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Reliability and Cost-Benefit Efficient in a Two-Dissimilar-Unit with Warm Unit Standby Case Subject to Arbitrary Repair and Replacement

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Abstract: In this paper, we deal with a warm case of standby system made up of two-dissimilar-units. One of these units is a high quality unit, while the other is a low quality one and may require repairs or replacement with a different lesser device in the event of failure, so the first unit is given priority in use. Assume that an arbitrary distribution for repairing the main unit and standby unit. We also examine all transition probabilities and calculate mean sojourn time, availability, and repair time. In the end, we use all of the previous measurments to evaluate the cost-benefit of the system.

Keywords: Warm standby system, Priority, Busy period, Cost benefit estimated

1 Introduction

Reliability system is becoming a key factor in production and industry, and redundancy is one of the most popular methods for enhancing reliability system. Due to the high prices of machines in some projects, the basic working units are replaced with other units of lower quality until the basic unit is repaired in order to save cost. Accordingly, our study focused on the priority of two dissimilar units' warm standby. Numerous researchers have investigated a variety of models for priority in various situations including[1,2,3,4,5,6,7]. [8,9] studied systems with non-similar units.
In a warm standby system, the spare parts are assumed to fail with a lower failure rate than the operating parts after being stored. The standby system is called warm if considered a better model for the spare parts degradation process. For example, [10] used different parameters of Poisson shocks for warm-repairable-system and availability for steady-state. The performance of a 3-unit warm-standby-system measured in [11], and used quadrivariate to estimate the lifetime and repair time. [12] deals with two similar- warm-standby-systems according to failure because of the melting of glaciers and severe storms due to global warming and failure rate as Gamma-distribution. As for [13], they economically analyzed a warm-standby-system with a single-server. In [14], using the supplementary variables technique, many reliability indexes were received through studied warm standby-systems with two-dissimilar-units. They also assumed that after repairing unit-one, it follows a geometric process. In addition, the research for warm- standby- systems continuously developed, including the papers [15,16,17] and others.
[18] discussed the analysis of planar arbitrarily curved microbeams with Euler-Bernoulli beam model. As for [19], they studied Mixed failure and shock-driven mission aborts in heterogeneous arbitrary systems.
We deal with a warm-standby-repairable-system made up of two-dissimilar-units. One of these units is of good quality, while the other is of lesser quality and may require repairs or replacement with a different lesser device in the event of failure, so the first unit is given priority in use. The following system reliability has explicit formulations: The mean time to system failure (MTSF), availability, repair time due to failure of the highest quality unit, failure of the lowest quality

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unit, or replacement of the lowest quality unit, the profit gain of the system is obtained, and the theoretical findings are shown using a numerical example.

2 Assumptions

- In this system we consider two dissimilar units . At first the highest quality unit is running, and the other is in standby mode (Warm standby).
- If the operating unit fails, we go to the standby unit and test it for work (which is called warm-standby).
- The highest quality unit has the priority for operating.
- When the unit fails, it goes directly to the repair.
- After the repair, the unit returns as new.
- The system completely fails if both units fail.
- All distributions are exponential except repair and replace times distributions are arbitrary.

3 Notations

$E,(T_{i,j})$	Regenerative states.
$\overline{E},(T^k)$	Non-regenerative states.
$q_{ij}(t),Q_{ij}(t)$	Transition from T_i to T_j probabilities and cumulative.
$q^k_{ij}(t),Q^k_{ij}(t)$	Transition from T_i to T_j passing through T^k, probabilities and cumulative.
P_{ij}	Transition probability from T_i to T_j.
$P^{(k)}_{ij}$	Probability that the system transit from T_i toT_j, Passing through T^k.
$F_i(t)(i=1,2)$	Cdf of the highest quality unit and the lowest quality unit failures respectively.
$G_i(t)(i=1,2)$	Cdf of the highest quality unit and the lowest quality unit repair times respectively .
$N(t)$	Cdf of replacement time.
λ_1,λ_2	parameter of the failure rate of the highest quality unit, the lowest quality unit respectively.
μ_1,μ_2	The parameter of repair rate of the highest quality unit, the lowest quality unit respectively.
β	parameter of unit 2 replacement rate.
τ	Probability that the Standby-unit is ready.
$(1-\tau)$	Probability that the Standby-unit is not ready.
θ	Probability that the lowest quality unit repair.
$(1-\theta)$	Probability that the lowest quality unit replacement.
m_{ij}	The mean sojourn time to transit fromT_i to T_j
$M_i(t)$	Probability that the system stay in T_i .
$M_i(s)$	Laplace transform of $M_i(t)$.
$\Psi_i(t)$	Cdf of time to system failure starting from state T_i.
$AV_i(t)$	p { The system is up at time t starting at state T_i}.
$O(t)$	The net revenue of the system in $(o,t]$.
$BP_i(t)$	Probability that the highest quality unit is in repair
$BP'_i(t)$	Probability that the lowest quality unit is in repair.
$BP''_i(t)$	Probability that the lowest quality unit is in replace.
©	Convolution.
$*$	Laplace transforms.

