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# A Coinfection Malaria and COVID-19 Model: The Conformable–Order Derivative Analysis

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**Abstract:** Confection infection diseases in recent times are frequent across the globe, particularly the sub-Saharan Africa. The coronavirus epidemic is not a new phenomenon and many individuals possess this disease with other diseases. In this regard, the study examines the existence and uniqueness of the malaria and COVID-19 model in the context of Fixed –point theorem and Picard iteration. The concept of Conformable-order derivative is employed to explore the dynamics of the coinfection model. Numerical simulation is carried out to support the theoretical analysis which yields a positive result.

**Keywords:** Conformable-order derivative, malaria, COVID-19, coinfection, fixed point theory.

## 1 Introduction

Environmental condition is increasingly becoming one of the critical factors contributing to the spread of many diseases. Malaria is a vector borne disease which thrives in a conducive climatic condition[1,2]. This disease accounts for the highest death in humans across the sub-Saharan Africa [3,4]. The infant mortality rate of malaria is high. Several activities through governmental and non- governmental agencies have been put in place to curtail the transmission of this disease. However, the incidence of malaria is still increasing every day, especially where the temperature keeps rising [4]. The economic burden of malaria is heavy and sometimes difficult to quantify in monetary terms. Malaria, as a singular condition or with other diseases in a geographic region causes severe harm to humanity [2].

Coronavirus, which is one of the deadliest diseases in the 21st century, came into being in 2019 from China particularly in Wuhan Province and was linked with environmental conditions of the region [5]. The spread of the disease is either through direct contact with an infected individual or indirect contact mode. The economic burden of the disease cannot be estimated across the globe and continue to pose major hardships on both developing and developed countries in the world[6]. The mortality rate of the disease has been extremely high such that the world finds it difficult to apprehend and respond to it appropriately. Various strategies including vaccination have been put in place, however, the spread of the disease is still on high ascendency [7]. Comorbidity and co-infectious with Coronavirus (COVID-19) enhance mortality rate. Malaria and Coronavirus are known in some geographical areas including sub-Sahara Africa to have similar environmental condition that they strive [8,9]. It is interesting to note the effect the coinfection poses on the lives of developing countries where medical care facilities do not exist or poorly exist [8].

Mathematical modelling is increasingly turning to be one of the most powerful tools for providing qualitative information on many phenomena [10]. The non-integer calculus is known to be a powerful tool in predicting accurately, a lot of scientific phenomena . The non-integer order is known to possess hereditary and memory which enhances effective predictions [1]. Some of the well-known non local and non-singular operations are Caputo-Fabrizio (CF) and Atangana-Baleanu operator in Caputo sense (ABC)[11].

Other non-local arbitrary operations which are being used in modelling include the following: Reimann-Lioville and Caputo, Weyl derivative, Hadamard derivative. There are other local derivatives which are different from the well-known

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classical calculus. Notably is the popular local derivative Conformable Dammed introduced by Khalil et al., [12]. It possesses properties closely related to the classical calculus including: linearity, constant derivative in zero, integration by part, product rule of derivative of function and inverse of function and many more.

In recent times, a lot of researchers have devoted substantial attention on utilising local derivative in studying real life issues including engineering, science, epidemiology and many others. Allahamou et al.,[13] examined Hantavirus of European mole in the light of Conformable-order derivative and obtained results which is similar to non-local arbitrary order derivatives. Ebaid et al., [14] investigated the phenomena of falling body with Conformable-order derivative and obtained sound findings as the case of classical order.

Srivastava and Günerhan [15] studied SIR model hinged on the concept of Conformable-order derivative and came out with results which is comparable to the non-local order derivate. Tong et al., [16] delved into Conformable SEIR model to provide quantitative information about the disease. Xie et al., [17] used Conformable-order derivative to study Grey model. Yavuz and Yokus [18] utilised Conformable-order derivative to obtain analytical and numerical solution to a nerve impulse mathematical model.

Kaya et al., [19] explored the dynamics of bacteria population using Conformable-order derivative. Bonyah [10] employed Conformable-order derivative in obtaining qualitative information on Coronavirus model. Atangana and Alkahtani investigated Rubella discourse with Conformable order derivative and obtain a good qualitative result.

This paper seeks to utilise the conformable order derivative to obtain qualitative information about coinfection model of malaria and corona virus.

## 2 Properties of Conformable Calculus

In this manuscript, the tools necessary for completion of this researched work is represented in this segment.

**Definition 1** Let $\Theta : [0,\infty) \longrightarrow \mathbb{R}$ a function with real values, the the Conformable derivative of $\Theta$ at $\nu$ of order $v \in (0,1]$, is given by [20,21]

$$T^{v}(\Theta)(\nu) = \lim_{r\longrightarrow 0} \frac{\Theta\left(\nu + r\nu^{1-v}\right) - \Theta(\nu)}{r}.$$

If $\Theta$ is $v$- differentiable in some $(0,\varphi), \varphi > 0$, and $\lim_{\nu\longrightarrow 0^{+}} \Gamma^{v}(\Theta)(\nu)$ exist, then let

$$\Gamma^{v}(\Theta)(0) = \lim_{\nu\longrightarrow 0} \Gamma^{v}(\Theta)(\nu)$$

The Conformable integral is described as the following

**Definition 2** Given $\varphi \geq 0$ and $\nu \geq \varphi$. Let $\Theta$ be a function with real values defined on $(\varphi,\nu]$ and $v > 0$. Then, the $v$ -Conformable integral of $\Theta$ is defined by [20,21],

$$I_{\varphi}^{v}(\Theta)(\nu) := \int_{\varphi}^{\nu} \Theta(x) d_{v}x = \int_{\varphi}^{\nu} x^{v-1}\Theta(x)dx,$$

if the Riemann improper integral occurs [21]. When $\varphi = 0$, we can write $I_{0}^{v}(\Theta)(\nu) = I^{v}(\Theta)(\nu)$.

Furthermore, $\Theta : [\varphi,\beta] \subset \mathbb{R}$ is $v$-integrable on $[\varphi,\beta]$ if and only if $\nu^{v-1}\Theta$ is integrable on $[\varphi,\beta]$.

