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Less Conservative Stability Criteria for a Class of Nonlinear Stochastic Hopfield Neural Networks with Time-varying Delays

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Abstract: In this paper, the problem of stability analysis of nonlinear stochastic Hopfield neural networks(FHNNs) with time-varying delays is investigated by using the Takagi-Sugeno(T-S) approach. Combined with both the fuzzy relaxed technique and an improved free-weighting matrix approach with weighting-dependent Lagrange multipliers, less conservative stability criteria is proposed via the Lyapunov-Krasovskii functional approach. Furthermore, related algebraic properties of the fuzzy membership functions in the unit simplex are considered in the process of stability analysis and the obtained stability criteria is in terms of Linear matrix inequalities. Finally, an illustrative example shows less conservatism of the proposed approaches.

Keywords: relaxed stability criteria, Hopfield neural networks, nonlinear systems, linear matrix inequality (LMI)

1. Introduction

Over the past decades, the Hopfield neural networks (HNNs) [1] have been extensively studied because of their important applications in various fields such as combinatorial optimization, signal processing, image processing and pattern recognition problems, see for examples [2-3]. These applications are built upon the stability of the equilibrium of neural networks. Thus, the stability analysis is a necessary step of the design and applications of neural networks. On the other hand, both in biological and artificial neural networks, the interactions between neurons are generally asynchronous which inevitably result in time delays [4]. Hence, there exist amounts of stability results for various neural network with several kinds of time delays [5-12]. Recently, fuzzy logic theory has shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear systems. In [13], Takagi and Sugeno proposed an effective way to transform a nonlinear dynamic system to a set of linear sub-models via some fuzzy models by defining a linear input-output relationship as its consequence of individual plant rule. More importantly, the authors in [14] have proved that the T-S fuzzy systems can be approximate to any continuous functions in a compact set of $\mathbb{R}^n$ at any preciseness. This allows the designers to take advantage of conventional linear systems to analyze the nonlinear systems [15-19]. Moreover, when performing the computation, there are many stochastic perturbations that affect the stability of neural networks. A neural network could be destabilized by certain stochastic inputs. It implies that the stability analysis of nonlinear stochastic neural networks also has primary significance in the research of neural networks.

Recently, the standard T-S fuzzy model has been used to analyze the stability analysis for various kinds of nonlinear neural networks with time delays and some stability conditions were presented in terms of linear matrix inequalities(LMIs). The problem of exponential stability for T-S fuzzy model, in which the consequent parts are composed of a set of stochastic HNNs with time-varying delays, has been addressed in [20], and the global asymptotic stability problem of fuzzy BAM neural networks with time-varying delays and parameter uncertainties has been investigated by means of T-S fuzzy modeling approach in [21]. New global stability criterion for T-S fuzzy Hopfield neural networks with time delays

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has also been provided in [22] by using a generalized Lyapunov functional and introducing a parameterized model transformation with free weighting matrices. Moreover, robust stability for uncertain delayed fuzzy Hopfield neural networks with markovian jumping parameters has also been investigated in [23] by considering both the lower and the upper value of the interval time delay. More recently, delay-dependent stability analysis for stochastic fuzzy neural networks has been investigated in [24-27]. To the best of our knowledge, the common Lyapunov-Krasovskii functional (one common Lyapunov matrix P for the overall fuzzy space) and conventional relaxed techniques were applied in the proof of [20-24] which tends to produce much conservatism.

In this paper, less conservative stability criteria for nonlinear stochastic Hopfield neural networks with time varying delays is proposed via the T-S fuzzy approach. To reduce the conservatism of previous results, we construct a parameter dependent Lyapunov-Krasovskii functional and derive less conservative stability criteria through using an improved free-weighting matrix approach with weighting-dependent Lagrange multipliers. Furthermore, new fuzzy relaxed techniques are also developed to further reduce the conservatism and algebraic properties of the fuzzy membership functions in the unit simplex are considered in the process of stability analysis. Finally, a numerical example is also given to demonstrate the effectiveness of the method proposed in this paper.

