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Wave Function Formulation for A Circular Motion

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Abstract: The particle that moving on a circular path under a certain constraint is studied using Lagrangian mechanics (Euler Lagrange equation). The action function S is obtained by integrating the Lagrangian through time interval t; from this function we can calculate the wave function ψ, the behavior for the action function S and the wave function ψ is described through illustrative graphs.

Keywords: Action Function, Circular Motion, Lagrangian, Wave Function.

1 Introduction

In physics, the wave function is a quantity that describe the quantum level of a particle as a function. The symbol used for the wave function is ψ. So that in quantum physics a particle behavior is described using wave function ψ. It was first presented by Schrödinger, the wave function describes the probability amplitude, also by squaring the absolute value of ψ, we can find the probability density for a particle to be found in specified locations, this was presented by Born.

For more accurate, there is another presentation of the wave function has been investigated by [1,2] hidden variables concepts are used to describe the developing of each system alone. Formulation of quantum mechanics is investigated by [3]. Foundation of the Schrödinger equation from Newtonian mechanics is studying by [4], the differences between the considered models and standard quantum mechanics is presented by [5], these works are followed by de Broglie Bohm theory which is a representation of quantum mechanics [6]. Recently wave function is used in quantization field using the action function for conservative and non-conservative systems [7,8].

In the second section, we explained the circular motion of the particle using Lagrangian mechanics. In the third section, we wrote the conclusion.

2 Lagrangian Treatment for the Motion of A particle on A circle

The elliptical motion is studied by many researchers [9,10]. The following Lagrangian describes the elliptical motion of a particle in two dimensions [11].

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) \tag{1}$$

The constraint equation is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

using the parametric equations,

$$x = a\cos\theta \tag{3}$$

$$y = b\sin\theta \tag{4}$$

If $a = b$, the parametric equations become

$$x = a\cos\theta \tag{5}$$

and

$$y = a\sin\theta \tag{6}$$

So that, equation (2) represented the circle equation as follows:

$$x^2 + y^2 = a^2 \tag{7}$$

Taking the first time derivative of equation (5) and (6), we obtain;

$$\dot{x} = -a\dot{\theta}\sin\theta \tag{8}$$

and

$$\dot{y} = a\dot{\theta}\cos\theta \tag{9}$$

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After squaring of equation (8) and (9) and substituting them into equation (1) we have:

$$L=\frac{1}{2}m\dot{\theta}^2(a^2\sin^2\theta+a^2\cos^2\theta) \tag{10}$$

The **final form** of our Lagrangian is:

$$L=\frac{1}{2}m\dot{\theta}^2a^2 \tag{11}$$

Now, we will use Euler Lagrange equation in the following form [12,13]:

$$\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=0 \tag{12}$$

Then using the generalized coordinate θ, Euler Lagrange equation becomes:

$$\frac{\partial L}{\partial \theta}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=0 \tag{13}$$

Thus;

$$\frac{\partial L}{\partial \theta}=ma^2\ddot{\theta} \tag{14}$$

and

$$\frac{\partial L}{\partial \theta}=0 \tag{15}$$

this means that the generalized coordinate θ is **a cyclic coordinate**

Equation (14) takes this form

$$ma^2\ddot{\theta}=0 \tag{16}$$

Integration of equation (16) we have,

$$\theta=Ct+D \tag{17}$$

Where C and D are constants of integration.

The first time derivative of equation (17) is:

$$\dot{\theta}=C \tag{18}$$

Squaring of equation (18) we get,

$$\dot{\theta}^2=C^2 \tag{19}$$

Now, using equation (19), the Lagrangian in equation (11) becomes:

$$L=\frac{1}{2}ma^2C^2 \tag{20}$$

Using equation (20) the action function is written as:

$$S=\int_0^t Ldt=\int_0^t\frac{1}{2}ma^2C^2dt=\frac{t}{2}ma^2C^2 \tag{21}$$

and the wave function is formulating as:

$$\psi=\exp\left(\frac{iS}{\hbar}\right) \tag{22}$$

Making use of equation (21), equation (22) becomes;

$$\psi=\exp\left(\frac{itma^2C^2}{2\hbar}\right) \tag{23}$$

Which represents the final form of the wave function.

