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Fuzzy Contra *gprw*-Continuous Mappings

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Abstract: In this manuscript new types of fuzzy mappings namely fuzzy contra gprw-continuous mappings have been introduced & investigated. Also we found out its relation with various other fuzzy contra mappings introduced earlier. We also introduced fuzzy contra gprw-open mappings and fuzzy contra gprw-closed mappings in this paper.

Keywords: Fuzzy contra gprw-continuous mappings; Fuzzy contra pre-continuous mappings; Fuzzy contra rw-continuous mappings; Fuzzy contra gprw-open mappings; Fuzzy contra gprw-closed mappings.

1 Introduction

The idea of fuzzy contra mappings was put forward by Ekici and Kerre in 2006 in [7]. Soon after that, based on various other types of fuzzy sets various fuzzy contra mappings were introduced like in 2011 fuzzy contra rw-continuous mappings were introduced by A.Vadivel,V. Chandrasekar and M.Saraswathi in [8]. In 2012 in [6] S.E. Abbas and I.M. Taha introduced the concepts of fuzzy contra-continuity, fuzzy almost contra-continuity, fuzzy contra μ continuity, fuzzy almost contra μ continuity, fuzzy contra semi-continuity and generalized fuzzy contra continuity in.

Based on fuzzy gprw-closed sets, we have introduced a new type of mappings namely fuzzy contra gprw-continuous mappings in this manuscript and have found out its relation with various other mappings introduced earlier. We found out that all fuzzy contra continuous mappings are fuzzy gprw-continuous mappings, All fuzzy contra pre-continuous mappings are fuzzy contra gprw-continuous mappings & all fuzzy contra rw-continuous mappings are fuzzy contra gprw-continuous mappings. The relationship of this new mapping with other mappings have been depicted via a table figure. Also we have introduced fuzzy contra gprw-open mappings and fuzzy contra gprw-closed mappings in this paper.

2 Preliminaries

Definition 2.1 "A mapping f is said to be a fuzzy continuous mapping if $f^{-1}(\lambda) \in \tau X$ for each $\lambda \in \tau Y$ or, equivalently $f^{-1}(\mu)$ is a fuzzy closed set of X for each fuzzy closed set μ of Y". [4]

Definition 2.2 "A function $f : X \to Y$ is said to be fuzzy contra pre continuous, if $f^{-1}(\lambda)$ is fuzzy pre-closed in X for every fuzzy open set λ of Y".[1]

Definition 2.3 "Suppose X and Y are fuzzy topological spaces. A map $f : X \to Y$ is called fuzzy contra rw -continuous if the inverse image of every fuzzy open set in Y is fuzzy rw -closed in X" . [2]

Definition 2.4 "A function $h : H \to K$ is called *fuzzy generalized pre regular weakly continuous* (briefly Fgprw-continuous) if inverse image of every fuzzy closed set in fuzzy topological space K is fuzzy generalized pre regular weakly closed (Fgprw-closed) in fuzzy topological space H". [5]

Definition 2.5 "A function $h : (H, \tau_1) \to (K, \tau_2)$ is said to be *fuzzy generalized pre regular weakly-irresolute* (briefly Fgprw-irresolute) if$h^{-1}(\{\psi\})$ is fuzzy gprw-closed for every fuzzy gprw-closed $\{\psi\}$ in (K, τ_2)". [5]

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Definition 2.6 "let $f:(X,\tau_1)\to(Y,\tau_2)$ be a mapping. Then f is fuzzy contra open mapping, if it maps every fuzzy open set in (X,τ_1) to a fuzzy closed set in (Y,τ_2)".[6]

Definition 2.7 "A function f from a fuzzy topological space (X,τ) to fuzzy topological space (Y,δ) is called fuzzy contra pre-continuous (fuzzy contra α-continuous, fuzzy contra semi-continuous) if $f^{-1}(\lambda)$ is fuzzy pre-closed (fuzzy α-closed, fuzzy semi-closed resp.) in X for every fuzzy open set λ of Y". [1]

