

Fuzzy Modeling and H_∞ Synchronization of Different Hyperchaotic Systems via T–S Models

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Abstract: In this paper, a unified fuzzy H_∞ control approach is proposed to deal with the problem of synchronization of two different hyperchaotic systems. The T–S fuzzy models with a small number of fuzzy IF-THEN rules are employed to represent many typical hyperchaotic systems exactly. Based on the T–S fuzzy hyperchaotic models, a fuzzy H_∞ synchronization controller is designed via parallel distributed compensation (PDC) techniques. The sufficient condition for the H_∞ synchronization of two different hyperchaotic systems is derived by applying Lyapunov stability theory. The results are expressed in terms of LMIs, and therefore it is very convenient to check in practice. This method is a universal one for synchronization of two different hyperchaotic systems. Numerical examples are given to demonstrate the validity of the proposed fuzzy modeling and hyperchaotic synchronization scheme.

Keywords: hyperchaotic systems, fuzzy modeling, T–S fuzzy model, H_∞ synchronization, linear matrix inequality(LMI)

1. Introduction

Since the pioneering work of Carroll and Pecora [1], synchronization of two chaotic dynamical systems has been paid more and more attention due to its potential applications in many fields. Recently, many new hyperchaotic systems have been found or created by researchers continually [2-5]. Moreover, synchronization of two hyperchaotic systems has been studied extensively in the last few years. Various control methods have been applied to synchronize two hyperchaotic (or chaotic) systems, such as robust control [6], adaptive control [7-11], linear feedback control [12], delayed feedback control [13], impulsive control [14-19], fuzzy control [20-21], etc. Among these synchronization methods, some only focus on the synchronization of two identical hyperchaotic systems. Others only dealt with one or two kinds of specified hyperchaotic systems. There are few unified methods suitable for synchronization of two different hyperchaotic systems.

At present, the Takagi–Sugeno (T–S) fuzzy model proposed in [22] is widely applied to many fields because of its simple structure with local dynamics [23-27]. Tanaka *et al* [28] established the accurate T–S fuzzy representation for many kinds of typical chaotic systems

and Lian *et al* [29] presented a synthesis approach for synchronization of chaotic systems based on T–S fuzzy models. But for hyperchaotic systems, to the best knowledge of authors, there are few results about this work. Therefore, in this paper, we will give a systematic scheme to represent many classes of well-known hyperchaotic systems by the T–S fuzzy models exactly and then we design a general fuzzy controller to realize the H_∞ synchronization of two different hyperchaotic systems by parallel distributed compensation (PDC) and LMI techniques.

2. T–S fuzzy modeling of hyperchaotic systems

2.1. T–S fuzzy model

Consider the continuous-time T–S fuzzy rule base described as follows:

R^i : IF $p_1(t)$ is M_{i1} , ..., and $p_q(t)$ is M_{iq} , THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + b,$$

where $i = 1, 2, \dots, r$ (r is the number of fuzzy rules), $x(t) \in R^n$ and $u(t) \in R^m$ denote the state vector and the

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input vector, respectively, $A_i \in R^{n \times n}$ and $B_i \in R^{n \times m}$ are known system matrix and input matrix with appropriate dimensions, respectively, $p_1(t), p_2(t), \dots, p_q(t)$ are premise variables, M_{ij} is a fuzzy set ($j = 1, 2, \dots, q$), b is a constant vector. By taking a standard fuzzy inference strategy, i.e., using a singleton fuzzifier, product fuzzy inference and center average defuzzifier, the final continuous-time fuzzy T-S system is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^r \omega_i(p(t)) (A_i x(t) + B_i u(t) + b)}{\sum_{i=1}^r \omega_i(p(t))}, \quad (1)$$

where

$$\omega_i(p(t)) = \prod_{j=1}^q M_{ij}(p_j(t)),$$

in which $M_{ij}(p_j(t))$ is the degree of membership of $p_j(t)$ in M_{ij} , with

$$\begin{cases} \sum_{i=1}^r \omega_i(p(t)) > 0, & i = 1, 2, \dots, r. \\ \omega_i(p(t)) \geq 0, \end{cases}$$

By using $\mu_i(p(t)) (= \omega_i(p(t)) / \sum_{i=1}^r \omega_i(p(t)))$ instead of $\omega_i(p(t))$, Eq. (1) is rewritten as:

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(p(t)) (A_i x(t) + B_i u(t) + b). \quad (2)$$

Note that

$$\begin{cases} \sum_{i=1}^r \mu_i(p(t)) = 1, & i = 1, 2, \dots, r, \\ \mu_i(p(t)) \geq 0, \end{cases}$$

in which $\mu_i(p(t))$ can be regarded as the firing strength of the i th IF-THEN rules.

