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A New Hybrid Root-Finding Algorithm to Solve Transcendental Equations Using Arcsine Function

Srinivasarao Thota[1,], Louai Maghrabi[2], P. Shanmugasundaram[3], Mohammad Kanan[4] and Ala'a Saeb Al-Sherideh[5]*

[1]Department of Mathematics, Amrita School of Physical Sciences, Amrita Vishwa Vidyapeetham, Amaravati, Andhra Pradesh – 522503, India
[2]Department of Software Engineering, College of Engineering, University of Business and Technology, 21448, Jeddah, Saudi Arabia
[3]Department of Mathematics, College of Natural & Computational Sciences, Mizan Tepi University, Mizan Tepi, Ethiopia
[4]Department of Industrial Engineering, College of Engineering, University of Business and Technology, 21448, Jeddah, Saudi Arabia
[5]Department of Cyber Security, Faculty of Information Technology, Zarqa University, Zarqa, Jordan

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Abstract: The objective of this paper is to propose a new hybrid root finding algorithms for solving non-linear equations (NLEs) or transcendental equations (TEs). The proposed algorithm is based on the trigonometrical algorithm using arcsine function to find a root. Several numerical examples are presented to illustrate the proposed algorithms, and comparisons are presented with other existing methods to show efficiency and accuracy. Implementation of the proposed algorithms is presented in a mathematical software tool Maple.

Keywords: Root-finding algorithm, arcsine function, Transcendental equations, Maple.

1 Introduction

The applications of NLEs of the type $f(x) = 0$ arise in numerous branches of pure sciences as well as applied sciences, such as computer science, chemical engineering, physics, etc. Getting the root of transcendental equations is of great importance. In the current scenario, many academicians have focused to solve NLEs numerically/ mathematically as well as logically/ analytically. In the research work of scientists, there are quite more new iterative methods/ algorithms and many hybrid methods existing. These hybrid algorithms are created with help of different techniques, see, for example, [1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 25, 26, 27]. In general, the roots of NLEs or TEs cannot be stated in a particular form or cannot be calculated systematically. The root-finding methods/algorithms offer us to calculate estimations/approximations to the roots; these estimations are stated either as small separate intervals or as floating-point numbers. The concept of creating a hybrid method combining two classic approaches is not new, but it has a long history.

2 Methodologies

In this paper, we develop a new mixed/hybrid root finding algorithm for solving TEs. This algorithm is created with the help of trigonometrical algorithm using arcsine function. Using numerical examples, we show that the proposed algorithms converge faster than the other related methods. The main idea of the proposed hybrid algorithm is based on the regula-falsi method and trigonometrical algorithm. A number of examples are presented to explain the proposed algorithm. The assessments are made to relate the calculated results using the proposed algorithm with other current algorithms/methods to show the efficiency and accuracy. Execution of the proposed algorithms is presented in Maple.

3 Main Results

In this section, we present the proposed hybrid algorithm. The regula-false method gives guarantee the existing of root, and the trigonometrical method gives faster convergent. The iterative formula used in trigonometrical method is as follows, more details about this method can be found in [14]. For $n = 0, 1, 2,\dots$

$$x_{n+1} = x_n\left(1+\arcsin\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)\right), \tag{1}$$

In regula-falsi method [20, 21], we take two initial guesses say a and b such that the product of $f(a)$ and $f(b)$ should be less than zero. The approximate root is calculated by discovering the point of intersection of the straight line combining

*Corresponding author e-mail: srinithota@ymail.com , t_srinivasarao@av.amrita.edu the coordinates $(a, f(a))$ and $(b, f(b))$ with the x-axis. Hence the estimated root can be calculated by the formula,

$$x_r = a - \frac{f(a)(b-a)}{f(b) - f(a)} \tag{2}$$

Now, we have to choose the appropriate interval to compute the second iteration. We have the following possible cases:

- If the product of $f(a)$ and $f(x_r)$ is less than zero, then the root exists in $[a, x_r]$, and set $b = x_r$ to find the second iteration using formula (2).
- If the product of $f(a)$ and $f(x_r)$ is greater than zero, then the root exists in $[x_r, b]$, and set $a = x_r$ to find the second approximate root using (2).
- If $f(x_r)$ is zero, then the required root is x_r and the process is stoped.

3.1 Algorithm

Input: The given function, interval, where the root lies in, $[a, b]$, number of iterations (n).