Remark 2.8 All fuzzy closed sets are fuzzy gprw-closed. [3]
Remark 2.9 All fuzzy pre-closed sets are fuzzy gprw-closed. [3]
Remark 2.10 All fuzzy rw-closed sets are fuzzy gprw-closed. [3]
Remark 2.11 All fuzzy open sets are fuzzy gprw-open. [3]

3 Fuzzy Contra gprw-Continuous Mappings

Definition 3.1 A mapping $r:(R,\tau_1)\to(S,\tau_2)$ is called *fuzzy contra gprw-continuous* if $r^{-1}(s): s\in\tau_2$ is fuzzy gprw-closed in R.

Theorem 3.2 A fuzzy contra continuous mapping $g:(G,\tau_1)\to(H,\tau_2)$ is always fuzzy contra gprw-continuous.

Proof: Consider $\psi\leq\tau_2$. Now, as g is fuzzy contra continuous implies $g^{-1}(\psi)$ is fuzzy closed in G. From *Remark 2.8* all fuzzy closed sets are fuzzy gprw-closed, so $g^{-1}(\psi)$ is fuzzy gprw-closed in G. Hence g is fuzzy contra gprw-continuous.

□

The other way round of the above theorem need not be true, as shown in the following example.

Example 3.3 Consider $G=H=\{l,m,n\}$ and function η ,ψ, $\chi: G\to[0,1]$be defined as

$$\eta(g)=\begin{cases}1 & \text{if } g=l\\ 0 & \text{otherwise}\end{cases} \qquad \psi(g)=\begin{cases}1 & \text{if } g=m\\ 0 & \text{otherwise}\end{cases}$$

$$\chi(g)=\begin{cases}1 & \text{if } g=m,n\\ 0 & \text{otherwise}\end{cases}$$

Suppose $\tau_1=\{0,1,\eta\}$, $\tau_2=\{0,1,\psi.\chi\}$. Now (G,τ_1) and (H,τ_2) are fuzzy topological spaces. Now define a function $f:(G,\tau_1)\to(H,\tau_2)$ by *f(l) = m, f(m) = n* and *f(n) = l*. Then f is fuzzy contra gprw-continuous & not fuzzy contra continuous as $f^{-1}(\psi)$ is η in (H,τ_2) & $\eta\in\tau_1$.

□

Theorem 3.4 A function $\zeta:(G,\tau_1)\to(H,\tau_2)$ is fuzzy contra gprw-continuous iff $\zeta^{-1}(\alpha)$ is fuzzy gprw-open in G for every $\alpha\in 1-\tau_2$.

Proof: Suppose $\alpha\in 1-\tau_2$, implying $1-\alpha\in\tau_2$. Now as ζ is fuzzy contra gprw-continuous, implies $\zeta^{-1}(1-\alpha)$ is fuzzy gprw-closed in G. Now as $\zeta^{-1}(1-\alpha)=1-\zeta^{-1}(\alpha)$ implies that $\zeta^{-1}(\alpha)$ is fuzzy gprw-open in G.
Contrarily, assume that $\zeta^{-1}(\alpha)$ is fuzzy gprw-open in G for every $\alpha\in 1-\tau_2$. Let $\beta\in\tau_2$, then $1-\beta$ is fuzzy closed in H. By hypothesis $\zeta^{-1}(1-\beta)=1-\zeta^{-1}(\beta)$ is fuzzy gprw open in G, implying $\zeta^{-1}(\beta)$ is fuzzy gprw-closed in G. Which proves the result.

□

Theorem 3.5 All fuzzy contra pre-continuous functions are fuzzy contra gprw-continuous.