3 Discussion

In this paper the motion of a particle on a circular path is studied using Euler Lagrange equation, then the equation of motion is obtained, we find that the generalized coordinate θ is **a cyclic coordinate.** Finally, we obtained the action function S which helps us to find the corresponding **wave function** ψ, the behavior of the particle is studied through the following figures.

The relation between action function S and the time t, through time interval equals to **10** seconds, is shown in figure **1**. Where $S=\frac{1}{2}tma^2C^2$, to simplify we assume that, $mC^2=1$, the radius of the circle $a=1$, then $S=\frac{t}{2}$

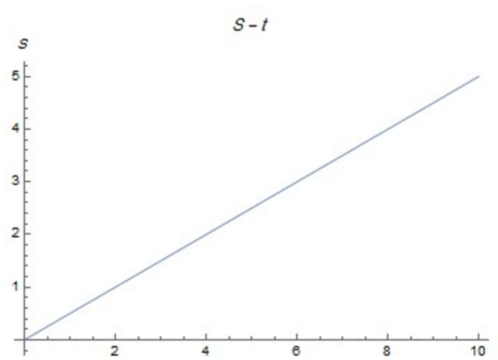

Fig -1-

Fig. 1: S-t

The relation between wave function ψ and the time t, through time interval equals to **0.5** second, is shown in figure **2.** Where, $\psi = \exp\left(\frac{itma^2C^2}{2\hbar}\right)$, to simplify we assume that, $\frac{mC^2}{2\hbar} = 1$, then $\psi = \exp(ia^2t)$. the radius of the circle $a = 1$,

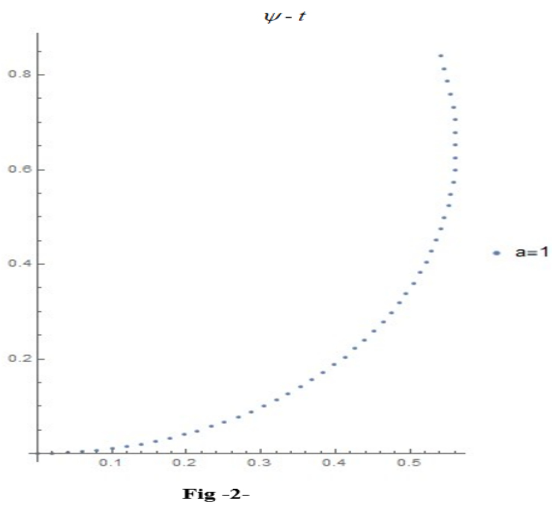

Fig.2: ψ - t

The relation between wave function ψ and the time t, through time interval equals to **2** seconds, is shown in figure **3.** Where $\psi = \exp(ia^2t)$, the radius of the circle $a = 1,2,3,4$

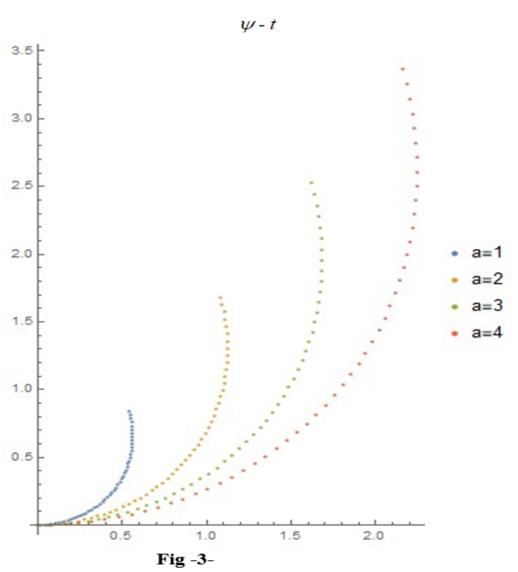

Fig.3: ψ - t

4 Conclusion

In conclusion, the study of a particle moving on a circular path under a specific constraint using Lagrangian mechanics (Euler Lagrange equation) has been conducted. By integrating the Lagrangian over a given time interval, the action function is obtained. This action function enables us to calculate the wave function and analyze the behavior of both the action function and the wave function through illustrative graphs.

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