

Efficient Circular Designs Partially Balanced for Residual Effects in Periods of Three Different Sizes

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Abstract: To estimate the treatment effects using repeated measurements designs (RMDs), residual effects often become the major source of bias. Minimal balanced RMDs are well known economical designs to minimize bias due to these effects for v odd, where v is the number of treatments to be compared. For v even, partially balanced RMDs are preferred. In this article, catalogues of two main classes of minimal circular partially balanced RMDs are compiled in periods of three different sizes. These designs possess high efficiency of residual effects and of Separability, therefore, our proposed designs are efficient.

Keywords: Carry over effects; RMDs; Minimal designs; Direct effects; Circular balanced designs.

Mathematics Subject Classification (2010): 05B05; 62K10; 62K05.

1 Introduction

Repeated measurements designs (RMDs) have application in many branches of scientific inquiry, therefore, choice of RMDs must be made in a way that the treatments can be efficiently compared after allowing for the residual effects. Minimal balanced RMDs are considered to be robust for residual effects. If minimal balanced RMDs could not be constructed then minimal partially balanced RMDs should be used to balance the residual effects. These designs are most economical. Following are some important definitions.

- RMD is minimal balanced if each treatment is immediately preceded once by all others treatments (excluding itself).

- RMD is minimal partially balanced if each treatment is immediately preceded by all others treatments (excluding itself), either (i) once or no time, or (ii) once or twice.

- If last period treatments are considered the preceding treatments of the first period then RMD is called circular.

- The effect of a treatment during its period of application (its direct effect) may persist into the following period(s) which is called residual effect or carry over effect.

[1] described the benefits of RMDs. [2], [3] and [4] gave some constructions of circular balanced RMDs (CBRMDs). [5] constructed balanced RMDs in periods of unequal sizes. [6] constructed RMDs in periods of equal as well as in different sizes. [7] constructed non-circular minimal BRMDs for v odd. [8], [9], [10] and [11] presented some generators to obtain MCBRMDs.

2 Statement of the Problem

MCBRMDs cannot be constructed for v even, therefore, in such situations minimal circular partially balanced RMDs (MCPBRMDs) should be used. [12] constructed these designs in periods of equal sizes for almost every possible cases of

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v and p . [13] constructed these designs in periods of two different sizes. MCPBRMDs in periods of three different sizes are not available in literature for v even and p_1 odd which should be constructed.

In this article, efficient MCPBRMDs-I and efficient MCPBRMDs-II are constructed in periods of three different sizes, using method of cyclic shifts (Rule I), where MCPBRMD-I is a design in which $v/2$ ordered pairs do not appear together while the remaining ones appear once, and MCPBRMD-II is a design in which $v/2$ ordered pairs appear twice together. Catalogues of these designs are also compiled for $v \leq 100$, $p_1 = 9$ and 11 , $7 \leq p_2 \leq 10$ and $6 \leq p_3 \leq 9$.

3 Method of Cyclic Shifts

Method of cyclic shifts introduced by [14] is described here for the construction of MCPBRMDs-I and MCPBRMDs-II.

MCPBRMDs-I and MCPBRMDs-II are constructed in periods of three different sizes $p_1 = r$ (odd), $p_2 = s$ and $p_3 = u$ from the following sets of shifts, using Rule I.

$$S_j = [q_{j1}, q_{j2}, \dots, q_{j(r-1)}]; \quad j = 1, 2, \dots, i.$$

$$S_{i+1} = [q_{(i+1)1}, q_{(i+1)2}, \dots, q_{(i+1)(s-1)}]; \quad S_{i+2} = [q_{(i+2)1}, q_{(i+2)2}, \dots, q_{(i+2)(u-1)}]$$

Where

- $1 \leq q_{j1}, q_{j2}, \dots, q_{j(r-1)}, q_{(i+1)1}, q_{(i+1)2}, \dots, q_{(i+1)(s-1)}, q_{(i+2)1}, q_{(i+2)2}, \dots, q_{(i+2)(u-1)} \leq v - 1$.
- S^* contains each of (i) $1, 2, \dots, v - 1$ once except $v/2$ which do not appear for MCPBRMDs-I, and (ii) $1, 2, \dots, v - 1$ once except $v/2$ which appear twice for MCPBRMDs-II.
- $S^* = [q_{j1}, q_{j2}, \dots, q_{j(r-1)}, v - (q_{j1} + q_{j2} + \dots + q_{j(r-1)}), q_{(i+1)1}, q_{(i+1)2}, \dots, q_{(i+1)(s-1)}, v - (q_{(i+1)1} + \dots + q_{(i+1)(s-1)}), q_{(i+2)1}, q_{(i+2)2}, \dots, q_{(i+2)(u-1)}, v - (q_{(i+2)1} + q_{(i+2)2} + \dots + q_{(i+2)(u-1)})]$

Example 3.1: $S_1 = [1, 2, 3, 4, 19, 18, 17]$, $S_2 = [5, 6, 7, 15, 14]$ and $S_3 = [8, 9, 12]$ MCPBRMD for $v = 20$, $p_1 = 8$, $p_2 = 6$ and $p_3 = 4$.

Proof: Here $S^* = [1, 2, 3, 4, 19, 18, 17, 16, 5, 6, 7, 15, 14, 13, 8, 9, 12, 11]$ and each of $1, 2, \dots, 19$ appears once except 10 which does not appear. Therefore, these sets produce MCPBRMD-I. Design from given sets is obtained as:

Take v experimental subjects for S_1 . Consider $0, 1, \dots, v - 1$ as period I values. To get period II values, add $1 \pmod{20}$ to period I values. To get period III values, add $2 \pmod{20}$ to period II values. Similarly, add $3, 4, 19, 18$ and 17 , see Table 1.