Proof: Let $g:(G,\tau_1)\to(H,\tau_2)$ be fuzzy contra pre-continuous and suppose $\lambda\in\tau_2$. So $g^{-1}(\lambda)$ is fuzzy pre-closed in G . Now by *Remark 2.9* $g^{-1}(\lambda)$ is fuzzy gprw-closed in G. Hence g is fuzzy contra gprw-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.6 Consider $G=H=\{l,m,n\}$ and function ω,η ,ψ, $\chi: G\to[0,1]$be defined as

$$\omega(g)=\begin{cases}1 & \text{if } g=l\\ 0 & \text{otherwise}\end{cases} \qquad \psi(g)=\begin{cases}1 & \text{if } g=l,n\\ 0 & \text{otherwise}\end{cases}$$

$$\chi(g)=\begin{cases}1 & \text{if } g=l,m\\ 0 & \text{otherwise}\end{cases} \qquad \eta(g)=\begin{cases}1 & \text{if } g=m\\ 0 & \text{otherwise}\end{cases}$$

Suppose $\tau_1=\{0,1,\omega,\psi\}$, $\tau_2=\{0,1,\omega,\chi\}$. Now (G,τ_1) and (H,τ_2) are fuzzy topological spaces. Now, we define a function $f:(G,\tau_1)\to(H,\tau_2)$ by *f(l) = l, f(m) = n* and *f(n) = m*. Then *f* is fuzzy contra gprw-continuous & not fuzzy contra pre-continuous as $f^{-1}(\omega)$ in (H,τ_2) is ω, which is fuzzy gprw- closed in (G,τ_1) but not fuzzy pre-closed.

□

Theorem 3.7 A fuzzy contra rw-continuous mapping $g:(G,\tau_1)\to(H,\tau_2)$ is fuzzy contra gprw-continuous also.

Proof: Consider $\alpha\leq\tau_2$, Now as g is fuzzy contra rw-continuos, implies $g^{-1}(\alpha)$ is fuzzy rw-closed in G. Now from *Remark 2.10* all fuzzy rw-closed sets are fuzzy gprw-closed, so $g^{-1}(\alpha)$ is fuzzy gprw-closed in G, implying g is fuzzy contra gprw-continuous.

□

The converse of the above theorem need not be true as shown in the following example.

Example 3.8 Consider $G = H = \{l,m,n,o,p\}$ are fuzzy spaces and functions $\eta,\alpha,\beta,\gamma : G \to [0,1]$ and $\delta : H \to [0,1]$ are defined as

$$\alpha(g) = \begin{cases} 1 & \text{if } g = l,m \\ 0 & \text{otherwise} \end{cases} \qquad \eta(g) = \begin{cases} 1 & \text{if } g = p \\ 0 & \text{otherwise} \end{cases}$$

$$\beta(g) = \begin{cases} 1 & \text{if } g = n,o \\ 0 & \text{otherwise} \end{cases} \qquad \gamma(g) = \begin{cases} 1 & \text{if } g = l,m,n,o \\ 0 & \text{otherwise} \end{cases}$$

$$\delta(h) = \begin{cases} 1 & \text{if } h = l \\ 0 & \text{otherwise} \end{cases}$$

Suppose $\tau_1 = \{0,1,\alpha,\beta,\gamma\}$, $\tau_2 = \{0,1,\delta\}$.With these topologies (G,τ_1) and (H,τ_2) are fuzzy topological spaces. Now, we define a function $f : (G,\tau_1) \to (H,\tau_2)$ by *f(l) = m, f(m) = n, f(n) = o*, f(o) = p and f(p)= l. Then *f* is fuzzy contra gprw-continuous but not fuzzy contra rw-continuous as $f^{-1}(\delta)$ in (H,τ_2) is η, & η is fuzzy gprw- closed in (G,τ_1) but not fuzzy rw-closed.

□

Remark 3.9 In the following examples we prove that Fuzzy contra gprw-continuous and fuzzy contra semi-continuous mappings are independent.

