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The Speed Mode Synergetic Control Approach for Magnetic Suspended Reaction Flywheel

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Abstract: Random torque disturbances generated by magnetic bearings eccentric magnetic force have negative effects on the output torque precision of the magnetic suspended reaction flywheel(MSRFW). In order to eliminate the negative effects, dynamic models of flywheel system including motor and the buck DC-DC converter are established and a synergetic control approach for speed mode operation is presented. This approach uses PI controller for speed operation and uses synergetic control theory to control the winding current of motor and the input current and output voltage of power converter, improving the robustness of the basic synergetic method in overcoming the random torque disturbance. Sufficient condition for synergetic control subsystem is certified by Lyapunov stability theory. Simulation and experimental results show that the presented approach has great robustness in torque disturbance and the output moment precision of the reaction flywheel system reaches up to $3 \times 10^{-5} N \cdot m$.

Keywords: Magnetic Suspended, Reaction Flywheel, Synergetic Control, Permanent-Magnet Brushless DC motor

1. Introduction

To make the satellite platform keep stable attitude, the reaction flywheel of three-axis satellite attitude control system through precisely tracking the torque command of the attitude control system adjust satellite attitude deviation or absorb disturbing torque. The space environment has small-magnitude disturbing torque, for example, the sun's radiation pressure on the earth synchronous satellites is about $10^{-4} N \cdot m$, which has the same or lower magnitude with the disturbing torque caused by the zero moment of mechanical bearing internal friction of the flywheel, in addition, the serious nonlinear of the mechanical bearing flywheel zero velocity nearby will lower the output control torque accuracy of the flywheel. To improve this, approaches of speed feedback compensation control, internal friction observation compensation, state feedback and attitude control system of the nonlinear control loop compensation are proposed to suppress the influence of the output torque accuracy produced by the internal friction in references [1] to [5]. However, thanks to the magnetic suspended reaction flywheel, non-contact bearing of flywheel rotor is realizing, and nonlinear friction effect produced while the flywheel works at low speed and zero passage of the rotate speed. Other factors such as random torque generated by magnetic bearings eccentric magnetic force and non-linear as well as non-modeling dynamic factors of the flywheel motor and PWM converter and so on restrict improving the accuracy of the output torque of the magnetic suspended reaction flywheel.

To improve this, an average state space model of MSR-FW system is established in this paper. What's more, synergetic control approaches for speed mode operation which uses PI controller for speed operation and uses synergetic control theory to control the winding current and the input current and output voltage of power converter is presented, improving the robustness of the basic synergetic method in overcoming the random torque disturbance. Then asymptotically stable sufficient condition of synergetic control subsystem is ensured by Lyapunov stability theory. Simulation and experimental results show that under the rate mode the presented

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approaches have great robustness in torque disturbance. The output moment precision of the flywheel system reaches up to $3 \times 10^{-5} N \cdot m$ in the range of four quadrant, the whole torque command and working speed.

2. Dynamic Modeling of MSRFW

The core controlling of the flywheel is motor control. Now the permanent magnetic brushless DC torque motor is generally used in the reaction flywheel at home and abroad, which taking advantage of motor to make the flywheel accelerating or braking, generating reaction torque as control torque , to change the attitude of the satellite or eliminate the disturbing torque to keep stability of the satellite attitude. To ensure the reliability of the space application requirements, the main circuit topology of MSRFW system structure is shown in Fig.1

As shown in Fig 1, it is assumed that the flywheel motor is three-phase symmetric, then the average state space model of flywheel.

$$\begin{cases} \dot{\omega} = \frac{K_T}{J} i_m - \frac{B}{J}\omega - \frac{1}{J}T_d \\ \dot{i}_m = \frac{U_o}{L_m} - \frac{\Delta V_T}{L_m} - \frac{30K_e}{\pi p L_m}\omega - \frac{R_m}{L_m} i_m \\ \dot{i} = -\frac{U_o}{L} - \frac{\Delta V_T}{L} + \frac{U_{DC}}{L} u_1 \\ \dot{U}_o = \frac{1}{C} i - \frac{1}{C} i_m \end{cases} \tag{1}$$