Output: The approximate root (x), function value ($f(x)$).

i = 0

while i == n **do**

i = i + 1;

$x_{rf} = a - \dfrac{f(a)\ (b - a)}{f(b) - f(a)}$

$x_{n+1} = x_n\left(1 + \arcsin\left(\dfrac{-f(x_n)}{x_n f'(x_n)}\right)\right)$

if $|f(x_i)| < |f(x)|$ and $x_i \in (a, b)$

if $f(x_i) * f(a) < 0$ then $b = x_i$ else $a = x_i$

else if $f(x_{rf}) * f(a) < 0$ then $b = x_{rf}$

else $a = x_{rf}$

end (while)

3.2 Flow-diagram

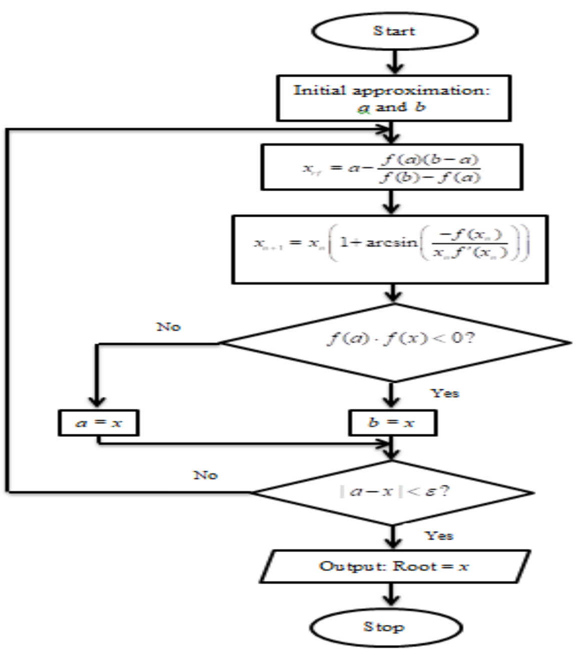

Fig. 1: Flow-chart of the algorithm

3.3 Maple Executions

```
RFARCSIN := proc (a, b, Eq, eps, n)
 local a1, b1, f, c, i, c1;
 i := 0;
 a1 := evalf(a);
 b1 := evalf(b);
 f := unapply(lhs(Eq), x);
 if f(a1) = 0 then
  return a1
 else if f(b1) = 0 then
  return b1
 else if 0 < f(a1)*f(b1) then
  error "Should be f(a)*f(b)<0"
 end if; end if; end if;
 do
  c1 :=(a1*f(b1)-b1*f(a1))/(f(b1)-f(a1));
  c :=c1*(1+arcsin(-f(c1)/(c1*(D(f))(c1))));
  i := i+1;
   if f(c) = 0 or c-a1 < eps or i = n then
     return c
   else if f(a1)*f(c) < 0 then
      b1 := c
   else a1 := c
   end if; end if;
   printf("Iteration %g : x = %g \n", i, c)
  end do
 end proc
```

4 Discussions with Numerical Examples

Example 1: Consider an equation $e^x - 3x - 2 = 0$ with initial approximations a = 2 and b = 3, to illustrate the proposed algorithm, as follows:

Iteration 1: x_{rf} = 2.063006766, x_1 = 2.128613403 and $f(x_1)$ = 0.017366659.

Iteration 2: x_{rf} = 2.125058497, x_2 = 2.125391285 and $f(x_2)$ = 4.63727E-07.

Iteration 3: x_{rf} = 2.12539119, x_3 = 2.125391199 and $f(x_3)$ = 0.

We obtain the required root in three iterations and the required root is 2.125391199.

Example 2: In this example we present the comparisons of the calculations with existing root finding methods. Consider an equation $\sin(x) - x^2 = 0$ with initial guesses a = 0.5 and b = 1. The exact root of the given equation is 0.8767262154. Using various numerical existing algorithms, we compute the root of this equation given in Table 1 up to correct ten decimal places. In the table, BM, NRF, RFM, Steffensen, Halley, EXP, TRIG and PA indicate bisection method, Newton-Raphson method, regula-falsi method, Steffensen's method, Helley's method, exponential method [13], trigonometric method [14] and proposed algorithm respectively.

Table 1: Number iterations required to compute a root with various methods

BM	RFM	NRM	Halley	Steffensen	EXP	TRIG	PA
30	12	5	4	4	4	5	3

Example 3: Recall the equation in Example 1, $e^x - 3x - 2 = 0$ with a = 2 and b = 3 for sample computations using maple, as follows.

```
> RFMARCSIN (2, 3, exp(x)-3*x-2 = 0, 10^(-10), 20);
```

Iteration 1 : x = 2.12861

Iteration 2 : x = 2.12539

Iteration 3 : x = 2.12539

Iteration 4 : x = 2.12539

2.125391198

5 Conclusions

In this paper, we planned a new hybrid root finding algorithms for solving the given NLEs or TEs. The proposed algorithm is mainly based on the arcsine function trigonometrical algorithm. To illustrate the proposed algorithm, we presented several numerical examples and also evaluations with existing methods are presented to show the efficiency and accuracy of the proposed method. Maple implementation is presented with sample computations.

6 Recommendations

This research recommends to compute a real root of a given TEs with fast converging rate. The idea of main result can be used to create new hybrid root-finding algorithms in the relevant research.

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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References

[1] B. Elsayed, A. Hala, G. Abdallah, Novel hybrid algorithms for root determining using advantages of open methods and bracketing methods, *Alexandria Engineering Journal*, **61**, 11579-11588, (2022).

[2] E. Badr, S. Almotairi, A.E. Ghamry, A Comparative Study among New Hybrid Root Finding Algorithms and Traditional Methods. *Mathematics*, **9**, 1306, (2021).

[3] A. Hasan, Numerical Study of Some Iterative Methods for Solving Nonlinear Equations. *Int. J. Eng. Sci. Invent.*, **5**, 1-10 (2016).

