Appl. Math. Inf. Sci. **17**, No. 5, 933-940 (2023)

Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/170520

Applications of the ARA-Residual Power Series Technique to Physical Phenomena

Aliaa Burqan

Department of Mathematics, Zarqa University, Zarqa, Jordan

Received: 7 Jun. 2023, Revised: 21 Jul. 2023, Accepted: 23 Aug. 2023
Published online: 1 Sep. 2023

Abstract: In this paper, a new analytical method called the ARA-Residual power series method (ARA- RPSM) is implemented to solve some fractional physical equations. The methodology of the proposed method based on applying the ARA-transform to the given fractional differential equations, followed by the creation of approximate series solutions using Taylor's expansion. Then the series solution is transformed using the inverse of the ARA-transform to get the solution in the original space. Accuracy, effectiveness, and validity of the suggested method are demonstrated through the discussion of three attractive applications. The solution obtained using ARA-RPSM demonstrates good agreement when compared to the solutions found using other methods.

Keywords: Approximate solutions; ARA-residual power series; Fractional calculus; physical equations.

1 Introduction

Many events in physics and other disciplines may be efficiently described using fractional calculus because accurate modeling of physical phenomena depends not only on immediate time but also on past time [1], [2], [3], [4], [5], [6], [7], [8], [9] and [10]. To solve fractional order differential equations (FODEs), various methods are used, including the fractional vibrational iteration method, the homotopy perturbation method, the exp-function method, the homotopy analysis method, the Adomian decomposition method, the adaptive finite element method, the sinc-collocation method and the residual power series method (RPSM) and other methods. The most often used techniques can be found in references [11], [12], [13], [14], [15], [16], [17] and [18]. The RPSM has been used to solve analytically a wide number of major models of linear and nonlinear equations that have appeared in numerous engineering and science fields. In a different evolution, the Laplace residual power series method (LRPSM), which is established in 2020, see [19] and [20], is created by combining the RPSM and the Laplace transform. The RPSM is further developed in this article by combining with the ARA transform (ARAT), [21], [22], [23], [24] and [25]. The advantage of the current method, the ARA residual power series method (ARA-RPSM), is that it is quick, requiring little computer memory, and not being influenced by computational round off errors. This paper is structured as follows: In the next section, we go through several definitions, concepts and properties associated to the ARAT and the fractional derivatives. In Section 3, the ARA- RPSM is used to formulate solutions of nonlinear FODEs. Section 4 illustrates how the current approach has been used to explore and solve several fractional physical equations. Finally, a summary of our findings appears in the conclusion section.

2 Materials and Methods

This section provides a definition of the Caputo fractional derivative. Concepts and properties associated with the ARA-RPSM are also supplied.

Definition 1. *The Caputo fractional derivative of order β, of the function $Q(x,t)$, with respect to the variable t, is given by*

$$D_t^{\beta} Q(x,t) = J_t^{m-\beta} D_t^m Q(x,t), 0 < m-1 < \beta \leq m,$$

where $m \in \mathbb{N}$, $D_t^m = \frac{\partial^m}{\partial t^m}$, and

$$J_t^{\gamma} Q(x,t) = \begin{cases} \frac{1}{\Gamma(\gamma)} \int_0^t (t-\tau)^{\gamma-1} Q(x,\tau) d\tau, t > \tau > 0, \ \gamma > 0, \\ Q(x,t), \gamma = 0. \end{cases}$$

* Corresponding author e-mail: aliaaburqan@zu.edu.jo

Definition 2. *[21] The ARAT of the continuous function $Q(x,t)$ of order n, for the variable t >, is given by*

$$\mathcal{G}_n[Q(x,t)](r) = r\int_0^\infty t^{n-1}e^{-rt}Q(x,t)dt, r>0.$$

Several of the ARAT properties that are essential in our analysis are covered in the next lemma.

Lemma 1. *[22], [23] If $Q(x,t)$ is a continuous function, then*

1. $lim_{r\to\infty} r\mathcal{G}_2[Q(x,t)] = Q(x,0),\ x\in I,\ r>0.$
2. $\mathcal{G}_2\left[D_t^\beta Q(x,t)\right] = r^\beta \mathcal{G}_2[Q(x,t)] - \beta r^{\beta-1}\mathcal{G}_1[Q(x,t)] + (\beta-1)r^{\beta-1}Q(x,0),\ r>0,\ 0<\beta\le 1.$
3. $\mathcal{G}_1\left[D_t^{n\beta}Q(x,t)\right](r) = r^{n\beta}\mathcal{G}_1[Q(x,t)](r) - \Sigma_{k=0}^n r^{(n-k)\beta}D_t^{k\beta}Q(x,0),\ r>0,\ 0<\beta\le 1.$
4. $\mathcal{G}_2\left[t^\beta\right] = \frac{\Gamma(\beta+2)}{r^{\beta+1}},\ r>0,\ \beta>0.$

Theorem 1. *[23] Suppose that the ARAT of the continuous function $Q(x,t)$ for the variable t exists and has the fractional series representation*

$$\mathcal{G}_2[Q(x,t)](r) = \Sigma_{n=0}^\infty \frac{\ell_n(x)}{r^{n\beta+1}}, r>0, 0<\beta\le 1. \quad (1)$$

Then

$$\ell_n(x) = (n\beta+1)D_t^{n\beta}Q(x,0). \quad (2)$$

Remark 1.