3.1 Symbols for the system states

- r I,r II the highest quality unit, the lowest quality unit are under repair respectively.
- R I,R II the highest quality unit, the lowest quality unit are under repair respectively from previous state.
- O I,O II the highest quality unit, the lowest quality unit are in operating state respectively.
- S II the lowest quality unit is in warm standby state.
- wr I,wr II the highest quality unit, the lowest quality unit are waiting for repair respectively.
- rep II the lowest quality unit is under replacement.

– REP II the lowest quality unitis under replacement from previous state.

The system can be in any one of the following states
$T_0 = (OI, SII)$, $T_1 = (rI, OII)$, $T_2 = (rI, wrII)$
$T_3 = (RI, wrII)$, $T_4 = (OI, rII)$, $T_5 = (OI, repII)$
$T_6 = (wrI, RII)$, $T_7 = (wrI, REPII)$.
Up states:T_0, T_1, T_4, T_5, **Down states:**T_2, T_3, T_6, T_7, **Non-regenrative states**:T_3, T_6, T_7

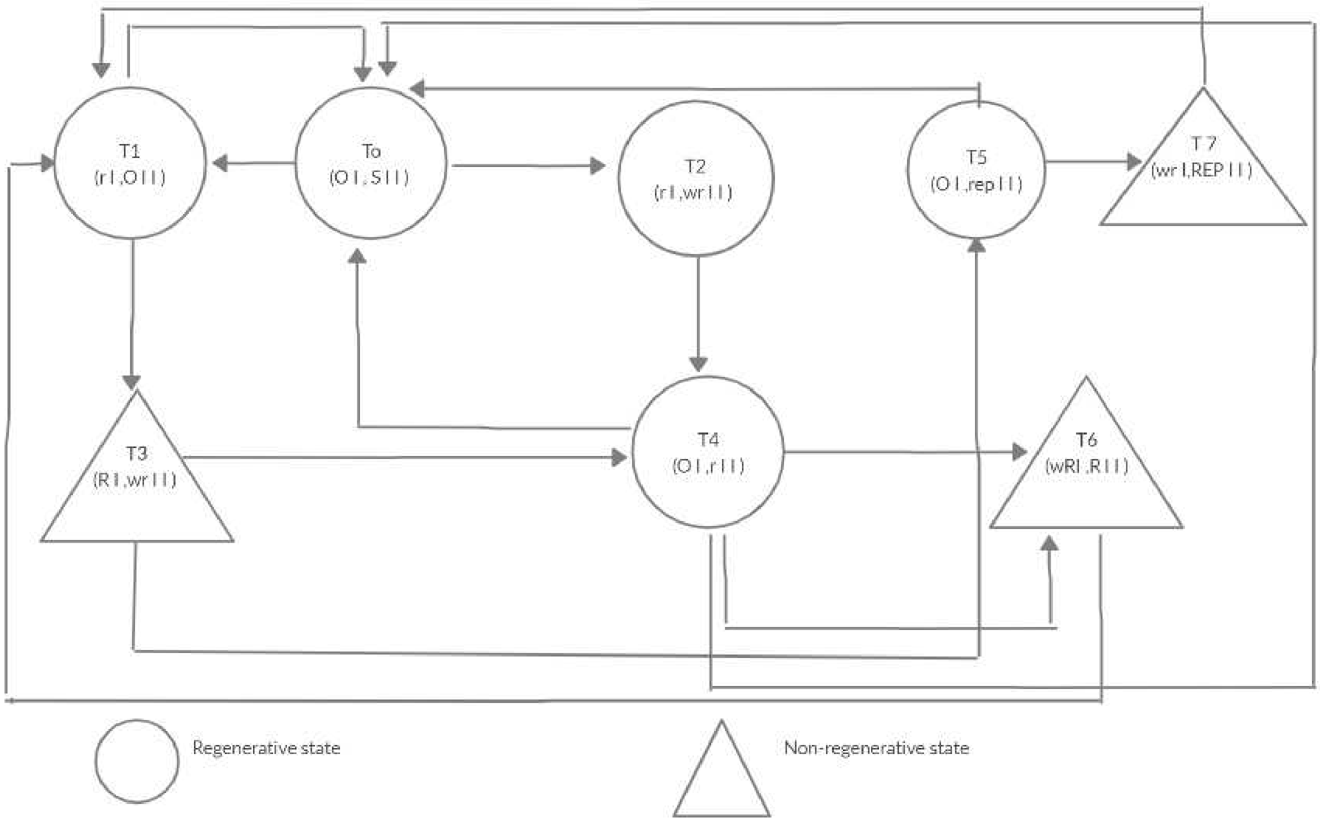

Fig. 1: System Transition

3.2 Transition probabilities and mean sojourn time

Simple probabilistic considerations yield the following expressions for the non-zero elements $P_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt$ as