**Lemma 1** Supposing $\Theta : [\varphi,\infty) \longrightarrow \mathbb{R}$ is continuous function and $0 < v \leq$ . Then, $\forall\ \nu > \varphi$ we obtain [20]

$$\Gamma^{v} I_{\varphi}^{v} \Theta(\nu) = \Theta(\nu).$$

**Lemma 2** Supposing $\Theta : [\varphi,\infty) \longrightarrow \mathbb{R}$ is differentiable and $0 < v \leq$ . Then, for $\nu > \varphi$ we obtain [20]

$$I_{\varphi}^{v} T^{v} \Theta(\nu) = \Theta(\nu) - \Theta(\varphi).$$

## 3 Model Formulation

In this work, the Tchoumi et al., [20] coinfection model of malaria and COVID -19 is modified to include recovery compartment for malaria and COVID -19 respectively. The total human population at time $t$ is denoted by $N(t)$ and the model is partitioned into the following: susceptible humans $S_h(t)$ individuals who have been infected with malaria $I_m(t)$

only, individuals who have malaria and exposed to COVID-19 $I_{mE_c}(t)$ individuals only exposed to COVID-19 $E_c(t)$ individuals infected with COVID-19 only, individuals coinfected with malaria and COVID-19 $I_{mc}(t)$, individuals recovered from only malaria $R_m(t)$ only $I_{mc}(t)$, individuals recovered from only COVID-19 $R_c(t)$. The total vector population at time t is denoted by $N(t)$ and subdivided into the following: susceptible mosquitoes $S_v(t)$ and infected mosquitoes $I_v(t)$.

$$\begin{aligned}
\frac{dS_h}{dt} &= \Pi_h + \omega_m R_m + \omega_c R_c - \beta_c (I_c + I_{mc}) S_h - \beta_m I_v S_h - \mu_h S_h,\\
\frac{dI_m}{dt} &= \beta_m I_v S_h - \beta_c (I_c + I_{mc}) I_m - (\phi_m + \mu_h) I_m,\\
\frac{dE_c}{dt} &= \beta_c (I_c + I_{mc}) S_h - \beta_m I_v E_c - (\phi_c + \mu_h) E_c,\\
\frac{dI_{mE}}{dt} &= \delta \beta_c (I_c + I_{mc}) I_m - (\alpha_1 + \sigma + \mu_h) I_{mE_c},\\
\frac{dI_c}{dt} &= \phi_c E_c - (\gamma_c + \mu_h) I_c,\\
\frac{dI_{mc}}{dt} &= \beta_m I_v E_m + \beta_c (I_c + I_{mc}) I_c + \sigma I_{mE_c} - (\mu_h + q) I_{mc},\\
\frac{dR_m}{dt} &= \phi_m I_m + \alpha_1 I_{mE_c} - (\omega_m + \mu_h) R_m,\\
\frac{dR_c}{dt} &= \gamma_c I_c - (\omega_c + \mu_h) R_c,\\
\frac{dS_v}{dt} &= \Pi_v - \beta_c (I_m + I_{mc} + I_{mE_c}) S_v - \mu_v S_v,\\
\frac{dI_v}{dt} &= \beta_c (I_m + I_{mc} + I_{mE_c}) S_v - \mu_v I_v.
\end{aligned} \tag{1}$$

with the following initial conditions

$$S_h(0) > 0, I_m(0) > 0, E_c(0) > 0, I_{mE_c}(0) > 0, I_c(0) > 0, I_{cm}(0) > 0, R_m(0) > 0, R_c(0) > 0, S_v(0) > 0, I_v(0) > 0$$

By substituting the classical derivative with Conformable-order derivative we have moment closure, which is a system of Conformable equations as shown in system equation2.

$$\begin{aligned}
T_t^{\upsilon} S_h(t) &= \Pi_h + \omega_m R_m + \omega_c R_c - \beta_c (I_c + I_{mc}) S_h - \beta_m I_v S_h - \mu_h S_h,\\
T_t^{\upsilon} I_m(t) &= \beta_m I_v S_h - \beta_c (I_c + I_{mc}) I_m - (\phi_m + \mu_h) I_m,\\
T_t^{\upsilon} E_c(t) &= \beta_c (I_c + I_{mc}) S_h - \beta_m I_v E_c - (\phi_c + \mu_h) E_c,\\
T_t^{\upsilon} I_{mE_c}(t) &= \delta \beta_c (I_c + I_{mc}) I_m - (\alpha_1 + \sigma + \mu_h) E_c,\\
T_t^{\upsilon} I_c(t) &= \phi_c E_c - (\gamma_c + \mu_h) I_c,
\end{aligned} \tag{2}$$

$$T_t^{\upsilon} I_{mc}(t) = \beta_m I_v E_m + \beta_c (I_c + I_{mc}) I_c + \sigma I_{mE_c} - (\mu_h + q) I_{mc},$$

$$T_t^{\upsilon} R_m(t) = \phi_m I_m + \alpha_1 I_{mE_c} - (\omega_m + \mu_h) R_m,$$

$$T_t^{\upsilon} R_c(t) = \gamma_c I_c - (\omega_c + \mu_h) R_c,$$

$$T_t^{\upsilon} S_v(t) = \Pi_v - \beta_c (I_m + I_{mc} + I_{mE_c}) S_v - \mu_v S_v,$$

$$T_t^{\upsilon} I_v(t) = \beta_c (I_m + I_{mc} + I_{mE_c}) S_v - \mu_v I_v.$$

taking the initial conditions, with $T_t^{\upsilon}$ being a Conformable-order derivative.

Table 1 shows symobols and parameter description of the coinfection model system equation 2.

**Table 1:** Parameters descriptions.

| Symbol | Parameter description |
|---|---|
| $\Pi_h$ | Recruitment rate |
| $\omega_m$ | Loss of immunity by humans recovered from malaria |
| $\omega_c$ | Loss of immunity of recovered individuals from COVID-19 |
| $\beta_c$ | The rate at which individuals transmit COVID-19 |
| $\beta_m$ | The rate at which mosquito transmit malaria |
| $\phi_m$ | The rate individuals infected with malaria move recovery class |
| $\phi_c$ | The rate at which exposed individuals to COVID-19 move to infected COVID-19 class |
| $\delta$ | Modification parameter for COVID-19 |
| $\alpha_1$ | Rate at wich infected individuals with malaria and exposed to COVID-19 move to recovered from malaria class |
| $\sigma$ | Rate at which infected individuals with malaria and exposed to COVID-19 move to coinfected class. |
| $\gamma_c$ | The rate coinfected individuals infected progress to recovered from COVID-19 class |
| $q$ | Disease induced mortality rate |
| $\Pi_v$ | Vector recruitment rate |
| $\mu_v$ | Natural human mortality rate |
| $\mu_h$ | Natural vector mortality rate |

## 4 Existence and Uniqueness of Solution

This study utilized the fixed point theorem to show the existence and the uniqueness of the solutions of the malaria and COVID-19 model in the context of the Conformable derivative.

By letting $P = (N([0,S]))^4$ and $N([0,S])$ be Banach spaces of all real-valued, continuous functions on the interval $[0,S]$ with the norm

$\| y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} \| = \| y_1 \| + \| y_2 \| + \| y_3 \| + \| y_4 \| + \| y_5 \| + \| y_6 \| + \| y_7 \| + \| y_8 \| + \| y_9 \| + \| y_{10} \|$

with $\| . \|$ representing the supremum norm in $N([0,S])$.

