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Dynamic Analysis of Rumor Spreading Models in Social Networks with Time Delay

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Abstract: The spread of rumors is essential to social interaction, significantly affecting work and daily life. In terms of transmission, rumors are similar to diseases, so a mathematical model of rumors can be constructed using the epidemic model. This study aims to develop and analyze a mathematical model for spreading rumors in the form of S, I, and R compartments. The experimental method is used by adding a delay time where the acceptance rate is constant. The analysis obtained two equilibrium points: the rumor-free equilibrium point and the rumor-endemic equilibrium point. The rumor-free equilibrium point will be asymptotically stable when R0 < 1, so rumors will not spread in the population. Furthermore, the rumor endemic equilibrium point will be asymptotically stable if R0 > 1. Based on mathematical analysis and simulation, it is obtained that if the delay time is more significant, the equilibrium points E0 and E* remain stable. The addition of the time delay in the system does not affect the stability of the equilibrium point. Furthermore, parameter value A significantly affects the spread of rumors. If the value of A increases, the effect on users of S, I, and R will also increase, it can also be seen at the peak of the number of users of S, I, and R increasing. Furthermore, the peak number of S, I, and R users will decrease if it increases.

Keywords: Rumor Spread, Social Network, Delay Time, Local stability.

1 Introduction

Rumor is an unproven exposition of an interpretation of news, events, or issues of public interest. Since rumors are unconfirmed information, it is difficult to determine whether they are true or false. [1] Stated that rumors can spread on a large scale in a short time through the communication chain. The process of spreading rumors has changed in the last 150 years. Rumors that initially spread by word of mouth then spread through various more sophisticated media, such as telephone, short message (SMS), e-mail, blogs, radio, TV, and other social networks. Media is now the main source of information, so it is very influential on a person's behavior in responding to rumors.

During this pandemic, many rumors have been circulating about the harmful Covid-19. The rumor about covid-19 was that this coronavirus had disappeared in Indonesian society in early 2021. However, even that rumor was broken when it was announced that hospitals in several areas and athletes ' homestays were filled with people exposed to COVID-19 The spread of rumors can shape public opinion, which can cause panic in society and has a certain impact on the stability of society [2]. Rumors can also affect a person's rationality in making decisions. People who spread rumors may have different goals, such as slandering others, seeking attention, distracting, and so on. It does not take a long time or much money to be able to get or provide information through online social networks. This information is also in line with what was conveyed by Zhu *et al.* [3] that spreading rumors is an important part of social interaction, which greatly affects work and daily life. In order to reduce and avoid the dangers of spreading rumors on online social networks, it is necessary to understand the dynamic characteristics of the spread of rumors.

Several studies on controlling the spread of rumors are described qualitatively in many cases. Daley and Kendall [4] are the experts who introduced the rumor spread model, namely the DK model with a sub-population that is Divided into three groups Ignorant or (Susceptible), group Spreader (Infected), and Stifler group (Recovered), Third, the subpopulation is a subpopulation where several users do not know about rumors (Ignorant), then groups who are vulnerable to spreading rumors (Spreaders), and Groups who know about rumors but do not intend to spread rumors (Stifler) In the DK Model, if a subpopulation Ignorant meets and interacts with a subpopulation Spreader, then without

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realizing it, people in the subpopulation Ignorant will change to a subpopulation spreader, as well as if the spreader group meets and interacts with a sub-population Stifler, then people in the subpopulation Spreader will change to Stifler.

Then this SIR model was developed by several experts such as [5] developed a model for spreading rumors based on the type of distribution media [6] and developed a mathematical model of spreading rumors through social networks by studying the classical SIR model of Dalley and Kendal after that adding the hibernator population (H) obtained from the human group that participated in spreading rumors because of the forgetting mechanism and can spread rumors again because of the remembering mechanism The same thing has been done [7] where the article reviews the spread of rumors in which two groups spread rumors, namely the group that spreads the rumor first and the group that spreads the next rumor Then this article also discusses two mathematical models, namely the model without control and the model with control in the form of addition of a parameter with giving response.

Developing a model for spreading rumors is similar to developing a model for spreading disease (epidemic). The spreading of rumors is similar to viruses or diseases, as mentioned by [8] and [9]. The concept of an epidemic was first proposed [10], namely introducing an epidemic model SIR (Susceptible, Infected, and Recovered), which describes the spread of a disease in a population divided into susceptible groups, the infected group (infected), and the healthy group of individuals (recovered) In this model it is assumed that the recovered individual has immunity to the disease.

The development of articles that discuss the disease spread model makes the SIR epidemic model a reference for experts to construct deeper than an epidemic model [11]. For the classic epidemic model, the cure rate from the infected is directly proportional to the number of people infected [6]. Their study analyzed the epidemic model of SIR on disease transmission and vaccination for newborns and their treatment capacity, in addition to discussing the local stability of the equilibrium point, as well as numerical simulations to illustrate the results of the analysis.

Having conducted a study on infectious disease epidemic models with non-linear events and time delays along with their analysis [12]. Two functions are obtained: the first function of disease transmission and the second function regarding the level of infected individuals recovering from the infected compartment. By constructing the Lyapunov function and using the Lyapunov-LaSalle invariant principle, the asymptotic stability of the disease-free and endemic equilibrium is obtained. This construction shows that the dynamics of the global property system depend on the properties of this general function and the basic reproduction number R_0.