2. System description and preliminaries

The model of Hopfield neural networks with time-varying delay can be expressed as follows:

$$\dot{u}_i(t) = -c_i u_i(t) + \sum_{j=1}^{n} a_{ij} g_j(u_j(t-\tau(t))) + J_i, \qquad (1)$$

where $i = 1,2,...,n$, $u_i(t)$ is the state variable of the ith neuron at time t; $c_i > 0$ represents the passive decay rate; a_{ij} is the synaptic connection weight; $g_j(\cdot)$ is the activation function of the neuron; J_i denotes the external input; $\tau(t)$ represents the time-varying delay of neural networks satisfying $0 < \tau(t) \le h$ and $\dot{\tau}(t) < \sigma$.

Throughout this paper, we make the following assumption about $g_j(u_j(t))$.

Assumption 2.1:

$$0 \le \frac{g_i(x) - g_i(y)}{x - y} \le L_q, q = 1,2,...,n,$$

for all $x, y \in \mathbb{R}, x \ne y$ and denotes $L = \{L_1, L_2, ..., L_n\}$.

It is reasonable to assume that the neural network (1) has only one equilibrium point[1], denoted by

$$u^* = (u_1^*, u_2^*, \ldots, u_n^*).$$

We shift the equilibrium to the origin by transformation $x(t) = u(t) - u^*$, which yields the following system:

$$\frac{dx(t)}{dt} = -Cx(t) + Af(x(t-\tau(t))), \qquad (2)$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$, $C = (c_1, c_2, \cdots, c_n)$, $A = (a_{ij})_{n\times n}$,

$$f(x) = [f_1(x_1), f_2(x_2), ..., f_n(x_n)] \in \mathbb{R}^n$$

with $f_i(x_i) = g_i(x_i + u_i^* - g_i(u_i^*))(i = 1,2,...,n)$. Under the above assumption, it is easy to get $|f_i(x_i(t))| \le l_i |x_i(t)| (i = 1,2,...,n)$.

As mentioned previously, stochastic perturbations in neural networks are always unavailable in practice. Therefore, the kth rule of the T-S fuzzy neural network with stochastic perturbations is of the following form:

Plant Rule k:
IF $\theta_1(t)$ is η_1^k, and ..., and $\theta_p(t)$ is η_p^k, **THEN**

$$\begin{aligned} dx(t) =& [-C_k x(t) + A_k f(x(t-\tau(t)))]dt \\ &+ [M_k x(t) + N_k x(t-\tau(t))]d\varpi(t), \end{aligned} \qquad (3)$$

where $k = 1,2,...,r$, η_i^k $(i = 1,2,...,p)$ is the fuzzy set, $\theta(t) = [\theta_1(t), ..., \theta_p(t)]^T$ is the premise variable vector, r is the number of IF-THEN rules. $\varpi(t)$ is a one-dimensional Brownian motion defined on

$$(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\ge 0}, \mathscr{P}).$$

$\phi \in L^2_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$ is the initial value of (3). C_k, A_k, M_k and N_k are constant known real matrices.

The defuzzified output of the stochastic T-S fuzzy system (3) is represented as follows:

$$\begin{aligned} dx(t) = & \sum_{k=1}^{r} \mu_k(\theta(t))\{[-C_k x(t) + A_k f(x(t-\tau(t)))]dt \\ & + [M_k x(t) + N_k x(t-\tau(t))]d\varpi(t)\}, \end{aligned} \qquad (4)$$

where

$$\mu_k(\theta(t)) = \frac{\nu_k(\theta(t))}{\sum_{j=1}^{r} \nu_j(\theta(t))}, \ \nu_k(\theta(t)) = \prod_{j=1}^{p} \eta_j^k(\theta_j(t))$$

in which $\eta_j^k(\theta_j(t))$ is the grade of membership of θ_j^k in η_j^k. According to the theory of fuzzy sets, we have:

$$\mu_k(\theta(t)) \ge 0, \sum_{k=1}^{r} \mu_k(\theta(t)) = 1, \sum_{k=1}^{r} \dot{\mu}_k(\theta(t)) = 0. \qquad (5)$$

In the existing work, the following assumption is usually given for facilitating the stability analysis:

Assumption 2.2 [17]: The time derivatives of the membership functions satisfy $|\dot{\mu}_k(\theta(t))| \le \phi_k$ $(k = 1,2,...,r)$, where ϕ_k is a given positive scalar.