$$P_{01} = \tau \int_0^\infty dF_1(t), \qquad P_{02} = (1-\tau) \int_0^\infty dF_1(t),$$

$$P_{01} + P_{02} = 1,$$

$$P_{10} = \int_0^\infty \overline{F_2}(t) dG_1(t), \qquad P_{13} = \int_0^\infty \overline{G}_1(t) dF_2(t),$$

$$P_{10} + P_{13} = 1,$$

$$P_{14}^3 = \theta \int_0^\infty F_2(t) dG_1(t), \qquad P_{15}^3 = (1-\theta) \int_0^\infty F_2(t) dG_1(t),$$

$$P_{13} = P_{14}^{3} + P_{15}^{3},$$

$$P_{24} = \theta \int_{0}^{\infty} dG_1(t), \qquad P_{25} = (1-\theta) \int_{0}^{\infty} dG_1(t),$$

$$P_{24} + P_{25} = 1,$$

$$P_{46} = \int_{0}^{\infty} \overline{G}_2(t) dF_1(t), \qquad P_{40} = \int_{0}^{\infty} \overline{F_1}(t) dG_2(t),$$

$$P_{40} + P_{46} = 1,$$

$$P_{41}^{6} = -\int_{0}^{\infty} F_1(t) d\overline{G_2}(t) = P_{46},$$

$$P_{50} = \int_{0}^{\infty} \overline{F_1}(t) dN(t), \qquad P_{57} = \int_{0}^{\infty} \overline{N}(t) dF_1(t),$$

$$P_{50} + P_{57} = 1,$$

$$P_{51}^{7} = \int_{0}^{\infty} F_1(t) dN(t) = P_{57}.$$

3.3 Mean sojourn times

The mean sojourn time to transit from T_i to T_j

$$m_{01} = \tau \int_{0}^{\infty} t dF_1(t), \qquad m_{02} = (1-\tau) \int_{0}^{\infty} t dF_1(t),$$

$$m_{10} = \int_{0}^{\infty} t\overline{F_2}(t) dG_1(t), \qquad m_{13} = \int_{0}^{\infty} t\overline{G}_1(t) dF_2(t),$$

$$m_{24} = \theta \int_{0}^{\infty} t dG_1(t), \qquad m_{25} = (1-\theta) \int_{0}^{\infty} t dG_1(t),$$

$$m_{46} = \int_{0}^{\infty} t\overline{G}_2(t) dF_1(t), \qquad m_{40} = \int_{0}^{\infty} t\overline{F_1}(t) dG_2(t),$$

$$m_{50} = \int_{0}^{\infty} t\overline{F_1}(t) dN(t), \qquad m_{57} = \int_{0}^{\infty} t\overline{N}(t) dF_1(t).$$

Mean sojourn time in state S_i which is given by $M_i(s) = \sum_j m_{ij}$

$$M_0(s) = \int_{0}^{\infty} \overline{F}_1(t) dt,$$

$$M_1(s) = \int_{0}^{\infty} \overline{G}_1(t) \overline{F}_2(t) dt,$$

$$M_4(s) = \int_{0}^{\infty} \overline{G}_2(t) \overline{F}_1(t) dt,$$

$$M_5(s) = \int_{0}^{\infty} \overline{N}(t) \overline{F}_1(t) dt.$$

3.4 Mean time to system failure MTSF

By using the regenerative technique, we obtain the following relation for $\overline{\Psi}_0(t)$

$$\overline{\Psi}_0(t) = \overline{F_2}(t) + q_{01}(t) \copyright \overline{\Psi}_1(t), \tag{1}$$

$$\overline{\Psi}_1(t) = \overline{G}_1(t)\overline{F_2}(t) + q_{10}(t) \copyright \overline{\Psi}_0(t), \tag{2}$$

$$\overline{\Psi}_4(t) = \overline{G}_1(t)\overline{F_2}(t) + q_{40}(t) \copyright \overline{\Psi}_0(t), \tag{3}$$

$$\overline{\Psi}_5(t) = \overline{N}(t)\overline{F_1}(t) + q_{50}(t) \copyright \overline{\Psi}_0(t). \tag{4}$$