[4] L. R. Burden, F.J. Douglas, *Numerical Analysis*, Prindle, Weber & Schmidt, 3rd ed.; Amazon: Seattle, WA, USA, 1 January 1985.

[5] S. Baskar, S.S. Ganesh, *Introduction to Numerical Analysis*, Department of Mathematics, Indian Institute of Technology Bombay, Powai: Mumbai, India, 2016.

[6] C.L. Sabharwal, Blended Root Finding Algorithm Outperforms Bisection and Regula Falsi Algorithms. *Mathematics*, **7(11)**, 1118 (2019).

[7] E. Novak, K. Ritter, H. Wozniakowski, Average-case optimality of a hybrid secant-bisection method. *Math. Comput*., **64(212)**, 1517-1539 (1995).

[8] S. Thota, V. K. Srivastav, An Algorithm to Compute Real Root of Transcendental Equations Using Hyperbolic Tangent Function, *Int. J. Open Problems Compt. Math*., **14(2)**, 1-14, (2021).

[9] S. Thota, A Numerical Algorithm to Find a Root of Non-linear Equations Using Householder's Method, *International Journal of Advances in Applied Sciences*, **10(2)**, 141-148 (2021).

[10] S. Thota, T. Gemechu, P. Shanmugasundaram, New Algorithms for Computing Non-linear Equations Using Exponential Series, *Palestine Journal of Mathematics*, **10(1)**, 128-134 (2021).

[11] T. Gemechu, S. Thota, On New Root Finding Algorithms for Solving Nonlinear Transcendental Equations, *International Journal of Chemistry, Mathematics and Physics*, **4(2)**, 18-24 (2020).

[12] S. Thota, T. Gemechu, A New Algorithm for Computing a root of Transcendental Equations Using Series Expansion, *Southeast Asian Journal of Sciences*, **7(2)**, 106-114 (2019).

[13] S. Thota, A New Root-Finding Algorithm Using Exponential Series, *Ural Mathematical Journal*, **5 (1)**, 83-90 (2019).

[14] V. K. Srivastav, S. Thota, M. Kumar, A New Trigonometrical Algorithm for Computing Real Root of Non-linear Transcendental Equations, *International Journal of Applied and Computational Mathematics*, **5:44** (2019).

[15] S. Thota, V. K. Srivastav, Quadratically Convergent Algorithm for Computing Real Root of Non-Linear Transcendental Equations, *BMC Research Notes*, **11:909** (2018).

[16] S. Thota, V. K. Srivastav, Interpolation based Hybrid Algorithm for Computing Real Root of Non-Linear Transcendental Functions, *International Journal of Mathematics and Computer Research*, **2 (11)**, 729-735 (2014).

[17] T. Parveen, S. Singh, S. Thota, V. K. Srivastav, *A New Hydride Root-Finding Algorithm for Transcendental Equations using Bisection, Regula-Falsi and Newton-Raphson methods*, National Conference on Sustainable & Recent Innovation in Science and Engineering (SUNRISE-19), (2019).

[18] S. Thota, *A New Hybrid Halley-False Position type Root Finding Algorithm to Solve Transcendental Equations*, Istanbul International Modern Scientific Research Congress-III, Istanbul Gedik University, Istanbul, Turkey (2022).

[19] E.M. Badr, H. Elgendy, A hybrid water cycle-particle swarm optimization for solving the fuzzy underground water confined steady flow. *Indones. J. Electr. Eng. Comput. Sci.*, **19**, 492-504 (2020).

[20] J.H. Mathews, K.D. Fink, *Numerical Methods Using Matlab*, 4th ed.; Prentice-Hall Inc.: Upper Saddle River, NJ, USA, (2004).

[21] D.H. Joe, *Numerical Methods for Engineers and Scientists*, 2nd ed.; CRC Press: Boca Raton, FL, USA (2001).

[22] S.C. Chapra, R.P. Canale, *Numerical Methods for Engineers*, 7th ed.; McGraw-Hill: Boston, MA, USA (2015).

[23] L.R. Burden, F.J. Douglas, *Numerical Analysis, Prindle*, Weber & Schmidt, 3rd ed.; Amazon: Seattle, WA, USA (1985).

[24] S. Thota, *A Blended Root-Finding Algorithm for Solving Transcendental Equations*, Research square, preprint, (2023). https://doi.org/10.21203/rs.3.rs-2956091/v1

[25] S. Thota, P. Shanmugasundaram, On New Sixth and Seventh Order Iterative Methods for Solving Non-Linear Equations Using Homotopy Perturbation Technique, *BMC Research Notes*, **15:267** (2022).

[26] S. Thota, Solution of Generalized Abel's Integral Equations by Homotopy Perturbation Method with Adaptation in Laplace Transformation, *Sohag Journal of Mathematics*, **9(2)**, 29-35 (2022).

[27] S. Thota, S. D. Kumar, A Symbolic Method for Finding Approximate Solution of Neutral Functional-Differential Equations with Proportional Delays, *Jordan Journal of Mathematics and Statistics*, *14(4),* 671-689 (2021).