2.2. Fuzzy modeling of hyperchaotic systems

In this subsection, we present a systematic T-S fuzzy modeling method for many typical hyperchaotic systems.

Note that most of hyperchaotic systems can be expressed as follows:

$$\dot{x}(t) = \tilde{A}x(t) + g(x(t)) + \tilde{b}, \quad (3)$$

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$, the linear term $\tilde{A}x(t)$ and the nonlinear term $g(x(t))$ represent the linear part and the nonlinear part of the hyperchaotic systems respectively, \tilde{b} is the constant vector of the hyperchaotic systems. According to the boundedness of hyperchaotic systems, one can assume that $x_i(t) \in [\tilde{c}_i - \tilde{d}_i, \tilde{c}_i + \tilde{d}_i]$, where $\tilde{d}_i > 0$. Next, According to various forms of the nonlinear part $g(x(t))$, the following T-S fuzzy modeling method for hyperchaotic systems is given below. First, we assume that there is a common factor in $g(x(t))$ of

hyperchaotic systems. In this situation, one can choose the common factor occurred in $g(x(t))$ as the premise variable of T-S fuzzy rules. For example, in the following hyperchaotic system [25], the nonlinear terms include $x_1(t)x_2(t)$ and $x_1(t)x_3(t)$. Among these two nonlinear terms, the common factor is $x_1(t)$. Therefore we can choose $x_1(t)$ as the premise variable.

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = dx_1(t) + cx_2(t) - x_1(t)x_3(t) - x_4(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - lx_3(t), \\ \dot{x}_4(t) = x_1(t) + k. \end{cases} \quad (4)$$

The membership functions are chosen as

$$F_{11}(x_1(t)) = \frac{1}{2} \left(1 - \frac{\tilde{c}_1 - x_1(t)}{\tilde{d}_1} \right),$$

$$F_{21}(x_1(t)) = \frac{1}{2} \left(1 + \frac{\tilde{c}_1 - x_1(t)}{\tilde{d}_1} \right).$$

And the fuzzy IF-THEN rules are defined as:

R^1 : IF $x_1(t)$ is F_{11} , THEN $\dot{x}(t) = A_1 x(t) + b$,

R^2 : IF $x_1(t)$ is F_{21} , THEN $\dot{x}(t) = A_2 x(t) + b$,

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$. For the above fuzzy rule base, by taking standard fuzzy inference strategy, i.e., using a singleton fuzzifier, product fuzzy inference and center average defuzzifier, we can get the following T-S fuzzy system:

$$\dot{x}(t) = \frac{\sum_{i=1}^2 F_{i1}(x_1(t)) (A_i x(t) + b)}{\sum_{i=1}^2 F_{i1}(x_1(t))}. \quad (5)$$

The rest work is to determine A_1, A_2 and b such that Eqs. (5) and (3) are equivalent completely. For the above hyperchaotic system 4, each part of (3) is presented below:

$$\tilde{A} = \begin{pmatrix} -a & a & 0 & 0 \\ d & c & 0 & -1 \\ 0 & 0 & -l & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$g(x(t)) = \begin{pmatrix} 0 \\ -x_1(t)x_3(t) \\ x_1(t)x_2(t) \\ 0 \end{pmatrix}, \quad \tilde{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k \end{pmatrix}.$$

it is easy to see that Eq. (5) is equal to (3) as long as $A_1, A_2(t)$ and b are chosen as follows:

$$A_1 = \begin{pmatrix} -a & a & 0 & 0 \\ d & c & -(\tilde{c}_1 + \tilde{d}_1) & -1 \\ 0 & \tilde{c}_1 + \tilde{d}_1 & -l & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -a & a & 0 & 0 \\ d & c & -(\tilde{c}_1 - \tilde{d}_1) & -1 \\ 0 & \tilde{c}_1 - \tilde{d}_1 & -l & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$b = (0 \ 0 \ 0 \ k)^T.$$

Consequently, the exact T-S fuzzy models of the above hyperchaotic system is obtained as (5).