The accompanying mathematical statement will be used in the reminder for this segment.

$$\Theta_0(t) = \Pi_h + \omega_m R_m + \omega_c R_c - \beta_c (I_c + I_{mc}) S_h - \beta_m I_v S_h - \mu_h S_h, \tag{3}$$

$$\Theta_1(t) = \beta_m I_v S_h - \beta_c (I_c + I_{mc}) I_m - (\phi_m + \mu_h) I_m, \tag{4}$$

$$\Theta_2(t) = \beta_c (I_c + I_{mc}) S_h - \beta_m I_v E_c - (+\phi_c + \mu_h) E_c, \tag{5}$$

$$\Theta_2(t) = \beta_c\left(I_c + I_{mc}\right)S_h - \beta_m I_v E_c - \left(+\phi_c + \mu_h\right)E_c, \tag{6}$$

$$\Theta_3(t) = \delta\beta_c\left(I_c + I_{mc}\right)I_m - \left(\alpha_1 + \sigma + \mu_h\right)I_{mE_c}, \tag{7}$$

$$\Theta_4(t) = \phi_c E_c - \left(\gamma_c + \mu_h\right)I_c, \tag{8}$$

$$\Theta_5(t) = \beta_m I_v E_m + \beta_c\left(I_c + I_{mc}\right)I_c + \sigma I_{mE_c} - \left(\mu_h + q\right)I_{mc}, \tag{9}$$

$$\Theta_6(t) = \phi_m I_m + \alpha_1 I_{mE_c} - \left(\omega_m + \mu_h\right)R_m, \tag{10}$$

$$\Theta_7(t) = \gamma_c I_c - \left(\omega_c + \mu_h\right)R_c, \tag{11}$$

$$\Theta_8(t) = \Pi_v - \beta_c\left(I_m + I_{mc} + I_{mE_c}\right)S_v - \mu_h S_v, \tag{12}$$

$$\Theta_9(t) = \beta_c\left(I_m + I_{mc} + I_{mE_c}\right)S_v - \mu_h I_v. \tag{13}$$

In further analysis, the study utilizes the definition of Conformable integral to obtain the Conformable operators properties

$$\begin{aligned}
S_h(t) - S_h(0) &= \int_0^t y^{\upsilon-1}\Theta_0\left(y, S_h\right)dy,\\
I_m(t) - I_m(0) &= \int_0^t y^{\upsilon-1}\Theta_1\left(y, I_m\right)dy,\\
E_c(t) - E_c(0) &= \int_0^t y^{\upsilon-1}\Theta_2\left(y, E_c\right)dy,\\
I_{mE_c}(t) - I_{mE_c}(0) &= \int_0^t y^{\upsilon-1}\Theta_3\left(y, I_{mE_c}\right)dy,\\
I_c(t) - I_c(0) &= \int_0^t y^{\upsilon-1}\Theta_4\left(y, I_c\right)dy,\\
I_{mc}(t) - I_{mc}(0) &= \int_0^t y^{\upsilon-1}\Theta_5\left(y, I_{mc}\right)dy,\\
R_m(t) - R_m(0) &= \int_0^t y^{\upsilon-1}\Theta_6\left(y, R_m\right)dy,\\
R_c(t) - R_c(0) &= \int_0^t y^{\upsilon-1}\Theta_7\left(y, R_c\right)dy,\\
S_v(t) - S_v(0) &= \int_0^t y^{\upsilon-1}\Theta_8\left(y, S_v\right)dy,
\end{aligned} \tag{14}$$

$$I_v(t) - I_v(0) = \int_0^t y^{\upsilon-1}\Theta_9(y, I_v)\,dy.$$

With an assumption that $\| S_h(t) \| \leq F_0, \| I_m(t) \| \leq F_1, \| E_c(t) \| \leq F_2, \| I_{mE_c}(t) \| \leq F_3, \| I_c(t) \| \leq F_4, \| I_{mc}(t) \| \leq F_5, \| R_m(t) \| \leq F_6, \| R_c(t) \| \leq F_7, \| S_v(t) \| \leq F_8, \| I_v(t) \| \leq F_9, \| S_h(t) \| \leq F_0$, we prove the theorems below.

for all $F_i, i = 0, ..., 10$ are positive constant and denote that:

$$Q_0 = \beta_c (I_c F_4 + I_{mc} F_5) S_h + \beta_m I_v F_9 S_h + \mu_h F_0 S_h, \tag{15}$$

$$Q_1 = \beta_c (I_c F_4 + I_{mc} F_5) I_m - (\phi_m + \mu_h) I_m F_1, \tag{16}$$

$$Q_2 = \beta_m I_v E_c F_2 - (\phi_c + \mu_h) E_c F_2 \tag{17}$$

$$Q_3 = (\alpha_1 + \sigma + \mu_h) I_{mE_c} F_3, \tag{18}$$

$$Q_4 = (\gamma_c + \mu_h) I_c F_4, \tag{19}$$

$$Q_5 = \beta_c I_{mc} I_c F_5 + (\mu_h + q) I_{mc} F_5, \tag{20}$$

$$Q_6 = (\omega_m + \mu_h) R_m F_6, \tag{21}$$

$$Q_7 = (\omega_c + \mu_h) R_c F_7, \tag{22}$$

$$Q_8 = \beta_c (I_m F_1 + I_{mc} F_5 + I_{mE_c} F_3) S_v + \mu_h F_8 S_v, \tag{23}$$

$$Q_9 = \beta_c (I_m F_1 + I_{mc} F_5 + I_{mE_c} F_3) S_v + \mu_h F_9 I_v.$$

**Theorem 1.** The kernels $\Theta_j, j = 0, ..., 10$ fulfill the Lipschitz criterion and are contraction provided the inequality set forth beneath holds

$$0 \leq F_i < 1, i = 0, ..., 10 \tag{24}$$

*Proof:* By letting $S_{h,1}$ and $S_{h,2}$ be two functions, we have

$$\begin{aligned} \left\|\Theta_0(t, S_{h,1}) - \Theta_0(t, S_{h,2})\right\| &= \left\|(-\beta_c(I_c + I_{mc}) - \beta_m I_v - \mu_h)(S_{h,1} - S_{h,2})\right\| \qquad (25) \\ &\leq (\beta_c(\|I_c\| + \|I_{mc}\|) + \beta_m \|I_v\| + \mu_h)\left\|S_{h,1}(t) - S_{h,2}(t)\right\| \qquad (26) \\ &\leq (\beta_c(I_c F_4 + I_{mc} F_5) + \beta_m I_v F_9 + \mu_h)\left\|S_{h,1}(t) - S_{h,2}(t)\right\| \end{aligned}$$

