 1: For M = 0 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2.

ii) For M = 0.1 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2 it is discovered that:-
There exist unstable domains

$$0 < x < 1.3310 \qquad\qquad 0 < x < 1.3258 \qquad\qquad \text{and } 0 < x < 1.8472$$

while domains that are stable are

$$1.3310 < x < \infty \qquad\qquad 1.3258 < x < \infty \qquad\qquad \text{and } 1.8472 < x < \infty$$

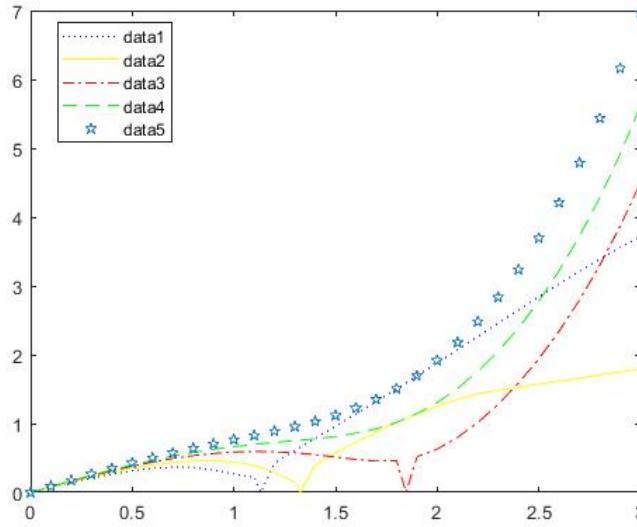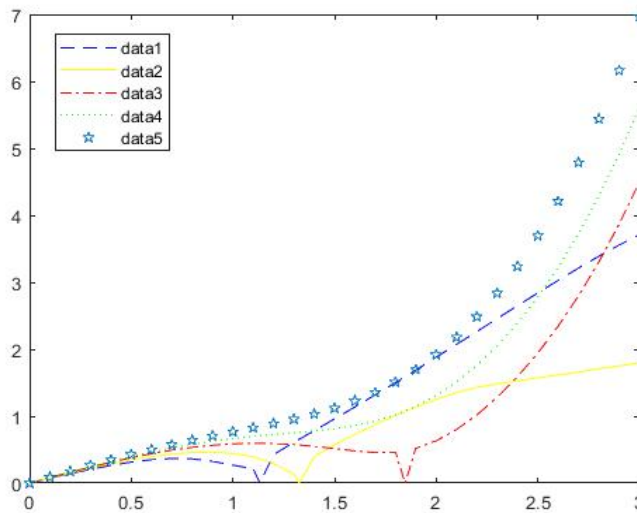Thus, as seen in Figure 2, The equalities line up with the states of marginal stability.

Fig. 2: For M = 0.1 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2.

iii) For M = 0.2 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2 it is discovered that:-
There exist unstable domains

| 0 < x < 1.1332 | 0 < x < 1.3260 | and 0 < x < 1.8472 |

while domains that are stable are

| 1.1332 < x < ∞ | 1.3260 < x < ∞ , | and 1.8472 < x < ∞ |

Thus, as seen in Figure 3, The equalities line up with the states of marginal stability.

Fig. 3: For M = 0.2 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2.

iv) For M = 0.4 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2 it is discovered that:-
There exist unstable domains

| 0 < x < 1.1334 | 0 < x < 1.3266 | and 0 < x < 1.8472 |

while domains that are stable are

| 1.1334 < x < ∞ | 1.3266 < x < ∞ | and 1.8472 < x < ∞ |

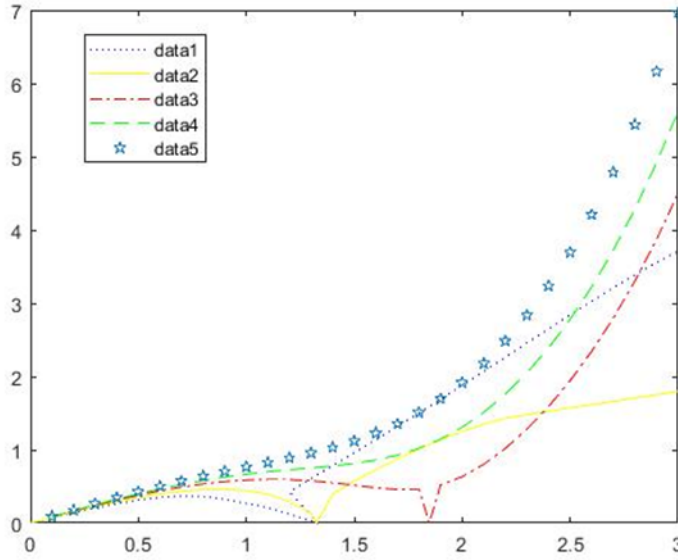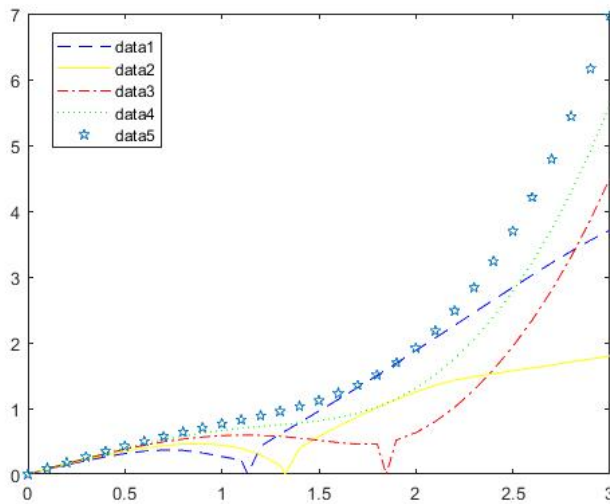Thus, as seen in Figure 4, The equalities line up with the states of marginal stability.

Fig. 4: For M = 0.4 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2.

v) For M = 0.8 , N = 0.1 , 0.4 , 0.7 , 0.9 and 1.2 it is discovered that:
There exist unstable domains

| 0 < x < 1.1386 | 0 < x < 1.3290 | nd 0 < x < 1.8471 |

while domains that are stable are

| 1.1386 < x < ∞ | 1.3290 < x < ∞ | and 1.8471 < x < ∞ |

Thus, as seen in Figure 5, The equalities line up with the states of marginal stability.

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Fig. 5: For $M = 0.8$, $N = 0.1$, 0.4, 0.7, 0.9 and 1.2

10 Conclusions

Using numerical analysis, we arrive at the following conclusion:

1- The model is stabilised when N is increased while the capillary force (M) remains unchanged, showing that the electric force has a stabilising effect.

2- Indicating that the capillary force significantly contributes to the self-gravitation destabilisation of the model, the unstable domain expands with increasing M values for a given value of N.

3- The self-gravitating instabilities of the model are stabilised by the electric.

4- The capillary force on the model significantly stabilizes the system.

Acknowledgement

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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