Table 1: Arrays obtained from S_1

Periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0
3	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2
4	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5
5	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9
6	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8
7	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6
8	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3

Take v more subjects for $S_2 = [5, 6, 7, 15, 14]$ and obtain arrays in similar way as of S_1 , see Table 2.

Table 2: Arrays obtained from S_2

Periods	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4
3	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9	10
4	18	19	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
5	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6

Take v more subjects for $S_3 = [12, 9, 8]$ and obtain arrays the similar way, see Table 3.

Table 3: Arrays obtained from S_3

Periods	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9	10	11
3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0
4	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8

Table 1, 2 and 3 jointly produce required MCPBRMD-I.

4 Efficiency measures

4.1 Efficiency of Residual Effects

Canonical efficiency factors are Eigen values (non-zero) of information matrix C^* , and efficiency factor for both direct and carryover effect is harmonic mean of Eigen values (non-zero) of their respective information matrix, see [15] and [16]. Design will be suitable to estimate residual effects when value of E_r is high.

4.2 Efficiency of Separability

[17] developed following efficiency of Separability (E_s) for BRMDs.

$$E_s = [1 - 1/(v\sqrt{v-1})] \times 100\%$$

5 Efficient MCPBRMDs-I in Periods of Three Different Sizes for p_1 odd and $p_3 > 5$

In this Section, efficient MCPBRMDs-I are constructed in periods of three different sizes when p_1 is odd and $p_3 > 5$.

Construction 5.1: Efficient MCPBRMDs-I for $v = p_{1i} + p_2 + 8$, i odd, $p_1 = 9 \& 11$, $p_2 = 7 \& 9$, $p_1 > p_2$ and $p_3 = 6$.

v	p_1	p_2	p_3	Sets	E_r	E_s
24	9	7	6	[1,2,3,4,5,6,7,8]+[11,18,13,14,17,16]+[15,19,20,21,22]	0.89	0.91
42	9	7	6	[3,2,4,5,6,7,8,9]+[10,11,12,13,14,16,15,17]+[41,36,22,23,24,25,26,27]+[30,29,31,32,33,20]+[34,37,38,39,1]	0.89	0.97
60	9	7	6	[1,2,18,4,5,6,7,8]+[19,11,12,13,14,15,16,17]+[20,21,22,23,24,25,27,26]+[29,28,31,38,32,34,35,36]+[51,40,41,42,53,45,44,46]+[47,48,49,50,39,10]+[54,55,56,43,33]	0.89	0.98
78	9	7	6	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+[19,20,21,22,23,24,25,26]+[28,29,30,31,32,33,58,35]+[37,38,48,41,42,43,44,45]+[10,49,51,50,46,53,27,57]+[77,75,1,76,60,61,62,63]+[65,66,67,59,69,68]+[72,73,64,55,56]	0.89	0.98
96	9	7	6	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+[20,21,22,28,24,25,26,27]+[23,30,31,37,33,34,35,36]+[32,58,39,40,41,42,43,44]+[47,46,49,51,50,75,53,54]+[57,38,77,60,61,62,63,64]+[66,19,67,86,69,70,71,72]+[73,1,10,78,79,80,81,82]+[83,84,59,87,88,89]+[90,91,85,93,56]	0.89	0.97
26	11	7	6	[3,1,4,5,6,8,9,7,11,10]+[12,2,20,16,17,18]+[15,21,22,23,24]	0.90	0.94
48	11	7	6	[2,3,4,15,6,7,8,9,10,11]+[23,13,14,44,16,17,18,19,20,34]+[12,39,26,27,28,29,32,30,31,33]+[46,36,37,38,47,43]+[42,40,5,45,35]	0.91	0.97
70	11	7	6	[2,3,4,5,6,7,12,9,10,11]+[14,13,15,16,17,18,20,19,21,24]+[23,22,34,25,27,28,29,30,31,32]+[68,53,3,38,40,39,41,42,43,44]+[62,47,48,49,50,51,52,36,54,55]+[57,58,65,67,46,64]+[61,59,66,60,8]	0.91	0.96
92	11	7	6	[2,3,4,5,6,7,8,9,10,11]+[23,14,15,16,17,18,19,20,21,22]+[13,24,25,26,1,28,29,30,31,32]+[35,36,33,38,39,40,41,42,44,43]+[86,48,49,50,51,53,54,55,90,56]+[85,84,59,60,61,62,63,64,65,66]+[34,68,58,71,72,73,74,75,76,78]+[79,80,81,82,83,70]+[47,87,88,89,45]	0.91	0.98
28	11	9	6	[2,3,5,4,6,7,8,9,10,11]+[27,21,12,13,16,18,15,1]+[22,23,24,25,26]	0.92	0.95

50	11	9	6	[2,3,4,5,6,7,8,9,10,11]+[12,14,15,16,17,18,19,20,21,26]+ [24,23,42,29,28,27,30,31,32,1]+[44,36,27,38,39,40,41,43]+[1,45,46,47,48]	0.91	0.94
72	11	9	6	[1,2,3,4,5,7,8,9,10]+[13,14,15,16,17,18,19,20,21,22]+ [23,24,6,25,27,28,29,30,31,32]+[34,35,46,38,39,40,26,42,43,44]+ [47,61,48,50,49,51,52,53,54,55]+[71,58,59,60,70,66,63,64]+[62,67,68,69,37]	0.85	0.92
94	11	9	6	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,20,21,]+ [22,24,25,26,27,28,1,30,34,32]+[75,35,36,37,38,39,40,42,43,44]+ [46,45,48,50,49,51,57,53,54,55]+[93,92,59,60,61,62,63,64,65,66]+ [91,90,87,86,72,73,74,31,76,77]+[69,80,81,82,83,68,85]+[70,71,79,84,78]	0.91	0.98