i. The j^{th} truncated series of the fractional representation (1) is

$$\mathcal{G}_2[Q(x,t)]_j(r) = \Sigma_{n=0}^j \frac{\ell_n(x)}{r^{n\beta+1}}. \quad (3)$$

ii. If $\mathcal{G}_2[Q(x,t)](r)$ has the fractional representation (1), then $\mathcal{G}_1[Q(x,t)](r)$ can be expressed as

$$\mathcal{G}_1[Q(x,t)](r) = \Sigma_{n=0}^\infty \frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \quad (4)$$

and the j^{th} truncated series is given by

$$\mathcal{G}_1[Q(x,t)]_j(r) = \Sigma_{n=0}^j \frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}. \quad (5)$$

iii. The inverse ARAT of the fractional representation (1) is given by

$$Q(x,t) = \mathcal{G}_2^{-1}\left[\Sigma_{n=0}^\infty{}^\infty \frac{\ell_n(x)}{r^{n\beta+1}}\right](t) = \Sigma_{n=0}^\infty \frac{D_t^{n\beta}Q(x,0)}{\Gamma(n\beta+1)}t^{n\beta}. \quad (6)$$

The next theorem, depending on the relationship between $\mathcal{G}_1[Q(x,t)](r)$ and $\mathcal{G}_2[Q(x,t)](r)$ and the characteristics of Taylor's series, includes the convergence conditions of the series representation introduced in Theorem 1.

Theorem 2. *Assume that $Q(x,t)$ is continuous on $I\times[0,\mu]$ where the ARAT for the variable t exists. Let $\mathcal{G}_1[Q(x,t)](r)$ has the expansion*

$$\mathcal{G}_1[Q(x,t)](r) = \Sigma_{n=0}^\infty \frac{D_t^{(n\beta)}Q(x,0)}{r^{n\beta}}.$$

If $|\mathcal{G}_1\left[D_t^{(n+1)\beta}Q(x,t)\right](r)| \le L(x)$ on $0<r\le d$, then the remainder $R_n(x,r)$ satisfies

$$|R_n(x,r)| \le \frac{L(x)}{r^{(n+1)\beta}}, x\in I, 0<r\le d.$$

Proof. Assume that $\mathcal{G}_1\left[D_t^{k\beta}Q(x,t)\right](r)$ exists on $0<r\le d$ for $k=0,\dots,n$. Then

$$R_n(x,r) = \mathcal{G}_1[Q(x,t)](r) - \Sigma_{k=0}^n \frac{D_t^{k\beta}Q(x,0)}{r^{k\beta}}.$$

Multiplying the last equation by $r^{(n+1)\beta}$, part (3) of Lemma 1 yields that

$$r^{(n+1)\beta}R_n(x,r) = r^{(n+1)\beta}\mathcal{G}_1[Q(x,t)](r) - \Sigma_{k=0}^n \frac{D_t^{k\beta}Q(x,0)}{r^{k\beta}}r^{(n+1)\beta}$$

$$= \mathcal{G}_1\left[D_t^{(n+1)\beta}Q(x,t)\right](r).$$

Thus,

$$|r^{(n+1)\beta}R_n(x,r)| = |\mathcal{G}_1\left[D_t^{(n+1)\beta}Q(x,t)\right](r)| \le L(x).$$

This yields that

$$|R_n(x,r)| \le \frac{L(x)}{r^{(n+1)\beta}}, 0<r\le d.$$

The objective of the ARA-RPSM is to find the power series solutions of FODEs related to physical phenomena. The following nonlinear differential equation will be used to illustrate the procedure of the ARA-RPSM

$$D_t^\beta Q(x,t) - N(Q) - L(Q) = 0 \quad (7)$$

subject to the initial condition

$$Q(x,0) = H(x), \quad (8)$$

where $N(Q)$ is a nonlinear term and $L(Q)$ is a linear term, and D_t^β denotes the Caputo derivative of order β, $0<\beta\le 1$, $t\ge 0$.

Applying the ARAT of order two on equation (7), we obtain

$$\mathscr{G}_2\left[D_t^\beta Q(x,t)\right](r)-\mathscr{G}_2\left[N(Q)\right](r)-\mathscr{G}_2\left[L(Q)\right](r)=0. \tag{9}$$

Using Lemma 1 part (2) and the initial condition (8), equation (9) becomes

$$\begin{aligned}\mathscr{G}_2\left[Q(x,t)\right](r)-\frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right](r)+\frac{\beta-1}{r}H(x)\\ -\frac{1}{r^\beta}\mathscr{G}_2\left[N(Q)\right](r)-\frac{1}{r^\beta}\mathscr{G}_2\left[L(Q)\right](r)=0.\end{aligned} \tag{10}$$

Assume that the ARA-residual power series solution (ARA-RPSS) of equation (10) is expanded as follows,

$$\mathscr{G}_1\left[Q(x,t)\right](r)=\Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}. \tag{11}$$

$$\mathscr{G}_2\left[Q(x,t)\right](r)=\Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{r^{(n\beta+1)}}. \tag{12}$$

The fact $lim_{r\to\infty}r\mathscr{G}_2\left[Q(x,t)\right](r)=Q(x,0)$ produces $\ell_0(x)=H(x)$. Hence, the ARA-RPSS of equation (10) can be written as,

$$\mathscr{G}_1\left[Q(x,t)\right](r)=H(x)+\Sigma_{n=1}^{\infty}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{13}$$

$$\mathscr{G}_2\left[Q(x,t)\right](r)=\frac{H(x)}{r}+\Sigma_{n=1}^{\infty}\frac{\ell_n(x)}{r^{(n\beta+1)}}. \tag{14}$$

Therefore, the j^{th} truncated series of equations (13) and (14) have the following forms

$$\mathscr{G}_1\left[Q(x,t)\right]_j(r)=H(x)+\Sigma_{n=1}^{j}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{15}$$

$$\mathscr{G}_2\left[Q(x,t)\right]_j(r)=\frac{H(x)}{r}+\Sigma_{n=1}^{j}\frac{\ell_n(x)}{r^{(n\beta+1)}}. \tag{16}$$