Taking Laplace transform (LT) for equations (1), (2), (3) and (4) and solving for $\overline{\Psi}_0^*(s)$ considering $S = 0$, We have the mean time to system failure MTSF as follows

$$MTSF = \frac{N_0}{D_0}, \tag{5}$$

where

$$D_0 = 1 - P_{01}P_{10},$$

and

$$N_0 = M_0(s) + M_1(s)P_{01}.$$

4 Availability Analysis

According to regenerative assumtion, the point wise availabilities $AV_i(t)$ where $i = 0,1,4,5$. we obtain the following relations.

$$AV_0(t) = M_0(t) + q_{01}(t) \copyright AV_1(t) + q_{02}(t) \copyright q_{24}(t) \copyright AV_4(t) + q_{02}(t) \copyright q_{25}(t) \copyright AV_5(t), \tag{6}$$

$$AV_1(t) = M_1(t) + q_{10}(t) \copyright AV_0(t) + q_{14}^{(3)}(t) \copyright AV_4(t) + q_{15}^{(3)}(t) \copyright AV_5(t), \tag{7}$$

$$AV_4(t) = M_4(t) + q_{41}^{(6)}(t) \copyright AV_1(t) + q_{40}(t) \copyright AV_0(t), \tag{8}$$

$$AV_5(t) = M_5(t) + q_{51}^{(7)}(t) \copyright AV_1(t) + q_{50}(t) \copyright AV_0(t). \tag{9}$$

Where

$$M_0(t) = \overline{F_1}(t),$$

$$M_1(t) = \overline{G_1}(t)\overline{F_1}(t),$$

$$M_4(t) = \overline{G_2}(t)\overline{F_2}(t),$$

$$M_5(t) = \overline{N}(t)\overline{F_1}(t).$$

Taking LT for equation (6), (7), (8) and (9) and solve for AV_0^*, then we get the steady state availability of the system AV_0 in the form,

$$AV_0 = AV_0(\infty) = \lim_{x \to 0} SAV_0^*(S) = \frac{N_1}{D_1}. \tag{10}$$

$$\frac{N_1}{D_1}=\frac{\begin{vmatrix} M_0(s) & -P_{01}(t) & -P_{02}(t)P_{24}(t) & -P_{02}(t)P_{25}(t) \\ M_1(s) & 1 & -P_{14}^{(3)}(t) & -P_{15}^{(3)}(t) \\ M_4(s) & -P_{41}^{(6)}(t) & 1 & 0 \\ M_5(s) & -P_{51}^{(7)}(t) & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -P_{01}(t) & -P_{02}(t)P_{24}(t) & -P_{02}(t)P_{25}(t) \\ -P_{10}(t) & 1 & -P_{14}^{(3)}(t) & -P_{15}^{(3)}(t) \\ -P_{40}(t) & -P_{41}^{(6)}(t) & 1 & 0 \\ -P_{50}(t) & -P_{51}^{(7)}(t) & 0 & 1 \end{vmatrix}}$$

$$a_1=\begin{vmatrix} 1 & -P_{14}^{(3)}(t) & -P_{15}^{(3)}(t) \\ -P_{41}^{(6)}(t) & 1 & 0 \\ -P_{51}^{(7)}(t) & 0 & 1 \end{vmatrix}$$

$$a_2=\begin{vmatrix} -P_{01}(t) & -P_{02}(t)P_{24}(t) & -P_{02}(t)P_{25}(t) \\ -P_{41}^{(6)}(t) & 1 & 0 \\ -P_{51}^{(7)}(t) & 0 & 1 \end{vmatrix}$$

$$a_3=\begin{vmatrix} -P_{01}(t) & -P_{02}(t)P_{24}(t) & -P_{02}(t)P_{25}(t) \\ 1 & -P_{14}^{(3)}(t) & -P_{15}^{(3)}(t) \\ -P_{51}^{(7)}(t) & 0 & 1 \end{vmatrix}$$

$$a_4=\begin{vmatrix} -P_{01}(t) & -P_{02}(t)P_{24}(t) & -P_{02}(t)P_{25}(t) \\ 1 & -P_{14}^{(3)}(t) & -P_{15}^{(3)}(t) \\ -P_{41}^{(6)}(t) & 1 & 0 \end{vmatrix}$$

$$N_1=M_0(s)a_1-M_1(s)a_2+M_4(s)a_3-M_5(s)a_4,$$

$$D_1=-(m_{01}+m_{02}P_{24}(t)+m_{24}P_{02}(t)+m_{02}P_{25}(t)+m_{25}P_{02}(t))a_1+(m_{10}+m_{14}^{(3)}+m_{15}^{(3)})a_2-(m_{40}+m_{41}^{(6)})a_3+(m_{50}+m_{51}^{(7))})a_4.$$

5 Busy Period Analysis

The probability that the repairman is busy due to repair of the failed unit.