Example 3.10 Consider $G = H = \{p,q,r,s\}$ are fuzzy spaces and functions $\alpha,\beta,\gamma,\delta : G \to [0,1]$ be defined as

$$\alpha(g) = \begin{cases} 1 & \text{if } g = p \\ 0 & \text{otherwise} \end{cases} \qquad \beta(g) = \begin{cases} 1 & \text{if } g = q \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma(g) = \begin{cases} 1 & \text{if } g = p,q \\ 0 & \text{otherwise} \end{cases} \qquad \delta(g) = \begin{cases} 1 & \text{if } g = p,q,r \\ 0 & \text{otherwise} \end{cases}$$

and $\psi,\eta : H \to [0,1]$ be defined as

$$\psi(h) = \begin{cases} 1 & \text{if } h = r \\ 0 & \text{otherwise} \end{cases} \qquad \eta(h) = \begin{cases} 1 & \text{if } h = r,s \\ 0 & \text{otherwise} \end{cases}$$

Suppose $\tau_1 = \{0,1,\alpha,\beta,\gamma,\delta\}$, $\tau_2 = \{0,1,\eta,\psi\}$.With these topologies (G,τ_1) and (H,τ_2) are fuzzy topological spaces. Now, we define a function $f : (G,\tau_1) \to (H,\tau_2)$ by *f(p) = r, f(q) = s, f(r) = p*, f(s) = q . Then *f* is fuzzy contra semi-continuous but not fuzzy contra gprw-continuous as $f^{-1}(\psi)$ in (H,τ_2) is α, which is fuzzy semi- closed in (G,τ_1) but not fuzzy gprw-closed.

□

Example 3.11 Consider fuzzy topological spaces (G,τ_1) and (H,τ_2) as defined in Example 3.10. Now, if we define a mapping $f : (G,\tau_1) \to (H,\tau_2)$ by *f(p) = r, f(q) = s, f(r) = q, f(s) = p* . Then *f* is fuzzy contra gprw-continuous & not fuzzy contra semi-continuous as $f^{-1}(\eta)$ in (H,τ_2) is γ, which is fuzzy gprw- closed in (G,τ_1) but not fuzzy semi-closed.

□

Theorem 3.12 Suppose $g : (G,\tau_1) \to (H,\tau_2)$ is fuzzy continuous and $h : (L,\tau_3) \to (G,\tau_1)$ is fuzzy contra gprw-continuous, then their composition map $goh : (L,\tau_3) \to (H,\tau_2)$ is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha \leq \tau_2$. Since g is fuzzy continuous, implies $g^{-1}(\alpha) \leq \tau_1$. Now h is fuzzy contra gprw-continuous, so $h^{-1}(g^{-1}(\alpha))$ is fuzzy gprw-closed in (L,τ_3). Since $(goh)^{-1}(\alpha) = h^{-1}(g^{-1}(\alpha))$. So $goh : (L,\tau_3) \to (H,\tau_2)$ is fuzzy contra gprw-continuous.

□

Remark 3.13 *In the following examples we prove that Fuzzy contra gprw-continuous and fuzzy contra generalized continuous mappings are independent.*

Example 3.14 Consider fuzzy topological spaces (G,τ_1) and (H,τ_2) as defined in Example 3.10. Now, if we define a mapping $l : (G,\tau_1) \to (H,\tau_2)$ by *l(p) = r, l(q) = p, l(r) = q* and *l(s) = s* . Then l is fuzzy contra generalized continuous mapping but not fuzzy contra gprw-continuous as $l^{-1}(\eta)$ in (H,τ_2) is $\chi : G \to [0,1]$ defined as

$$\chi(g) = \begin{cases} 1 & \text{if } g = p,s \\ 0 & \text{otherwise} \end{cases}$$

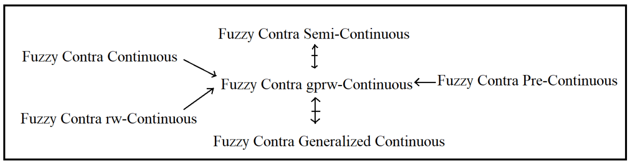
which is fuzzy generalizedclosed in (G,τ_1) but not fuzzy gprw-closed.