Among the studies on the spread of rumors described previously, the writer wants to develop the Dalley and Kendal model by using the delay time in the SIR compartment model because this DK model does not consider any delay time to receive then or even propagate rumors. So the writer wants to analyze a model of spreading rumors on social networks with a time delay $(t - \tau)$.

Rumors can be analogous to an "infection of the mind" whose spread resembles the transmission of a disease, so a mathematical model of spreading rumors can be built by adopting an epidemic model. Among the many rumor-spreading models, namely the SEI, SIR, and IMGT rumor-spreading models, the author chooses one model. The SIR epidemic model became the author's reference to model the spread of tumors in online social networks. In disease transmission, the study of the SIR epidemic model with a time delay and its analysis is one of the authors' references in this study. Therefore, the authors find out how the model for spreading rumors with a time delay, along with their analysis of the formulation of the problem formulation is as follows:

1. How is the construction model mathematics spreading rumors?
2. How is the analysis of the point equilibrium local stability of the equilibrium point of the mathematical model of the spread of the rumor?
3. How do results simulation solution mathematical model rumors spread?

2 Methodologies

The research method used is a literature study, namely studying literature books by collecting information from reference books and journals about similar research that has been done before.

The topics in this research include:

1. Studying books, journals and articles related to rumors, dissemination, dynamic models, time delays, equilibrium points.
2. Determining the constraints or assumptions with a time delay in order to obtain a model.
3. Drawing model transfer diagrams to form models' mathematics Diagram. transfer aims to find the system

differential.

4. Solving systems of differential equations.
5. Determining the local stability equilibrium point equations.
6. Analyzing the stability equilibrium point. Then, stability analysis of point free equilibrium rumor and endemic rumor to analyze the properties of point stability, equilibrium is carried out Linearization of the system by determining the matrix Jacobian at the point equilibrium is carried out Then by using definition of polynomial characteristics, we get the eigenvalues of the matrix and determine its stability.
7. Performing Numerical Simulation of the rumor spread model with time delay.
8. The final section establishes the conclusions of the research conducted.

3 Results

3.1 Rumor Spreading Model with Delayed Time

The model that will be discussed and developed in this research is the rumor spreading model introduced [4] which is commonly known as the DK model In this model the closed population and homogeneous mixture are divided into three groups, those who are not aware of the rumor, those who have heard about it and are actively spreading it, and those who have heard the rumor but no longer spread it These groups are called fools, spreaders and stiflers, respectively It is assumed that rumors spread through the population through direct contact between the spreader and the fools Every time a spreader interacts with a fool, the fool becomes a spreader When a spreader contacts a stifler, the spreader turns into a stifler and when a spreader meets another spreader, the spreader that started it becomes a stifler Another basic assumption is that someone who knows the rumor will continue to tell it until he or she decides that the rumor is no longer worthy of being called "news" In the [13] rumors spread through direct contact of the disseminator with other individuals Therefore, when a spreader contacts another spreader, only the initiating spreader becomes the stifler [14] have worked on the process of disseminating various ideas [15] studied the process of spreading rumors with denial and skepticism, two models were established to accommodate skepticism Many models in the literature represent the dynamics of spreading rumors by a system of ordinary differential without any time delay To reflect the real dynamic behavior of the model that depends on the system's past history, it makes sense to incorporate a time delay into this system In fact, the inclusion of a delay in the rumor-spreading model makes it more realistic by allowing a description of the effect of the situation when a fool contacts the spreader, the fool may experience a latent period before becoming a propagator.

In this study, the authors propose a distribution model in the presence of a time delay (t-τ), the population is classified into three groups: people who are not aware of rumors (S), spreaders (I), and stiflers (R). The spread of rumors can be seen in the following diagram with the assumption:

1. The acceptance rate of vulnerable people exposed to rumors is constant and occurs at a constant A, and the displacement rate is.
2. Whenever a spreading user infects a susceptible user, there is an incubation period of spread during which the infectious agent develops in the network; the fool becomes a spreader at a level, i.e., the spread rate; however, transmission takes some time for the individual to move from hearing rumors to spreading status That susceptible individuals first go through a latency period.
3. When a spreader is in contact with another or a stifler, only the initial spreader becomes a stifler at the rate, i.e., the stifling rate.

Therefore, determining the delay for the incubation period of spread is more appropriate usually, when a rumor spreads, the government will take effective measures to control and remove the users who spread it.

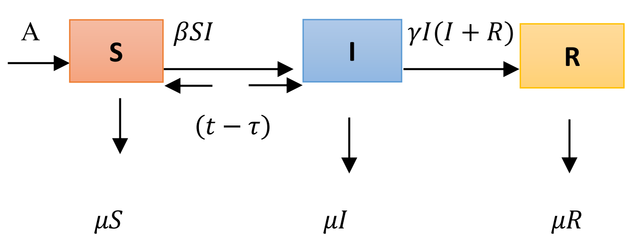

Fig. 1: SIR Model Diagram with Time Delay

The assumptions above can be written in the form of the following differential system 1:

$$\frac{dS}{dt} = A - \mu S - \beta S(t)I(t)$$
$$\frac{dI}{dt} = \beta S(t-\tau)I(t-\tau) - \gamma I(t)\big(I(t) + R(t)\big) - \mu I(t)$$
$$\frac{dR}{dt} = \gamma I(t)\big(I(t) + R(t)\big) - \mu R(t) \quad (4.2))$$

(1)

Where S is Susceptible, I is infected, and R is stifler (Recovered). A is the rate of increase in susceptibility to rumors, β probability transition from population vulnerable to dangerous population, μ rate of natural birth/death, and γ Probability transition from infected to latent, i.e., control rate and τ is a constant non-negative which represents the spread of the incubation period.