Construction 5.2: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 8$, i even, $p_1 = 9$ & 11 , $p_2 = 8$ & 10 , $p_1 > p_2$ and $p_3 = 6$.

v	p_1	p_2	p_3	Sets	Er	Es
34	9	8	6	[2,3,4,5,6,7,8,9]+[10,11,12,13,14,16,15,18]+[33,21,22,23,1,25,26]+ [28,29,30,31,32]	0.90	0.96
62	9	8	6	[2,3,4,5,6,7,8,16]+[10,11,12,13,14,15,17,18]+[39,21,22,23,24,25,27,28]+ [44,30,31,32,22,34,35,36]+[50,20,40,41,42,29,43]+[9,45,48,49,38]	0.89	0.91
70	9	8	6	[2,3,4,5,6,7,8,9]+[12,11,13,14,15,16,17,18]+[19,20,21,22,23,10,43,25]+ [48,30,32,31,33,34,35,36]+[39,40,41,42,1,44,45,46]+[67,68,49,50,51,38,53]+ [28,57,58,29,60,61,62]+[64,63,66,47,54]	0.89	0.96
38	11	8	6	[12,3,4,5,6,9,7,8,10,11]+[13,2,14,16,15,18,17,20,22,21]+ [33,24,27,26,28,29,30]+[32,16,34,35,36]	0.91	0.93
60	11	8	6	[2,3,4,5,6,7,8,9,53,10]+[12,59,14,15,16,17,18,19,27,21]+ [52,24,25,26,46,29,28,31,33,32]+[58,36,54,39,38,40,41,42,43,44]+ [47,57,48,35,50,51,11]+[20,37,56,55,49]	0.91	0.98
82	11	8	6	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,20,21]+ [43,24,25,26,27,28,29,30,31,32]+[55,35,36,37,39,38,40,81,42,44]+ [46,47,48,50,49,51,52,53,54,68]+[80,78,59,60,74,62,63,64,66,65]+ [79,69,70,71,72,77,61]+[1,73,58,34,57]	0.91	0.97
40	11	10	6	[2,3,4,5,6,7,8,9,11,10]+[12,13,14,25,17,16,18,19,23,21]+ [26,1,27,28,29,34,31,32,33]+[30,35,36,37,38]	0.92	0.96
62	11	10	6	[2,3,4,5,6,7,8,9,10,11]+[14,25,15,16,17,18,21,20,19,22]+ [23,24,13,26,28,29,38,30,33,32]+[36,37,46,39,40,41,42,43,44,12]+ [60,56,50,55,57,51,27,52,35]+[47,45,48,49,1]	0.91	0.98
84	11	10	6	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,83,20,21]+ [51,24,25,27,26,28,29,30,31,33]+[67,74,36,37,39,40,38,41,44,43]+ [82,47,68,49,52,22,50,53,54,55]+[80,58,59,60,61,62,63,64,65,66]+ [48,69,70,71,35,73,78,75,76]+[72,79,57,81,46]	0.91	0.98

Construction 5.3: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 9$, i odd, $p_1 = 9$ & 11 , $p_2 = 8$ & 10 , $p_1 > p_2$ and $p_3 = 7$.

v	p_1	p_2	p_3	Sets	Er	Es
26	9	8	7	[1,2,10,5,4,6,7,8]+[11,24,12,15,14,19,17]+[16,20,21,23,22,3]	0.90	0.94
44	9	8	7	[2,3,4,5,6,7,8,10]+[9,11,13,14,15,16,17,18]+ [12,34,21,24,23,25,26,27]+[29,30,31,32,33,38,35]+[37,20,39,40,41,42]	0.89	0.97
62	9	8	7	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,1,17]+[19,20,21,22,28,24,26,27]+ [29,44,30,33,32,34,35,36]+[38,39,40,41,42,43,23,60]+ [59,58,57,50,51,52,53]+[55,56,49,48,47,45]	0.90	0.97
80	9	8	7	[2,3,4,5,6,7,8,9]+[11,12,14,13,15,16,17,18]+[52,20,21,22,23,24,25,26]+ [28,30,29,31,32,33,34,35]+[39,37,38,41,42,69,43,46]+ [47,48,49,50,51,73,53,54]+[72,57,58,59,60,61,62,67]+ [65,66,63,10,79,70,71]+[19,74,75,76,77,78]	0.89	0.98
98	9	8	7	[1,3,4,5,6,7,8,9]+[11,80,12,13,14,15,16,17]+[29,21,22,23,24,25,27,26]+ [43,20,30,31,32,33,34,35]+[75,39,40,41,42,28,44,45]+ [96,94,48,51,50,52,53,54]+[66,95,58,59,60,61,62,63]+ [65,56,67,68,69,47,71,72]+[74,37,76,77,78,88,93,81]+ [83,84,85,86,87,70,91]+[89,92,10,79,57,46]	0.89	0.99

