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Statistical Inference for the Modified Weibull Model Based on the Generalized Order Statistics

M. Maswadah and M. Seham *

Department of Mathematics, Faculty of Science, Aswan University, Aswan, Egypt

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Abstract: In recent years, a new family of distributions has been proposed to exhibit bathtub-shaped failure rate functions. The modified Weibull is one of these models, which is a generalization for the Weibull distribution and is capable of modeling bathtub-shaped and increasing failure rate lifetime data. In this paper, conditional inference has been applied to constructing the confidence intervals for its parameters based on the generalized order statistics. For measuring the performance of this approach compared to the Asymptotic Maximum Likelihood estimates (AMLEs), simulations studies have been carried out for different values of sample sizes and shape parameters. The simulation results indicated that the conditional intervals possess good statistical properties and they can perform quite well even when the sample size is extremely small compared to the AMLE intervals. Finally, a numerical example is given to illustrate the confidence intervals developed in this paper.

Keywords: Modified Weibull Model; Weibull Extension Model; Weibull distribution; Burr-type XII distribution; Lomax distribution; Generalized Pareto model; Progressive type-II censored samples with binomial random removals; Asymptotic Maximum Likelihood estimates.

1 Introduction

The Modified Weibull distribution has been considered by Lai et al. [1], as a new lifetime distribution, they have shown its cabiblity of describing the lifetime variables of bathtub-shaped hazard rate function, with distribution function given by

$$F(x) = 1 - exp(-\frac{x^{\alpha} exp(\lambda x)}{\beta}) \, , \alpha \geq 0, \beta, x > 0, \lambda \geq 0. \quad (1)$$

Moreover, it is one of the models, which has some distributions as special cases such as the ordinary Weibull ($\lambda = 0$) and the type I extreme value distribution ($\alpha = 0$). Sometimes, it is referred to as a log Weibull model. The main objective of this work is to apply the conditional inference on the modified Weibull for constructing the confidence intervals for the unknown parameters based on the generalized order statistics. The conditional approach as proposed by Sir Fisher [2], has been applied for many lifetime distributions belonging to the location-scale family, see Lawless [3,4,5,6,7,8,9] or those can be converted to this family, see Maswadah [10,11]. In this section, we will give a new application for this approach to cover the situation in which the distribution does not belong to the location-scale family, via converting it to a Generalized Life Model (GLM), with scale and shape parameters, which has distribution function given by

$$F(x) = 1 - exp(-(g(x)^{\alpha})/\beta), \alpha, \beta, x > 0, \quad (2)$$

where α and β are shape and scale parameters respectively. The family of the GLM includes among others the Modified Weibull model, Weibull Extension model, Weibull distribution, Pareto distribution, Burr-type-XII distribution, Generalized Pareto, and Lamox models according to the values of $g^{\alpha}(x)$. The conditional and the classical approaches have been applied to the Modified Weibull distribution based on the generalized order statistics (GOS), that introduced by Kamps [12] as a unified approach to the ordinary OS, record values and k-th record values, which can be outlined as:

* Corresponding author e-mail: seham _ mohamed@sci.aswu.edu.eg

The random variables $X(1,n,\widetilde{m},k),....,X(n,n,\widetilde{m},k)$ be GOS from an absolutely continuous (cdf) $F(x)$ and (pdf) f(x), with noting that $X(0,n,\widetilde{m},k)=0, n\in N,\ \ k\geq 1, and\ \widetilde{m}\in\ R^{n-1}$. Then their joint pdf can be written in the form:

$$f(x_1,x_2,...,x_n)=C\prod_{i=1}^{n-1} f(x_i)\left[1-F(x_i)\right]^{m_i}\left[1-F(x_n)\right]^{k-1} f(x_n) \tag{3}$$

on the cone $F^{-1}(0)<x_1<\cdots<x_n<F^{-1}(1)$ of R^n, where $C=\prod_{i=1}^{n}\gamma_i$, $\gamma_i=k+n-i+M_i$, $M_i=\sum_{j=i}^{n-1} m_j$, , $\gamma_n=k>0$, and $\widetilde{m}=(m_1,m_2,...,m_{n-1})$.