For stochastic systems, It$\hat{o}$ formula plays an important role in the stability analysis of stochastic systems. Consider a general stochastic system $dx(t) = f(x(t),t)dt + g(x(t),t)d\varpi(t)$ on $t \ge t_0$ with initial value $x(t_0) = x_0 \in \mathbb{R}^n$, where $f :\in \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n\times m}$ and $g :\in \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n\times m}$. Let $\mathscr{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+; \mathbb{R}^+)$ denote the family of all nonnegative functions $V(x,t)$ on $\mathbb{R}^n \times \mathbb{R}^+$ which are continuously twice differentiable in x and once differentiable in t, an stochastic operator $\mathscr{L}V(x,t)$ is defined from $\mathbb{R}^n \times \mathbb{R}^+$ to $\mathbb{R}$ by

$$\begin{aligned}\mathscr{L}V(x,t) =& V_t(x,t) + V_x(x,t)f(x,t)\\ &+ \frac{1}{2}\mathtt{trace}[g^T(x,t)V_{xx}(x,t)g(x,t)],\end{aligned}$$

where

$$\begin{aligned}V_t(x,t) =& \frac{\partial V(x,t)}{\partial t},\ V_x(x,t)\\ =& \left(\frac{\partial V(x,t)}{\partial x_1}, \cdots, \frac{\partial V(x,t)}{\partial x_n}\right),\\ V_{xx}(x,t) =& \left(\frac{\partial^2 V(x,t)}{\partial x_i \partial x_j}\right)_{n\times n}.\end{aligned}$$

We end this section with an useful lemma.

Lemma 2.1: [30] For any positive definite symmetric constant matrix $M \in \mathbb{R}^{n\times n}$, the scalars $r_1 < r_2$ and vector function $w : [r_1, r_2] \to \mathbb{R}^n$ such that the concerned integrations are well defined, then the following inequality holds:

$$\left(\int_{r_1}^{r_2} w(s)ds\right)^T M \int_{r_1}^{r_2} w(s)ds \le r_{12}\int_{r_1}^{r_2} w^T(s)Mw(s)ds$$

where $r_{12} = r_2 - r_1$.

3. Main results

In this section, based on the Lyapunov stability theorem and the stochastic analysis approach, less conservative stability criteria for the stochastic fuzzy HNNs (3) in the mean square sense will be proposed by applying both a parameter-dependent Lyapunov-Krasovskii functional and some fuzzy relaxed techniques. For simplicity, we use x, μ_k instead of $x(t), \mu_k(\theta(t))$ respectively in the following sections. For convenience, we set

$$\begin{aligned}&\bar{C} = \sum_{k=1}^{r}\mu_k C_k,\ \bar{A} = \sum_{k=1}^{r}\mu_k A_k,\ \bar{M} = \sum_{k=1}^{r}\mu_k M_k,\\ &\bar{N} = \sum_{k=1}^{r}\mu_k N_k,\ x_\tau = x(t-\tau(t)),\ \tau = \tau(t),\end{aligned}$$

then the system (4) can be rewritten as:

$$dx = [-\bar{C}x + \bar{A}f(x_\tau)]dt + [\bar{M}x + \bar{N}x_\tau]d\varpi(t). \tag{6}$$