Now, define the ARA-residual function and the j^{th} ARA-residual function of equation (10), respectively as follows

$$\begin{aligned}\mathscr{G}_2Res(x,r)=\;&\mathscr{G}_2\left[Q(x,t)\right](r)-\frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right](r)\\ &+\frac{(\beta-1)}{r}H(x)\;-\frac{1}{r^\beta}\mathscr{G}_2\left[N(Q)\right](r)-\frac{1}{r^\beta}\mathscr{G}_2\left[L(Q)\right](r),\end{aligned} \tag{17}$$

$$\begin{aligned}\mathscr{G}_2Res_j(x,r)=\mathscr{G}_2\left[Q(x,t)\right]_j(r)-\frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right]_j(r)+\frac{(\beta-1)}{r}H(x)\\ -\frac{1}{r^\beta}\mathscr{G}_2\left[N(Q)\right]_j(r)-\frac{1}{r^\beta}\mathscr{G}_2\left[L(Q)\right]_j(r),\; j=2,3,\ldots\end{aligned} \tag{18}$$

Now, we introduce some facts that are necessary to get the ARA-RPSS.

- $\mathscr{G}_2Res(x,r)=0,\ x\in I,\ r>0.$
- $lim_{r\to\infty}\mathscr{G}_2Res_j(x,r)=\mathscr{G}_2Res(x,r),\ x\in I,\ r>0.$
- $lim_{r\to\infty}r\mathscr{G}_2Res(x,r)=lim_{r\to\infty}r\mathscr{G}_2Res_j(x,r)=0,$ $x\in I,\ r>0.$
- $lim_{r\to\infty}r^{j\beta+1}\mathscr{G}_2Res(x,r)=lim_{r\to\infty}r^{j\beta+1}\mathscr{G}_2Res_j(x,r)$ $=0,\ x\in I,\ r>0.$

To determine the coefficients $\ell_n(x), n\geq 2$, substitute $\mathscr{G}_1\left[Q(x,t)\right]_j,\mathscr{G}_2\left[Q(x,t)\right]_j$ into equation (18), multiply both sides by $r^{j\beta+1}$, $j=2,3,\cdots$, then solve the equations

$$lim_{r\to\infty}r^{j\beta+1}\mathscr{G}_2Res_j(x,r)=0,\ j=2,3,\cdots.$$

Finally, after substituting the coefficients in the series solution (12), operate the inverse ARAT of order two to the resulting series to obtain the solution of the initial value problem (7), (8).

3 Results and Discussion

Three examples connected to physical phenomena are given in this section to illustrate the effectiveness, precision, and simplicity of the ARA-RPSM.

Example 1. Consider the radioactive decay FODE

$$D_t^\beta W(t)=-\mu W(t),\ 0<\beta\leq 1, \tag{19}$$

subject to the initial condition

$$W(0)=W_\circ. \tag{20}$$

Now, applying the procedures of ARA-RPSM as described in Section 2, we obtain

$$\mathscr{G}_2\left[D_t^\beta W(t)\right](r)=-\mu\mathscr{G}_2\left[W(t)\right](r). \tag{21}$$

According to Lemma 1 part (2) and the initial condition (20), equation (21) can be written on the form

$$\begin{aligned}\mathscr{G}_2\left[W(t)\right](r)-\frac{\beta}{r}\mathscr{G}_1\left[W(t)\right](r)+\frac{(\beta-1)}{r}W_\circ\\ +\frac{\mu}{r^\beta}\mathscr{G}_2\left[W(t)\right](r)=0.\end{aligned} \tag{22}$$

Suppose that the expansions of the solution to equation (22) are as follows,

$$\mathscr{G}_1\left[W(t)\right](r)=\Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{23}$$

$$\mathscr{G}_2\left[W(t)\right](r)=\Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{r^{n\beta+1}}. \tag{24}$$

According to the given initial condition in equation (20) and using Lemma 1 part (2), the j^{th}-truncated series of (23) and (24) can be written as

$$\mathcal{G}_1[W(t)]_j(r) = W_\circ + \Sigma_{n=1}^{j} \frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \quad (25)$$

$$\mathcal{G}_2[W(t)]_j(r) = \frac{W_\circ}{r} + \Sigma_{n=1}^{j} \frac{\ell_n(x)}{r^{n\beta+1}}. \quad (26)$$

For the purpose of determining the series expansions' coefficients in equations (25) and (26), we define the ARA-residual function of equation (22) as

$$\begin{aligned}\mathcal{G}_2Res(r) = \mathcal{G}_2[W(t)](r) - \frac{\beta}{r}\mathcal{G}_1[W(t)](r) \\ + \frac{(\beta-1)}{r}W_\circ + \frac{\mu}{r^\beta}\mathcal{G}_2[W(t)](r),\end{aligned} \quad (27)$$

and the j^{th} ARA-residual function is

$$\begin{aligned}\mathcal{G}_2Res_j(r) =& \mathcal{G}_2[W(t)]_j(r) - \frac{\beta}{r}\mathcal{G}_1[W(t)]_j(r) + \frac{(\beta-1)}{r}W_\circ \\ &+ \frac{\mu}{r^\beta}\mathcal{G}_2[W(t)]_j(r), \; j = 1,2,3,\cdots.\end{aligned} \quad (28)$$

The first unknown coefficient $\ell_1(x)$ is determined by substituting $\mathcal{G}_1[W(t)]_1(r)$ and $\mathcal{G}_2[W(t)]_1(r)$ into $\mathcal{G}_2Res_1(r)$ to obtain

$$\begin{aligned}\mathcal{G}_2Res_1(r) = \frac{W_\circ}{r} + \frac{\ell_1(x)}{r^{\beta+1}} - \frac{\beta}{r}\left(W_\circ + \frac{\ell_1(x)}{(\beta+1)r^\beta}\right) \\ + \frac{\beta-1}{r}W_\circ + \frac{\mu}{r^\beta}\left(\frac{W_\circ}{r} + \frac{\ell_1(x)}{r^{\beta+1}}\right).\end{aligned} \quad (29)$$