Therefore, for all values of $\Theta_0$ to $\Theta_9$, the Lipschitz criterion can be confirmed and presented below.

$$\left\|\Theta_0(t, S_{h,1}) - \Theta_0(t, S_{h,2})\right\| \leq Q_0 \left\|S_{h,1}(t) - S_{h,2}(t)\right\|$$

$$\|\Theta_1(t, I_{m,1}) - \Theta_1(t, I_{m,2})\| \leq Q_1 \|I_{m,1}(t) - I_{m,2}(t)\|$$

$$\|\Theta_2(t, E_{c,1}) - \Theta_2(t, E_{c,2})\| \leq Q_2 \|E_{c,1}(t) - E_{c,2}(t)\|$$

$$\begin{aligned}
\|\Theta_3(t,I_{mE_c,1})-\Theta_3(t,I_{mE_c,2})\| &\leq Q_3\|I_{mE_c,1}(t)-I_{mE_c,2}(t)\| \\
\|\Theta_4(t,I_{c,1})-\Theta_4(t,I_{c,2})\| &\leq Q_4\|I_{c,1}(t)-I_{c,2}(t)\| \\
\|\Theta_5(t,I_{mc,1})-\Theta_5(t,I_{mc,2})\| &\leq Q_5\|I_{mc,1}(t)-I_{mc,2}(t)\| \\
\|\Theta_6(t,R_{m,1})-\Theta_6(t,R_{m,2})\| &\leq Q_6\|R_{m,1}(t)-R_{m,2}(t)\| \\
\|\Theta_7(t,R_{c,1})-\Theta_7(t,R_{c,2})\| &\leq Q_7\|R_{c,1}(t)-R_{c,2}(t)\| \\
\|\Theta_8(t,S_{v,1})-\Theta_8(t,S_{v,2})\| &\leq Q_8\|S_{v,1}(t)-S_{v,2}(t)\| \\
\|\Theta_9(t,I_{v,1})-\Theta_9(t,I_{v,2})\| &\leq Q_9\|I_{v,1}(t)-I_{v,2}(t)\|
\end{aligned} \tag{27}$$

This completes the proof.

System (3) can be reoganised in a recursive form as follows:

$$\begin{aligned}
S_{h,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_0(y,S_{h,n-1})dy, \\
I_{m,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_1(y,I_{m,n-1})dy, \\
E_{c,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_2(y,E_{c,n-1})dy, \\
I_{mE_{c,n}}(t) &= \int_0^t y^{\upsilon-1}\Theta_3(y,I_{mE_c,n-1})dy, \\
I_{c,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_4(y,I_{c,n-1})dy, \\
I_{mc,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_5(y,I_{mc,n-1})dy, \\
R_{m,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_6(y,R_{m,n-1})dy, \\
R_{c,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_7(y,R_{c,n-1})dy, \\
S_{v,n}(t) &= \int_0^t y^{\upsilon-1}\Theta_8(y,S_{v,n-1})dy, \\
I_{v,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_9(y,I_{v,n-1})dy.
\end{aligned} \tag{28}$$

with the following initial conditions

$S_{h,0}(t) = S_h(0), I_{m,0}(t) = I_m(0), E_{c,0}(t) = E_c(0), I_{mE_c,0}(t) = I_{mE_c,}(0), I_{c,0}(t) = I_c(0), I_{mc,0}(t) = I_{mc}(0), R_{m,0}(t) = R_m(0), R_{c,0}(t) = R_c(0), S_{v,0}(t) = S_v(0), I_{v,0}(t) = I_v(0)$

The difference between the successive terms are expressed as:

$$\begin{aligned}
A_{0,n}(t) = S_{h,n}(t) - S_{h,n-1}(t) &= \int_0^t y^{\upsilon-1}(\Theta_0(y,S_{h,n-1}) - \Theta_0(y,S_{h,n-2}))dy,\\
A_{1,n}(t) = I_{m,n}(t) - I_{m,n-1}(t) &= \int_0^t y^{\upsilon-1}(\Theta_1(y,I_{m,n-1}) - \Theta_1(y,I_{m,n-2}))dy,\\
A_{2,n}(t) = E_{c,n}(t) - E_{c,n-1}(t) &= \int_0^t y^{\upsilon-1}(\Theta_2(y,E_{c,n-1}) - \Theta_2(y,E_{c,n-2}))dy,\\
A_{3,n}(t) = I_{mE_c,n}(t) - I_{mE_c,n-1}(t) &= \int_0^t y^{\upsilon-1}(\Theta_3(y,I_{mE_c,n-1}) - \Theta_3(y,I_{mE_c,n-2}))dy,\\
A_{4,n}(t) = I_{c,n}(t) - I_{c,n-1}(t) &= \int_0^t y^{\upsilon-1}(\Theta_4(y,I_{c,n-1}) - \Theta_4(y,I_{c,n-2}))dy,\\
A_{5,n}(t) = I_{mc,n}(t) - I_{mc,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_5(y,I_{mc,n-1}) - \Theta_5(y,I_{mc,n-2}))dy,\\
A_{6,n}(t) = R_{m,n}(t) - R_{m,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_6(y,R_{m,n-1}) - \Theta_6(y,R_{m,n-2})dy,\\
A_{7,n}(t) = R_{c,n}(t) - R_{c,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_7(y,R_{c,n-1}) - \Theta_7(y,R_{c,n-2})dy,\\
A_{8,n}(t) = S_{v,n}(t) - S_{v,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_8(y,S_{v,n-1}) - \Theta_8(y,S_{v,n-2})dy,\\
A_{9,n}(t) = I_{v,n}(t) - I_{v,n-1}(t) &= \int_0^t y^{\upsilon-1}\Theta_9(y,I_{v,n-1}) - \Theta_9(y,I_{v,n-2})dy.
\end{aligned} \tag{29}$$

taking norm from equation (7) while utilising the Lipschitz criterion establised in equation (5) and likewise, using the triangular inequality, we have

$$\|A_{0,n}(t)\| \leq \left\|S_{h,n}(t) - S_{h,n-1}(t)\right\| \leq Q_1 \int_0^t y^{\upsilon-1} \left\|S_{h,n-1} - S_{h,n-2}\right\| dy.$$

we obtain,

$$\|A_{0,n}(t)\| \leq Q_0 \frac{t^{\upsilon}}{\upsilon} \|A_{0,n-1}(t)\| \tag{30}$$