□

Example 3.15 Consider fuzzy topological spaces (G,τ_1) and (H,τ_2) as defined in Example 3.10. Now, if we define a mapping $h : (G,\tau_1) \to (H,\tau_2)$ by *h(p) = r, h(q) = s, h(r) = p* and *h(s) = q* . Then h is fuzzy contra gprw-continuous mapping but not fuzzy contra generalized continuous as $h^{-1}(\eta)$ in (H,τ_2) is γ in (G,τ_1), which is fuzzy gprw-closed in (G,τ_1) but not fuzzy generalized closed.

□

Remark 3.16: From the above discusion of Results we have the following diagram of implications.Here
$A \to B$ means A implies B.
$A \nleftrightarrow B$ means A & B are independent of each other.



Definition 3.17 Suppose (G,τ_1) and (H,τ_2) be two fuzzy topological spaces. Then a function $g:(G,\tau_1)\to(H,\tau_2)$ is called fuzzy contra gprw-contra irresolute map if $g^{-1}(h)$ is fuzzy gprw-closed in (G,τ_1) for every fuzzy gprw-open set h in (H,τ_2).

Theorem 3.18 If $g:(G,\tau_1)\to(H,\tau_2)$ is fuzzy contra gprw-irresolute, then it is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha\leq\tau_2$, Now from *Remark 2.11* α is fuzzy gprw-open in (H,τ_2). Since g is fuzzy contra gprw-irresolute, implying $g^{-1}(\alpha)$ is fuzzy gprw-closed in (G,τ_1). Thus g is fuzzy contra gprw-continuous.

□

Theorem 3.19 Let (L,τ_1), (M,τ_2) and (N,τ_3) are fuzzy topological spaces. If $l:(L,\tau_1)\to(M,\tau_2)$ is fuzzy contra gprw-irresolute and $k:(M,\tau_2)\to(N,\tau_3)$ is fuzzy gprw-continuous, then their composition $kol:(L,\tau_1)\to(N,\tau_3)$ is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha\leq\tau_3$, Now as k is fuzzy gprw-continuous means $k^{-1}(\alpha)$ is fuzzy gprw open set in (M,τ_2). Now as l is fuzzy contra gprw-irresolute, implies $l^{-1}(k^{-1}(\alpha))$ is fuzzy gprw closed set in (L,τ_1). But $l^{-1}(k^{-1}(\alpha))=(kol)^{-1}(\alpha)$, implies kol is fuzzy contra gprw-continuous.

□

Theorem 3.20 Let (L,τ_1), (M,τ_2) and (N,τ_3) are fuzzy topological spaces. If $l:(L,\tau_1)\to(M,\tau_2)$ is fuzzy gprw-irresolute and $m:(M,\tau_2)\to(N,\tau_3)$ is fuzzy contra gprw-irresolute, then their composition $mol:(L,\tau_1)\to(N,\tau_3)$ is fuzzy contra gprw-irresolute.

Proof: Suppose α is fuzzy gprw-open in (N,τ_3). Since m is fuzzy contra gprw-irresolute, implies $m^{-1}(\alpha)$ is fuzzy gprw-closed in (M,τ_2). Now as l is fuzzy gprw-irresolute, implies $l^{-1}(m^{-1}(\alpha))$ is fuzzy gprw-closed in (L,τ_1). Now $(mol)^{-1}(\alpha)=l^{-1}(m^{-1}(\alpha)$, implying $mol:(L,\tau_1)\to(N,\tau_3)$ is fuzzy contra gprw-irresolute.