According to (1), K_T is electromagnetic torque coefficient of the flywheel motor, J is rotary inertia of flywheel rotor, i_m is winding current of the flywheel motor, $B$is damping coefficient, T_d is uncompensated random torque disturbances, $p$is pole number of the flywheel motor rotor, ω is angular rate of the flywheel, K_e is back-EMF coefficient of the flywheel motor, U_{dc} is power supply voltage, u_1 is controlling input, I is input current of the buck converter, U_o is output voltage of the buck converter, L ,C for filtering inductances and capacitance of the buck converter respective, L_m is winding phase inductance, R_m is winding phase resistance,$\Delta V_T$is state voltage drop of power switch tube.

At controllable dynamic braking section, power switch tube VT4 shut off the power supply circuit, so that through the power resistance R_p and power switch tube VT5 shown in Fig.1 the motor back-EMF consists of a current loop, forming controllable reverse winding current, and that when the motor working in power generation condition, the flywheel produces braking torque. the effect of transformation between winding phase current is neglected, an average state space model of the system is gotten as following:

$$\begin{cases} \dot{\omega} = -\frac{K_T}{J} i_m - \frac{B}{J}\omega - \frac{1}{J}T_d \\ \dot{i}_m = -\frac{R_m + R_p}{L_m} i_m - \frac{\Delta V_T + \Delta V_D}{L_m} + \frac{30K_e\omega}{\pi p L_m} u_2 \end{cases} \tag{2}$$

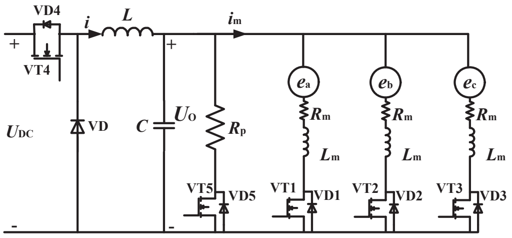

Figure 1 The main circuit topology of MSRFW system

where u_2 is control input, R_P is power resistance, $\Delta V_D$is power diode state voltage drop. In reverse connect braking section, braking power is still provided by power supply, and the buck DC-DC converter output constant voltageU_o. In order to avoid inner circulation generated between winding phases.

$U_o > K_e\omega + \Delta V_T$is required. Converter power switch tube VT1,VT2,VT3 in Fig.1 adjust the flywheel motor winding current in their conduction area respective, making the flywheel output braking reaction torque. Then, the average state space model of the system is set as:

$$\begin{cases} \dot{\omega} = -\frac{K_T}{J} i_m - \frac{B}{J}\omega - \frac{1}{J}T_d \\ \dot{i}_m = -\frac{R_m}{L_m} i_m - \frac{\Delta V_T}{L_m} + \left(\frac{U_o}{L_m} + \frac{30K_e\omega}{\pi p L_m}\right) u_3 \\ \dot{i} = -\frac{1}{L} U_o - \frac{\Delta V_T}{L} + \frac{U_{DC}}{L} u_4 \\ \dot{U}_o = \frac{1}{C} i - \frac{1}{C} i_m \end{cases} \tag{3}$$

where,u_3, u_4 for control in Reaction flywheel output torque is proportional to the flywheel differential angular rate ω:

$$T_o = J\dot{\omega}, \tag{4}$$

According to Newton's third law of motion, reaction torque is equal to the flywheel rotor torque T_o which is produced by reaction flywheel of the satellite, but they have opposite directions.

3. MSRFW Rate Model With PI Synergetic Control

Synergetic control is a new nonlinear control theory put forward in recent years, which is based on the modern mathematics and synergetic of the state space method [7]. On the principle of self-organization and control target or dynamic performance indicators of the system, the approa-ches of synergetic control form manifold in the state space of the controlled object, and using manifold to solve the control law of the system. Synergetic control theory has been widely used in many areas, such as DC-DC power converter [8-11], motor [12-14], and HVDC transmission system [15].