If there is no common factor in the nonlinear part $g(x(t))$, we need to choose any two state variables occurring in $g(x(t))$ as the premise variables. The rest modeling process is similar to the above, and therefore is omitted.

To summarize, most existing hyperchaotic systems can be represented by the following T-S fuzzy models exactly:

R^i : IF $p_1(t)$ is F_{i1} and ... and $p_q(t)$ is F_{iq} , THEN $\dot{x}(t) = A_i x(t) + b$,

where $i = 1, 2, \dots, r$, r is the number of fuzzy IF-THEN rules, the premise variable $p_j(t)$ is a proper state variable and b is a constant vector. Then, applying the product-inference rule, singleton fuzzifier, and the center of gravity defuzzifier to the above fuzzy rule base, the overall fuzzy hyperchaotic system can be inferred as:

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(p(t))(A_i x(t) + b), \quad (6)$$

$$\text{where } \mu_i(p(t)) = \frac{\prod_{j=1}^q F_{ij}(p_j(t))}{\sum_{i=1}^r \left(\prod_{j=1}^q F_{ij}(p_j(t)) \right)}.$$

3. Fuzzy H_∞ synchronization of two different hyperchaotic systems

In this section, we will investigate the synchronization of two different hyperchaotic systems based on the T-S fuzzy models. Firstly, an assumption is given as follows.

Assumption 3.1: The numbers of fuzzy rules of two different fuzzy hyperchaotic systems to synchronize are identical.

Consider the fuzzy hyperchaotic system (6) as the drive system, and the following fuzzy hyperchaotic system as the response system:

$$\dot{\hat{x}}(t) = \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t)) (\hat{A}_i \hat{x}(t) + \hat{b}) + u(t). \quad (7)$$

Based on PDC technique, a fuzzy controller can be designed to realize the synchronization as follows:

Subcontroller $u_d(t)$:

R^i : IF $p_1(t)$ is F_{i1} and ... and $p_q(t)$ is F_{iq} , THEN $u_d(t) = K_i x(t)$, $i = 1, 2, \dots, r$.

Subcontroller $u_r(t)$:

R^i : IF $\hat{p}_1(t)$ is \hat{F}_{i1} and ... and $\hat{p}_q(t)$ is \hat{F}_{iq} , THEN $u_r(t) = -\hat{K}_i \hat{x}(t)$, $i = 1, 2, \dots, r$.

The overall fuzzy controller is constructed by the parallel connection, i.e.,

$$u(t) = u_d(t) + u_r(t) = \sum_{i=1}^r \mu_i(p(t)) K_i x(t) - \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t)) \hat{K}_i \hat{x}(t). \quad (8)$$

Denote the error signal as $e(t) = x(t) - \hat{x}(t)$. Substituting (8) into (7), we can derive the closed-loop synchronization error system as follows:

$$\begin{aligned} \dot{e}(t) = & \sum_{i=1}^r \mu_i(p(t))(A_i - K_i)e(t) + \\ & \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t)) (\hat{A}_i - \hat{K}_i) e(t) + \varpi, \end{aligned} \quad (9)$$

where

$$\varpi(t) = \varpi_1(t) + \varpi_2(t) + (b - \hat{b}),$$

$$\varpi_1(t) = \sum_{i=1}^r \mu_i(p(t))(A_i - K_i)\hat{x}(t),$$

$$\varpi_2(t) = \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t)) (\hat{A}_i - \hat{K}_i)x(t).$$

For the synchronization error system (9), consider the following H_∞ performance index:

$$\int_0^{t_f} e^T(t)e(t)dt < \frac{1}{2}e^T(0)Pe(0) + \frac{\gamma_1^2 + \gamma_2^2}{2} \int_0^{t_f} \varpi^T(t)\varpi(t)dt, \quad (10)$$

where γ_1 and γ_2 are prescribed attenuation level, t_f is terminal time.