After simple computations, we get

$$\mathcal{G}_2Res_1(r) = \frac{\ell_1(x)}{r^{\beta+1}} - \frac{\beta}{r}\left(\frac{\ell_1(x)}{(\beta+1)r^\beta}\right) + \frac{\mu}{r^\beta}\left(\frac{W_\circ}{r} + \frac{\ell_1(x)}{r^{\beta+1}}\right). \quad (30)$$

By taking the limit as $r \to \infty$ after multiplying equation (30) by $r^{\beta+1}$, the fact $lim_{r\to\infty}\left(r^{\beta+1}\mathcal{G}_2Res_1(r)\right) = 0$ yields that

$$\ell_1(x) = -\mu(\beta+1)W_\circ. \quad (31)$$

In a similar manner, to find the next coefficient $\ell_2(x)$, substitute $\mathcal{G}_1[W(t)]_2(r) = \ell_0(x) + \frac{\ell_1(x)}{(\beta+1)r^\beta} + \frac{\ell_2(x)}{(2\beta+1)r^{2\beta}}$ and $\mathcal{G}_2[W(t)]_2(r) = \frac{\ell_0(x)}{r} + \frac{\ell_1(x)}{r^{\beta+1}} + \frac{\ell_2(x)}{r^{2\beta+1}}$ into $\mathcal{G}_2Res_2(r)$ to obtain

$$\begin{aligned}\mathcal{G}_2Res_2(r) = \frac{\ell_1(x)}{r^{\beta+1}} + \frac{\ell_2(x)}{r^{2\beta+1}} - \frac{\beta}{r}\left(\frac{\ell_1(x)}{(\beta+1)r^\beta} + \frac{\ell_2(x)}{(2\beta+1)r^{2\beta}}\right) \\ + \frac{\mu}{r^\beta}\left(\frac{\ell_0(x)}{r} + \frac{\ell_1(x)}{r^{\beta+1}} + \frac{\ell_2(x)}{r^{2\beta+1}}\right).\end{aligned} \quad (32)$$

Again, by taking the limit as $r \to \infty$ after multiplying equation (32) by $r^{2\beta+1}$, then the fact that $lim_{r\to\infty}\left(r^{2\beta+1}\mathcal{G}_2Res_2(r)\right) = 0$, yields that

$$\ell_2(x) = \mu^2(2\beta+1)W_\circ. \quad (33)$$

If we proceed in the same manner, we arrive to the conclusion that the j^{th} coefficient of the series (25) and (26) has the following general form

$$\ell_j(x) = (-\mu)^j(j\beta+1)W_\circ, \; j = 1,2,\cdots. \quad (34)$$

According to what was presented, the series solution of equation (22) is

$$\mathcal{G}_2[W(t)]_2(r) = \frac{W_\circ}{r} - \mu\frac{(\beta+1)W_\circ}{r^{\beta+1}} + \mu^2\frac{(2\beta+1)W_\circ}{r^{2\beta+1}} + \cdots \quad (35)$$

So, the series solution of the radioactive decay FODE (19) can be obtained by transforming the solution in equation (35), using the inverse ARAT of order two to get

$$\begin{aligned}W(t) = W_\circ - \mu\frac{W_\circ}{\Gamma(\beta+1)}t^\beta + \mu^2\frac{W_\circ}{\Gamma(2\beta+1)}t^{2\beta} \\ - \mu^3\frac{W_\circ}{\Gamma(3\beta+1)}t^{3\beta} + \cdots\end{aligned} \quad (36)$$

$$\begin{aligned}W(t) = W_\circ(1 - \frac{\mu}{\Gamma(\beta+1)}t^\beta + \frac{\mu^2}{\Gamma(2\beta+1)}t^{2\beta} \\ - \frac{\mu^3}{\Gamma(3\beta+1)}t^{3\beta} + \cdots),\end{aligned} \quad (37)$$

which is equivalent to

$$W(t) = W_\circ E_\beta(-\mu t^\beta), \quad (38)$$

where $E_\beta(-\mu t^\beta) = \Sigma_{k=0}^{\infty}\frac{(-\mu t^\beta)^k}{\Gamma(k\beta+1)}$ is the Mittag-Leffler function. In case $\beta = 1$, the solution will be

$$W(t) = W_\circ e^{-\mu t} \quad (39)$$

which coincides with exact solution obtained in [26] as in Figure 1.

Example 2. Consider the Rosenau-Hyman FODE

$$\begin{aligned}D_t^\beta Q(x,t) - Q(x,t)Q_{xxx}(x,t) - Q(x,t)Q_x(x,t) \\ - 3Q_x(x,t)Q_{xx}(x,t) = 0, \; t > 0, \; 0 < \beta \leq 1,\end{aligned} \quad (40)$$

subject to the initial condition

$$Q(x,0) = -\frac{8c}{3}cos^2\left(\frac{x}{4}\right). \quad (41)$$

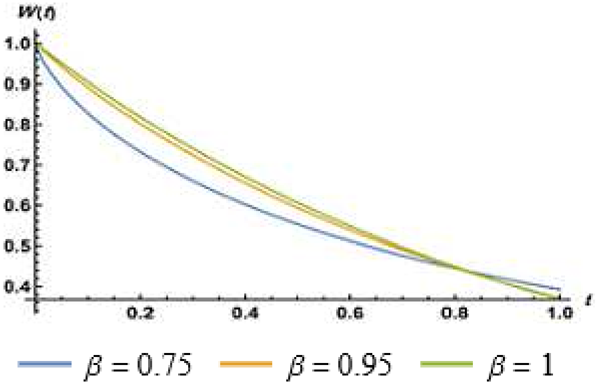

Fig. 1: The ARA-RPSS plot of equation (19) with $W_\circ = 1, \mu = 1$ for different values of β.