In this paper, magnetic suspended reaction flywheel controlled by rate mode consists of a current inner loop of synergetic control, a speed outer loop of PI control, rotate speed given by torque command calculating through

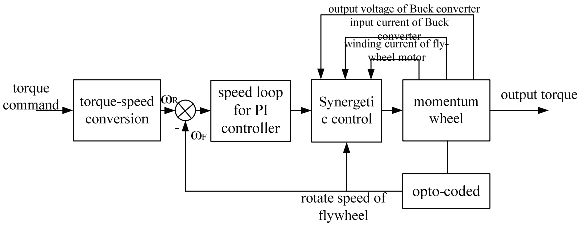

Figure 2 Diagram model of the control system

torque-speed conversion, and flywheel speed obtained by photoelectric encoder feedback which is using PI control to output a variable which is used as the current reference of the current synergetic controller. MSRFW rate model with PI synergetic control system is shown in Fig.2

In Fig.2, speed loop for PI controller is set as

$$u = K_p e + K_i \int e dt, \tag{5}$$

where K_p is scale factor, K_i integral coefficient $e = \omega_R - \omega_F$ Under the torque command, when the flywheel accelerated running, macro functions of synergetic control subsystem can be defined as a linear combination of the motor winding current, input current of the buck converter and its reference value.

$$\psi_1 = K_1(i_m - i_{mr}) + K_2(i - i_r), \tag{6}$$

where K_1 and K_2 are weight coefficient, i_m is reference value of the flywheel motor winding current, i_r is input current reference value,$i_{mr} = u(k)$.

The expected dynamic characteristics equation value of the manifold is defined as:

$$T_1(K_1 i_m + K_2 i_r) + \psi_1 = 0, \tag{7}$$

Where, the near rate T_1 is determined by ψ_1,making the system move to manifold $\psi_1 = 0$ and eventually reach the convergence rate. Usually, T_1 is the smaller, the state variables of the manifold reach time is shorter, the response speed of system is faster, and dynamic performance is better. By substituting (1) into (7), one can obtain the synergetic control law while the flywheel accelerating, with

$$\begin{gathered} u_1 = \frac{30k_1k_eL}{\pi pk_2L_mU_{DC}}\omega + \frac{k_1R_mL}{k_2L_mU_{DC}}i_m + \left(\frac{k_1L}{U_{DC}} - \frac{k_1L}{k_2L_mU_{DC}}\right)U_o + \\ \left(\frac{k_1L}{k_2L_mU_{DC}} + \frac{1}{U_{DC}}\right)\Delta V_T - \frac{k_1L}{T_1k_2U_{DC}}(i_m - i_{mr}) - \frac{L}{T_1U_{DC}}(i - i_r) \end{gathered} \tag{8}$$

By substituting (8) into (1), one can obtain the system matrix of synergetic control subsystem state equation, with

$$A_{s1} = \begin{bmatrix} \frac{k_T}{J} & -\frac{B}{J} & 0 & 0 \\ -\frac{R_m}{L_m} & -\frac{30k_e}{\pi pL_m} & 0 & \frac{1}{L_m} \\ \frac{k_2}{k_1L_m} - \frac{k_2}{T_1k_1} & \frac{30k_2k_e}{\pi pk_1L_m} & -\frac{1}{T_1} & -\frac{k_2}{k_1L_m} \\ -\frac{1}{C} & 0 & \frac{1}{C} & 0 \end{bmatrix}$$

By Lyapunov stability theory, when flywheel accelerating, one can get the sufficient conditions of equilibrium state asymptotic stability of synergetic control subsystem, with $T_1K_1(1 - R_m) - (K_1 + K_2)L_m/T_1K_1 < 0$ Macro function of synergetic control subsystem at controllable braking energy period is defined as:

$$\psi_2 = k_3(i_m - i_{mr}) \tag{9}$$

where K_3 is weight coefficient.

