Single-Source Multi-Period Problem Model with Active Constraints-Based Approach Algorithm

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Abstract: In this paper, we introduce the multi-period single sourcing problem as an assignment problem. The multi-period single-sourcing problem in this research is seen as a problem of finding assignments, from time to time to obtain the minimum possible total transportation and inventory costs for distributing goods to customers. The case considered in this problem is the case of placing inventory items that are distributed to customers online, so this case is seen as a non-polynomial or NP hard problem that requires a solution algorithm, and the algorithm we offer is a direct search algorithm to solve the problem. multi period single sourcing. The direct search algorithm offered is the Branch and Price algorithm which was developed for Generalized Assignment Problems (GAP) to a much more complete class of problems, called CAP (Convex Assignment Problems). We offer this algorithm because the results it will obtain are more optimal, the computing time is superior, and it shows greater stability, that is, fewer outliers are observed. Specifically, we generalize the strategy of separating nonbasic variables from their constraints, combined with using active constraint methods to solve the Generalized Assignment Problem (GAP) into a Convex Assignment problem. Then, identification of important subclasses of the problem is carried out, which contains many variations of multi-period single sourcing problems, as well as GAP variants. The final result we found is an active depth-based single source multi-period model that can minimize the damage to the optimal integer solution for solving the MPSSP convex problem.

Keywords: General Assignment Problem; Convex Assignment; Active Constraint

1 Introduction

Multi Period Single Sourcing Problem is the problem of assigning warehouses from each retailer to each particular facility at the start of planning with the aim of minimizing placement costs, inventory acquisition and unfulfilled orders by taking into account the fulfillment of planning requests and limited production capacity at the facility [1]. The network configuration problem referred to in MPSSP is not only limited to the movement of goods from suppliers to consumers, but also needs to consider the retrieval of each item from the downstream side of the supply chain, such as damaged goods, packaged materials and other types of goods [2,3].

It is further stated that the facility in question is described as a production factory with a connected warehouse [4]. Each retailer must be linked to a specific facility. Then, each production factory has a limited and known capacity. It is also assumed that the storage warehouse has unlimited capacity so that it is able to store all production even though the factory produces the maximum in each period.

In logistics distribution management, a network system is very necessary for determining and placing appropriate warehouses as well as preparing logistics distribution plans to obtain the shortest delivery routes [5,6], so that in the logistics distribution process, the nominal costs incurred are as minimum as possible [7]. The logistics distribution network is very important for companies to be able to deliver their products to users where the intended distribution network consists of producers, service providers, distributors, sales channels such as retailers and customers [8]. On the other hand, a logistics distribution network is created so that the storage warehouses that are built can guarantee the fulfillment of retailer demand which is associated with a fairly large capacity [9].

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The obstacles faced in the logistics goods distribution process are the number of requests for goods that differ from each consumer, vehicle capacity, delivery time limits [10], on the other hand, there are also problems of distance and travel time, production time, inventory location, and customer assignment. to the warehouse where the company delivers its products to customers using a logistics distribution network [11]. All of these problems require a distribution network in the form of a flow diagram or algorithm that connects producers to consumers through transfer points, distribution centers (warehouses), and retailers [12] due to the dynamic environmental conditions in which the expected supply chain must develop and the demands for obtaining a short product distribution cycle period requires companies to redesign the logistics distribution network [13].

In this paper, some of the problems discussed in this paper are the problem of warehouse location placement and location routing by considering inventory, such as [14, 15]. Each plant is positioned to have a known, limited, and possibly time-varying capacity, each customer needs to be served by a unique facility within the considered planning horizon, and customers demand to follow seasonal patterns [16, 25]. We will also introduce convex assignment problems (CAP), and propose the branch-and-price algorithm as a precise and practical solution to mathematical problems. This approach generalizes to the branch-and-price algorithm procedure that fits the general GAP assignment problem [17][18].