Operating ARAT of order two on equation (40), we get

$$\begin{aligned}\mathscr{G}_2\left[D_t^\beta Q(x,t)\right] &- \mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-3\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &=0.\end{aligned} \tag{42}$$

Which is equivalent to

$$\begin{aligned}r^\beta\mathscr{G}_2\left[Q(x,t)\right] &- \beta r^{\beta-1}\mathscr{G}_1\left[Q(x,t)\right]+(\beta-1)r^{\beta-1}Q(x,0)\\ &-\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-3\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &=0.\end{aligned} \tag{43}$$

Simplifying equation (43), we have

$$\begin{aligned}\mathscr{G}_2\left[Q(x,t)\right] &- \frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right]+\frac{(\beta-1)}{r}Q(x,0)\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\frac{3}{r^\beta}\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &=0.\end{aligned} \tag{44}$$

Consider expanding the ARA-RPSS of equation (44) as follows,

$$\mathscr{G}_1\left[Q(x,t)\right](r) = \Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{45}$$

$$\mathscr{G}_2\left[Q(x,t)\right](r) = \Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{r^{n\beta+1}}. \tag{46}$$

The j^{th} truncated series of the expansions (45) and (46) are

$$\mathscr{G}_1\left[Q(x,t)\right]_j(r) = \Sigma_{n=0}^{j}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{47}$$

$$\mathscr{G}_2\left[Q(x,t)\right]_j(r) = \Sigma_{n=0}^{j}\frac{\ell_n(x)}{r^{n\beta+1}}. \tag{48}$$

By taking the limit as $r\to\infty$ after multiplying both sides of equation (48) by r, we get

$$lim_{r\to\infty}r\mathscr{G}_2\left[Q(x,t)\right]_j(r) = \ell_0(x)+lim_{r\to\infty}\Sigma_{n=1}^{j}\frac{\ell_n(x)}{r^{n\beta}}.$$

Using the fact

$$lim_{r\to\infty}r\mathscr{G}_2\left[Q(x,t)\right]_j(r) = Q(x,0),$$

and the initial condition in equation (41), we get

$$\ell_0(x) = -\frac{8c}{3}cos^2\left(\frac{x}{4}\right).$$

Hence, the series representations (47) and (48) become

$$\mathscr{G}_1\left[Q(x,t)\right]_j(r) = -\frac{8c}{3}cos^2\left(\frac{x}{4}\right)+\Sigma_{n=1}^{j}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{49}$$

$$\mathscr{G}_2\left[Q(x,t)\right]_j(r) = -\frac{8c}{3r}cos^2\left(\frac{x}{4}\right)+\Sigma_{n=1}^{j}\frac{\ell_n(x)}{r^{n\beta+1}}. \tag{50}$$

The ARA-residual function of equation (44) is now given by

$$\begin{aligned}\mathscr{G}_2Res(x,r) = \mathscr{G}_2\left[Q(x,t)\right] &- \frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right]+\frac{(\beta-1)}{r}\ell_0(x)\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right]\\ &-\frac{3}{r^\beta}\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right].\end{aligned} \tag{51}$$

The j^{th} ARA-residual function is

$$\begin{aligned}\mathscr{G}_2Res_j(x,r) = \mathscr{G}_2\left[Q(x,t)\right]_j &- \frac{\beta}{r}\mathscr{G}_1\left[Q(x,t)\right]_j+\frac{(\beta-1)}{r}\ell_0(x)\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\right]\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\right]\\ &-\frac{3}{r^\beta}\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]_j\right]\right].\end{aligned} \tag{52}$$

To determine the first unknown coefficient $\ell_1(x)$ in equation (49) and equation (50), we substitute $\mathscr{G}_1[Q(x,t)]_1(r)$ and $\mathscr{G}_2[Q(x,t)]_1(r)$ into $\mathscr{G}_2 Res_1(x,r)$ to obtain,

$$\begin{aligned}\mathscr{G}_2 Res_1(x,r) &= \mathscr{G}_2[Q(x,t)]_1 - \frac{\beta}{r}\mathscr{G}_1[Q(x,t)]_1 + \frac{(\beta-1)}{r}\ell_0(x)\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\,\partial_{xxx}\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\right]\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\,\partial_x\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\right]\\ &-\frac{3}{r^\beta}\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\,\partial_{xx}\mathscr{G}_2^{-1}[\mathscr{G}_2[Q(x,t)]_1]\right].\end{aligned} \quad (53)$$

Substituting $\mathscr{G}_1[Q(x,t)]_1(r) = \ell_0(x) + \frac{\ell_1(x)}{(\beta+1)r^\beta}$ and $\mathscr{G}_2[Q(x,t)]_1(r) = \frac{\ell_0(x)}{r} + \frac{\ell_1(x)}{r^{\beta+1}}$ in equation (53). After simple computations, we have

$$\mathscr{G}_2 Res_1(x,r) = \frac{\ell_1(x)}{r^{\beta+1}} - \frac{\beta}{(\beta+1)}\frac{\ell_1(x)}{r^{\beta+1}}$$

$$\begin{aligned}&-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\partial_{xxx}\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\right]\\ &-\frac{1}{r^\beta}\mathscr{G}_2\left[\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\partial_{x}\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\right]\\ &-\frac{3}{r^\beta}\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\partial_{xx}\mathscr{G}_2^{-1}\left[\frac{\ell_0(x)}{r}+\frac{\ell_1(x)}{r^{\beta+1}}\right]\right].\end{aligned} \quad (54)$$