28	11	8	7	[2,3,5,4,6,7,8,9,10,11]+[12,13,15,17,16,18,1]+[21,22,23,24,25,26]	0.84	0.95
50	11	8	7	[2,3,4,5,6,7,8,9,10,11]+[12,26,14,15,16,18,17,19,20,21]+ [24,23,42,27,28,29,30,31,33,32]+[36,49,37,38,39,40,48]+[43,44,45,46,47,41]	0.91	0.97
72	11	8	7	[1,2,3,4,5,7,8,9,10,11]+[13,14,15,16,17,18,19,20,21,22]+ [24,26,25,27,28,29,34,31,32,33]+[35,63,37,38,39,40,6,42,43,44]+ [47,61,48,49,50,51,52,53,54,55]+[69,68,59,66,46,62,70]+[65,60,67,58,57,30]	0.91	0.98
94	11	8	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,20,21]+ [22,24,25,26,27,28,1,30,34,32]+[75,35,36,37,38,39,40,41,42,43]+ [46,45,48,50,49,51,57,53,54,55]+[93,92,59,60,61,62,63,64,65,66]+ [91,90,89,87,80,73,74,31,76,77]+[79,72,81,88,83,84,85]+[71,82,70,69,68,58]	0.91	0.98
30	11	10	7	[25,2,5,3,4,6,7,9,10,8]+[12,13,24,14,17,18,19,20,22]+[23,16,1,26,27,28]	0.92	0.95
52	11	10	7	[3,2,4,5,6,7,8,9,10,11]+[12,13,15,14,37,17,18,19,20,21]+ [25,51,27,28,29,50,31,32,33,34]+[49,48,23,38,1,35,43,46,40]+ [44,47,36,42,30,16]	0.92	0.97
74	11	10	7	[1,2,3,4,5,6,7,9,10,15]+[13,14,11,16,17,18,19,21,20,22]+ [36,25,26,27,28,29,30,31,32,33]+[53,35,24,38,39,40,41,42,43,44]+ [61,8,48,49,50,47,52,34,54,55]+[71,57,59,56,72,62,63,64,65]+ [67,68,69,70,58,46]	0.91	0.98
96	11	10	7	[2,3,4,5,6,7,8,9,10,11]+[12,,13,14,15,16,17,23,19,20,21]+ [18,39,25,26,27,28,29,1,30,32]+[95,35,36,37,38,24,45,41,42,43]+ [46,63,47,50,49,51,52,53,54,55]+[70,58,59,60,61,62,40,64,65,66]+ [94,81,93,71,72,73,74,75,76,77]+[79,91,86,92,90,83,85,89,87]+ [84,80,34,57,68,88]	0.89	0.98

Construction 5.4: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 9$, i even, $p_1 = 11$, $p_2 = 9$ and $p_3 = 7$.

v	p_1	p_2	p_3	Sets	Er	Es
40	11	9	7	[2,3,4,5,6,7,8,9,11,10]+[12,13,14,17,16,18,19,22,23,21]+ [36,1,26,27,28,29,30,31]+[33,34,35,24,37,38]	0.92	0.96
62	11	9	7	[2,3,4,5,6,7,8,9,10,11]+[14,25,15,16,17,18,21,19,20,22]+ [23,35,26,28,27,29,30,33,34,32]+[24,36,37,38,39,40,46,42,44,43]+ [60,58,57,55,56,51,52,53]+[49,50,48,47,1,41]	0.91	0.97
84	11	9	7	[2,3,4,5,6,7,8,9,10,11]+[12,43,14,16,15,17,18,54,20,21]+ [24,51,25,27,26,28,29,30,31,32]+[52,35,37,36,38,39,40,41,13,44]+ [78,57,48,49,50,23,34,53,1,55]+[47,58,59,60,61,62,63,64,65,66]+ [83,69,70,80,72,73,74,75]+[77,46,79,71,81,82]	0.91	0.98

Construction 5.5: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 9$, i odd, $p_1 = 11$, $p_2 = 9$ and $p_3 = 8$.

v	p_1	p_2	p_3	Sets	Er	Es
30	11	9	8	[1,4,3,6,5,7,10,9,8,11]+[14,13,16,12,17,19,18,20]+[22,23,24,25,27,28,29]+ [36,1,26,27,28,29,30,31]+[33,34,35,24,37,38]	0.92	0.95
52	11	9	8	[3,2,4,5,6,7,8,9,10,11]+[13,14,15,16,17,18,19,20,21,22]+ [25,24,27,29,28,30,31,32,12,34]+[36,37,38,1,23,41,42,43]+ [44,45,46,47,48,49,50]	0.92	0.97
74	11	9	8	[1,2,3,5,6,7,8,9,10,11]+[13,15,14,17,16,18,19,20,21,22]+ [24,25,27,26,28,29,30,31,32,45]+[36,35,38,40,39,41,42,43,44,33]+ [4,48,50,49,51,52,34,54,55,56]+[57,58,59,60,63,62,64,72]+ [66,67,68,69,70,71,46]	0.91	0.98
96	11	9	8	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,21,22]+ [23,24,35,26,27,28,30,29,1,32]+[34,36,37,38,39,40,41,42,43,44]+ [47,46,49,51,50,52,53,54,55,56]+[57,58,59,68,61,62,45,64,65,66]+ [93,80,71,72,73,74,75,76,77,78]+[70,81,82,83,84,85,20,88]+ [89,90,91,92,69,94,60]	0.91	0.99

Construction 5.6: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 10$, i even, $p_1 = 11, p_2 = 10$ and $p_3 = 8$.