2   Theoretical Basis

Model Multi Period Single sourcing (MPSS)

The multi-period single-source problem is a problem of assigning each particular facility to each retailer's location at the beginning of planning with the aim of making the logistics distribution process more efficient [19,20], thereby minimizing placement costs, inventory acquisition and unfulfilled orders by paying attention to fulfillment planning demands and production capacity limitations at the facility [21]. The logistics network design that has been designed as shown in Figure 1 is a cost optimization design for placing the things needed for production and maintenance in one warehouse area with the aim of minimizing transportation costs [22].
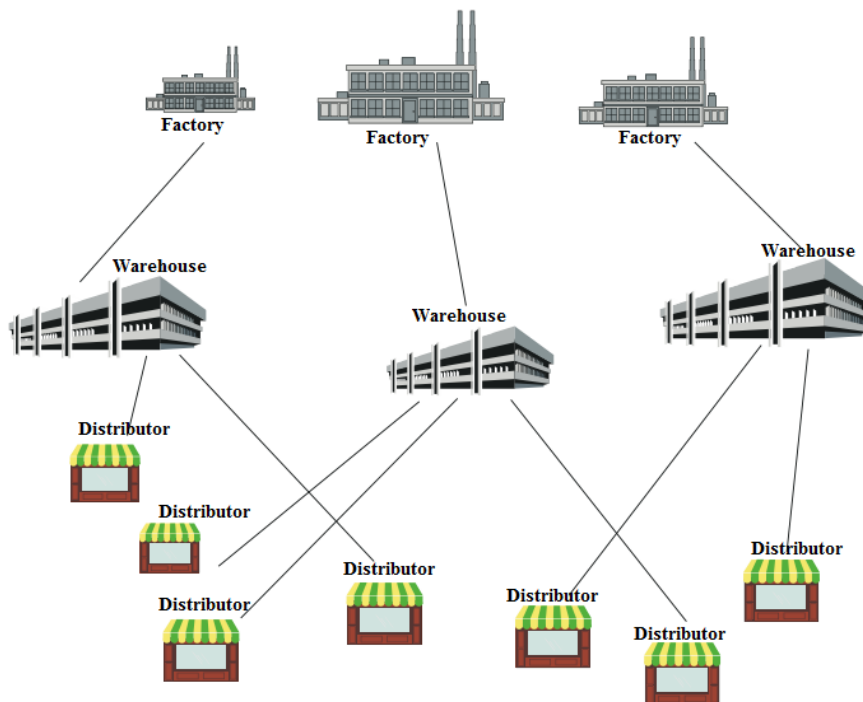

Figure 1. Logistics distribution network and its allocation without retailers [1]

Figure 1 above is a picture of a logistics network design for optimizing the costs of placing inventory and warehouse items so that production and logistics distribution transportation costs produced can be optimized by looking at the production party sending goods to distributors, so that the distributor distributes the goods to customers. Next, we will look at the design of the logistics distribution network which was developed taking into account current conditions.
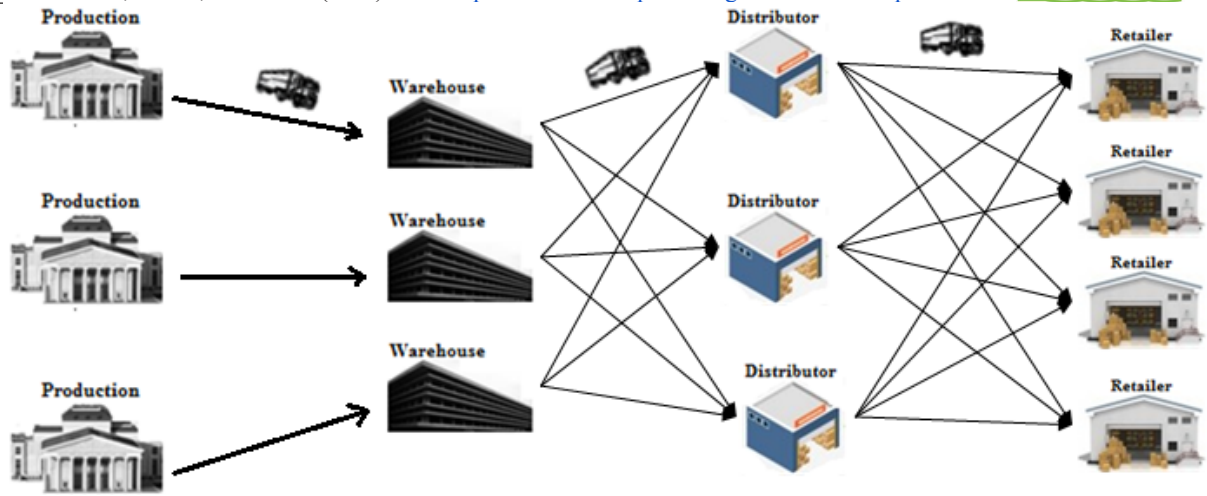
Figure 2. Logistics distribution network and its allocation through retailers