The expectation dynamic characteristics equation of the system of manifold is set as:

$$\psi_2 + T_2k_3i_m = 0 \tag{10}$$

By substituting (2) into (10) one can obtain the synergetic control law of MSRFW at controllable braking energy period, with

$$\begin{gathered} u_2 = \frac{\pi pi_m\left(R_m+R_p\right)}{30k_e\omega} + \frac{\pi p(\Delta V_T+\Delta V_D)}{30k_e\omega} \\ - \frac{\pi pL_m\left(R_m+R_p\right)(i_m-i_{mr})}{30T_2k_e\omega} \end{gathered} \tag{11}$$

The value range of parameter T_2 is determined by the synergetic control subsystem stability condition for there is no obvious relation between u_2 and K_3 . For this, by substituting (11) into (2), one can obtain the system matrix of synergetic control subsystem state equation at controllable braking energy period, with

$$A_{s2} = \begin{bmatrix} -\frac{1}{T_2} & 0 \\ -\frac{k_T}{T_2} & -\frac{B}{J} \end{bmatrix}$$

According to Lyapunov theory, sufficient conditions of equilibrium state asymptotic stability is set as: $T_2 > 0$ .The macro function while the flywheel has a plug braking operation is set as:

$$\boldsymbol{\psi_3} = \begin{bmatrix} k_4(i_m - i_{mr}) \\ k_5(i - i_r) + k_6(U_o - U_or) \end{bmatrix} \tag{12}$$

where K_4,K_5,K_6 are weight coefficient.The expectation dynamic characteristics equation of manifold is set as:

$$\begin{bmatrix} T_3 & 0 \\ 0 & T_4 \end{bmatrix} \begin{bmatrix} k_4\dot{i}_m \\ k_5\dot{i} + k_6\dot{U}_o \end{bmatrix} + \boldsymbol{\psi_3} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

Substituting (3) into (13) one can obtain the control law ,with

$$u_3 = \frac{\pi p(R_mi_m + \Delta V_T)}{\pi pU_o + 30k_e\omega} - \frac{\pi pL_m(i_m - i_{mr})}{T_3(\pi pU_o + 30k_e\omega)} \tag{14}$$

$$\begin{gathered} u_4 = \frac{U_o}{U_{DC}} + \frac{\Delta V_T}{U_{DC}} - \frac{k_6L}{k_5CU_{DC}}i + \frac{k_6L}{k_5CU_{DC}}i_m \\ - \frac{L}{T_4U_{DC}}(i - i_r) - \frac{k_6L}{T_4k_5U_{DC}}(U_o - U_or) \end{gathered} \tag{15}$$

Combined (14) and (15), we can get the synergetic control law of magnetic suspended reaction flywheel

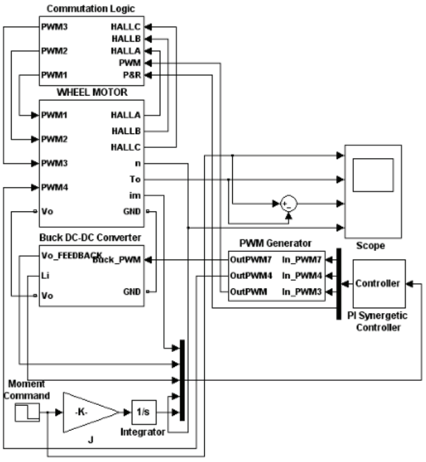

Figure 3 Simulation model of the control system

motor system at the moment of plug braking. Also at this time, braking power is still supplied by power source, and buck converter output constant voltage U_o, requiring U_o greater than the maximum back-EMF, in order to avoid generating inner circulation between the winding phases. The value range of parameter T_3, T_4 and weight coefficient k_3, k_4 are determined by the synergetic control subsystem asymptotically stability condition for there is no obvious relation between u_2 and k_4. By substituting (14) and (15) into (3) one can obtain the system matrix of synergetic control subsystem state equation, with

$$A_{s1} = \begin{bmatrix} \frac{k_T}{J} & -\frac{B}{J} & 0 & 0 \\ -\frac{R_m}{L_m} & -\frac{30k_e}{\pi p L_m} & 0 & \frac{1}{L_m} \\ \frac{k_2}{k_1 L_m} - \frac{k_2}{T_1 k_1} & \frac{30k_2 k_e}{\pi p k_1 L_m} & -\frac{1}{T_1} & -\frac{k_2}{k_1 L_m} \\ -\frac{1}{C} & 0 & \frac{1}{C} & 0 \end{bmatrix}$$

According to Lyapunov first method, sufficient conditions of equilibrium state asymptotic stability is set as: $T_3 > 0, k_6/T_4 \cdot k_5 > 0$.