3.2 Stability analysis

In determining the answer or solution, an equation differential that does not change with time is used in Equilibrium point analysis, the equilibrium the differential (1) above will be obtained by determining $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, $\frac{dR}{dt} = 0$ So we get:

$$\frac{dS}{dt} = A - \mu S - \beta S(t)I(t) = 0$$
$$\frac{dI}{dt} = \beta S(t-\tau)I(t-\tau) - \gamma I(t)\big(I(t) + R(t)\big) - \mu I(t) = 0$$
$$\frac{dR}{dt} = \gamma I(t)\big(I(t) + R(t)\big) - \mu R(t) = 0$$

(2)

When there is no spread of rumors in a population, the point-free equilibrium rumor occurs. The equilibrium point is obtained when there are no rumored individuals in the population or I = 0 Substitute I = 0 into the system (1). By solving simultaneously, the equilibrium point is obtained as $E_0 = \big(S_{0,} I_0, R_0\big) = \left(\frac{A}{\mu}, 0{,}0\right)$ Stability Analysis

System (1) always has one rumor-free equilibrium point $E_0 = (\frac{A}{\mu}, 0, 0)$ The basic reproduction number of the system of system (1) is

$$R_0 = \frac{A(\beta+\gamma)}{\gamma A + \mu^2}.$$

If $R_0>1$, then the system of system (1) has a valid balance based on rumors $E^* (S^*, I^*, R^*)$,

$$\text{Where} \begin{cases} S^* = \frac{A}{\mu R_0}, \\ I^* = \frac{\mu(R_0 - 1)}{\beta}, \\ R^* = \frac{\gamma A (R_0 - 1)^2}{\mu \beta R_0}. \end{cases}$$

We will now show the local stability of the rumor equilibrium point E_0

Let $X = S - \frac{A}{\mu}$, $Y = I$ and $Z = R$ The form of a linear system from system (1) around E_0 is as follows:

$$\begin{cases} \frac{dX}{dt} = -\mu X(t) - \frac{\beta A}{\mu} Y(t), \\ \frac{dY}{dt} = \frac{\beta A}{\mu} Y(t-\tau) - \mu Y(t), \\ \frac{dZ}{dt} = -\mu Z(t). \end{cases} \quad (3)$$

The characteristic equation for the system of system (1) is:

$$\Delta(\lambda, \tau) = (\lambda + \mu)\left(\lambda + \mu - \frac{\beta A}{\mu} e - \lambda\tau\right) = 0 \quad (4)$$

Theorem 1: If $R_0 < 1$, *then the rumor-free equilibrium E_0 is locally asymptotically stable And if $R_0 > 1$, then the equilibrium at E_0 is unstable*

Proof: true that, if $r = 0$, then Equation (4) becomes

$$\Delta(\lambda, 0) = (\lambda + r)\left[\lambda + \left(\frac{\gamma A + \mu^2}{\mu}\right)(1 - R^0)\right] = 0. \quad (5)$$

It is easy to see that equation 1 has two roots

$\lambda_1 = - r < 0$ dan $\lambda_2 = \left(\frac{\gamma A + \mu^2}{\mu}\right)(R_0 - 1)$.

So if $R_0 < 1$, the equilibrium E_0 is asymptotically stable and if $R_0 > 1$ equilibrium is unstable.

By Rouché's Theorem, if instability occurs for a certain delay value, the characteristic root of Equation (4) must intersect with the imaginary axis Suppose that system (4) has a pure imaginary root $i\omega$, where $\omega > 0$. Then, by separating the real and imaginary parts of system (3), we get:

Therefore,

$$\begin{cases} \frac{\beta A}{\mu}\cos(\omega\tau) = \mu, \\ \frac{\beta A}{\mu}\sin(\omega\tau) = -\omega. \end{cases} \quad (6)$$

Because of that,

$$\omega^2 = \left(\frac{\beta A + \mu}{\mu}\right)\left(\frac{\gamma A}{\mu}\right)(R - 1). \quad (7)$$

So, if $R_0 < 1$ Equation (4) has no pure imaginary roots, and because E_0 is asymptotically stable for $\tau = 0$ and remains asymptotically stable for all *If* $\tau \geq 0$

If $R_0 < 1$, the rumor-free equilibrium E_0 is unstable for $\tau = 0$ By Kuang's Theorem, then E_0 is not stable for all $\tau \geq 0$ Proven

Suppose, establish the local stability of the prevailing rumor balance, E^* Let $X = S - S^*$, $Y = I - I^*$ and $Z = R - R^*$ Form the linear system of system (2) around $E\ (S^*, I^*, R^*)$ *is* as follows:

$$\begin{cases} \frac{dX}{dt} = -(\mu + \beta S^*)X(t) - \beta I^*\ Y(t), \\ \frac{dY}{dt} = \beta I^* X(t - \tau) + \beta I^* Y(t - \tau) - [\gamma(2S^* + R) + \mu]Y(t) \\ \qquad\qquad - \gamma S^*\ Z(t), \\ \frac{dZ}{dt} = \gamma(2S + R)Y(t) + (\gamma S - \mu)Z(t). \end{cases} \quad (8)$$

And the characteristic equation is given by:

$$\Delta(\lambda, \tau) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 + (b_1\lambda^2 + b_2\lambda + b_3)(-\lambda\tau) \quad (9)$$

Where

$a_1 = 2\mu + \beta(S + I)$, $\quad b_1 = -\beta I$,

$a_2 = 2\mu\beta I + \mu + \beta SI + \mu\beta S$, $\quad b_2 = \beta I\ (-2\mu + \gamma S)$,

$a_3 = \mu\beta I\ (\mu + \beta S)$, $\quad b_3 = \mu\beta I\ (\gamma S - \mu)$.