v	p_1	p_2	p_3	Sets	Er	Es
42	11	10	8	[19,2,3,4,5,6,7,8,10,9]+[12,13,30,15,16,18,17,22,24,20]+ [1,25,26,27,34,29,14,31,32]+[28,35,36,37,38,39,40]	0.92	0.97
64	11	10	8	[63,2,3,4,5,6,7,8,9,10]+[12,13,14,15,16,17,18,19,25,21]+ [23,24,20,26,27,28,29,31,45,33]+[35,36,37,38,39,40,41,1,43,44]+ [46,47,48,49,50,51,52,53,61]+[56,57,58,59,60,54,62]	0.92	0.98
86	11	10	8	[62,2,3,4,5,6,7,8,9,55]+[45,33,32,15,16,17,18,19,20,21]+ [23,12,25,27,26,28,29,30,31,14]+[34,85,36,37,38,39,77,42,60,44]+ [46,47,48,49,50,51,52,53,54,10]+[57,58,59,41,61,63,1,64,65,66]+ [68,69,70,71,72,73,74,75,76]+[78,79,80,81,82,83,84]	0.92	0.98

Construction 5.7: Efficient MCPBRMDs-I for $v = p_1i + p_2 + 11$, i odd, $p_1 = 11, p_2 = 10$ and $p_3 = 9$.

v	p_1	p_2	p_3	Sets	Er	Es
32	11	10	9	[2,34,5,6,7,8,9,10,11]+[1,14,13,15,18,17,20,19,21]+ [23,24,25,26,27,28,29,30]	0.92	0.91
54	11	10	9	[2,3,4,5,6,7,8,9,11,10]+[14,13,15,16,17,18,20,19,21,22]+ [23,26,24,25,28,29,39,31,32,33]+[35,36,38,37,30,49,42,1,12]+ [45,46,47,48,40,50,51,52]	0.91	0.97
76	11	10	9	[1,12,3,4,5,6,7,8,9,10]+[13,14,15,16,17,18,19,20,21,22]+ [49,25,26,27,28,29,34,31,32,33]+[36,35,37,40,39,42,41,43,44,45]+ [46,47,48,24,50,51,52,73,30,55]+[75,70,59,60,61,62,63,64,71]+ [67,68,69,65,69,72,2,58,74]	0.92	0.97
98	11	10	9	[22,2,13,4,5,6,7,8,9,10]+[93,60,14,15,16,17,18,19,20,21]+ [23,24,25,28,27,26,29,81,32,31]+[35,34,36,27,38,39,3,4,42,43]+ [45,46,48,47,50,70,52,53,54,67]+[57,58,59,11,61,62,63,65,64,33]+ [68,69,51,71,72,73,74,76,75,77]+[79,80,30,82,83,84,85,86,87]+ [89,90,91,92,40,94,95,96]	0.92	0.97

6 Efficient MCPBRMDs-II in Periods of Three Different Sizes for p_1 odd and $p_3 > 5$

In this Section, efficient MCPBRMDs-II are constructed in periods of three different sizes when p_1 is odd and $p_3 > 5$.

Construction 6.1: Efficient MCPBRMDs-II for $v = p_1i + p_2 + 6$, i odd, $p_1 = 9 \& 11, p_2 = 7 \& 9, p_1 > p_2$ and $p_3 = 6$.

v	p_1	p_2	p_3	Sets	Er	Es
22	9	7	6	[21,2,3,4,5,6,8,7]+[9,11,12,13,17,15]+[16,14,18,19,20]	0.90	0.94
40	9	7	6	[2,3,4,5,6,7,8,9]+[39,11,13,12,14,15,22,16]+[19,20,17,21,28,24,25,26]+ [27,38,29,10,31,32]+[34,35,1,37,23]	0.90	0.96
58	9	7	6	[2,3,4,5,6,7,8,9]+[10,11,12,13,1,15,19,17]+[20,21,22,23,24,25,26,27]+ [37,29,30,31,32,33,34,35]+[36,53,38,39,40,41,42,43]+ [57,46,47,48,49,50]+[52,28,54,55,56]	0.89	0.98
76	9	7	6	[2,3,4,5,6,7,8,9]+[36,11,12,13,14,15,16,17]+ [20,21,22,23,24,25,26,27]+[28,29,30,31,74,33,34,35]+ [37,38,58,39,41,42,43,44]+[46,47,48,49,50,51,52,53]+ [54,55,64,57,75,59,45,61]+[63,56,65,66,67,68]+[70,69,72,73,1]	0.89	0.98
94	9	7	6	[2,3,4,5,6,7,8,9]+[72,11,12,13,14,15,16,17]+[20,21,22,23,24,26,25,28]+ [29,54,31,32,33,34,35,36]+[37,38,46,40,41,42,43,44]+ [39,47,48,49,84,51,52,53]+[91,55,56,57,58,65,59,61]+ [89,64,90,66,68,67,69,74]+[30,73,88,75,76,77,78,79]+ [81,27,83,1,85,80]+[70,63,60,10,86]	0.90	0.99
24	11	7	6	[1,2,3,4,5,12,7,8,9,10,11]+[6,12,13,14,18,16,17]+[15,19,20,21,22,23]	0.61	0.94
46	11	7	6	[2,3,4,5,6,7,8,9,10,11]+[45,13,24,15,16,17,18,19,20,21]+ [14,23,39,26,1,28,29,30,32,31]+[12,40,34,44,37,38]+[35,41,42,43,36]	0.91	0.97
68	11	7	6	[2,3,4,5,6,7,8,9,12,11]+[13,14,15,16,17,18,19,20,21,22]+ [23,24,25,26,27,28,61,30,31,32]+[58,67,35,36,37,38,39,40,41,42]+ [44,45,46,47,48,49,55,51,52,53]+[64,56,57,34,59,60]+[62,63,50,65,66]	0.91	0.98