Accordingly, other system of equation that remained can be gotten as follows

$$\|A_{1,n}(t)\| \leq Q_1 \frac{t^{\upsilon}}{\upsilon} \|A_{1,n-1}(t)\| \tag{31}$$

$$\|A_{2,n}(t)\| \leq Q_2 \frac{t^{\upsilon}}{\upsilon} \|A_{2,n-1}(t)\| \tag{32}$$

$$\|A_{3,n}(t)\| \leq Q_3 \frac{t^{\upsilon}}{\upsilon} \|A_{3,n-1}(t)\| \tag{33}$$

$$\|A_{4,n}(t)\| \leq Q_4 \frac{t^{\upsilon}}{\upsilon} \|A_{4,n-1}(t)\| \tag{34}$$

$$\left\|A_{5,n}(t)\right\| \leq Q_5 \frac{t^{\upsilon}}{\upsilon} \left\|A_{5,n-1}(t)\right\| \tag{35}$$

$$\left\|A_{6,n}(t)\right\| \leq Q_6 \frac{t^{\upsilon}}{\upsilon} \left\|A_{6,n-1}(t)\right\| \tag{36}$$

$$\|A_{7,n}(t)\| \leq Q_7 \frac{t^{\upsilon}}{\upsilon} \|A_{7,n-1}(t)\| \tag{37}$$

$$\|A_{8,n}(t)\| \leq Q_8 \frac{t^{\upsilon}}{\upsilon} \|A_{8,n-1}(t)\| \tag{38}$$

$$\|A_{9,n}(t)\| \leq Q_9 \frac{t^{\upsilon}}{\upsilon} \|A_{9,n-1}(t)\| \tag{39}$$

In line with the results obtained above, we established the accompanying theorem as

**Theorem 2.** The uniqueness and the existence of the Conformable-order model (2) solution exists if a real number $t$ satisfies

$$\frac{t^{\upsilon}}{\upsilon} Q_i < 1, \text{ for } i \in \{0,1,2,3,4,5,6,7,8,9\}, \tag{40}$$

*Proof:* By taking into consideration equation (8) - (17) the following relations are obtained:

$$\|A_{0,n}(t)\| \leq \left(Q_0 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{0,1}\| \tag{41}$$

$$\|A_{1,n}(t)\| \leq \left(Q_1 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{1,1}\| \tag{42}$$

$$\|A_{2,n}(t)\| \leq \left(Q_2 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{2,1}\| \tag{43}$$

$$\|A_{3,n}(t)\| \leq \left(Q_3 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{3,1}\| \tag{44}$$

$$\|A_{4,n}(t)\| \leq \left(Q_4 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{4,1}\| \tag{45}$$

$$\left\|A_{5,n}(t)\right\| \leq \left(Q_5 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \left\|A_{5,1}\right\| \tag{46}$$

$$\left\|A_{6,n}(t)\right\| \leq \left(Q_6 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \left\|A_{6,1}\right\| \tag{47}$$

$$\|A_{7,n}(t)\| \leq \left(Q_7 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{7,1}\| \tag{48}$$

$$\|A_{8,n}(t)\| \leq \left(Q_8 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{8,1}\| \tag{49}$$

$$\|A_{9,n}(t)\| \leq \left(Q_9 \frac{t^{\upsilon}}{\upsilon}\right)^{n-1} \|A_{9,1}\| \tag{50}$$

In other to show that the functions are solutions of the system (2), we let

$$\begin{aligned}
S_h(t)-S_h(0)&=S_{h,n}(t)-P_n^0(t),\\
I_m(t)-I_m(0)&=I_{m,n}(t)-P_n^1(t)\\
E_c(t)-E_c(0)&=E_{c,n}(t)-P_n^2(t),\\
I_{mE_c}(t)-I_{mE_c}(0)&=I_{mE_{c,}}(t)-P_n^3(t),\\
I_c(t)-I_c(0)&=I_{c,n}(t)-P_n^4(t),\\
I_{mc}(t)-I_{mc}(0)&-I_{mc,n}(t)-P_n^5(t),\\
R_m(t)-R_m(0)&=R_{m,n}(t)-P_n^6(t),\\
R_c(t)-R_c(0)&=R_{c,n}(t)-P_n^7(t),\\
S_v(t)-S_v(0)&=S_{v,n}(t)-P_n^8(t),\\
I_v(t)-I_v(0)&=I_{v,n}(t)-P_n^9(t).
\end{aligned}$$

$\overline{S}_h(t),\overline{I}_m(t),\overline{E}_c(t),\overline{I}_{mE_c}(t),\overline{I}_c(t),\overline{I}_{mc}(t),\overline{R}_m(t),\overline{R}_c(t),\overline{S}_v(t),\overline{I}_v(t)$

where

$$P_n^0(t)=\int_0^t y^{\upsilon-1}(\Theta_0(y,S_h(y))-\Theta_0(y,S_{h,n-2}))dy. \tag{51}$$

Hence, we obtain

$$\begin{aligned}
\left\|P_n^0(t)\right\| &\le \int_0^t y^{\upsilon-1}\left\|\Theta_0(y,S_h(y))-\Theta_0(y,S_{h,n-2})\right\|dy \\
&\le Q_0\int_0^t y^{\upsilon-1}\left\|S_h(t)-S_{h,n-1}\right\|dy
\end{aligned} \tag{52}$$

By recursive process, we get

$$\left\|P_n^0(t)\right\|\left(Q_0\frac{t^{\upsilon}}{\upsilon}\right)^n f_1$$

At $t$, we obtain

$$\left\|P_n^0(t)\right\|\left(Q_0\frac{\tau^{\upsilon}}{\upsilon}\right)^n f_1$$

Applying limits as $n$ approaches infinity, we have

$$\left\|P_n^0(t)\right\|\to 0$$

We get the same result by repeating the technique.

$$\left\|P_n^1(t)\right\|\to 0,\ \text{for } i=1,2,3,4,5,6,7,8,9,10$$

Therefore, existence is proved

In proofing the uniqueness, we assume that the system (2) concedes double solutions, taking the second solution as $\overline{S}_h(t),\overline{I}_m(t),\overline{E}_c(t),\overline{I}_{mE_c}(t),\overline{I}_c(t),\overline{I}_{mc}(t),\overline{R}_m(t),\overline{R}_c(t),\overline{S}_v(t),\overline{I}_v(t)$ then

$$S_h(t)-\overline{S}_h(t)=\int_0^t y^{\upsilon-1}(\Theta_0(y,S_h)-\Theta_0(y,\overline{S}_h(t))dy$$

taking the norm from the equation above, we have

$$\left\|S_h(t)-\overline{S}_h(t)\right\|\le\int_0^t y^{\upsilon-1}\left\|\Theta_0(y,S_h)-\Theta_0(y,\overline{S}_h)\right\|dy$$

When we use the Lipschitz criterion of kernel, we get

$$\left\|S_h(t)-\overline{S}_h(t)\right\| \leq \frac{t^{\upsilon}}{\upsilon}Q_0\left\|S_h(t)-\overline{S}_h(t)\right\|$$

This gives

$$\left\|S_h(t)-\overline{S}_h(t)\right\| \leq \left(1-\frac{\tau^{\upsilon}}{\upsilon}Q_0\right) \leq 0.$$

Distinctly $S_h(t)=\overline{S}_h(t)$ if

$$1-\frac{\tau^{\upsilon}}{\upsilon}Q_0>0. \quad (53)$$

Applying the same technique, we have $S_h(t) = \overline{S}_h(t), I_m(t) = \overline{I}_m(t), E_c(t) = \overline{E}_c(t), I_{mE_c}(t) = \overline{I}_{mE_c}(t), I_c(t)-\overline{I}_c(t), I_{mc}(t)-\overline{I}_{mc}(t), R_m(t)=\overline{R}_m(t), R_c(t)=\overline{R}_c(t), S_v(t)=\overline{S}_v(t), I_v(t)=\overline{I}_v(t)$

As a result, if condition (18) is met, the solution is unique.