5.1 Expected busy period with first unit failure

By using probabilistic arguments, we obtain

$$BP_0(t)=q_{01}(t)©BP_1(t)+q_{02}(t)©q_{24}(t)©BP_4(t)+q_{02}(t)©q_{25}(t)©BP_5(t)+q_{02}(t)©\overline{G_1}(t), \tag{11}$$

$$BP_1(t)=q_{10}(t)©BP_0(t)+q_{14}^{(3)}(t)©BP_4(t)+q_{15}^{(3)}(t)©BP_5(t)+\overline{G_1}(t), \tag{12}$$

$$BP_4(t)=q_{41}^{(6)}(t)©BP_1(t)+q_{40}(t)©BP_0(t), \tag{13}$$

$$BP_5(t)=q_{51}^{(7)}(t)©BP_1(t)+q_{50}(t)©BP_0(t). \tag{14}$$

Using LT to solve equations (11), (12), (13) and (14) for $R_0^*(s)$, we calculate the expectation of repair in steady state as follows

$$BP_0=BP_0(\infty)=\frac{N_2}{D_1}, \tag{15}$$

where

$$N_2 = \overline{G_1}^*(0)\{(P_{02})a_1 - a_2\},$$

and

$$\overline{G_1}^*(0) = \frac{1}{\mu_1}.$$

5.2 Expected busy period with second unit failure

By using probabilistic arguments, we obtain

$$BP_0'(t) = q_{01}(t)©BP_1'(t) + q_{02}(t)©q_{24}(t)©BP_4'(t) + q_{02}(t)©q_{25}(t)©BP_5'(t), \tag{16}$$

$$BP_1'(t) = q_{10}(t)©BP_0'(t) + q_{14}^{(3)}(t)©BP_4'(t) + q_{15}^{(3)}(t)©BP_5'(t), \tag{17}$$

$$BP_4'(t) = q_{41}^{(6)}(t)©BP_1'(t) + q_{40}(t)©BP_0'(t) + \overline{G_2}(t), \tag{18}$$

$$BP_5'(t) = q_{51}^{(7)}(t)©BP_1'(t) + q_{50}(t)©BP_0'(t). \tag{19}$$

Using LT to solve equations (16), (17), (18) and (19) for $R_0'^*(s)$, we calculate the expectation of repair in steady state as follows

$$BP_0' = BP_0'(\infty) = \frac{N_3}{D_1}, \tag{20}$$

where

$$N_3 = \overline{G_2}^*(0)\{a_3\},$$

and

$$\overline{G_2}^*(0) = \frac{1}{\mu_2}.$$

5.3 Expected busy period due to replacement

$$BP_0''(t) = q_{01}(t)©BP_1''(t) + q_{02}(t)©q_{24}(t)©BP_4''(t) + q_{02}(t)©q_{25}(t)©BP_5''(t), \tag{21}$$

$$BP_1''(t) = q_{10}(t)©BP_0''(t) + q_{14}^{(3)}(t)©BP_4''(t) + q_{15}^{(3)}(t)©BP_5''(t), \tag{22}$$

$$BP_4''(t) = q_{41}^{(6)}(t)©BP_1''(t) + q_{40}(t)©BP_0''(t), \tag{23}$$

$$BP_5''(t) = q_{50}(t)©BP_0''(t) + q_{51}^{(7)}(t)©BP_1''(t) + \overline{N}(t). \tag{24}$$

Using LT to solve equations 21, 22, 23 and 24 for $BP_0''^*(s)$, we calculate the expectation of replacement in steady state as follows

$$BP_0'' = BP_0''(\infty) = \frac{N_4}{D_1}, \tag{25}$$

where

$$N_4 = \overline{N}^*(0)\{-a_4\},$$

and

$$\overline{N}^*(0) = \frac{1}{\beta}.$$

5.4 Cost benefit analysis

This section, we calculate the expected profit to the system in the period $(0,t]$ by calculating the deference between total revenue and total cost of repair

$$O(t) = K_1\omega_{up}(t) - K_2\omega_{bp}(t) - K_3\omega_{bp'}(t) - K_4\omega_{bp''}(t), \tag{26}$$

Where, K_1 is the revenue at the time the system works, K_2 is the cost of repairing the highest quality unit, K_3 is the cost of repairing the lowest quality unit, and K_4 is the cost of replacing the lowest quality unit .