90	11	7	6	[2,3,4,5,6,7,8,9,10,11]+[13,14,15,16,17,18,19,26,21,22]+ [23,24,83,20,27,28,29,30,32,31]+[55,36,35,37,38,39,40,41,42,43]+ [46,45,47,48,49,50,51,52,53,54]+[56,84,59,58,60,61,62,63,64,65]+ [85,67,68,69,80,71,73,72,74,75]+[77,78,79,86,81,82]+[1,66,70,87,34]	0.91	0.98
26	11	9	6	[3,2,5,4,6,7,8,9,10,11]+[1,14,13,20,16,17,18,19]+[21,22,23,24,25]	0.91	0.94
48	11	9	6	[2,3,4,5,6,7,8,9,10,11]+[13,15,14,16,17,18,19,25,21,22]+ [23,24,44,26,28,27,29,30,1,32]+[34,35,36,37,38,39,43,41]+[40,20,45,46,47]	0.91	0.97
70	11	9	6	[2,3,4,5,6,7,12,9,10,11]+[14,13,16,15,17,18,19,20,21,22]+ [66,26,27,25,28,29,30,31,32,33]+[8,35,37,36,38,39,40,41,42,43]+ [45,47,46,48,49,50,51,52,53,56]+[54,64,58,59,34,60,62,44]+[65,24,67,68,69]	0.91	0.98
92	11	9	6	[90,3,4,5,6,7,8,9,12,11]+[13,14,15,16,17,18,30,20,21,23]+ [24,25,26,27,28,1,19,39,32,33]+[35,36,70,38,88,40,41,42,43,44]+ [83,46,48,47,50,86,58,52,53,54]+[85,84,81,87,80,61,62,68,64,65]+ [2,63,57,69,37,72,73,74,55,76]+[78,79,49,51,45,46,71,56]+[60,59,10,34,82]	0.91	0.91

Construction 6.2: Efficient MCPBRMDs-II for $v = p_{1i} + p_2 + 6$, i even, $p_1 = 9 \& 11$, $p_2 = 8 \& 10$, $p_1 > p_2$ and $p_3 = 6$.

v	p_1	p_2	p_3	Sets	Er	Es
32	9	8	6	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,21]+[17,22,23,24,25]	0.89	0.87
50	9	8	6	[2,3,4,5,11,7,8,9]+[35,12,13,14,15,16,17,18]+[20,49,36,22,24,23,25,26]+ [28,29,30,31,33,32,34,6]+[48,47,38,46,45,41,42]+[40,39,37,21,19]	0.89	0.97
68	9	8	6	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+ [40,21,22,23,1,25,26,27]+[44,67,30,31,33,32,34,35]+ [36,37,38,10,39,41,42,43]+[60,47,48,50,49,51,52,53]+ [55,56,57,58,59,64,62]+[63,46,29,45,28]	0.89	0.97
86	9	8	6	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+ [20,21,22,24,23,25,28,27]+[29,30,31,32,33,34,35,36]+ [55,38,39,41,40,1,43,44]+[83,79,47,48,49,50,51,52]+ [37,72,53,58,59,60,61,62]+[64,65,85,77,19,69,70]+[73,74,75,76,67]	0.89	0.89
36	11	8	6	[2,3,4,5,6,9,8,7,16,11]+[12,13,14,10,15,17,18,19,20,24]+ [23,35,25,26,27,28,30]+[31,32,33,34,21]	0.91	0.96
58	11	8	6	[2,3,4,5,6,7,8,9,10,11]+[13,14,17,16,15,18,19,20,21,22]+ [50,41,26,25,27,28,29,30,31,32]+[33,34,35,36,37,38,39,40,24,42]+ [45,46,47,43,56,55,12]+[53,1,23,49,54]	0.91	0.97
80	11	8	6	[2,3,4,5,6,7,8,9,10,11]+[13,14,68,16,17,18,19,20,21,22]+ [24,25,26,27,28,29,30,31,32,33]+[73,36,37,38,39,40,41,42,43,51]+ [46,45,47,48,49,50,72,52,53,54]+[71,78,58,59,60,61,62,63,64,65]+ [66,1,69,74,56,34,23]+[75,76,67,57,55]	0.91	0.98
38	11	10	6	[2,3,4,5,6,8,7,9,10,11]+[28,3,14,15,17,16,18,19,20,21]+ [22,23,24,25,26,37,12,36,30]+[33,34,35,29,27]	0.86	0.93
60	11	10	6	[2,4,3,5,6,7,8,9,10,11]+[13,14,15,23,16,19,18,20,21,22]+ [17,24,25,26,27,28,29,30,31,33]+[34,35,36,47,38,39,40,42,41,12]+ [58,46,57,37,48,49,50,51,52]+[1,56,45,41,43]	0.91	0.95
82	11	10	6	[2,3,4,5,6,7,8,9,10,11]+[12,13,15,14,16,76,19,18,20,21]+ [43,24,25,26,27,28,29,30,31,32]+[34,35,36,37,38,39,40,41,46,23]+ [44,45,81,47,48,49,50,51,52,53]+[74,79,71,60,59,61,62,63,64,65]+ [66,68,67,69,70,58,72,73,56]+[75,77,78,1,55]	0.91	0.95

Construction 6.3: Efficient MCPBRMDs-II for $v = p_{1i} + p_2 + 7$, i odd, $p_1 = 9 \& 11$, $p_2 = 8 \& 10$, $p_1 > p_2$ and $p_3 = 7$.