Theorem 3.1. For system (6), suppose Assumption 2 holds. Given scalar h,σ and $\phi_i (i = 1,2,...,r)$, the stochastic T-S fuzzy system (6) is globally asymptotically stable in the mean square sense, if there exist two positive diagonal matrix X and Y, real matrices $P_i > 0 (i = 1,...,r), Q > 0, R > 0, S > 0, W_{ik}, V_{ik} (i = 1,2,...,6; k = 1,2,...,r)$, $,U_{iii} = U_{iii}^T (i = 1,2,...,r)$, $U_{iij} = U_{jii}^T$ and $U_{iji} = U_{iji}^T (i = 1,2,...,r, i \ne j, j = 1,2,...,r)$, $U_{ijl} = U_{lji^T}$, $U_{jil} = U_{lij}^T$ and $U_{ilj} = U_{jli}^T (i = 1,2,...,r-2, j = i+1,...,r-1, l = j+1,...,r)$, such that the following LMIs hold:

$$\begin{aligned}&\Pi_{iii} < U_{iii},\ i = 1,2,...,r;\\ &\Pi_{iij} + \Pi_{iji} + \Pi_{jii} < U_{iij} + U_{iji} + U_{iij}^T,\\ &\textit{where } i = 1,2,...,r, i \ne j, j = 1,2,...,r;\\ &\Pi_{ijl} + \Pi_{lji} + \Pi_{ilj} + \Pi_{jli} + \Pi_{jil} + \Pi_{lij}\\ &\quad < U_{ijl} + U_{ijl}^T + U_{ilj} + U_{ilj}^T + U_{jil} + U_{jil}^T,\\ &\textit{where } i = 1,2,...,r-2, j = i+1,...,\ r-1,\\ &l = j+1,...,r;\\ &\begin{bmatrix} U_{1i1} & U_{1i2} & \cdots & U_{1ir}\\ U_{2i1} & U_{2i2} & \cdots & U_{2ir}\\ \vdots & \vdots & \ddots & \vdots\\ U_{ri1} & U_{ri2} & \cdots & U_{rir}\end{bmatrix} < 0, i = 1,2,...,r;\end{aligned}$$

where $\Pi_{ijl} = \begin{bmatrix} \Pi_{ijl}^{11} & \Pi_{ijl}^{12} & \Pi_{ijl}^{13} & \Pi_{ijl}^{14} & \Pi_{ijl}^{15} & \Pi_{ijl}^{16} & V_{1i}\\ * & \Pi_{ijl}^{22} & \Pi_{ijl}^{23} & \Pi_{ijl}^{24} & \Pi_{ijl}^{25} & \Pi_{ijl}^{26} & V_{2i}\\ * & * & \Pi_{ijl}^{33} & \Pi_{ijl}^{34} & 0 & \Pi_{ijl}^{36} & V_{3i}\\ * & * & * & \Pi_{ijl}^{44} & \Pi_{ijl}^{45} & \Pi_{ijl}^{46} & V_{4i}\\ * & * & * & * & \Pi_{ijl}^{55} & \Pi_{ijl}^{56} & V_{5i}\\ * & * & * & * & * & \Pi_{ijl}^{66} & V_{6i}\\ * & * & * & * & * & * & -S\end{bmatrix}$, and

$\Pi_{ijl}^{11} = \sum_{k=1}^{r}\phi_k(P_k + E_{ijl}) + W_{1i}C_l + C_i^T W_{1l}^T + V_{1i} + V_{1i}^T + hM_i^T SM_l + M_i^T P_j M_l$,
$\Pi_{ijl}^{12} = W_{2i} + C_i^T W_{2l}^T + V_{2i}^T - V_{1i} + hM_i SN_l + M_i^T P_j N_l$,
$\Pi_{ijl}^{13} = XL + W_{3i} + C_i^T W_{3l}^T + V_{3i}^T$,
$\Pi_{ijl}^{14} = W_{4i} + C_i^T W_{4l}^T - W_{1i}A_l + V_{4i}^T$,
$\Pi_{ijl}^{15} = P_i + W_{1i} + C - i^T W_{5l}^T + V_{5i}^T$,
$\Pi_{ijl}^{16} = C_i^T W_{6l}^T + V_{6i}^T - V_{1i}$,
$\Pi_{ijl}^{22} = -V_{2i} - V_{2i}^T + hN_i^T SN_i + N_i^T P_j N_l$,
$\Pi_{ijl}^{23} = -V_{3i}^T$,
$\Pi_{ijl}^{24} = YL - W_{2i}A_l - V_{4i}^T$,
$\Pi_{ijl}^{25} = -V_{5i}^T$,
$\Pi_{ijl}^{26} = -V_{6i}^T - V_{2i}$, $\Pi_{ijl}^{33} = Q - 2X$,
$\Pi_{ijl}^{34} = -W_{3i}A_l$, $\Pi_{ijl}^{36} = -V_{3i}$,
$\Pi_{ijl}^{44} = -(1-\sigma)Q - 2Y - W_{4i}A_l - A_i^T W_{4l}^T$,
$\Pi_{ijl}^{45} = -A_i^T W_{5l}^T$,
$\Pi_{ijl}^{46} = -A_i^T W_{6l}^T - V_{4i}$,
$\Pi_{ijl}^{55} = hR + W_{5i} + W_{5i}^T$,
$\Pi_{ijl}^{56} = W_{6i}^T - V_{5i}$,
$\Pi_{ijl}^{66} = -\frac{1-\sigma}{h}R - V_{6i} - V_{6i}^T$.