$$\omega_{up}(t) = \int_0^t AV_0(t)dt, \tag{27}$$

$$\omega_{bp}(t) = \int_0^t BP_0(t)dt, \tag{28}$$

$$\omega_{bp'}(t) = \int_0^t BP'_0(t)dt, \tag{29}$$

$$\omega_{bp''}(t) = \int_0^t BP''_0(t)dt. \tag{30}$$

using 27, 28, 29 and 30 we obtain

$$O^*(s) = K_1\omega^*_{up}(s) - K_2\omega^*_{bp}(s) - K_3\omega^*_{bp'}(s) - K_4\omega^*_{bp''}(s).$$

Therefore the expected revenue per unit time in steady state is given by

$$O = \lim_{t\to\infty}\frac{O(t)}{t} = \lim_{s\to 0} s^2 O^*(s) = \frac{K_1N_1 - K_2N_2 - K_3N_3 - K_4N_4}{D_1}. \tag{31}$$

6 Numerical Example

Let us consider that

$$F_i(t) = 1 - e^{\lambda_i t} \quad ,G_i(t) = 1-(1-\mu_i t)e^{-\mu_i t} \quad i = 1,2. \quad N(t) = 1-(1-\beta t)e^{-\beta t}.$$

in 5, 10, 15, 20, 25, 31. By setting $K_1 = 500, K_2 = 10, K_3 = 5, K_4 = 2$, figures display the variation of MTSF, availability, busy period1, busy period2, busy period due to replacement and Cost benefit, for different values of $\theta, \tau, \beta, \mu_1, \mu_2, \lambda_1$ *and* λ_2.

Table 1: MTSF with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$.

	λ_2								
λ_1	0.40	0.45	0.5	0.55	0.60	0.65	0.70	0.75	0.80
0.05	22.6225	22.3387	22.1103	21.9226	21.7655	21.6321	21.5175	21.4179	21.3306
0.10	12.0317	11.8126	11.6362	11.491	11.3695	11.2663	11.1776	11.1005	11.0328
0.15	8.50144	8.30395	8.14482	8.01387	7.90423	7.81108	7.73097	7.66134	7.60026
0.20	6.73631	6.5496	6.39913	6.27528	6.17157	6.08345	6.00766	5.94177	5.88397
0.25	5.67723	5.497	5.35172	5.23213	5.13198	5.04688	4.97367	4.91003	4.8542
0.30	4.97118	4.79526	4.65344	4.5367	4.43892	4.35582	4.28435	4.22221	4.16769
0.35	4.46686	4.29402	4.15468	4.03996	3.94387	3.86222	3.79197	3.7309	3.67732
0.40	4.08862	3.91809	3.7806	3.66741	3.57259	3.49201	3.42269	3.36242	3.30954
0.45	3.79443	3.6257	3.48965	3.66741	3.28381	3.20407	3.13547	3.07583	3.0235
0.50	3.55908	3.39178	3.25689	3.14583	3.05279	2.97372	2.9057	2.84655	2.79466