4. Computer Simulations

The simulation model of MSRFW rate model synergetic control system is shown in Fig 3.

Rotate speed of the flywheel motor is obtained by calculating torque command according to (4) and the output speed of PI controller act as a reference of motor winding current. Input current reference i_r of the buck converter can be gotten by solving $\psi_1 = 0$ and $\psi_3 = 0$ when the flywheel accelerating and braking segment. The setting input current of the buck DC-DC converter when the flywheel acceleration is gotten. Input voltage of the buck DC-DC converter is offered by DC power supply of 28V which is used in aerospace area. When the flywheel is during plug braking, output voltage of the buck converter is a given value for $U_{or} = 12V$.In the range of whole work speed, disturbance torque T_d takes for maximum value $0.005N \cdot m$ which is a Gaussian random disturbance. Parameters of MSRFW system selected according to designing and practical measured values shown in Table 1.

According to Lyapunov stability condition of MSRFW speeding up, we can select $T_1 = 0.001$, $T_2 = 0.01$, $T_3 = 0.001$, and $T_4 = 0.02$, $K_1 = 0.01$, $K_2 = 0.04$, $K_4 = 0.04$, $K_5 = -0.4$, $K_6 = 0.04$.The proportion coefficient of speed PI controller is select as $K_p = 1$, integral factor $K_i = 0.7$. Both the state voltage drop of power switch tube $\triangle V_T$ and power diode $\triangle V_D$ all take for $0.7V$.

Table 1 Major Parameter value of the wheel system

Major parameter	**Parameter value**
rotary inertia of flywheel rotor$J$	0.0287kg·m^2
damping coefficient$B$	2×10^{-5}N·m/ rad·s^{-1}
pole number of the flywheel motor rotor$p$	8
torque coefficientK_T	0.04966N·m·A^{-1}
back-EMF coefficientK_e	0.0052V/r·min^{-1}
winding phase inductance of flywheel motor L_m	0.06mH
winding phase resistance of flywheel motorR_m	2Ω
power resistance R_p	10Ω/4W
filtering inductances of Buck converter respective$L$	0.6mH
filtering capacitance of Buck converter respective$C$	47μF

According to Lyapunov stability condition of MSRFW speeding up, we can select $T_1 = 0.001$, $T_2 = 0.01$, $T_3 = 0.001$, and $T_4 = 0.02$, $K_1 = 0.01$, $K_2 = 0.04$, $K_4 = 0.04$, $K_5 = -0.4$, $K_6 = 0.04$.The proportion coefficient of speed PI controller is select as $K_p = 1$, integral factor $K_i = 0.7$. Both the state voltage drop of power switch tube $\triangle V_T$ and power diode $\triangle V_D$ all take for $0.7V$.

Fig.4 shows the response curve of flywheel rotate speed and output torque which is set at $0.055N \cdot m$ and $-0.055N \cdot m$ when the MSRFW in synergetic control function.

Under the synergetic control action, output torque deviation of MSRFW four quadrant, the whole torque command and working speed is $[1.47 \times 10^{-5} N \cdot m$, $1.46 \times 10^{-5} N \cdot m]$.

5. Experimental Verification

MSRFW experimental platform consists of 15 Nms magnetic reaction flywheel, motor digital control system, magnetic bearing digital control system, sealed cowling,

Table 2 The control logic of MSRFW

torque command	Steering	Speed valuer/min	Running state
positive	Positive Steering	0→5000	Positive accelerated
	Reverse Steering	-5000 →-2000	Controllable energy braking
		-2000→0	Reverse connect braking
negative	Reverse Steering	0→-5000	Reverse accelerated
	Positive Steering	5000→2000	Controllable energy braking
		2000→0	Reverse connect braking