Proof. For simplicity, let us denote

$$g(t) = -\bar{C}x + \bar{A}f(x_\tau),\ y(t) = \bar{M}x + \bar{N}x_\tau. \tag{7}$$

Choosing a parameter-dependent Lyapunov-Krasovskii functional as follows:

$$V(x,t) = V_1(x,t) + V_2(x,t) + V_3(x,t) + V_4(x,t), \quad (8)$$

where

$$V_1(x,t) = x^T \left(\sum_{i=1}^{r} \mu_i P_i\right) x,$$

$$V_2(x,t) = \int_{t-\tau}^{t} f^T(x(s)) Q f(x(s)) ds,$$

$$V_3(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} g^T(s) R g(s) ds d\theta,$$

$$V_4(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s) S y(s) ds d\theta.$$

By the Itô formula, we can calculate $\mathscr{L}V(x,t)$ along (6), then we have

$$\begin{aligned}\mathscr{L}V(x,t) = {} & 2x^T \left(\sum_{i=1}^{r} \mu_i P_i\right) g(t) + x^T \left(\sum_{k=1}^{r} \dot{\mu}_k P_k\right) x \\ & + y^T(t) \left(\sum_{i=1}^{r} \mu_i P_i\right) y(t) + f^T(t) Q f(t) \\ & -(1-\dot{\tau}) f^T(t-\tau) Q f(t-\tau) + \tau g^T(t) R g(t) \\ & -(1-\dot{\tau}) \int_{t-\tau}^{t} g^T(s) R g(s) ds + \tau y^T(t) S y(t) \\ & -(1-\dot{\tau}) \int_{t-\tau}^{t} y^T(s) S y(s) ds. \end{aligned} \quad (9)$$

By using Lemma 1, we obtain

$$\begin{aligned} & -\int_{t-\tau}^{t} g^T(s) R g(s) ds \\ & \quad \leq -\frac{1}{\tau}\left(\int_{t-\tau}^{t} g(s) ds\right)^T R \left(\int_{t-\tau}^{t} g(s) ds\right). \end{aligned} \quad (10)$$

Noting two diagonal positive definite matrices X and Y and using Assumption (A), we can have that

$$2f^T(x(t)) X L x(t) - 2f^T(x(t)) X f(x(t)) \geq 0, \quad (11)$$

$$\begin{aligned} & 2f^T(x(t-\tau)) Y L x(t-\tau) \\ & \quad - 2f^T(x(t-\tau)) Y f(x(t-\tau)) \geq 0. \end{aligned} \quad (12)$$

Recalling (6) and (7), it is easy to see that the following equalities hold

$$0 = 2\xi^T(t) W \cdot (g(t) + \bar{C}x - \bar{A} f(x_\tau)), \quad (13)$$

$$\begin{aligned} 0 = {} & 2\xi^T(t) V \cdot \\ & \left(x - x_\tau - \int_{t-\tau}^{t} g(s) ds - \int_{t-\tau}^{t} y(s) d\varpi(s)\right), \end{aligned} \quad (14)$$

where

$$\xi(t) = \left[x^T, x_\tau^T, f^T(x(t)), f^T(x(t-\tau)), g^T(s), \upsilon^T\right]^T$$