**Conformable Predictor- Corrector Scheme**

The Multi-step techniques are well-known for their efficiency in solving differential equations numerically. For integer-order differential equations, such approaches have been establishment. They have recently been expanded to arbitrary order derivatives. We would develop a Conformable version of multi-step techniques. If the order of derivation is one, the classical predictor-corrector approaches are recovered. As a result, the suggested technique can be thought of as a generalisation of the classical multi-step procedures.
By considering the Conformable-order differential equation

$$T_t^{\upsilon}y(t)=U(t,y(t)),\ \ T>t\geq 0,\ \ 1\geq\upsilon>0. \quad (54)$$
$$y(0)=y_0.$$

With Conformable integral definition and likewise, applying the second lemma, we have

$$y(t)=y(0)+\int_0^t \tau^{\upsilon-1}U(\tau,y(\tau))d\tau,$$

Using $t_n$ and $t_{n+1}$ we obtain

$$y(t_n)=y(0)+\int_0^{t_n} \tau^{\upsilon-1}U(\tau,y(\tau))d\tau \quad (55)$$

$$y(t_{n+1})=y(0)+\int_0^{t_{n+1}} \tau^{\upsilon-1}U(\tau,y(\tau))d\tau \quad (56)$$

Subtracting equation (32) from (33), we have

$$y(t_{n+1})-y(t)=\int_{t_n}^{t_{n+1}} \tau^{\upsilon-1}U(\tau,y(\tau))d\tau$$

The predictor technique presented in earlier works is used to discretize equation (23) and by taking into consideration that, $h=T/N, t_j=jh, j=0,1,2,3,4,5,...,N$ for the interval of $[0,T]$ and $N$ steps. The function $U(t,y(t))$ can be assumed closed to $[t_n,t_{n+1}]$, for $n=0,1,2,3,4,5,...,$ applying Lagrange polynomial interpolation ( of degree m). This becomes

$$U(t,y(t))\approx B_m(t,y(t))=\sum_{j=m-n}^{n} L_j(t)U(t_j,y_j) \quad (57)$$

where $L_j$ are the Lagrange functions at the $(m+1)$ points $\{t_{n-m},...,t_{n-1},t_n\}$ and written as

$$L_j(t)=\prod_{\substack{k=n-m\\k\neq j}}^{n}\frac{t-t_k}{t_j-t_k}.$$

Hence, we read the integral as

$$\begin{aligned}\int_{t_n}^{t_{n+1}} t^{\upsilon-1}U(t,y(t))dt &\approx \int_{t_n}^{t_{n+1}} t^{\upsilon-1}B_m(t,y(t))dt\\ &= \int_{t_n}^{t_{n+1}} t^{\upsilon-1}\sum_{j=m-n}^{n} L_j(t)t^{\upsilon-1}(t)U(t_j,y_j)dt,\\ &= \sum_{j=m-n}^{n} U(t_j,y_j)\int_{t_n}^{t_{n+1}} t^{\upsilon-1}L_j(t)dt.\end{aligned}$$

The generalised Conformable Adams-Bashforth method is thus defined as follows:

$$y_{n+1} = y_n + \sum_{j=m-n}^{n} b_j^{\upsilon}U(t_j,y_j), \tag{58}$$

for $y_n$ representing the approximate solution at $t_n$ and $b_j^{\upsilon} = \int_{t_n}^{t_{n+1}} t^{\upsilon-1}L_j(t)dt$

Explicitly, conformable Euler method for $m = 0$ was obtain as

$$y_{n+1} = y_n + \frac{t_{n+1}^{\upsilon} - t_n^{\upsilon}}{\upsilon}U(t_n,y_n),$$

where $t_n = nh$ and $t_{n+1} = (n+1)h$, it implies that

$$y_{n+1} = y_n + \frac{(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}h^{\upsilon}(t_n,y_n),$$

for $m = 1$ we obtain

$$\begin{aligned}B_m &\approx \frac{t-t_{n-1}}{t_n-t_{n-1}}U(t_n,y_n) + \frac{t-t_{n-1}}{t_{n-1}-t_n}U(t_{n-1},y_{n-1})\\ &= \frac{U(t_n,y_n)}{h}(t,t_{n-1}) - \frac{U(t_{n-1},y_{n-1})}{h}(t,t_n)\end{aligned} \tag{59}$$

That is

$$y_{n+1} = y_n + \frac{U(t_n,y_n)}{h}\int_{t_n}^{t_{n+1}} \tau^{\upsilon-1}(\tau-t_{n-1})d\tau - \frac{U(t_{n-1},y_{n-1})}{h}\int_{t_n}^{t_{n+1}} \tau^{\upsilon-1}(\tau-t_n)d\tau$$

Conformable Adams-Bashforth Method which is two-step is as follows:

$$\begin{aligned}y_{n+1} = y_n &+ \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1} - \frac{(n-1)(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)h^{\upsilon}U(t_n,y_n)\\ &- \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1}\frac{-n(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)h^{\upsilon}U(t_{n-1},y_{n-1})\end{aligned} \tag{60}$$