□

4 Fuzzy Contra gprw-open Mappings and Fuzzy Contra gprw-closed Mappings

Definition 4.1 A mapping $g:(G,\tau_1)\to(H,\tau_2)$ is *fuzzy contra gprw-open* if the image of $\lambda\leq\tau_1$ in (G,τ_1) is fuzzy gprw-closed in (H,τ_2).

Example 4.2 All fuzzy contra open mappings are fuzzy contra gprw-open mappings.

Proof: Suppose $l:(G,\tau_1)\to(H,\tau_2)$ be a fuzzy contra open mapping and $\alpha\leq\tau_1$, then $l(\alpha)\leq 1-\tau_2$. Now remark 2.8 implies $l(\alpha)$ is fuzzy gprw-closed set in (H,τ_2). Hence l is fuzzy contra gprw-open mapping.

□

The other way round of the above theorem need not be true, as shown in the following example.

Example 4.3 Suppose $G=H=\{l,m,n,o\}$ are fuzzy spaces and functions $\alpha,\beta,\gamma,\delta:G\to[0,1]$ be defined as

$$\alpha(g)=\begin{cases}1 & \text{if } g=l\\ 0 & \text{otherwise}\end{cases}\qquad \beta(g)=\begin{cases}1 & \text{if } g=m\\ 0 & \text{otherwise}\end{cases}$$

$$\gamma(g)=\begin{cases}1 & \text{if } g=l,m\\ 0 & \text{otherwise}\end{cases}\qquad \delta(g)=\begin{cases}1 & \text{if } g=l,m,n\\ 0 & \text{otherwise}\end{cases}$$

Consider $\tau_1=\{0,1,\alpha,\beta,\gamma\}$, $\tau_2=\{0,1,\alpha,\beta,\gamma,\delta\}$.With these topologies (G,τ_1) and (H,τ_2) are fuzzy topological spaces. Now, we define a mapping $f:(G,\tau_1)\to(H,\tau_2)$ by $f(l)=l, f(m)=m, f(n)=o$ & f(o) = n . Then f is fuzzy contra gprw-open mapping but not fuzzy contra open mapping, as image of $\gamma\leq\tau_1$ in (G,τ_1) is fuzzy set γ in (H,τ_2) which is fuzzy gprw- closed in (H,τ_2) but not fuzzy closed.

□

Example 4.4 If $l:(G,\tau_1)\to(H,\tau_2)$ is a fuzzy open map and $m:(H,\tau_2)\to(K,\tau_3)$ is fuzzy contra gprw-open, then the composition map $mol:(G,\tau_1)\to(K,\tau_3)$ is fuzzy contra gprw-open map.

Proof: Suppose $\alpha\leq\tau_1$. Now, as l is a fuzzy open map implies $l(\alpha)\leq\tau_2$. Since m is a fuzzy contra gprw-open map $m(l(\alpha))$ is fuzzy gprw-closed set in (K,τ_3). Now $m(l(\alpha))=(mol)(\alpha)$, implying mol is fuzzy contra gprw-open map.

□

Definition 4.5 Let (G,τ_1) and (H,τ_2) be two fuzzy topological spaces. A mapping $g:(G,\tau_1)\to(H,\tau_2)$ is called fuzzy contra gprw-closed if the image of $\gamma\leq 1-\tau_1$ in (G,τ_1) is fuzzy gprw-open in (H,τ_2).

Theorem 4.6 If $l:(G,\tau_1)\to(H,\tau_2)$ is a fuzzy contra closed mapping, then it is fuzzy contra gprw-closed mapping also.

Proof: Suppose $\lambda\leq 1-\tau_1$, Now as l is fuzzy contra closed mapping, implies $l(\lambda)\leq\tau_2$. Now, as all fuzzy open sets are fuzzy gprw-open implying that $l(\lambda)$ is fuzzy gprw-open in (H,τ_2). Hence l is a fuzzy contra gprw-closed mapping.