Theorem 3.1: Considering the fuzzy error system (9), for the given constants γ_1 and γ_2 , if there exist a matrix $P = P^T > 0$, and real matrices M_i, N_i with appropriate dimensions, such that the following LMIs are satisfied:

$$\begin{pmatrix} PA_i + A_i^T P - M_i^T - M_i + I & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_1^2 I \end{pmatrix} < 0, \quad (11)$$

and

$$\begin{pmatrix} P\hat{A}_i + \hat{A}_i^T P - N_i^T - N_i + I & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_2^2 I \end{pmatrix} < 0, \quad (12)$$

where $i = 1, 2, \dots, r$. Then we can choose $K_i = P^{-1}M_i$, $\hat{K}_i = P^{-1}N_i$ in (8), such that the robust performance (10) is guaranteed.

Proof: Now, we select the Lyapunov function as:

$$V(t) = e^T(t)Pe(t),$$

where P is a positive definite matrix. The time derivative of $V(t)$ along the trajectory of (9) is given by

$$\begin{aligned} \dot{V}(t) = & \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\ = & e^T(t)(X+Y)^T Pe(t) + e^T(t)P(X+Y)e(t) \\ & + \varpi^T(t)Pe(t) + e^T(t)P\varpi(t) \\ = & \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix}^T \Psi \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix}, \end{aligned}$$

where

$$\Psi = \begin{pmatrix} X^T P + P X + Y^T P + P Y & P \\ P & 0 \end{pmatrix},$$

$$X = \sum_{i=1}^r \mu_i(p(t))(A_i - K_i),$$

$$Y = \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t))(\hat{A}_i - \hat{K}_i).$$

Based on (10), define

$$J = \int_0^{t_f} (2e^T(t)e(t) - (\gamma_1^2 + \gamma_2^2)\varpi^T(t)\varpi(t) + \dot{V}(t)) dt - \int_0^{t_f} \dot{V}(t) dt.$$

And we have

$$\begin{aligned} J &= \int_0^{t_f} (2e^T(t)e(t) - (\gamma_1^2 + \gamma_2^2)\varpi^T(t)\varpi(t) + \dot{V}(t)) dt - V(t_f) + V(0) \\ &\leq \int_0^{t_f} \left\{ \sum_{i=1}^r \mu_i(p(t)) \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix}^T \Xi \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix} + \sum_{i=1}^r \hat{\mu}_i(\hat{p}(t)) \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix}^T \Gamma \begin{pmatrix} e(t) \\ \varpi(t) \end{pmatrix} \right\} dt + V(0), \end{aligned}$$

where $\Xi = \begin{pmatrix} G & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_1^2 I \end{pmatrix}$, $\Gamma = \begin{pmatrix} H & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_2^2 I \end{pmatrix}$, $G = (A_i - K_i)^T P + P(A_i - K_i) + I$, $H = (\hat{A}_i - \hat{K}_i)^T P + P(\hat{A}_i - \hat{K}_i) + I$.

It is easy to see that if the following inequalities hold:

$$\begin{pmatrix} G & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_1^2 I \end{pmatrix} < 0, \tag{13}$$

and

$$\begin{pmatrix} H & \frac{1}{2}P \\ \frac{1}{2}P & -\gamma_2^2 I \end{pmatrix} < 0, \tag{14}$$

then we have

$$J = \int_0^{t_f} (2e^T(t)e(t) - (\gamma_1^2 + \gamma_2^2)\varpi^T(t)\varpi(t)) dt \leq V(0). \tag{15}$$

From (15), it is obvious that the H_∞ performance index (10) is guaranteed. Denoting $M_i = PK_i$, $N_i = P\hat{K}_i$, then (13) and (14) are equivalent to the LMIs (11) and (12). This completes the proof.

Remark 3.1: If the numbers of fuzzy rules of two different hyperchaotic systems to synchronize are not identical, there is another method we can adopt in [23] to solve the synchronization problem. The authors in [23] developed a switching controller to synchronize the two different fuzzy chaotic systems with the non-identical numbers of fuzzy rules.