Figure 2 above is a logistics network design to optimize the costs of placing the things needed for production and maintenance in one warehouse area with the aim of minimizing total transportation and inventory costs to a minimum for distributing goods to customers designed as in Figure 2. Distribution of goods. what is meant is from the production site to the warehouse and from the warehouse to distributors and customers and from distributors to customers.

For this case, let n be the number of customers, m the number of production and warehouses, and T the planning time. The demand from customer j in period t is d_{jt}, while the production capacity at facility i in period t is equal to b_{it}. The cost of assigning customer j to facility i in period t is c_{ijt} (called a function of production and transportation costs). The costs of production and procurement of inventory items at facility i for period t are and , which are assumed to be non-negative. Customer service considerations may require some or all customers to be assigned to the same facility in any given period. To incorporate these possibilities in the model, a set $S\{1,...,n\}$ is introduced that represents the subset of customers (called repeat customers) that need to be assigned to the same facility in all periods. Let $D = \{1,\rho,n\}\backslash S$ be the remaining set of customers (called non-regular customers).

MPSSP is formulated as follows:

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ijt}x_{ijt} + \sum_{t=1}^{T}\sum_{i=1}^{m}h_{it}I_{it} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{k=1}^{n}k_{ikt}y_{ikt} \qquad (5.1)$$

With constraints

$$y_{ijk} \leq b_{ikt}. \qquad i = 1,....,m; k \in S = 1,......,n\ t = 1,......,T \qquad (5.2)$$

$$\sum_{j=1}^{n}d_{jt}x_{ijt} + I_{it} = y_{it} + I_{i,t-1} \qquad i = 1,....,m; t = 1,......,T \qquad (5.3)$$

$$\sum_{j=1}^{n}d_{jt}x_{ijt} = q_{it} \qquad i = 1,....,m; t = 1,......,T \qquad (5.4)$$

$$\sum_{j=1}^{n}x_{ijt} = 1 \qquad i = 1,....,m; j \in S, t = 1,......,T \qquad (5.5)$$

$$x_{ikt} = x_{ik1} \qquad i = 1,....,m; t = 1,......,T \qquad (5.6)$$

$I_{i0} = 0 \qquad i = 1,....,m \qquad (5.7)$

$X_{ikt} \in \{0,1\}, y_{it}, I_{it} \geq 0 \qquad i = 1,....,m; k \in S = 1,....,n; t = 1,......,T \qquad (5.8)$

Where x_{ijt} is equal to 1 if customer j is assigned to facility i in period t, and 0 otherwise. is the production quantity at facility i in period t, and is the storage level at facility i at the end of period t. Objective function (5.1) to minimize the

total costs of production, placement and procurement of inventory. Production at facility i in period t is limited by (5.2), and constraint (5.3) is a balancing constraint equation that ensures that production demand matches production capacity q. Equations (5.5) and (5.6) guarantee that each customer is assigned exactly to one facility in each period. Equation (5.6) ensures that each customer remains assigned to the same facility through planning. Equation (6) determines that inventory at the beginning of planning is equal to zero for each facility. Since h_{it} is non-negative, inventory at the end of planning, without reducing optimality, is equal to zero.