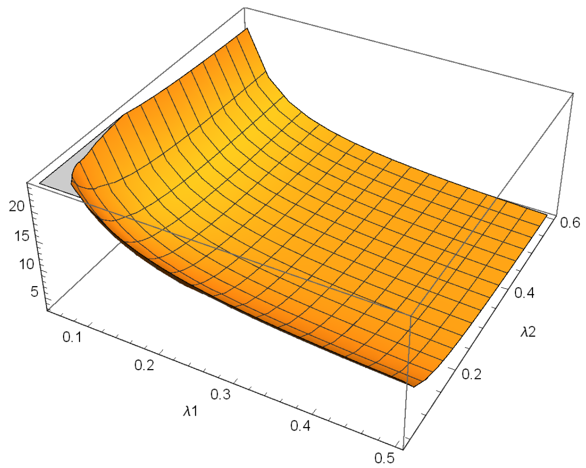

Fig. 2: MTSF with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

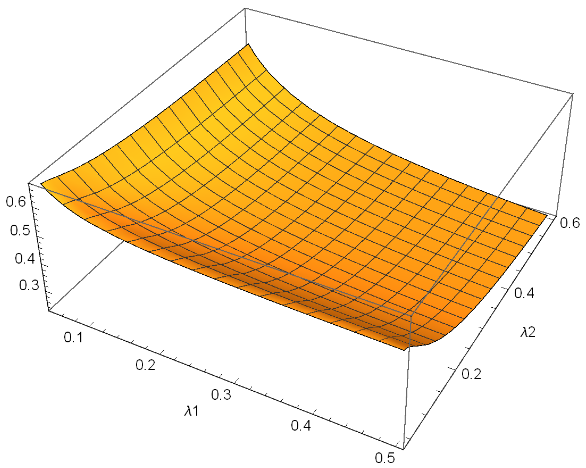

Fig. 3: Availability with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

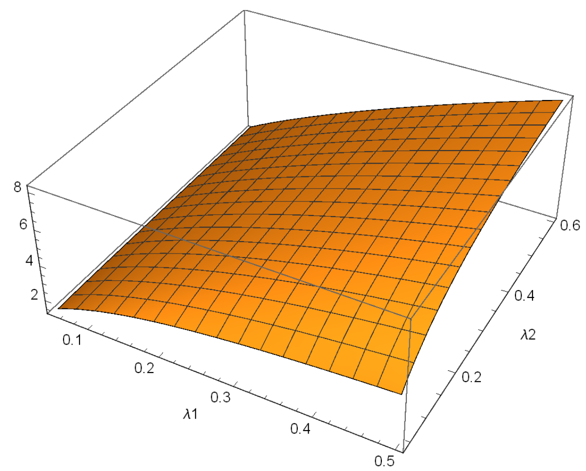

Fig. 4: Busy period due to unit 1 failure with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

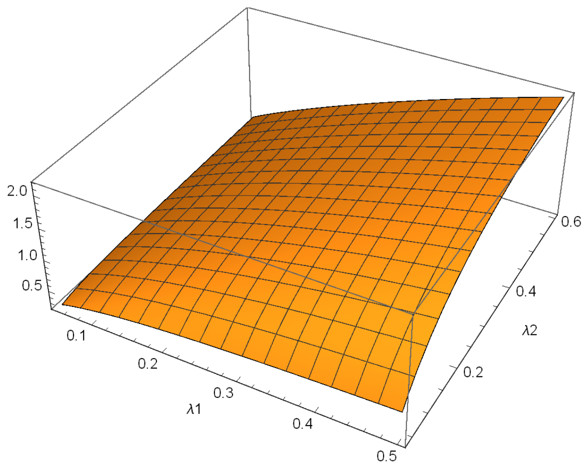

Fig. 5: Busy period due to unit 2 failure with$\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

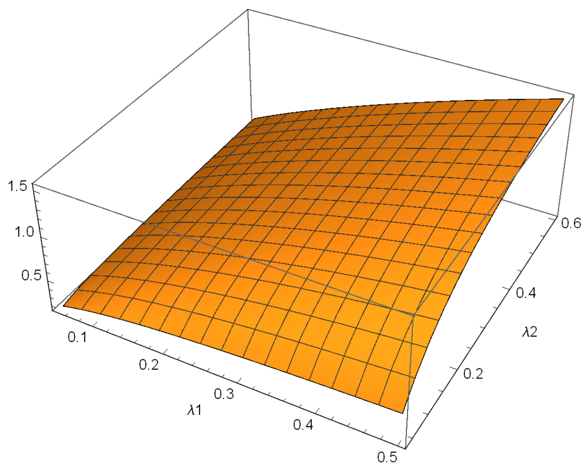

Fig. 6: Busy period due to replacement with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = .05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

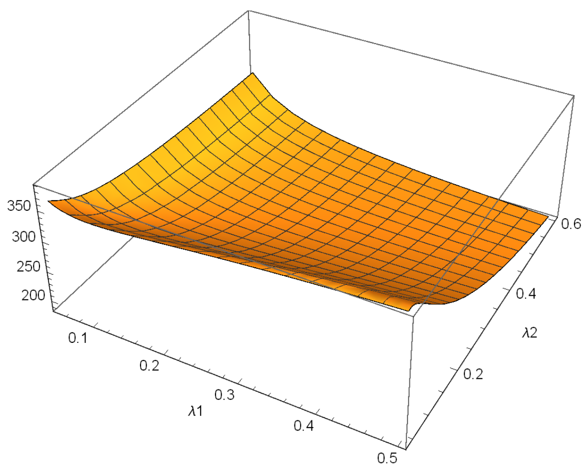

Fig. 7: Cost benefit with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.04$ *to* 0.6.

Table 2: Availability with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$.