□

The other way round of the above theorem need not be true, as shown in the following example.

Example 4.7 Suppose $X = Y = \{l,m,n,o,p\}$ are fuzzy topological spaces with topologies $\tau_1 = \{0,1,\chi\}$and $\tau_2 = \{0,1,\alpha,\beta,\gamma\}$ where $\chi : X \to [0,1]$ and $\alpha,\beta,\gamma : Y \to [0,1]$ are defined as

$$\chi(x) = \begin{cases} 1 & \text{if } x = p \\ 0 & \text{otherwise} \end{cases} \qquad \beta(y) = \begin{cases} 1 & \text{if } y = n,o \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha(y) = \begin{cases} 1 & \text{if } y = l,m \\ 0 & \text{otherwise} \end{cases} \qquad \gamma(y) = \begin{cases} 1 & \text{if } y = l,m,n,o \\ 0 & \text{otherwise} \end{cases}$$

Let the function $f : (X,\tau_1) \to (Y,\tau_2)$ be defined as $f(l) = m$, $f(m) = n$, $f(n) = o$, $f(o) = p$ and $f(p) = l$. Then f is fuzzy contra gprw-closed map but not fuzzy contra closed map as image of $\psi \le 1-\tau_1$ in X defined as

$$\psi(x) = \begin{cases} 1 & \text{if } x = l,m,n,o \\ 0 & \text{otherwise} \end{cases}$$

is μ in Y defined as

$$\mu(y) = \begin{cases} 1 & \text{if } y = m,n,o,p \\ 0 & \text{otherwise} \end{cases}$$

which is fuzzy gprw-open in (Y,τ_2) but not fuzzy open.

□

Theorem 4.8 If $l : (G,\tau_1) \to (H,\tau_2)$ and $m : (H,\tau_2) \to (K,\tau_3)$ be two maps. Then $mol : (G,\tau_1) \to (K,\tau_3)$ is fuzzy contra gprw-closed map if l is a fuzzy closed mapping and m is fuzzy contra gprw-closed mapping.

Proof: Let $\alpha \le 1-\tau_1$ Since l is a fuzzy closed mapping, so $l(\alpha) \le 1-\tau_2$. Now m is a fuzzy contra gprw-closed map, implies $m(l(\alpha))$ is fuzzy gprw-open in (K,τ_3). But $m(l(\alpha)) = (mol)(\alpha)$, implying mol is fuzzy contra gprw-closed mapping.

□

Theorem 4.9 Let $l : (G,\tau_1) \to (H,\tau_2)$ and $m : (H,\tau_2) \to (K,\tau_3)$ be two mappings such that $mol : (G,\tau_1) \to (K,\tau_3)$ is fuzzy contra gprw-closed map then,

(I)Suppose l is fuzzy continuous and onto then m is fuzzy contra gprw-closed.
(II)Suppose m is fuzzy gprw-irresolute and one-one then l is fuzzy contra gprw-closed.

Proof: (I) Let $\alpha \le \tau_2$. Since l is fuzzy continuous, $l^{-1}(\alpha) \le 1-\tau_1$. Since mol is fuzzy contra gprw-closed map, $(mol)(l^{-1}(\alpha))$ is fuzzy gprw-open set in I. But $(mol)(l^{-1}(\alpha)) = m(\alpha)$, as l is surjective. Thus m is fuzzy contra gprw-closed.

(II) Let $\mu \le 1-\tau_1$. Now mol is fuzzy contra gprw-closed, implies $mol(\mu)$ is fuzzy gprw-open in I. Since m is fuzzy gprw-irresolute, so $m^{-1}(mol)(\mu)$ is fuzzy gprw-open in (H,τ_2). But $m^{-1}(mol)(\mu) = l(\mu)$ as m is injective. Thus l is fuzzy contra gprw-closed mapping.

□

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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