3 Method

In previous research, the MPSSP problem has been studied as a problem of placing inventory and warehouse items so that the production and logistics distribution transportation costs produced can be optimized by looking at the production party sending the goods to the distributor, so that the distributor distributes the goods to customers. For problems like this, the MPSSP problem model is found, as follows.

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ijt}x_{ijt} + \sum_{t=1}^{T}\sum_{i=1}^{m}h_{it}I_{it}$$

With constraints:

$$\sum_{j=1}^{n}d_{jt}x_{ijt} + I_{it} \leq b_{it} + I_{i,t-1} \qquad i = 1,....,m; t = 1,......,T$$

$$\sum_{i=1}^{m}x_{ijt} = 1 \qquad i = 1,....,m; t = 1,......,T$$

$I_{i0} = I_{iT}$ $\qquad i \in \wp$
$I_{i0} = 0$ $\qquad i \notin \wp$
$x_{ijt} \in \{0,1\},$ $\qquad i = 1, ...,m; j = 1,....,n; t = 1,......,T$
$I_{it} \geq 0$ $\qquad i = 1,....,m; t = 1,......,T$

Furthermore, this research will develop a logistics network to optimize production and maintenance costs in one area to minimize total transportation and inventory costs to a minimum for distributing goods to customers, in which case the placement of inventory items and the warehouse will distribute production goods to customers in an orderly manner. online. By consulting this problem into a multi-period general assignment problem (GAP), it is seen as a non-polynomial problem or NP hard problem that requires a solution algorithm. Next, this problem is directed to CAP (Convex Assignment Problems) to find a more complete solution. Because the results of the CAP process will lead to more complete problems, a solution algorithm is needed, which in this research we use the Branch and Price algorithm so that the expected final results are obtained.

4 Result and Discussion

Equations 5.1 to 5.8 explain that every problem of precise assignment of customers to warehouses so that in a certain time period each customer can be linked to exactly one warehouse in each period to obtain minimized total transportation and inventory costs can be sought using the convex assignment method. In [13] it has been shown that the MPSSP inventory variable can be removed, by using convexity in the objective function, that is, with the same formulation as the existing convex function. In this case, this MPSSP reformulation produces a Single Source Problem (hereinafter SSP) with a convex objective function. First, the problem (MP) will be formulated with an equivalent model, which can be written as:

Minimizing:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{t=1}^{T}c_{ijt}\right)x_{ij} + \sum_{i=1}^{m}K_i\sum_{j=1}^{n}\left(d_jx_{ij}\right) \qquad (5.9)$$

With constraints

$$\sum_{j=1}^{n} d_j x_{ij} \leq \min_{t=1,\dots,T} \left(\frac{\sum_{\tau=1}^{t} b_{i\tau}}{\sum_{\tau=1}^{t} \sigma_\tau} \right) \qquad i = 1, \dots, m \qquad (5.10)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, \dots, n \qquad (5.11)$$

$$x_{ij} \in \{0, 1\} \qquad i = 1, \dots, m; \ j = 1, \dots, n \qquad (5.12)$$

Where $H_i(\alpha)$ is the convex function given by the optimum value of the following problem:
Minimizing:

$$\sum_{t=1}^{T} k_{it} I_t$$

With constraints:

$$I_t - I_{t-1} \leq b_{it} - \sigma_t \alpha \qquad t = 1, \dots, T$$
$$I_0 = 0$$
$$I_t \geq 0 \qquad t = 1, \dots, T.$$

Proof: Let F be the feasible region for (5.9). By decomposing (5.9) we obtain the following equation:

$$\min_{(x,I)\in F} \left(\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijt} x_{ijt} + \sum_{t=1}^{T}\sum_{i=1}^{m} k_{it} I_{it} \right)$$

$$= \min_{x:\exists'(x,I)\in F} \left(\sum_{i=1}^{m}\sum_{j=1}^{n} \left(\sum_{t=1}^{T} c_{ijt} \right) x_{ijt} + \min_{(x,I)\in F} \sum_{t=1}^{T}\sum_{i=1}^{m} k_{it} I_{it} \right)$$

$$= \min_{x:\exists'(x,I)\in F} \left(\sum_{i=1}^{m}\sum_{j=1}^{n} \left(\sum_{t=1}^{T} c_{ijt} \right) x_{ijt} + K(x) \right)$$