The author starts by considering the case without delay $\tau = 0$. Then found the proposition applies:

Proposition 1 *if $R_0 > 1$then the equilibrium point E is stable for $\tau = 0$, Proof is true if* $\tau = 0$, then Equation (9) becomes:

$$\Delta(\lambda, 0) = \lambda^3 - (+b_1)^2 + (a_2 + b_2) + (a_3 + b_3) = 0 \quad (10)$$

With

$a1 + b1 = 2\mu + \beta S > 0$,

a2 + b2 $= \beta S\left((\gamma+\beta)I+\mu\right)+\mu>0,$

a3 + b3 $= \mu\beta I\,(\gamma+\beta)S>0$

(a1 + b1) (a2 +b2) – (a3 + b3) $= \mu\beta S\left((\gamma+\beta)I+2\mu+2\mu\right)+\beta S\,(\beta S\left((\gamma+\beta)I+\mu\right)$
$+\mu\gamma S(\mu-\gamma S)$

Because $(\mu-\gamma S^{*}) = (\mu/\beta A)$, $R_0>1$ then $(a_1+b_1)(a_2+b_2)-(a_3+b_3)>0$. So according to Hurwitz's criterion, all roots of Equation (10) have negative real numbers Therefore Asymptotically stable The following is a positive delay problem $\tau>0$ And the results can be seen in the following table:

Table 1: Model Dynamics for R_0 and _

Threshold value	Latent period	Equilibrium	
		E_0	E^*
$R_0<1$	For all τ	Stable	None
$R_0>1$	For all τ	Unstable	Stable

Theorem 2 If $R_0>1$, *then the equilibrium point* E^* *is asymptotically stable for all* $\tau\geq 0$ *The proof* is true that, in the following, if instability occurs for a given value of time delay, the characteristic root of equation (9) must intersect with the imaginary axis If equation (9) has a pure imaginary root $i\omega$, where $i\omega>0$, then, separating the real and imaginary parts of Equation (7) we get:

$$a_1\omega^2-a_3=(b_3-b_1\omega^2)\cos(\omega\tau)+b_2\omega\sin(\omega\tau) \quad (11)$$

$$\omega^3-a^2\omega=b^2\omega\cos(\omega\tau)-(b^3-b^1\omega)\sin(\omega\tau) \quad (12)$$

Squaring and adding equation (11) and (12), we get

$$(b_3-b_1\omega^2)^2+b_2{}^2\omega^2=(a^1\omega^2-a^3)^2+(\omega^3-a^2\omega)^2$$

So the final result

$$\rho^3+A^1\rho^2+A^2\rho+A_3=0 \quad (13)$$

Where

$$\rho=\omega^2,\qquad A_1=a_1{}^2-2a_2-b_1{}^2=\mu^2+\left(\mu+\beta S^{*}\right)^2>0,$$

$$A_2=a_2{}^2-b_2{}^2-2a_1a_3+2b_1b=\left(\mu^2+\mu\beta S^{*}\right)^2+\beta^2(\beta+\gamma)\left(2\mu+(\beta+\gamma)S^{*}\right)S^{*}I^{*},$$

$$A^3=a_3{}^2-b_3{}^2=(a^3+b^3)(a^3-b^3)=\mu^2\beta^2I^{*2}S^{*}(\gamma+\beta)(2\mu+(\beta-\gamma)S^{*}.$$

It should be noted that:

$$2\mu+(\beta-\gamma)S^{*}=\mu R_0\left(\frac{\beta A+\mu^2}{\beta A}\right)>0$$

indicates $A_2>0$ and $A_3>0$ Therefore, according to Descartes' sign rule, equation (13) has no real positive roots Thus, the characteristic equation (9) does not admit a pure imaginary root and since E^* is asymptotically stable for $\tau=0$, then E^* is asymptotically stable for all $\tau\geq 0$

Furthermore, from the system of system (1), the equilibrium point will be determined, and the analysis will be carried out.

3.3 Numerical Simulation

This section uses several numerical simulations to describe the theoretical results, and the authors discuss parameter A's impact and bring R0 below one. First, to understand the stability of the RFE, the author uses the parameter values seen in Table 1. It can be seen that the value of R0 = 0,8889 < 1, then RFE E0 = (0,6, 0,0 0,0) is asymptotically stable locally for all. But if we choose the parameter values defined in Table 2. In this case we see that the value of R0 = 1,7081 > 1, then the system (1) has a valid equilibrium E* = (0,46, 0,125, 0,201) and is locally asymptotically stable for all