	λ_2								
λ_1	0.40	0.45	0.5	0.55	0.60	0.65	0.70	0.75	0.80
0.05	0.574764	0.571095	0.568076	0.565548	0.563402	0.561557	0.559954	0.558548	0.557306
0.10	0.455363	0.449689	0.445018	0.441106	0.437784	0.434928	0.432446	0.430271	0.428348
0.15	0.396042	0.388944	0.383096	0.378197	0.374035	0.370455	0.367344	0.364616	0.362204
0.20	0.359247	0.351064	0.344318	0.338663	0.333857	0.329722	0.326127	0.322974	0.320186
0.25	0.333721	0.324678	0.317219	0.310964	0.305646	0.301069	0.297089	0.293597	0.290508
0.30	0.314784	0.305041	0.297002	0.290257	0.28452	0.279582	0.275288	0.271519	0.268185
0.35	0.300089	0.289766	0.281244	0.274092	0.268007	0.262768	0.258211	0.254212	0.250673
0.40	0.288312	0.2775	0.26857	0.261074	0.254695	0.249201	0.244422	0.240227	0.236515
0.45	0.27864	0.267409	0.25813	0.250339	0.243708	0.237996	0.233026	0.228663	0.224803
0.50	0.270542	0.258948	0.249367	0.241321	0.234471	0.22857	0.223435	0.218927	0.214937

Table 3: Busy period due to unit 1 failure with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$.

	λ_2								
λ_1	0.40	0.45	0.5	0.55	0.60	0.65	0.70	0.75	0.80
0.05	1.44753	1.4492	1.45056	1.45168	1.45264	1.45345	1.45415	1.45477	1.45531
0.10	2.38808	2.3964	2.40322	2.40891	2.41373	2.41786	2.42144	2.42458	2.42735
0.15	3.21632	3.23587	3.25198	3.26548	3.27697	3.28685	3.29544	3.30299	3.30966
0.20	3.97835	4.01296	4.04162	4.06575	4.08634	4.10412	4.11963	4.13328	4.14537
0.25	4.68891	4.74172	4.78568	4.82285	4.85467	4.88224	4.90635	4.9276	4.94649
0.30	5.35557	5.42918	5.49074	5.54298	5.58789	5.62689	5.66108	5.6913	5.7182
0.35	5.98339	6.07989	6.16097	6.23004	6.2896	6.34147	6.38706	6.42744	6.46346
0.40	6.57621	6.69731	6.79949	6.88687	6.96244	7.02844	7.08658	7.13819	7.1843
0.45	7.13717	7.28423	7.40884	7.51575	7.6085	7.68971	7.76142	7.8252	7.88229
0.50	7.66894	7.84305	7.99114	8.11864	8.22955	8.32692	8.41308	8.48987	8.55873

Table 4: Cost benefit with $\theta = 0.7$, $\tau = 0.6$, $\beta = .03$, $\mu_1 = .02$, $\mu_2 = 0.05$.

	λ_2								
λ_1	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
0.05	295.875	293.853	292.195	290.811	289.639	288.633	287.76	286.996	286.322
0.10	253.247	249.854	247.055	244.706	242.709	240.989	239.493	238.179	237.017
0.15	236.892	232.278	228.449	225.223	222.467	220.086	218.008	216.18	214.559
0.20	229.096	223.359	218.574	214.523	211.05	208.04	205.406	203.082	201.017
0.25	225.062	218.285	212.603	207.772	203.615	200.001	196.831	194.027	191.53
0.30	222.974	215.227	208.701	203.129	198.318	194.123	190.434	187.164	184.245
0.35	221.996	213.343	206.019	199.742	194.305	189.55	185.358	181.634	178.304
0.40	221.695	212.192	204.113	197.164	191.125	185.83	181.15	176.984	173.253
0.45	221.523	211.521	202.727	195.136	188.52	182.703	177.55	172.954	168.83
0.50	221.232	211.178	201.705	193.5	186.328	180.007	174.395	169.38	164.871

7 Conclusion

This paper provides the reliability analysis for a warm-standby-repairable-system consisting of two-dissimilar-units. One of them is a high quality unit, and the other is a low quality one unit that might require repairs or replacement by another substandard unit upon failure. The repair and replacement times are assumed to be arbitrarily distributed. We were successful in obtaining some system reliability measurements, such as the MTSF, the availability analysis, the expected busy period, and the expected profit of the system.

- The mean time to system failure increases with decreasing the failure rate of the highest quality unit(λ_1) and the failure rate of the lowest quality unit (λ_2).
- The Availability increases when the failure rate of the highest quality unit(λ_1) and the lowest quality unit (λ_2) decrease.
- The busy period with first and second unit increases with increasing the failure rate of the highest quality unit(λ_1) and the failure rate of the lowest quality unit (λ_2).
- The busy period due to the replacement of the second unit increases with increasing the failure rate of the highest quality unit(λ_1) and the failure rate of the lowest quality unit (λ_2).
- The cost-benefit increases when the failure rate of the highest quality unit(λ_1) and the lowest quality unit (λ_2) decrease.

Therefore, we recommend introducing preventive maintenance to improve the performance of this system.

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