4. Simulation study

To visualize the effectiveness of the theoretical analysis and design, some examples are included for illustration. Firstly, we investigate the T-S fuzzy modeling of Liu hyperchaotic system and Lorenz hyperchaotic system.

Liu hyperchaotic system [2]:

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = lx_1(t) - kx_1(t)x_3(t) + x_4(t), \\ \dot{x}_3(t) = -cx_3(t) + gx_1^2(t), \\ \dot{x}_4(t) = -dx_1(t), \end{cases} \tag{16}$$

where $a = 10$, $l = 40$, $c = 2.5$, $d = 10.6$, $k = 1$ and $g = 4$. Here, Liu hyperchaotic system has two positive Lyapunov exponents $\lambda_1 = 1.1491$ and $\lambda_2 = 0.12688$. The attractor of Liu hyperchaotic system is shown in Fig. 1 and Fig. 2. The initial state vector is chosen as $x(0) = (1, 1, -1, 0)^T$. From the simulation we can get $x_1 \in [\tilde{c}_1 - \tilde{d}_1, \tilde{c}_1 + \tilde{d}_1]$, where $\tilde{c}_1 = 1.2347$, $\tilde{d}_1 = 18.6833$. Obviously, the nonlinear terms are $-kx_1(t)x_3(t)$ and $gx_1^2(t)$ in (16). there exists a common factor $x_1(t)$ in the nonlinear terms. Then we can derive the T-S fuzzy models of Liu hyperchaotic system as follows:

R^1 : IF $x_1(t)$ is F_1 , THEN $\dot{x}(t) = A_1x(t) + b_1$,
 R^2 : IF $x_1(t)$ is F_2 , THEN $\dot{x}(t) = A_2x(t) + b_2$,
 where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$,

$$A_1 = \begin{pmatrix} -10 & 10 & 0 & 0 \\ 40 & 0 & -19.9180 & 1 \\ 79.6720 & 0 & -2.5 & 0 \\ -10.6 & 0 & 0 & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -10 & 10 & 0 & 0 \\ 40 & 0 & 17.4486 & 1 \\ -69.7944 & 0 & -2.5 & 0 \\ -10.6 & 0 & 0 & 0 \end{pmatrix},$$

$$b_1 = b_2 = (0 \ 0 \ 0 \ 0)^T.$$

The membership functions are chosen as:

$$F_1(x_1(t)) = \frac{1}{2} \left(1 - \frac{1.2347 - x_1(t)}{18.6833} \right),$$

$$F_2(x_1(t)) = \frac{1}{2} \left(1 + \frac{1.2347 - x_1(t)}{18.6833} \right).$$

Lorenz hyperchaotic system [3]:

$$\begin{cases} \dot{\hat{x}}_1(t) = -a(\hat{x}_1(t) - \hat{x}_2(t)) + \hat{x}_4(t), \\ \dot{\hat{x}}_2(t) = l\hat{x}_1(t) - \hat{x}_2(t) - \hat{x}_1(t)\hat{x}_3(t), \\ \dot{\hat{x}}_3(t) = \hat{x}_1(t)\hat{x}_2(t) - k\hat{x}_3(t), \\ \dot{\hat{x}}_4(t) = -\hat{x}_1(t)\hat{x}_3(t) + d\hat{x}_4(t), \end{cases} \tag{17}$$

where $a = 10$, $l = 28$, $k = 8/3$ and $d = 1.3$. Here, Lorenz hyperchaotic system has two positive Lyapunov exponents $\lambda_1 = 0.39854$ and $\lambda_2 = 0.24805$. The attractor of Lorenz hyperchaotic system is shown in Fig. 3 and Fig. 4. The initial state vector is chosen as $\hat{x}(0) = (1, 2, 1, 2)^T$. From the simulation we can get $\hat{x}_1 \in [\tilde{c}_1 - \tilde{d}_1, \tilde{c}_1 + \tilde{d}_1]$, where