Thus,

$$\begin{aligned}\mathscr{G}_2 Res_1(x,r) &= \frac{\ell_1(x)}{r^{\beta+1}} - \frac{\beta}{(\beta+1)}\frac{\ell_1(x)}{r^{\beta+1}} - \frac{\ell_0(x)\ell_0'''(x)}{r^{\beta+1}}\\ &-\frac{\ell_0(x)\ell_1'''(x)}{r^{2\beta+1}} - \frac{\ell_0'''(x)\ell_1(x)}{r^{2\beta+1}} - \frac{\Gamma(2\beta+2)\ell_1(x)\ell_1'''(x)}{\Gamma^2(\beta+2)r^{3\beta+1}}\\ &-\frac{\ell_0(x)\ell_0'(x)}{r^{\beta+1}} - \frac{\ell_0(x)\ell_1'(x)}{r^{2\beta+1}} - \frac{\ell_0'(x)\ell_1(x)}{r^{2\beta+1}}\\ &-\frac{\Gamma(2\beta+2)\ell_1(x)\ell_1'(x)}{\Gamma^2(\beta+2)r^{3\beta+1}} - 3\frac{\ell_0'(x)\ell_0''(x)}{r^{\beta+1}}\\ &-3\left(\frac{\ell_0'(x)\ell_1''(x)}{r^{2\beta+1}} + \frac{\ell_0''(x)\ell_1'(x)}{r^{2\beta+1}} + \frac{\Gamma(2\beta+2)\ell_1'(x)\ell_1''(x)}{\Gamma^2(\beta+2)r^{3\beta+1}}\right).\end{aligned} \quad (55)$$

By taking the limit as $r \to \infty$ after multiplying equation (55) by $r^{\beta+1}$, the fact $lim_{r\to\infty}\left(r^{\beta+1}\mathscr{G}_2 Res_1(r)\right) = 0$, yields that

$$\ell_1(x) = (\beta+1)\left(\ell_0(x)\ell_0'''(x) + \ell_0(x)\ell_0'(x) + \ell_0'(x)\ell_0''(x)\right). \quad (56)$$

Substituting $\ell_0(x) = -\frac{8c}{3}cos^2\frac{x}{4}$ in equation (56), we get

$$\ell_1(x) = -(\beta+1)\frac{2c^2}{3}sin\left(\frac{x}{2}\right).$$

Similarly, to find $\ell_2(x)$, we substitute $\mathscr{G}_1[Q(x,t)]_2(r) = \ell_0(x) + \frac{\ell_1(x)}{(\beta+1)r^\beta} + \frac{\ell_2(x)}{(2\beta+1)r^{2\beta}}$ and $\mathscr{G}_2[Q(x,t)]_2(r) = \frac{\ell_0(x)}{r} + \frac{\ell_1(x)}{r^{\beta+1}} + \frac{\ell_2(x)}{r^{2\beta+1}}$ into $\mathscr{G}_2 Res_2(s)$ and solve the equation $lim_{r\to\infty}r^{2\beta+1}\mathscr{G}_2 Res_2(x,r) = 0$ to get

$$\begin{aligned}\ell_2(x) = (2\beta+1)[&\ell_0(x)\ell_0'''(x) + \ell_1(x)\ell_0'''(x) + \ell_1(x)\ell_1'(x) +\\ &\ell_0(x)\ell_0'(x) + \ell_1(x)\ell_0'(x) + \ell_1(x)\ell_1'(x) + 3(\ell_1'(x)\ell_1''(x) +\\ &\ell_1'(x)\ell_0''(x) + \ell_1'(x)\ell_1''(x))].\end{aligned} \quad (57)$$

Substituting $\ell_0(x) = -\frac{8c}{3}cos^2\left(\frac{x}{4}\right)$ and $\ell_1(x) = -(\beta+1)\frac{2c^2}{3}sin\left(\frac{x}{2}\right)$ in (2), we get

$$\ell_2(x) = (2\beta+1)\left(\frac{c^3}{3}cos\left(\frac{x}{2}\right)\right).$$

Repeating the same arguments as before, we get the solution of equation (44) as

$$\begin{aligned}\mathscr{G}_2[Q(x,t)] = &-\frac{8c}{3r}cos^2\left(\frac{x}{4}\right) - \frac{2c^2(\beta+1)}{3r^{\beta+1}}sin\left(\frac{x}{2}\right)\\ &+\frac{(2\beta+1)}{r^{2\beta+1}}\left(\frac{c^3}{3}cos\left(\frac{x}{2}\right)\right) + \cdots\end{aligned} \quad (58)$$

Applying the inverse ARAT on equation (58), the solution of the problem (40) and (41), is obtained as follows

$$\begin{aligned}Q(x,t) = &-\frac{8c}{3}cos^2\left(\frac{x}{4}\right) - \frac{2c^2}{3}sin\left(\frac{x}{2}\right)\frac{t^\beta}{\Gamma(\beta+1)}\\ &+\frac{c^3}{3}cos\left(\frac{x}{2}\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \cdots.\end{aligned} \quad (59)$$

It is worth mentioning that when $\beta = 1$, we get from equation (59) that

$$Q(x,t) = -\frac{8c}{3}cos^2\left(\frac{x}{4}\right) - \frac{2c^2}{3}sin\left(\frac{x}{2}\right)t + \frac{c^3}{6}cos\left(\frac{x}{2}\right)t^2 + \cdots \quad (60)$$

which coincides with the exact solution of the given problem. Moreover, it is completely confirmed by the solution found in [26], [27] and [28], as in Figure 2.