Table 2: Parameter Values for $R_o<1$

Parameter	A	μ	β	γ	τ
Value	0,18	0,3	0,4	0,4	0/5/10

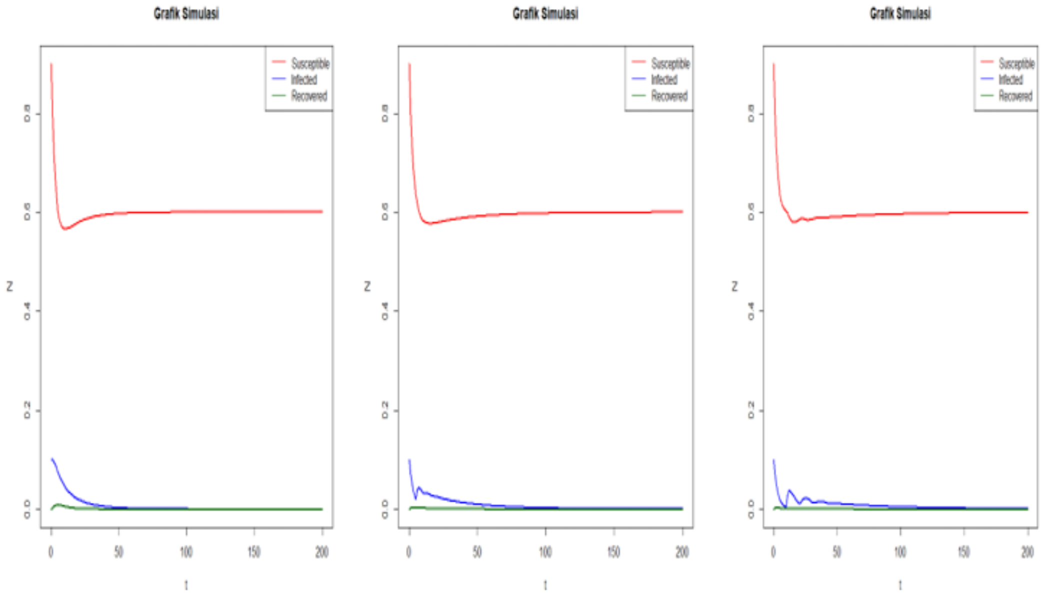

Fig. 2: The dynamics of the SIR model with τ = 05 and 10

If we separate each function S, I and R, we will obtain a dynamics model of the SIR model as shown below

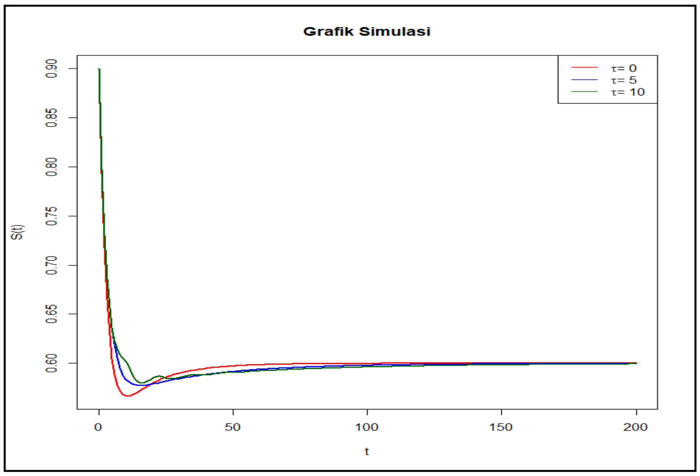

Fig. 2a: Dynamics of Graph S

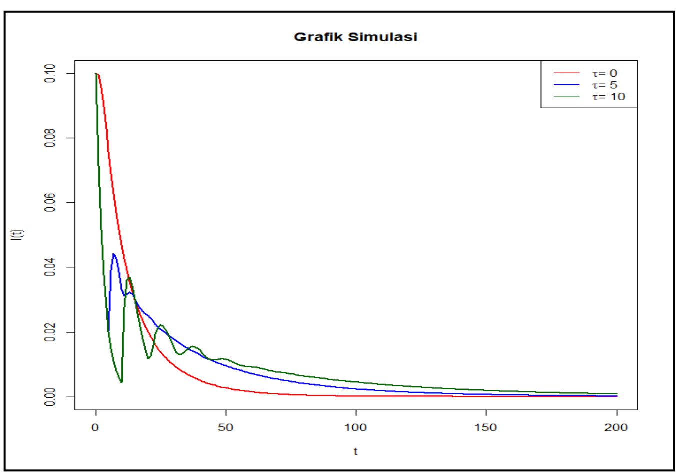

Fig. 2b: Dynamics of Graphs I

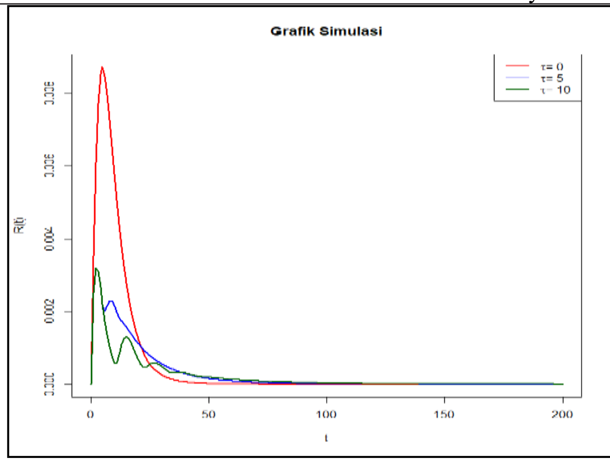

Fig. 2c: Dynamics of Graphs R

In Figure 2 above, given the parameter A = 0,18 β = 0,4., μ = 0,3,, γ= 0,4 with initial value S(0) = 0,9, I(0) = 0,1, R(0) = 0,0 with R_0 = 0,8889 look that curve *S, I* and *R* will go to the fixed point ie (0,6, 0,0, 0,0). The picture shows the balance of rumors This means that if $R_0 < 1$ the rumor-free equilibrium point E_0 is locally asymptotically stable Thus the numerical simulation results obtained support the stability analysis results that if the value of $R_0 < 1$ the equilibrium point E_0 is locally asymptotically stable, which meant that the rumors would stop spreading.