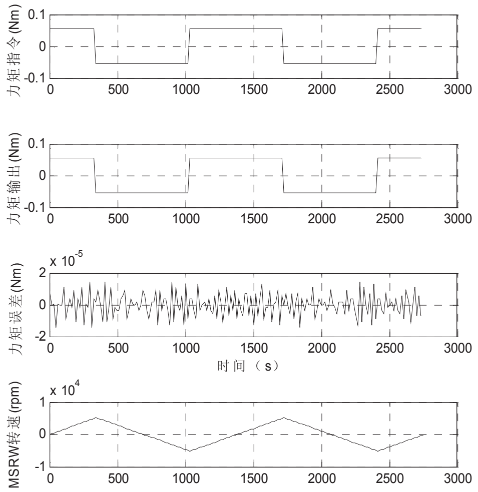

Figure 4 Simulation results of MSRFW PI synergetic control

28V dc power supply used in aerospace and so on. Photoelectrical encoder is used to detect rotate speed of reaction flywheel. The main technical index of 15Nms MSRFW used in the experiment is shown in Table 3.

Table 3 The key technical indexes of MSRFW

key technical indexes	Index value
Maximum working speed	±5000r/min
Maximum accelerate torque	0.055N·m
Maximum loss torque	¡0.01N·m
Maximum starting torque	¡1×10^{-4}N·m

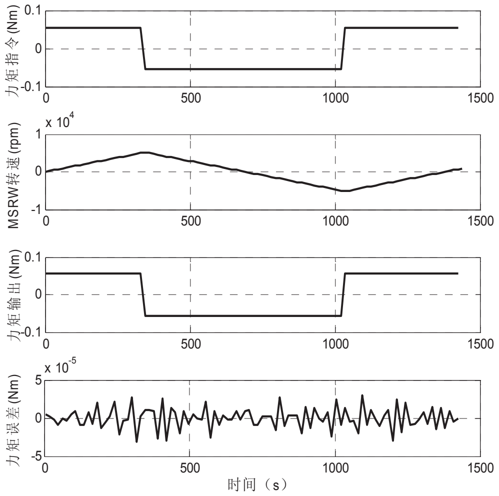

Figure 5 Experimental results of MSRFW synergetic control

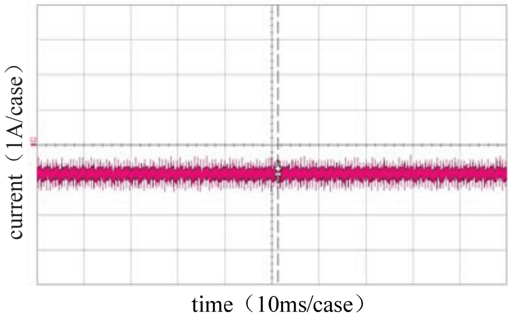

Figure 6 Neutral current in dynamic braking

Fig.5 shows the experimental results of MSRFW synergetic control and the deviation of output torque is $[2.9\times10^{-5}N\cdot m, 2.7\times10^{-5}N\cdot m]$.

Under the synergetic control action, it output neutral current of the motor at maximum braking torque in dynamic braking, and the average winding current of reaction flywheel motor is 1.12 A, as shown in Fig.6.

A-phase winding current of plug breaking motor under the synergetic control law is shown in Fig.7, and the average winding current of reaction flywheel motor is 1.28A.

6. Conclusions

An average state space model of MSRFW system which includes the PWM converter and motor is established in this paper, putting forward a approach of speed rate model synergetic control, improving the robustness

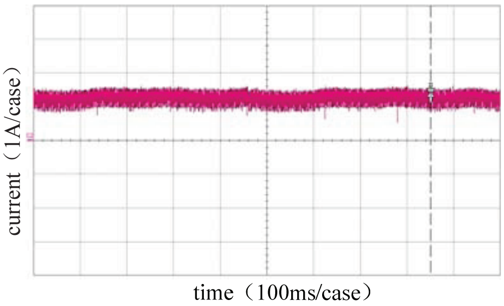

Figure 7 Neutral current in plug braking

resulted by containing non-compensatory random torque disturbance in the basic synergetic control law, and ensuring the value range of controller parameters for synergetic control subsystem by Lyapunov stability theory. Simulation and experimental results show that under the rate mode the present approach has great robustness in torque disturbance and the output moment precision of the flywheel reaches up to 310-5Nm in the range of four quadrant, the whole torque command and working speed, verifying that it is effectiveness in restraining random torque disturbance.

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