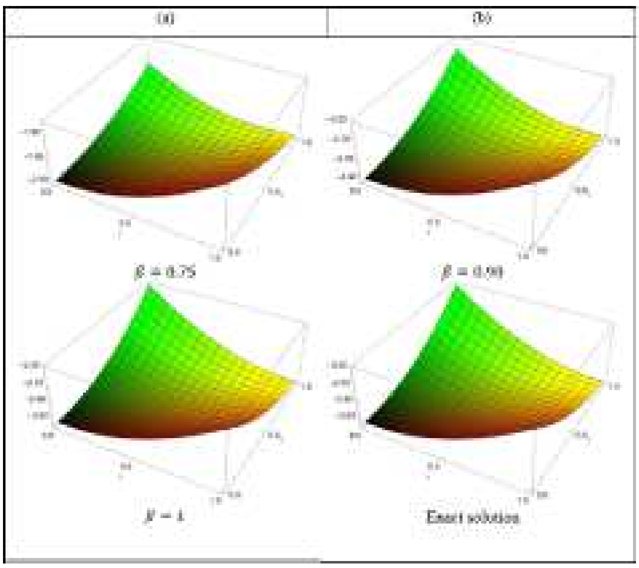

Figure 2. The solution of equation (40) for different values of β with $c = 1$.

Example 3. Consider the backward Kolmogorov FODE,

$$D_t^{\beta}Q(x,t) - (x+1)Q_x(x,t) - x^2e^tQ_{xx}(x,t) = 0, \quad t>0,\ 0<\beta\leq 1, \tag{61}$$

with the initial condition

$$Q(x,0) = x+1, x\in\mathbb{R}. \tag{62}$$

When we apply the ARAT on equation (61), we obtain

$$\begin{aligned} r^{\beta}\mathscr{G}_2\left[Q(x,t)\right] - \beta r^{\beta-1}\mathscr{G}_1\left[Q(x,t)\right] + (\beta-1)r^{\beta-1}Q(x,0) \\ -(x+1)\mathscr{G}_2\left[\partial_x\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right] \\ -x_2\mathscr{G}_2\left[e^t\partial_{xx}\mathscr{G}_2^{-1}\left[\mathscr{G}_2\left[Q(x,t)\right]\right]\right] = 0. \end{aligned} \tag{63}$$

Suppose that the ARA-RPSS of equation (63) has the Laurent expansions as follows

$$\mathscr{G}_1\left[Q(x,t)\right](r) = \Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{(n\beta+1)r^{n\beta}}, \tag{64}$$

$$\mathscr{G}_2\left[Q(x,t)\right](r) = \Sigma_{n=0}^{\infty}\frac{\ell_n(x)}{r^{n\beta+1}}. \tag{65}$$

Similar to the previous applications using ARA-RPSM, we get

$$\ell_0(x) = x+1, \ell_1(x) = (\beta+1)(x+1), \ell_2(x) = (2\beta+1)(x+1),$$

and so on ... .

Thus, the series solution of equation (63) is the following,

$$\mathscr{G}_2\left[Q(x,t)\right] = (x+1)\left(1+\frac{(\beta+1)}{r^{\beta+1}}+\frac{(2\beta+1)}{r^{2\beta+1}}+...\right) \tag{66}$$

So, the series solution of the backward Kolmogorov FODE (61) is obtained by transforming the solution in equation (66) using the inverse ARAT of order two. Therefore, the ARA-RPSS has the following expression,

$$Q(x,t) = (x+1)\left(1+\frac{t^{\beta}}{\Gamma(\beta+1)}+\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\frac{t^{3\beta}}{\Gamma(3\beta+1)}+...\right), \tag{67}$$

which is equivalent to

$$Q(x,t) = (x+1)E_{\beta}(t^{\beta}). \tag{68}$$

For $\beta = 1$ the solution will be

$$Q(x,t) = (x+1)e^t,$$

that is the exact solution of the given problem which is completely confirmed by the solution found in [29], [30] and [31].

4 Conclusion

In this article, a novel technique to determine accurate solutions of physical phenomena in the fractional order was successfully implemented. The results attained demonstrate great agreement with both the exact and other known approaches. The ARA-RPSM performance demonstrates its efficiency, accuracy, and ability for obtaining analytical and numerical solutions to a wide range of fractional physical phenomena that occur in engineering and physics. In later work, we plan to solve fractional integral equations both linearly and nonlinearly using the ARA-RPSM.

Acknowledgement

The author thanks the editor and referees for their insightful criticism, which helped to improve the paper's final version.

References

[1] K. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*, Wiley, (1993).

[2] K. Oldham and J. Spanier, *The fractional calculus theory and applications of differentiation and integration to arbitrary order*, Elsevier, (1974).

[3] J. Sabatier, O. Agrawal and J. Machado, *Advances in fractional calculus* (Vol. 4, No. 9), Dordrecht: Springer, (2007).

[4] R. Magin, C. Ingo, L. Colon-Perez, W. Triplett and T. Mareci, Characterization of anomalous diffusion in porous biological tissues using fractional order derivatives and entropy, *Microporous and Mesoporous Materials*, **178**, 39-43 (2013).

[5] S. Cifani and E. Jakobsen, Entropy solution theory for fractional degenerate convection- diffusion equations, *In Annales de l'IHP Analyse non linéaire*, **28(3)**, 413-441 (2011).

[6] S. Zhang and H. Zhang, Fractional sub-equation method and its applications to nonlinear fractional PDEs, *Physics Letters A*, **375(7)**, 1069-1073 (2011).

[7] S. Hasan, M. Al-Smadi, A. El-Ajou, S. Momani, S. Hadid and Z. Al-Zhour, Numerical approach in the Hilbert space to solve a fuzzy Atangana-Baleanu fractional hybrid system, *Chaos Solitons Fractals*, **143**: 110506 (2021).