Table 3: Parameter value table for R_o>1

Parameter	A	μ	β	γ	τ
Value	0,055	0,07	0,4	0,35	0/5/10

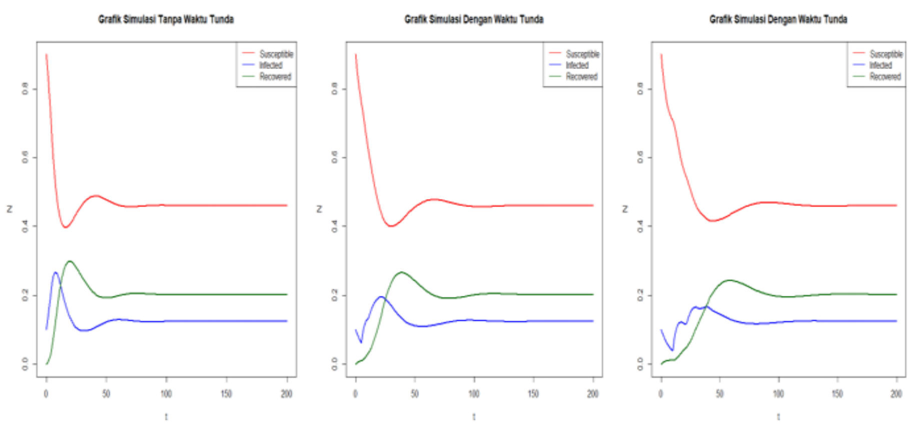

Fig. 3: The dynamics of the SIR model with τ = 05 and 10

If we separate each function S, I and R, we will obtain a dynamics model of the SIR model as shown below:

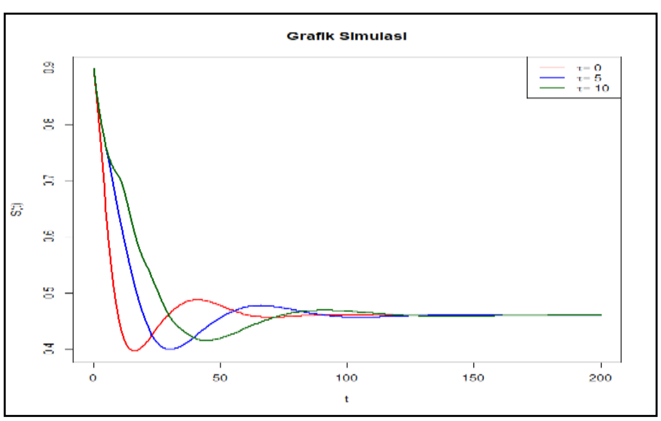

Fig. 3a: Dynamics of Graph S

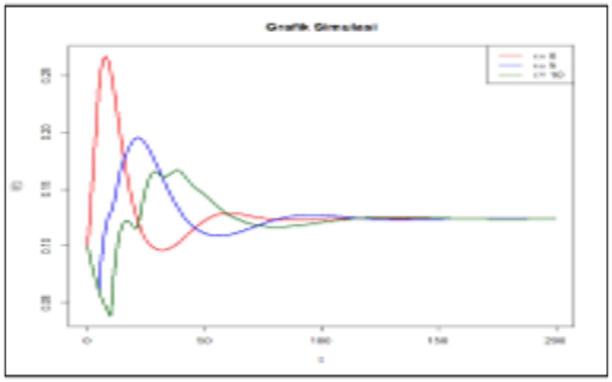

Fig. 3b: Dynamics of Graphs I

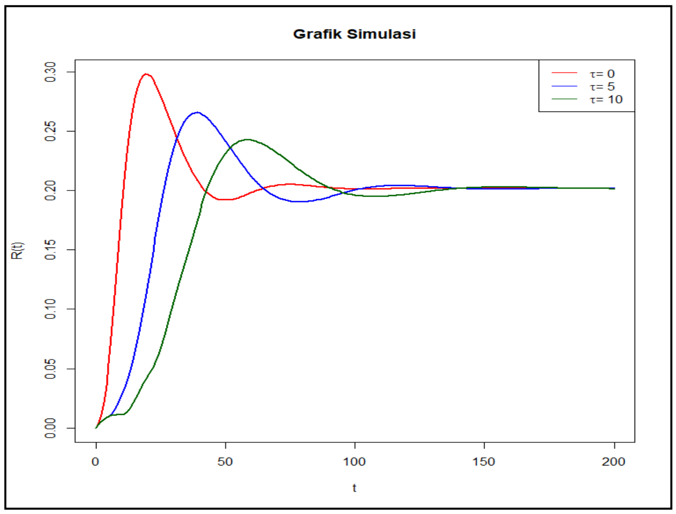

Fig. 3c: Dynamics of Graphs R

Based on Figure 3 above, given the parameter A = 0.055 μ = 0.07, β = 0.4, γ =0.35 with initial value S(0) = 0,9, I(0) = 0,1, R(0) = 0,0 with R_0 = 1,70809 curves *S, I* and *R* will go to their fixed points, namely (0,46, 0,125, 0,201) the picture shows towards the rumor equilibrium point E^* And no graph moves toward the rumor-free equilibrium point E_0 This means that E^* is asymptotically stable Which means that rumors will persist and spread within the population.