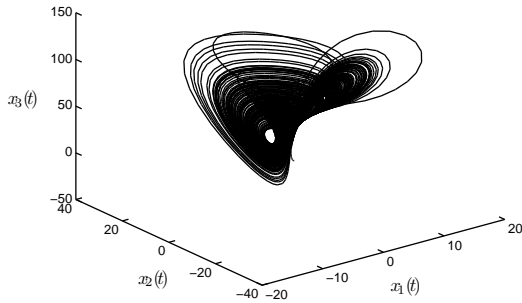


Figure 1 Projection of Liu hyperchaotic attractor on $x_1 - x_2 - x_3$

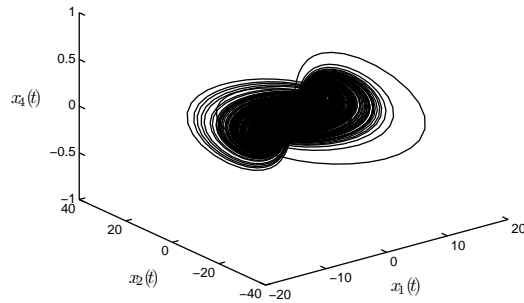


Figure 2 Projection of Liu hyperchaotic attractor on $x_1 - x_2 - x_4$

$\tilde{c}_1 = -1.4366, \tilde{d}_1 = 28.4241$. Then we can derive the T-S fuzzy models of Lorenz hyperchaotic system as follows:

R¹: IF $\hat{x}_1(t)$ is \hat{F}_1 , THEN $\hat{x}(t) = \hat{A}_1\hat{x}(t) + \hat{b}_1$,

R²: IF $\hat{x}_1(t)$ is \hat{F}_2 , THEN $\hat{x}(t) = \hat{A}_2\hat{x}(t) + \hat{b}_2$,

where $\hat{x}(t) = (\hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t), \hat{x}_4(t))^T$,

$$\hat{A}_1 = \begin{pmatrix} -10 & 10 & 0 & 1 \\ 28 & -1 & -26.9875 & 0 \\ 0 & 26.9875 & -8/3 & 0 \\ 0 & 0 & -26.9875 & 1.3 \end{pmatrix},$$

$$\hat{A}_2 = \begin{pmatrix} -10 & 10 & 0 & 1 \\ 28 & -1 & 29.8607 & 0 \\ 0 & -29.8607 & -8/3 & 0 \\ 0 & 0 & 29.8607 & 1.3 \end{pmatrix},$$

$$\hat{b}_1 = \hat{b}_2 = (0 \ 0 \ 0 \ 0)^T.$$

The membership functions are chosen as:

$$\hat{F}_1(\hat{x}_1(t)) = \frac{1}{2} \left(1 - \frac{-1.4366 - \hat{x}_1(t)}{28.4241} \right),$$

$$\hat{F}_2(\hat{x}_1(t)) = \frac{1}{2} \left(1 + \frac{-1.4366 - \hat{x}_1(t)}{28.4241} \right).$$

In the following, based on the above T-S fuzzy hyperchaotic models, we investigate the synchronization between Liu hyperchaotic system and Lorenz hyperchaotic system. We consider Liu hyperchaotic system as the drive system, and Lorenz hyperchaotic system as the response system. The initial conditions for the drive system and the response system are

$$x(0) = (15, 25, 100, 0)^T,$$

$$\hat{x}(0) = (-25, -20, 30, 100)^T.$$

According to Theorem 1, choosing $\gamma_1 = \gamma_2 = 0.1$, we can get the positive definite matrix P and feedback gains K_i ,