[8] F. Mainardi, M. Raberto, R. Gorenflo and E. Scalas, Fractional calculus and continuous-time finance II: the waiting-time distribution, *Physica A: Statistical Mechanics and its Applications*, **287(3-4)**, 468-481 (2000).

[9] M. Caputo, Linear models of dissipation whose Q is almost frequency independent, II, *Geophysical Journal International*, **13(5)**, 529-539 (1967).

[10] M. Khandaqji and A. Burqan, Results on sequential conformable fractional derivatives with applications, *J. Comput. Anal. Appl*, **29(6)**, 1115-1125 (2021).

[11] A. El-Ajou, Z. Odibat, S. Momani and A. Alawneh, Construction of analytical solutions to fractional differential equations using homotopy analysis method, *International Journal of Applied Mathematics*, **40(2)**, (2010).
[12] G. Adomian, A review of the decomposition method in applied mathematics, *Journal of mathematical analysis and applications*, **135(2)**, 501-544 (1988).
[13] S. Das, Analytical solution of a fractional diffusion equation by variational iteration method. *Computers and Mathematics with Applications*, **57(3)**, 483-487 (2009).
[14] H. J. He, Approximate analytical solution for seepage flow with fractional derivatives porous media, *Computer Methods in Applied Mechanics and Engineering*, **167(1-2)**, 57-68 (1998).
[15] S. Momani and Z. Odibat, Comparison between the homotopy perturbation method and the variational iteration method for linear fractional partial differential equations, *Computers and Mathematics with Applications*, **54(7-8)**, 910-919 (2007).
[16] M. Shqair, A. El-Ajou and M. Nairat, Analytical solution for multi-energy groups of neutron diffusion equations by a residual power series method, *Mathematics*, **7(7)**: 633 (2019).
[17] Q. Wang, Numerical solutions for fractional KdV-Burgers equation by Adomian decomposition method, *Applied Mathematics and Computation*, **182(2)**, 1048-1055 (2006).
[18] G. Ismail, H. Abdl-Rahim, H. Ahmad and Y. Chu, Fractional residual power series method for the analytical and approximate studies of fractional physical phenomena, *Open Physics*, **18(1)**, 799-805 (2020).
[19] T. Eriqat, A. El-Ajou, N. Moa'ath, Z. Al-Zhour and S. Momani, A new attractive analytic approach for solutions of linear and nonlinear neutral fractional pantograph equations, *Chaos, Solitons and Fractals*, **138**, 109957 (2020).
[20] A. El-Ajou, Adapting the Laplace transform to create solitary solutions for the nonlinear time-fractional dispersive PDEs via a new approach, *The European Physical Journal Plus*, **136(2)**, 1-22 (2021).
[21] R. Saadeh, A. Qazza and A. Burqan, A new integral transform: ARA transform and its properties and applications, *Symmetry*, **12(6)**, 925 (2020).
[22] A. Qazza, A. Burqan and R. Saadeh, A new attractive method in solving families of fractional differential equations by a new transform, *Mathematics* , **9(23)**, 30-39 (2021).
[23] A. Burqan, R. Saadeh and A. Qazza, A novel numerical approach in solving fractional neutral pantograph equations via the ARA integral transform, *Symmetry*, **14(1)**, 50 (2021).
[24] A. Burqan, A novel scheme of the ARA transform for solving systems of partial fractional differential equations, *Fractal and Fractional*, **7**, 306 (2023).
[25] A. Burqan, R. Saadeh, A. Qazza and S. Momani, ARA-residual power series method for solving partial fractional differential equations, *Alexandria Engineering Journal*, **62**, 47-62 (2023).
[26] P. Goswami and R. Alqahtani, Solutions of fractional differential equations by Sumudu transform and variational iteration method, *Journal of Nonlinear Science and its Applications*, **9(4)**, 1944-1951 (2016).
[27] A. El-Ajou, O. Abu Arqub, A. Batanieh and I. Hashim, A representation of the exact solution of generalized Lane-Emden equations using new analytical method, *Abstr Appl Anal.*, **2013**, (2013). doi: 10.1155/2013/378593.
[28] R. Molliq and M. Noorani, Solving the fractional Rosenau-Hyman equation via fractional iteration method and Homotopy perturbation method, *Int J Differ Equ.*, **14**, 472030 (2012). doi:10.1155/2012/472030.
[29] J. Biazar, K. Hosseini and P. Gholamin, Homotopy perturbation method Fokker-Planck equation, *Int Math Forum*, **3(19)**, 945–954 (2008).
[30] A. Sadhigi, D. Ganji and Y. Sabzehmeidavi, A study on Fokker-Planck equation by variational iteration method, *Int J Nonlinear Sci.*, **4**, 92–102 (2007).
[31] M. Tatari, M. Dehghan and M. Razzaghi, Application of A domain decomposition method for the Fokker-Planck equation, *Math Comput Model*, **45**, 639–650 (2007).

Aliaa Burqan is an Associate Professor in the Mathematics Department at Zarqa University, Zarqa, Jordan. She is the Dean of the Faculty of Science at Zarqa University since 2018. She holds a PhD in Mathematics from the University of Jordan, Amman, Jordan. Her areas of research interests are functional analysis, matrix analysis, operator theory and fractional Calculus. She has several research papers published in reputed international journals and conferences. Dr. Aliaa is an active member in the organizing committee of all Mathematical Conferences that held at Zarqa University from 2006 up to now and a Chairman of the last three, IACMC 2019, 2022 and 2023. She is also serving as scientific and advisory board member of some educational bodies. During the last few years, she has supervised several Master students and some of them are continuing their PhD study abroad.