In the following we will see how one of several factors will affect the rumors. Here the author chooses factor A and μ so it will be shown how the graph is for each Variable S, I and R

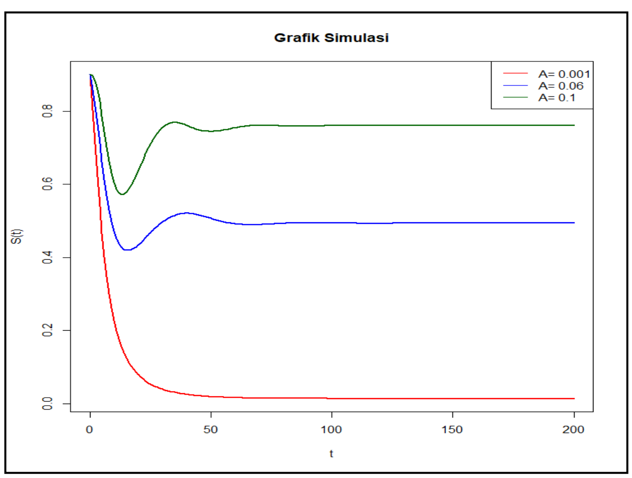

Fig. 4a: Dynamics of Graph S for *A* different

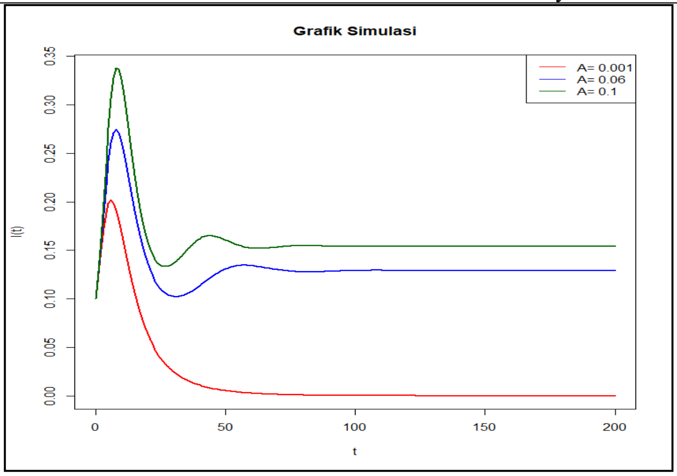

Fig. 4b: Dynamics of graphs I for A different

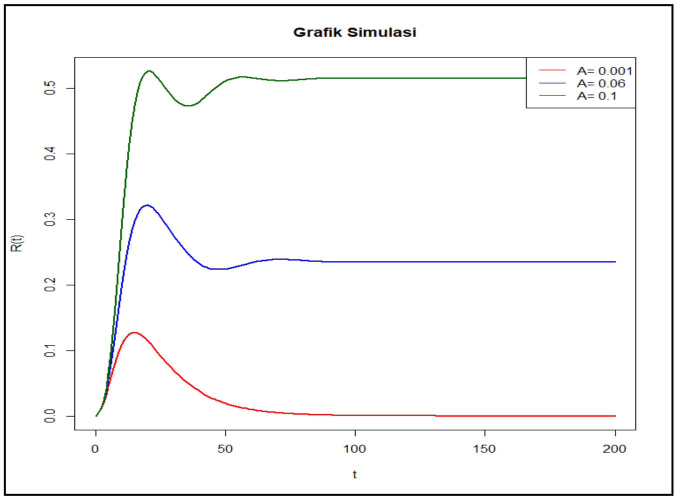

Fig. 4c: Dynamics of graphs of R for A different

Based on Figure 4 above with parameter $\mu = 0{,}07$, $\beta = 0{,}4$, $\gamma = 0{,}35$ with an initial value of S (0) = 0,9, I(0) = 0,1, R(0) = 0,0 with A , it can be seen that the spread of rumors will increase if the A is higher.