For m=2 we obtain the three-steps Conformable Adam-Bashforth method

$$y_{n+1} = y_n + h^{\upsilon}\left(b_{n-2}^{\upsilon}U(t_{n-2},y_{n-2}) + b_{n-1}^{\upsilon}U(t_{n-1},y_{n-1}) + b_n^{\upsilon}U(t_n,y_n), \right. \tag{61}$$

where

$$b^{\upsilon}_{n-2} = \frac{(n+1)^{\upsilon+2} - n^{\upsilon+2}}{2(\upsilon+2)} - \frac{(2n+1)((n+1)^{\upsilon+1} - n^{\upsilon+1})}{2(\upsilon+1)} + \frac{n(n-1)((n+1)^{\upsilon} - n^{\upsilon})}{2\upsilon}, \tag{62}$$

$$b^{\upsilon}_{n-1} = -\frac{(n+1)^{\upsilon+2} - n^{\upsilon+2}}{\upsilon+2} + \frac{(2n-2)((n+1)^{\upsilon+1} - n^{\upsilon+1})}{\upsilon+1} - \frac{n(n-2)((n+1)^{\upsilon} - n^{\upsilon})}{2\upsilon}, \tag{63}$$

$$b^{\upsilon}_{n} = \frac{(n+1)^{\upsilon+2} - n^{\upsilon+2}}{2(\upsilon+2)} - \frac{(2n-3)((n+1)^{\upsilon+1} - n^{\upsilon+1})}{2(\upsilon+1)} + \frac{(n-1)(n-1)((n+1)^{\upsilon} - n^{\upsilon})}{2\upsilon}$$

**The Predictor-Corrector Scheme**

In this section, we will look at the predictor-corrector technique in the situation of conformable derivation utilising 2-step AB and 1-step AM methods. We will begin with the predictor, which is the Conformable 2-steps AB technique in our case.

$$\begin{aligned}\overline{y}_{n+1} = y_n &+ \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1} - (n-1)\frac{(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)\frac{U(t_n, y_n)}{h} \\ &- \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1} - n\frac{(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)\frac{U(t_{n-1}, y_{n-1})}{h}\end{aligned} \tag{64}$$

However, we utilise this output in the implicit term on the Conformedable one step AM method to rectify it. i.e.,

$$\begin{aligned}y_{n+1} = y_n &+ \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1} - (n+1)\frac{(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)\frac{U(t_n, y_n)}{h} \\ &+ \left(\frac{(n+1)^{\upsilon+1} - n^{\upsilon+1}}{\upsilon+1} - n\frac{(n+1)^{\upsilon} - n^{\upsilon}}{\upsilon}\right)\frac{U(t_{n-1}, y_{n-1})}{h}\end{aligned} \tag{65}$$

We can express the numerical solution of model (2) with the Conformable derivative utilising this scheme

$$Y(t) = \begin{bmatrix} S_h(t) \\ I_m(t) \\ E_c(t) \\ I_{mE_c}(t) \\ I_c(t) \\ I_{mc}(t) \\ R_m(t) \\ R_c(t) \\ S_v(t) \\ I_v(t) \end{bmatrix}, \quad Y(0) = \begin{bmatrix} S_{h,0}(t) \\ I_{m,0}(t) \\ E_{c,0}(t) \\ I_{mE_c,0}(t) \\ I_{c,0}(t) \\ I_{mc,0}(t) \\ R_{m,0}(t) \\ R_{c,0}(t) \\ S_{v,0}(t) \\ I_{v,0}(t) \end{bmatrix} \quad \text{and } H(t, Y(t)) = \begin{bmatrix} \Theta_0(t, Y(t)) \\ \Theta_1(t, Y(t)) \\ \Theta_2(t, Y(t)) \\ \Theta_3(t, Y(t)) \\ \Theta_4(t, Y(t)) \\ \Theta_5(t, Y(t)) \\ \Theta_6(t, Y(t)) \\ \Theta_7(t, Y(t)) \\ \Theta_8(t, Y(t)) \\ \Theta_9(t, Y(t)) \end{bmatrix}, \tag{66}$$

The numerical scheme below is obtain as first equation of the model (2):

$$\begin{cases} Y_0 = Y(0), \\ Y_1 = Y_0 + \frac{h^{\upsilon}}{\upsilon} H(t_0, Y_0) (\text{Conformable Euler method}) \\ \text{Preditor} \\ \tilde{Y}_{k+1} = Y_k + \tilde{b}^{\upsilon}_k H(t_k, Y_k) + b^{\upsilon}_{k-1} H(t_{k-1}, Y_{k-1}) (\text{Conformable AB2 method}), \\ \text{Corrector} \\ Y_{k+1} = Y_k + \tilde{b}^{\upsilon}_k H(t_k, Y_k) + b^{\upsilon}_{k-1} H(t_{k-1}, Y_{k-1}) (\text{Conformable AM1 method}), \end{cases} \tag{67}$$

## 5 Numerical Results and Discussions

This section described the numerical simulation results with the associated step size $h = 0.001$. The numerical scheme utilized was hinged on Adam-type predictor-corrector as the details can be found in [20]. The parameter values used some were mostly obtained in [4] and others few ones were estimated: $\Pi_h = 0.46$, $\Pi_v = 0.047$, $\beta_m = 0.0125$, $\beta_c = 0.4531$, $\beta_v = 0.48$, $\omega_m = 0.25$, $\omega_c = 0.3$, $\beta_m = 0.001$, $\mu_h = 0.05$, $\phi_m = 0.33$, $\phi_c = 0.2$, $\delta = 0.005$, $\alpha_1 = 0.05$, $q = 0.2$, $\sigma = 0.02$, $\gamma_c = 0.08$.

Figure 1(a) is the number of susceptible individuals $(S_h)$ and as the Conformable-order derivative rises from 0.65 towards 1 the these individuals reduce drastically. This is more common to most epidemiology because more individuals get infected as the susceptible individuals $(S_h)$ reduce in the community. In Figure 1(b), the number of infected individuals with malaria $I_m(t)$ only reduces when the Conformable-order derivative moves up towards 1. Thus, in this instance the individuals solely infected with malaria $I_m(t)$ reduces in the community. Figure 1(c) depicts the individuals exposed to COVID-19 $E_c(t)$ and reducing the Conformable-order derivative from 1 towards 0.65 resulted to a rise in the number of individuals in this respective class. Figure 1(d) indicates the number of individuals infected with malaria but exposed to COVID-19 $I_{mE}(t)$ and as the Conformable-order derivative moving up towards 1, the number of individuals in this class reduces. This shows that the number of individuals in this compartment reduce in the community with time. Figure 1(e) is the individuals infected with COVID-19 $I_c(t)$ only and as the Conformable-order derivative increase from 0.65 to 1 these individuals in the class reduce. The increase or decrease of the Conformable-order derivative in the system has a direct consequence on the spread of the disease.