Where K(x) is equal to
Minimizing:

$$\sum_{i=1}^{m}\sum_{t=1}^{T} k_{it} I_{it}$$

With constraints

$$I_{it} - I_{i,t-1} \leq b_{it} - \sigma_t \cdot \sum_{j=1}^{n} d_j x_{ij} \qquad i = 1, \dots, m; \ t = 1, \dots, T$$
$$I_{i0} = 0 \qquad i = 1, \dots, m$$
$$I_{it} \geq 0 \qquad i = 1, \dots, m; \ t = 1, \dots, T$$

This problem is separated to i, so that for each $i = 1,\dots,m$ only depends on $\sum_{j=1}^{n} d_j x_{ij}$, so, $K(x) = \sum_{i=1}^{m} K_i \sum_{j=1}^{n} \left(d_j x_{ij} \right).$

Next, it will be shown that the feasible area of the decomposed problem is the same as the feasible area for (5.1). Suppose there exists x such that there is exactly one feasible solution (x,I) to (5.9). For each facility i, adding up the capacity constraints in each period, we get:

$$\sum_{t=1}^{T} \left(\sigma_t \cdot \sum_{j=1}^{n} d_j x_{ij} + I_u \right) \leq \sum_{t=1}^{T} \left(b_{it} + I_{i,t-1} \right)$$

$$\left(\sum_{t=1}^{T} \sigma_t \right) \cdot \sum_{j=1}^{n} d_j x_{ij} + \sum_{t=1}^{T} I_u \sum_{t=1}^{T} b_{it} + \sum_{t=1}^{T} I_{i,t-1}$$

$$\left(\sum_{t=1}^{T}\sigma_t\right).\sum_{j=1}^{n}d_j x_{ij} + I_{iT} \leq \sum_{t=1}^{T}b_{it} + I_{i0}$$

which is equivalent to

$$\left(\sum_{t=1}^{T}\sigma_t\right).\sum_{j=1}^{n}d_j x_{ij} + I_{iT} \leq \sum_{t=1}^{T}b_{it}$$

And resulted

$$\left(\sum_{t=1}^{T}\sigma_t\right).\sum_{j=1}^{n}d_j x_{ij} \leq \sum_{t=1}^{T}b_{it}$$

The previous inequality shows that x is feasible in (5.1). Assuming x is a feasible solution to (5.1), then there exists a vector $y \in R^{mT}$, such that:

$$y_{it} \leq b_{it} \qquad\qquad i = 1, ..., m; \; t = 1, ..., T$$

and

$$\sum_{t=1}^{T}y_{it} = \left(\sum_{t=1}^{T}\sigma_t\right).\sum_{j=1}^{n}d_j x_{ij} \qquad\qquad i = 1, ..., m;$$

(Note that y can be interpreted as a set of production levels corresponding to (x,I) in formulation (5.9)). Next, I_{it} is defined as:

$$I_{it} = \sum_{t=1}^{T}y_{i\tau} - \left(\sum_{t=1}^{T}\sigma_t\right)\cdot\sum_{j=1}^{n}d_j x_{ij} \qquad \text{for all } i = 1, ..., m \text{ and } t = 1, ..., T. \; n \qquad\qquad (5.13)$$

and I_{it} is not negative, $(x,I) \in F$.