Then it will also show the graph of the spread of rumors for μ different values

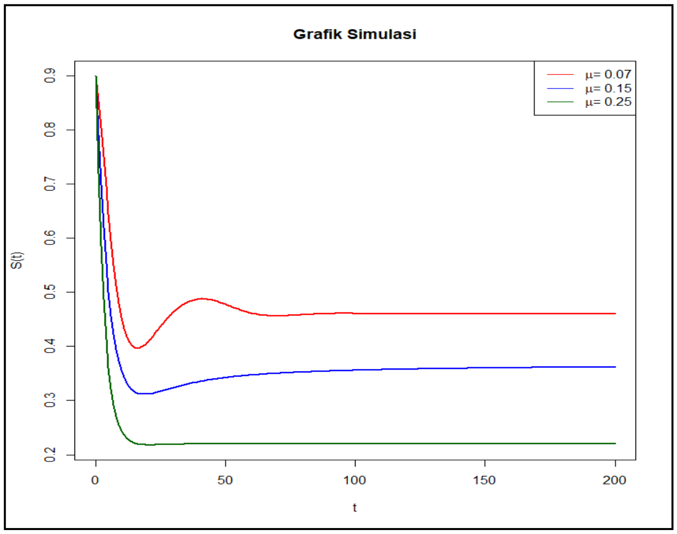

Fig. 5a: Dynamics of Graph S for μ different

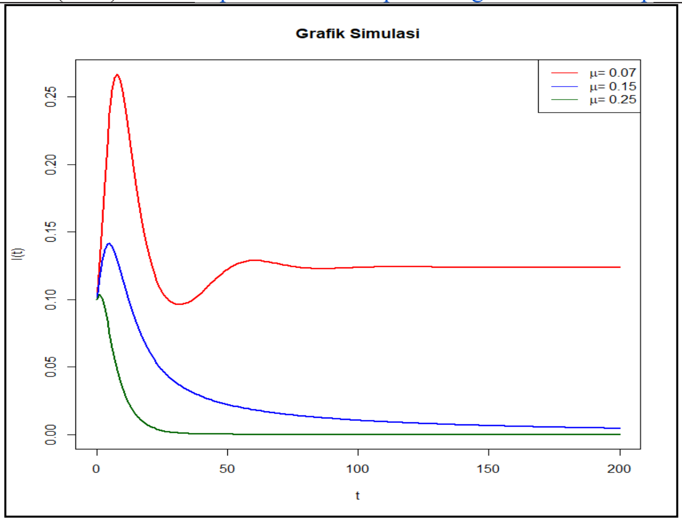

Fig. 5b: Dynamics of Graph I for μ different

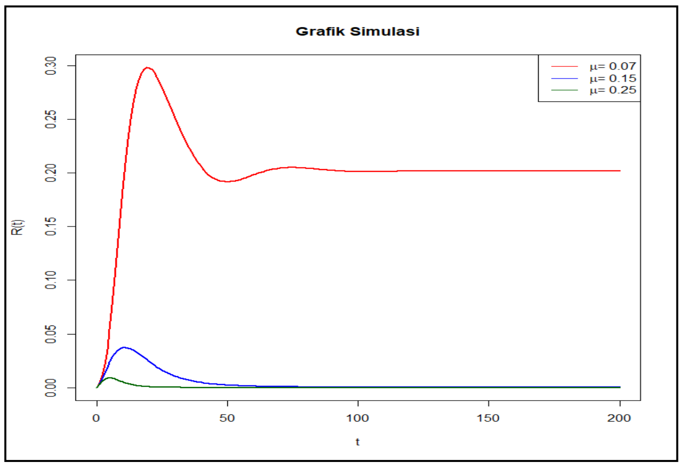

Fig. 5c: Dynamics of R graphs for μ different

Based on Figure 5 above with parameters A = 0,055, β = 0.4, γ = 0.35 with an initial value of S (0) = 0.9, I(0) = 0.1, R(0) = 0.0 with a different value of , it can be seen that the spread of rumors will increase if the value μ decreases.

From the results of mathematical analysis and simulation, it is found that if the delay time is greater, the equilibrium points E0 and Remain stable. The addition of the time delay in the system does not affect the stability of the equilibrium point. If the value is getting bigger (the delay time is getting longer) with the same initial value, then the time needed to reach the equilibrium point will be longer. From the analysis results, parameter A's value greatly affects the spread of rumors, where A is the rate of increase in susceptible people and the rate of natural births/deaths. If the value of A increases, the effect on users of S, I, and R will also increase. It can also be seen at the peak of the number of users of S, I, and R increasing parameter value inverse. As the death or birth rate increases, the peak number of S, I and R users will decrease.

4 Conclusions

Based on the discussion above, the researchers concluded that:

1. The time-delay model of rumor spreading in social networks is a nonlinear autonomous system consisting of three classes: individuals who do not know the rumor (S), individuals who actively spread the rumor (I), and individuals who know the rumor but do not spread it stifler (R). A time-delay model for the dynamics of rumor transmission with constant recruitment and incubation period is developed, a rumor-free equilibrium point is found.

2. The researchers conducted a model stability analysis and analyzed the influence of various parameters on the rumor transmission process. The model for spreading rumors on social networks with a time delay has 2 equilibrium points, namely the rumor-free equilibrium points and the endemic equilibrium point. The basic reproduction

number determines the existence of the equilibrium point Ro. If R0 < 1 then point. equilibrium that exists only point rumor free equilibrium or when the basic reproduction number is less than one then the rumor free equilibrium will be asymptotically stable, and rumors can be eradicated for all $\tau \geq 0$, and it can be interpreted that rumors will not spread in the population or eventually rumors will disappear from the population. If R0 > 1 then there are two equilibrium points that exist, namely the rumor-free equilibrium points and the endemic equilibrium point. Or it can be said that if the basic reproduction number is more than one then the rumor-free equilibrium will be asymptotically stable for all $\tau \geq 0$, which means that rumors will persist and spread in the population. The rumor-free equilibrium point is locally asymptotically stable if R0<1. If the endemic equilibrium point satisfies the Routh-Hurwitz criterion, then it is locally asymptotically stable.

3. Based on mathematical analysis and simulation, it is found that if a larger delay time τ is given, the equilibrium points E0 and E^* remain stable. Adding a delay time to the system does not affect the stability of the equilibrium point. And the values of parameters A and μ greatly influence the spread of rumors. If the value of A increases, the influence on users of S, I and R will increase. This can also be seen at the peak of the number of users of S, I and R increasing. And as μ increases, the peak number of S, I and R users will decrease.

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Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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