v	p_1	p_2	p_3	Sets	Er	Es
24	9	8	7	[2,3,4,6,5,10,8,9]+[23,11,12,13,18,15,16]+[17,14,19,20,21,22]	0.83	0.94
42	9	8	7	[3,2,4,5,6,7,8,9]+[10,11,12,14,15,16,17,18]+[19,20,21,30,22,23,24,25]+ [28,27,38,29,31,32,33]+[35,36,37,21,1,39]	0.90	0.97
60	9	8	7	[2,3,4,5,6,7,8,9]+[11,10,12,13,24,14,1,17]+[19,20,22,21,23,57,25,26]+ [28,29,30,48,31,32,33,34]+[36,37,39,38,40,41,42,43]+ [59,58,47,54,49,50,51]+[53,30,55,15,56,45]	0.90	0.98
78	9	8	7	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+[20,21,22,23,25,24,26,27]+ [28,29,30,31,32,33,58,35]+[37,38,39,67,41,42,43,44]+ [45,73,47,48,49,51,50,52]+[54,55,56,57,1,59,63,61]+ [77,76,75,74,68,69,10]+[71,72,70,66,64,65]	0.89	0.98

96	9	8	7	[2,3,4,5,6,7,8,9]+[11,12,13,14,15,16,17,18]+[20,21,19,23,24,29,30,27]+ [28,26,25,31,32,39,33,38]+[37,35,54,40,41,42,43,44]+ [46,45,47,48,49,50,51,88]+[1,69,53,57,58,59,60,61]+ [92,80,65,66,67,68,93,70]+[87,85,86,75,10,77,78,83]+ [90,82,79,84,73,74,72]+[89,81,64,63,55,34]	0.89	0.99
26	11	8	7	[3,2,5,4,6,7,8,9,10,11]+[13,12,25,14,15,16,17]+[19,20,21,22,23,24]	0.92	0.95
48	11	8	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,25,21,22]+ [12,13,15,14,16,17,18,19,25,21]+[23,43,24,26,27,28,29,30,1]+ [47,46,45,44,37,38,39]+[41,42,20,36,35,34]	0.88	0.94
70	11	8	7	[2,3,4,5,6,7,8,9,10,15]+[12,13,14,16,38,17,18,19,20,21]+ [65,24,25,27,26,28,29,31,30,32]+[54,35,36,37,11,39,47,41,42,43]+ [62,60,61,40,48,49,50,51,52,53]+[55,56,57,58,59,46,45]+[63,64,23,66,67,68]	0.91	0.98
94	11	8	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,20,21]+ [22,24,25,26,27,28,1,30,34,32]+[75,35,36,37,38,39,40,41,42,43]+ [67,46,47,48,49,50,51,52,53,54]+[93,92,57,58,59,60,61,62,63,64]+ [91,45,69,68,70,71,72,73,74,31]+[55,56,79,80,81,76,90]+[78,82,77,65,86,87]	0.91	0.97
38	11	9	7	[2,3,4,5,6,8,7,9,10,11]+[28,37,14,15,16,17,18,19,24,21]+ [22,23,20,25,26,13,2,29]+[31,32,33,34,35,36]	0.98	0.94
60	11	9	7	[2,4,3,5,6,7,8,9,10,11]+[13,14,15,16,17,18,19,20,28,21]+ [23,24,25,26,27,22,29,30,31,33]+[44,58,36,35,37,38,39,40,56,54]+ [52,45,46,47,48,49,51,50]+[53,42,1,41,57,34]	0.91	0.98
82	11	9	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,76,19,18,21,20]+ [43,24,25,26,27,28,29,30,31,32]+[34,35,36,37,38,39,40,41,46,23]+ [44,45,81,47,48,49,50,51,52,53]+[74,73,72,58,59,60,1,62,63,64]+ [66,67,68,69,65,71,57,56]+[75,61,77,78,79,80]	0.91	0.98
28	11	10	7	[2,3,5,4,6,7,8,9,10,11]+[12,13,14,15,17,16,18,1,20]+[21,22,23,24,25,26]	0.92	0.93
50	11	10	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,33,21]+ [23,24,25,26,28,27,29,30,31,32]+[49,20,1,36,37,38,48,39,40]+ [43,44,45,46,47,41]	0.92	0.97
72	11	10	7	[2,3,4,5,12,7,8,9,10,11]+[13,14,15,16,17,18,19,20,21,22]+ [24,26,25,27,28,29,34,31,32,33]+[30,36,35,38,37,39,40,56,42,43]+ [45,47,46,48,49,50,51,52,53,65]+[55,56,57,58,59,60,61,62,63]+ [54,66,67,68,69,44]	0.91	0.97
94	11	10	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,20,21]+ [22,24,25,26,27,28,1,30,34,32]+[35,36,37,38,39,40,41,42,43,44]+ [46,47,49,48,50,51,52,53,54,67]+[93,92,57,58,59,60,61,62,63,64]+ [45,69,68,70,71,72,73,74,31,88]+[55,56,79,80,81,76,90,84,85]+ [78,82,77,65,86,87]	0.91	0.98

Construction 6.4: Efficient MCPBRMDs-II for $v = p_1i + p_2 + 7$, i even, $p_1 = 11$, $p_2 = 9$ and $p_3 = 7$.

v	p_1	p_2	p_3	Sets	Er	Es
38	11	9	7	[2,3,4,5,6,8,7,9,10,11]+[28,37,14,15,16,17,18,19,24,21]+ [22,23,20,25,26,13,2,29]+[31,32,33,34,35,36]	0.98	0.94
60	11	9	7	[2,4,3,5,6,7,8,9,10,11]+[13,14,15,16,17,18,19,20,28,21]+ [23,24,25,26,27,22,29,30,31,33]+[44,58,36,35,37,38,39,40,56,54]+ [52,45,46,47,48,49,51,50]+[53,42,1,41,57,34]	0.91	0.98
82	11	9	7	[2,3,4,5,6,7,8,9,10,11]+[12,13,14,15,16,76,19,18,21,20]+ [43,24,25,26,27,28,29,30,31,32]+[34,35,36,37,38,39,40,41,46,23]+ [44,45,81,47,48,49,50,51,52,53]+[74,73,72,58,59,60,1,62,63,64]+ [66,67,68,69,65,71,57,56]+[75,61,77,78,79,80]	0.91	0.98