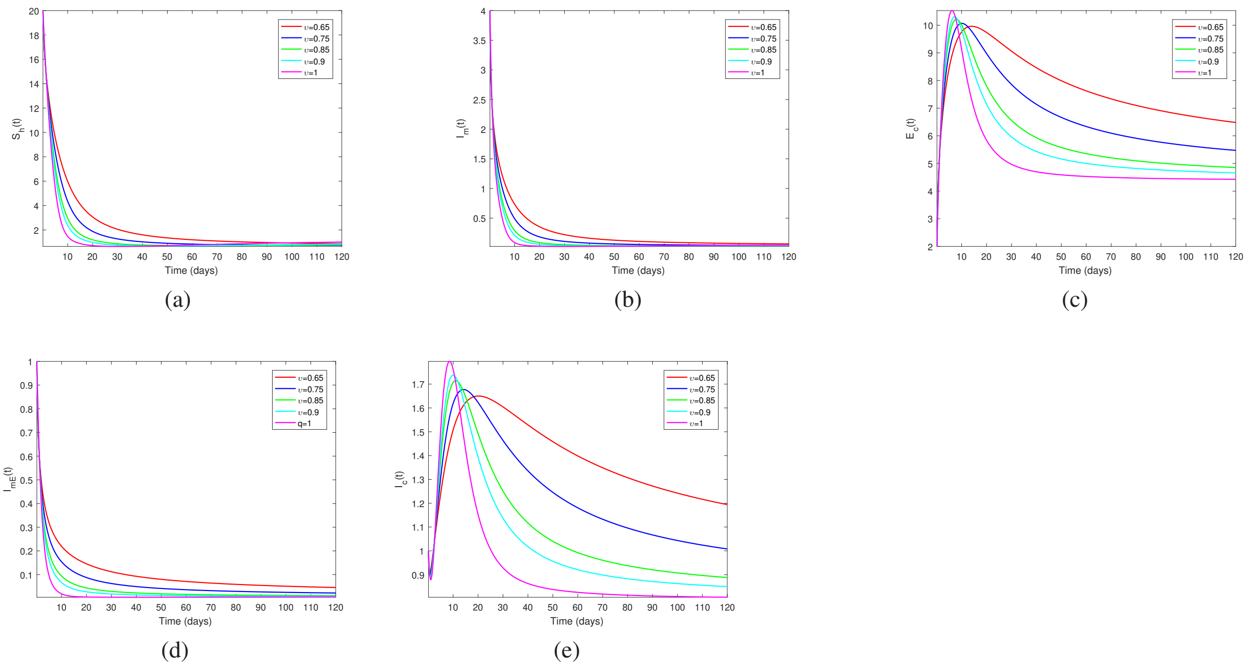


**Fig. 1:** Numerical simulation results of model 2 based on Conformable -order derivative for five arbitrary values of the order $\upsilon$

Figure 2(a) depicts the number of individuals infected with Malaria and COVID-19 $I_{mc}(t)$ and as the Conformable-order derivative increase from 0.65 to 1, the number of the individuals in the class also increase. This indicates that the coinfected individuals can be reduced by reducing the Conformable-order derivative. In Figure 2(b), the individuals recovered who from Malaria decreased as the Conformable-order derivative increased towards the integer order. Practically, if we want to increase the recovery rate the derivative order ought be reduced. Figure 2(c) is the individuals recovered from COVID-19 $(R_c)$ and as the Conformable-order derivative moved towards the integer order the number of individuals recovered increased in the community. Figure 2(d) shows the susceptible mosquitoe individuals $(S_v)$ and naturally, as more vectors get infected the susceptible individuals reduce in the environment. In this instance, Conformable-order derivative increases towards integer the susceptible vector individuals $(S_v)$ decrease. In Figure 2(e) the number of infected mosquitoes $(I_v)$ increases as the Conformable-order derivative increases toward the integer. This is expected in epidemiological dynamics since more mosquitoes get infected in the community.

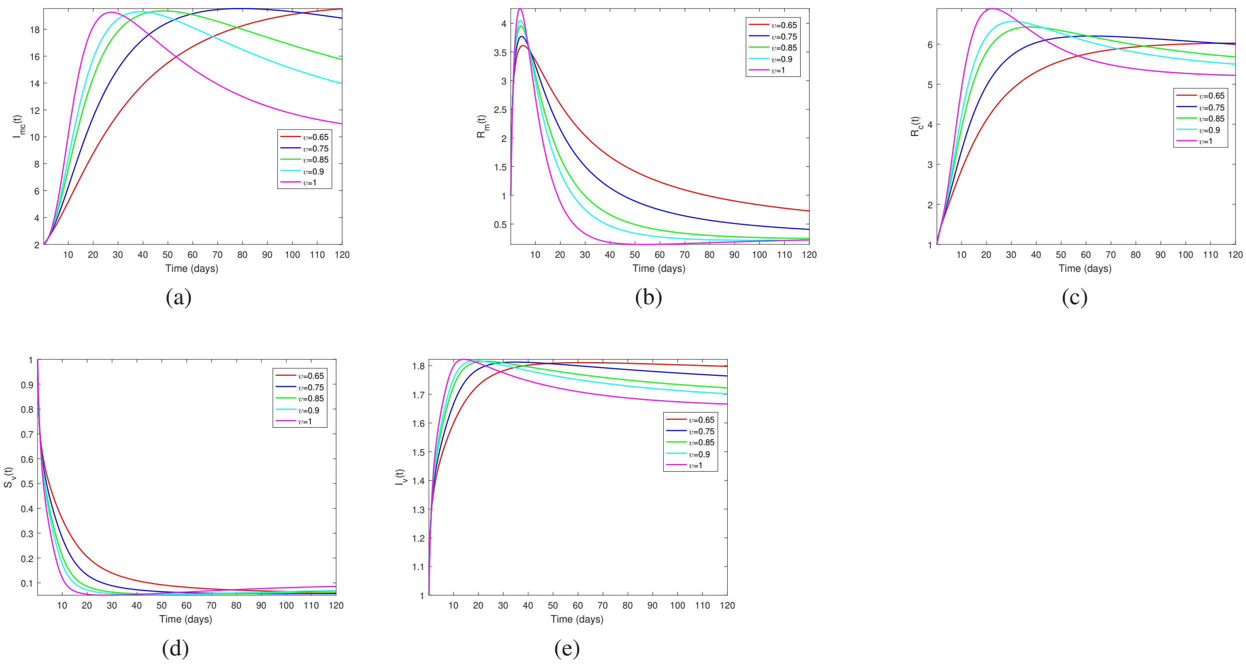


**Fig. 2:** Numerical simulation results of model 2 based on Conformable -order derivative for five arbitrary values of the order $\upsilon$

## 6 Conclusion

In this work, a coinfection model of Malaria and COVID-19 formulated in the context of Conformable -order derivative had been investigated. The fixed-point theorem and Picard iterative method were fully utilised to establish the uniqueness and existence of solutions of the coinfected model. In this study, the 2-steps Adams-Bashforth and one-step Adams-Moulton predictor corrector methods were employed in stimulating the numerical results. The Conformable-order derivative played a major role in the dynamics of the coinfected model by either increasing or decreasing infection in the various compartments. The result obtained in this work can be compared with other methodologies such as fractional order derivatives. It is therefore suggested that other complex coinfected models can be analysed using the Conformable-order derivative.

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