This means that x is a feasible solution to the decomposed problem. By considering the function Ki(u) has a finite value and due to the duality of LP we obtain:

$$H_i(\alpha) = \min\left\{\sum_{t=1}^{T}k_{it}I_t : I_t - I_{t-1} \leq b_{it} - \sigma_t\alpha, I_0 \geq 0, \qquad t = 1,2,.....,T\right\}$$

$$= \max\left\{\sum(\sigma_t\alpha - b_{it})w_t : w \in W_i\right\}$$

Where:

$$W_i = \min\{w \in R^T : -w_t + w - t + 1 \geq h_{it}, t = 1, ..., T-1; w_t \geq 0, t = 1, ..., T\}.$$

Let $\mu \in [0, 1]$ and determine α, then:

$$\max\left\{\sum_{t=1}^{T}((\mu\alpha + (1-\mu)\alpha)\sigma_t - b_{it})w_t : w \in W_i\right\}$$

$$\max\left\{\mu\sum_{t=1}^{T}(\sigma_t\alpha - b_{it})w_t + (1-\mu)\sum_{t=1}^{T}(\sigma_t\alpha' - b_{it})w_t : w \in W_i\right\}$$

$$\leq \mu\max\left\{\sum_{t=1}^{T}(\sigma_t\alpha - b_{it})w_t : w \in W_i\right\} + (1-\mu)\max\left\{\sum_{t=1}^{T}(\sigma_t\alpha' - b_{it})w_t : w \in W_i\right\}$$

Which shows the convexity of $K_i(\alpha)$.

The function obtained is a general assignment problem function and is NP-Hard, so then we can solve the LP(MP) problem above with the following algorithm:

Step 0. Building a set of columns, denote $\subseteq \{(l, i) : l = 1, \ldots, L_i; i = 1, \ldots, m\}$, so LP(MP($N_0$)) has a feasible solution. Set $N = N_0$..

Step 1. Solving LP(MP(N)), resulting $y^*(N)$

Step 2. If $y^*(N)$, expanded to LP(MP) solution by setting the remaining variables to zero, optimal for LP(MP): STOP.

Step 3. Find a column (or set of columns) such that the new goal value is at least as good as the goal value $y^*(N)$ and add this column (or set of columns) to N. Go to Step 1.

The main thing in such integer optimization problems is the problem of minimizing damage when searching for continuous optimization solutions. So that the solution steps for the problem are obtained as follows:

Step 0. runs $L = \{1, \ldots, n\}$, $=$ _ and $x_{ij}^G = 0$ for each $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

Step 1. Set $F_j = \{i = 1, \ldots m : x^G \cup \{(\hat{\imath}, \hat{\jmath}) \in X_i\}$ for $j \in L$.

If $F_j = \varnothing$ for $j \in L$. the algorithm cannot assign object j; then set L = $L \backslash \{j\}$, $\alpha\theta2\omega\alpha$.

$$\rho_j = \min_{\substack{S \in F_t \\ S \neq i_t}} f(s, j) - f(i_j, j) \text{ untuk } j \in L.$$

Step 2. Provide:

$\hat{\jmath} \in \arg\max_{j \in L} \rho_j$, then set:

$$x_{i,\hat{\jmath}}^G = 1$$
$$L = L \backslash \left\{ \hat{\jmath} \right\}$$

Step 3. If $L = \varnothing$: STOP. If $= \varnothing$, x^G can be used as an assignment for (CAP). Otherwise, xG can be used as a partial assignment to (CAP). If not, go to Step 1.

5   Conclusion

From the results of the explanation above, it can be concluded that the multi-period single-sourcing problem is a problem of finding assignments, from time to time to obtain the minimum total transportation and inventory costs for the distribution of goods to customers which is a non-polynomial problem or NP hard problem so that the steps What must be done is to generalize the Branch and Price algorithm developed for Generalized Assignment Problems (GAP) to a much more complete class of problems, called CAP (Convex Assignment Problems). The viability of this approach largely depends on the possibility of solving the pricing problem efficiently. An important subclass of problems has been identified, containing many variants of multi-period single source problems (MPSSP), as well as several variants of GAP, which is the case. The results of applying the method to certain variants of MPSSP show that the Branch and Price algorithm is very useful for solving problems where the ratio between the number of customers and the number of warehouses is at most 10. For this problem the Branch and Price algorithm is more successful in finding the optimal solution, the computing time is superior (or comparable for large ratios between the number of customers and the number of warehouses) to the computational time obtained using the MIP solver of CPLEX, and shows greater stability, i.e., fewer outliers are observed.

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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