Construction 6.5: Efficient MCPBRMDs-II for $v = p_1i + p_2 + 8$, i odd, $p_1 = 11$, $p_2 = 9$ and $p_3 = 8$.

v	p_1	p_2	p_3	Sets	Er	Es
28	11	9	8	[2,3,5,4,6,7,8,9,10,11]+[27,13,14,20,16,17,18,1]+[15,21,22,23,24,25,26]	0.93	0.95
50	11	9	8	[2,3,4,5,6,7,8,9,10,11]+[13,14,15,16,18,17,19,20,21,22]+ [23,24,12,25,26,28,40,29,30,31]+[47,34,1,36,37,38,27,39]+ [49,42,43,44,45,46,33]	0.92	0.97

72	11	9	8	[2,3,4,5,12,7,8,9,10,11]+[41,13,14,15,16,17,18,19,20,21]+ [24,26,25,27,28,29,34,31,32,33]+[70,35,36,37,48,39,40,6,42,43]+ [63,62,57,47,38,49,50,51,52,53]+[63,62,57,47,38,49,50,51]+ [56,55,66,58,59,60,61]	0.91	0.93
94	11	9	8	[29,2,3,4,5,6,7,8,9,10]+[12,13,14,15,16,17,18,19,20,21]+ [22,24,25,26,27,28,40,88,31,32]+[34,35,36,37,38,39,81,41,42,43]+ [45,46,47,48,49,50,51,52,75]+[53,56,57,58,59,60,61,62,63,64]+ [84,83,68,69,70,71,72,73,85,76]+[77,78,79,80,74,82,67,66]+ [86,87,30,89,90,91,92]	0.91	0.98

Construction 6.6: Efficient MCPBRMDs-II for $v = p_1i + p_2 + 8$, i even, $p_1 = 11$, $p_2 = 10$ and $p_3 = 8$.

v	p_1	p_2	p_3	Sets	Er	Es
40	11	10	8	[1,2,3,4,5,6,39,8,31,10]+[12,13,14,15,16,30,18,19,20,21]+ [23,24,25,26,27,28,29,17,9]+[33,34,35,36,37,38,7]	0.92	0.96
62	11	10	8	[1,2,3,4,5,6,7,8,9,35]+[11,13,14,15,16,17,18,19,20,21]+ [23,24,25,26,27,28,29,30,31,34]+ [32,10,36,37,38,39,40,41,42,45]+ [43,46,47,48,49,50,51,52,56]+[55,53,57,58,59,60,61]	0.86	0.98
84	11	10	8	[1,2,21,4,5,6,7,8,9,10]+[12,13,70,15,16,17,18,19,47,3]+ [23,24,25,26,27,28,29,39,50,32]+[34,35,36,37,38,30,40,41,42,43]+ [45,46,20,48,49,31,51,52,53,54]+[56,57,58,59,60,61,62,63,64,65]+ [66,68,83,14,71,72,73,74,75]+[77,78,79,80,81,82,69]	0.81	0.98

Construction 6.7: Efficient MCPBRMDs-II for $v = p_1i + p_2 + 9$, i odd, $p_1 = 11$, $p_2 = 10$ and $p_3 = 9$.

v	p_1	p_2	p_3	Sets	Er	Es
30	11	10	9	[25,2,6,3,4,5,7,9,8,10]+[29,13,14,15,16,17,22,19,20]+ [21,18,23,24,1,26,27,28]	0.93	0.95
52	11	10	9	[3,2,4,5,6,7,8,9,10,11]+[12,13,14,15,16,17,18,19,41,21]+ [34,24,25,26,27,28,29,30,31,32]+[33,23,35,36,37,38,1,47,20]+ [43,44,45,46,40,48,49,50]	0.92	0.97
74	11	10	9	[2,3,4,5,14,7,8,9,10,11]+[13,12,15,49,16,17,18,19,21,20]+ [73,24,25,27,26,28,29,42,31,32]+[34,35,36,37,38,39,41,40,43,64]+ [44,45,46,47,48,6,72,51,52,53]+[71,65,57,58,59,66,61,62,63]+ [70,60,67,68,69,56,55,50]	0.92	0.98
96	11	10	9	[1,2,3,4,5,6,7,8,9,40]+[12,13,14,15,16,17,18,19,20,26]+ [23,24,10,21,27,28,29,30,31,32]+[87,35,36,37,38,39,25,54,42,43]+ [45,46,47,48,49,76,51,52,66,41]+[56,57,58,59,60,61,62,63,64,65]+ [53,68,69,70,84,72,73,74,75,50]+[78,79,77,81,82,95,71,85,86]+ [88,89,90,91,92,93,94,83]	0.84	0.99

7 Results and Conclusion

In this article, efficient MCPBRMDs-I and efficient MCPBRMDs-II are constructed in three different period sizes, where $v/2$ ordered pairs (i) do not appear in MCPBRMD-I, and (ii) appear twice together in MCPBRMD-II while the remaining ones appear once. Catalogues of efficient MCPBRMDs-I and efficient MCPBRMDs-II are also compiled for $v \leq 100$, $p_1 = 9$ & 11 , $7 \leq p_2 \leq 10$ and $6 \leq p_3 \leq 9$. These catalogues provide the readymade solution to the researchers and experimenters to estimate the direct effects and residual effects independently.

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