and $\upsilon = \left(\int_{t-\tau}^{t} g(s) ds\right)$

$$W = \left[W_1^T(t)\ W_2^T(t)\ W_3^T(t)\ W_4^T(t)\ W_5^T(t)\ W_6^T(t) \right]^T,$$

$$V = \left[V_1^T(t)\ V_2^T(t)\ V_3^T(t)\ V_4^T(t)\ V_5^T(t)\ V_6^T(t) \right]^T,$$

with $W_i(t) = \sum_{k=1}^{r} \mu_k W_{ik}, V_i(t) = \sum_{k=1}^{r} \mu_k V_{ik} (i = 1,2,...,6)$.

Based on (5), it follows that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{k=1}^{r} \mu_i \mu_j \mu_l \dot{\mu}_k E_{ijl} = \bar{E} = 0,$$

where E_{ijl} any symmetric matrices of proper dimensions. Recalling the assumption that $|\dot{\mu}_i| \leq \phi_i$, thus the following inequality holds:

$$\begin{aligned} & x^T \left(\sum_{k=1}^{r} \dot{\mu}_k P_k\right) x \\ & = x^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{k=1}^{r} \mu_i \mu_j \mu_l \dot{\mu}_k (P_k + E_{ijl})\right) x \\ & \leq x^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{k=1}^{r} \phi_k (P_k + E_{ijl})\right) x. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} & -2\xi^T(t) V \int_{t-\tau}^{t} y(s) d\varpi(s) \\ & \leq \frac{1}{1-\sigma} \xi^T(t) V S^{-1} V^T \xi(t) \\ & +(1-\sigma)\left(\int_{t-\tau}^{t} y(s) d\varpi(s)\right)^T S \left(\int_{t-\tau}^{t} y(s) d\varpi(s)\right). \end{aligned} \quad (15)$$

From (13-15), we can obtain

$$\begin{aligned} & \mathscr{L}V(x,t) \leq \xi^T(t) \left(\Xi + V S^{-1} V^T\right) \xi(t) \\ & -(1-\sigma) \int_{t-\tau}^{t} y^T(s) S y(s) ds \\ & +(1-\sigma)\left(\int_{t-\tau}^{t} y(s) d\varpi(s)\right)^T S \left(\int_{t-\tau}^{t} y(s) d\varpi(s)\right), \end{aligned} \quad (16)$$

where $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ * & * & \Xi_{33} & \Xi_{34} & 0 & \Xi_{36} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\ * & * & * & * & \Xi_{55} & \Xi_{56} \\ * & * & * & * & * & \Xi_{66} \end{bmatrix}$, here

$$\begin{aligned} \Xi_{11} = & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{k=1}^{r} \phi_k (P_k + E_{ijl}) \\ & + W_1(t)\bar{C} + \bar{C}^T W_1^T(t) + V_1(t) + V_1^T(t) \\ & + h\bar{M}^T S \bar{M} + \bar{M}^T (\sum_{i=1}^{r} \mu_i P_i) \bar{M}, \end{aligned}$$

$$\begin{aligned} \Xi_{12} = & W_2(t) + \bar{C}^T W_2^T(t) + V_2^T(t) - V_1(t) \\ & + h\bar{M} S \bar{N} + \bar{M}^T (\sum_{i=1}^{r} \mu_i P_i) \bar{N}, \end{aligned}$$

$\Xi_{13} = XL + W_3(t) + \bar{C}^T W_3^T(t) + V_3^T(t),$
$\Xi_{14} = W_4(t) + \bar{C}^T W_4^T(t) - W_1(t)\bar{A} + V_4^T(t),$
$\Xi_{15} = \sum_{i=1}^r \mu_i P_i + W_1(t) + \bar{C}^T W_5^T(t) + V_5^T(t),$
$\Xi_{16} = \bar{C}^T W_6^T(t) + V_6^T(t) - V_1(t),$

$$\Xi_{22} = -V_2(t) - V_2^T(t) + h\bar{N}^T S\bar{N} + \bar{N}^T(\sum_{i=1}^r \mu_i P_i)\bar{N},$$