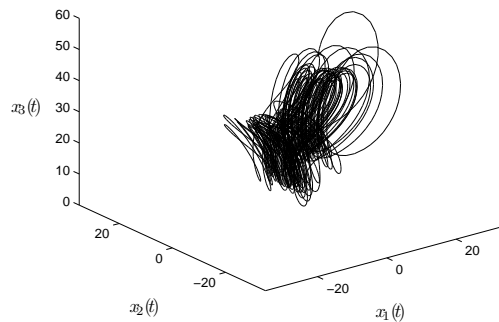


Figure 3 Projection of Lorenz hyperchaotic attractor on $x_1 - x_2 - x_3$

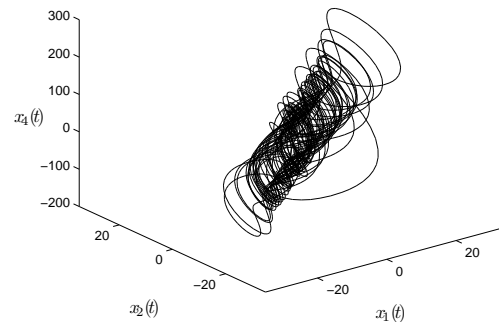


Figure 4 Projection of Lorenz hyperchaotic attractor on $x_1 - x_2 - x_4$

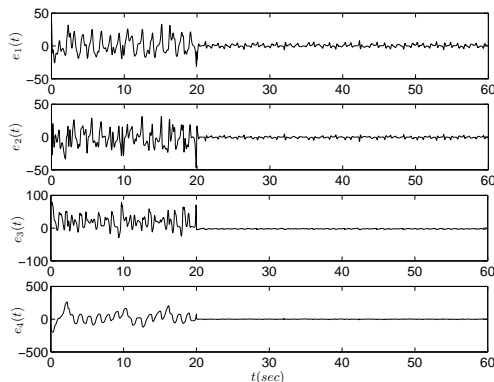


Figure 5 Synchronization error $e(t)$ between Liu hyperchaotic system and Lorenz hyperchaotic system

\hat{K}_i ($i = 1, 2$) computed by LMI toolbox in Matlab 7.0 as follows:

$$P = \begin{pmatrix} 1.6093 & -0.0410 & -0.0022 & -0.3284 \\ -0.0410 & 0.3789 & -0.0044 & 0.0092 \\ -0.0022 & -0.0044 & 0.1059 & -0.0003 \\ -0.3284 & 0.0092 & -0.0003 & 0.2155 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} 39.3794 & 13.5798 & -0.2381 & 76.9472 \\ 40.6736 & 142.2773 & -1.6448 & -6.2164 \\ 29.4006 & -22.1059 & 516.3327 & -25.0879 \\ 68.0431 & 5.3914 & -13.0605 & 371.6015 \end{pmatrix},$$

$$K_2 = \begin{pmatrix} 38.9603 & 14.2497 & 2.9000 & 77.1718 \\ 40.6733 & 143.8961 & 14.6389 & -5.0789 \\ -27.7512 & 37.8918 & 514.4715 & 31.3370 \\ 66.1391 & 7.3065 & 18.7104 & 372.9590 \end{pmatrix},$$

$$\hat{K}_1 = \begin{pmatrix} 35.7096 & 13.3922 & 2.1372 & 72.7527 \\ 39.6357 & 134.9720 & -3.9529 & -5.4449 \\ 51.7616 & -29.9640 & 487.2763 & -22.7680 \\ 62.5319 & 6.6327 & -8.1879 & 350.9440 \end{pmatrix},$$

$$\hat{K}_2 = \begin{pmatrix} 35.3859 & 13.8039 & 0.4756 & 72.9124 \\ 38.7566 & 136.4886 & 16.1391 & -4.6764 \\ -50.9872 & 44.6941 & 485.6173 & 29.9071 \\ 61.7018 & 7.5811 & 14.2738 & 351.4143 \end{pmatrix}.$$

The synchronization control signals are imposed when $t = 20\text{sec}$. The synchronization error curves are presented in Fig. 5. From Fig. 5, we can see that H_∞ synchronization between the above two different hyperchaotic systems is derived rapidly.

Remark 4.1: If a pair of smaller γ_1 and γ_2 are chosen, then we can derive more satisfactory synchronization results. But the higher gains of the controller are needed at the same time.

5. Conclusions

In this work, a T-S-fuzzy-model-based synchronization method for two different hyperchaotic systems is proposed. Based on the exact fuzzy hyperchaotic models, a universal fuzzy H_∞ synchronization controller is designed via the Lyapunov stability theory and PDC technique. The relevant results are presented in the form of LMIs. Numerical simulation results are given to demonstrate the effectiveness of the proposed scheme. This implies that fuzzy-model-based synchronization approach for two different hyperchaotic systems is very flexible and useful in practical applications.

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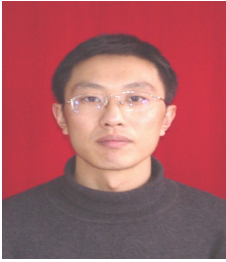
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