$\Xi_{23} = -V_3^T(t),\ \Xi_{24} = YL - W_2(t)\bar{A} - V_4^T(t),$
$\Xi_{25} = -V_5^T(t),\ \Xi_{26} = -V_6^T(t) - V_2(t),$
$\Xi_{33} = Q - 2X,\ \Xi_{34} = -W_3(t)\bar{A},\ \Xi_{36} = -V_3(t),$
$\Xi_{44} = -(1-\sigma)Q - 2Y - W_4(t)\bar{A} - \bar{A}^T W_4^T(t),$
$\Xi_{45} = -\bar{A}^T W_5^T(t),\ \Xi_{46} = -\bar{A}^T W_6^T(t) - V_4(t),$
$\Xi_{55} = hR + W_5(t) + W_5^T(t),\ \Xi_{56} = W_6^T(t) - V_5(t),$
$\Xi_{66} = -\frac{1-\sigma}{h}R - V_6(t) - V_6^T(t).$

Using the well-known Schur complement lemma, it is easy to see that $\Xi + VS^{-1}V^T < 0$ is equivalent to:

$$\Pi = \begin{bmatrix} \Xi & V \\ * & -S \end{bmatrix} < 0, \tag{17}$$

On the other hand, reordering the expression of Π, we can obtain:

$$\Pi = \sum_{i=1}^r \mu_i^3 \Pi_{iii} + \sum_{i=1}^r \sum_{j=1, j\neq i}^r \mu_i^2 \mu_j (\Pi_{iij} + \Pi_{iji} + \Pi_{jii}) + \sum_{i=1}^{r-2} \sum_{j=i+1}^{r-1} \sum_{l=j+1}^r \mu_i \mu_j \mu_l \Pi, \tag{18}$$

where $\Pi = \Pi_{ijl} + \Pi_{lji} + \Pi_{ilj} + \Pi_{jli} + \Pi_{jil} + \Pi_{lij}$, Π_{ijl} have been defined in Theorem 1.

Recalling (7-10), we have

$$\begin{aligned}\Pi &< \sum_{i=1}^r \mu_i^3 U_{iii} + \sum_{i=1}^r \sum_{j=1, j\neq i}^r \mu_i^2 \mu_j (U_{iij} + U_{iji} + U_{iij}^T) + \sum_{i=1}^{r-2} \sum_{j=i+1}^{r-1} \sum_{l=j+1}^r \mu_i \mu_j \mu_l (U) \\ &= [\mu \otimes I]^T \left(\sum_{i=1}^r \mu_i \begin{bmatrix} U_{1i1} & U_{1i2} & \cdots & U_{1ir} \\ U_{2i1} & U_{2i2} & \cdots & U_{2ir} \\ \vdots & \vdots & \ddots & \vdots \\ U_{ri1} & U_{ri2} & \cdots & U_{rir} \end{bmatrix} \right) [\mu \otimes I] < 0,\end{aligned} \tag{19}$$

where $U = U_{ijl} + U_{ijl}^T + U_{ilj} + U_{ilj}^T + U_{jil} + U_{jil}^T$, $\mu = [\mu_1, \mu_2, \cdots, \mu_r]^T$.

Using the property of Itô isometry, we have

$$\begin{aligned}&\mathbb{E}\left\{ \left(\int_{t-\tau}^t y(s) d\varpi(s) \right)^T S \left(\int_{t-\tau}^t y(s) d\varpi(s) \right) \right\} \\ &= \mathbb{E}\left\{ \int_{t-\tau}^t y^T(s) S y(s) ds \right\}.\end{aligned} \tag{20}$$

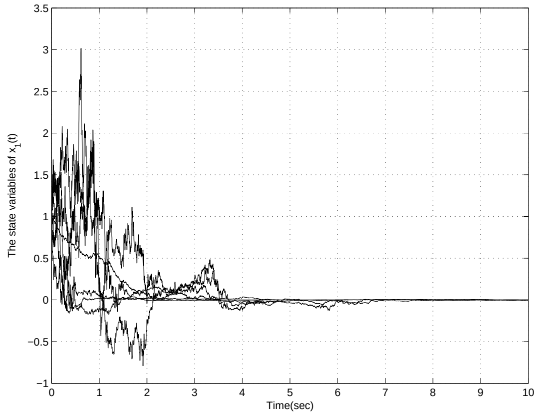

Figure 1 Six path trajectories of $x_1(t)$.

Taking the mathematical expectation of both sides of (21) and considering (24) and (25), there exists

$$\mathbb{E}[\mathscr{L}V(x,t)] \leq -\gamma \mathbb{E}|x(t)|^2. \tag{21}$$

Thus, it follows the stochastic stability theory that the stochastic T-S fuzzy system (6) is globally asymptotically stable in the mean square sense.

This completes the proof.

4. Numerical examples

Example 1. Consider the stochastic fuzzy HNNs (3) with $r = 2$. The fuzzy T-S fuzzy model of fuzzy Hopfield neural network is of the following form:

Plant Rules:

Rule 1: IF $\theta_1(t)$ are M_{k1}, THEN
$dx(t) = (-C_1 x(t) + A_1 f(x(t-\tau(t))))dt + [M_1 x(t) + N_1 x(t-\tau(t))]d\varpi(t),$

Rule 2: IF $\theta_2(t)$ are M_{k2}, THEN
$dx(t) = (-C_2 x(t) + A_2 f(x(t-\tau(t))))dt + [M_2 x(t) + N_2 x(t-\tau(t))]d\varpi(t),$

with $f(x) = \tanh(x)$. The membership functions for rules 1 and 2 are $M_{k1} = \frac{1}{e^{-2\theta_1(t)}}, M_{k2} = 1 - M_{k1}$, and
$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$
$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0.88 & 0.30 \\ 0.26 & -0.25 \end{bmatrix},$
$M_1 = \begin{bmatrix} 2.7 & 0 \\ 0 & 2.6 \end{bmatrix}, N_1 = \begin{bmatrix} 1.8 & 0 \\ 0 & 2.5 \end{bmatrix}.$

Combining with the above membership functions, we set the Assumption 2 as $|\dot{\mu}_1| < 10$ and $|\dot{\mu}|_2 < 10$. Using Theorem 1, the allowable value of h with $\dot{\tau}(t) = 0.5$ is calculated as $h = 2.6$s which is larger than those existing approaches' results. This fact illustrates that less conservative stability criteria is provided by using the method proposed in this paper. Next, choosing system

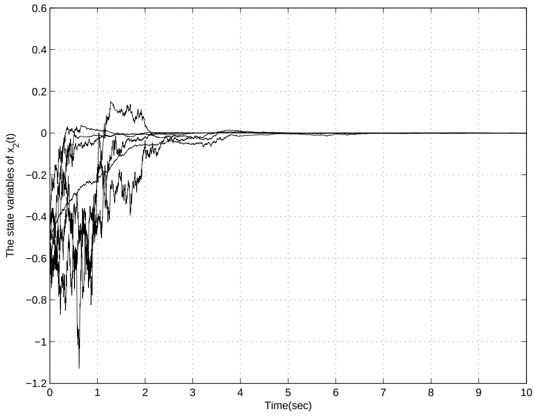

Figure 2 Six path trajectories of $x_2(t)$.

initial condition as $x(0) = (1,-0.5)^T$ and $h = 2.6$s, Fig. 1 and Fig. 2 show six path trajectories of $x_1(t)$ and $x_2(t)$ respectively. From these two figures, one can conclude that the above stochastic fuzzy Hopfield neural networks with time-varying delays is globally asymptotically stable in the mean square sense.

5. Conclusion

Less conservative stability criteria for nonlinear stochastic Hopfield neural networks with time-varying delays has been derived by using a parameter-dependent Lyapunov functional. The proposed condition is given in terms of LMIs and thus can be readily solved via standard numerical software. With the purpose of reducing the conservatism, new fuzzy relaxed techniques are also developed. Finally, numerical example has been provided to demonstrate the effectiveness of the proposed criterion.

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