- If $\widetilde{m}=0$ and k=1 then (3) is the joint pdf of the ordinary order statistics.
- If $\widetilde{m}=0$ except $m_n=N-n=k-1$ then (3) is the joint pdf of the type-II censored order statistics.
- If $\widetilde{m}\neq 0$,$m_n=k-1$ and $N=n+\sum_{i=1}^{n} m_i$ then (3) is the joint pdf of the type-II progressively censored order statistics.

2 Conditional inference methodology

In this section, a new application for the conditional approach has been introduced to distributions belong to shape-scale family, such as the two-parameter GLM (2). Given a set of GOS $X(1,n,\widetilde{m},k),....,X(n,n,\widetilde{m},k)$ with sampling density function belonging to the GLM (2), which can be substituted in (3) to derive the joint pdf as:

$$f(x_1,...,x_n)=C\alpha^n\beta^{-n}\prod_{i=1}^{n} g^{\alpha-1}(x_i)g'(x_i)exp[-(\sum_{i=1}^{n}(1+m_i)g^{\alpha}(x_i)+(k-m_n-1)g^{\alpha}(x_n))/\beta] \tag{4}$$

Let $\hat{\alpha}$ and $\hat{\beta}$ be any equivariant estimators such as the MLEs of α and β. Suppose $Z_1=\alpha/\hat{\alpha}$ and $Z_2=\hat{\beta}\beta^{-1/z_1}$ are pivotal quantities and $a_i=g^{\hat{\alpha}}(x_i)/\hat{\beta}$, $i=1,2,...,n$ form a set of ancillary statistics. Thus, based on the following theorem, we can derive the conditional densities for the pivotal quantities and thus the confidence intervals can be constructed which can be converting them for α and β fiducially.

Theorem:

Let $\hat{\alpha}$ and $\hat{\beta}$ be any equivariant estimators of α and β, based on the generalized order statistics $X(1,n,\widetilde{m},k),....,X(n,n,\widetilde{m},k)$. Then the conditional pdf of Z_1 and Z_2 given $A=(a_1,a_2,...,a_{n-2})$ can be derived in the form

$$g(z_1,z_2|A)=D\cdot z_1^{n-1}z_2^{nz_1-1}\prod_{i=1}^{n} a_i^{z_1-1}a_i' exp(-z_2^{z_1}U), \tag{5}$$

D is a normalizing constant depends on A only, a_i' is the derivative of a_i and $U=\sum_{i=1}^{n}(1+m_i)a_i^{z_1}+(k-m_n-1)a_n^{z_1}$.

Proof:

Make the change of variables from $X(1,m,k),...,X(n,\widetilde{m},k)$ with pdf (4) to $(\hat{\alpha},\hat{\beta},\ a_1,...,a_{n-2})$.
This transformation can be written as:
$g(x_i)=(\hat{\beta}a_i)^{1/\hat{\alpha}}$, $i=1,2...,n-2$, $g(x_{n-1})=(\hat{\beta}a_{n-1})^{1/\hat{\alpha}}$, and $g(x_n)=(\hat{\beta}a_n)^{1/\hat{\alpha}}$.

The jacobian of this transformation is

$\hat{\beta}^{n-2}h(A)$. Thus, the joint pdf of $(\hat{\alpha},\hat{\beta},a_1,...,a_{n-2})$ can be written in the form:

$$f(\hat{\alpha},\hat{\beta},a_1,...,a_{n-2})\propto\alpha^n\beta^{-n}\prod_{i=1}^{n}(a_i\hat{\beta})^{\alpha/\hat{\alpha}}(a_i'/a_i)exp[-(\sum_{i=1}^{n}(1+m_i)(a_i\hat{\beta})^{\alpha/\hat{\alpha}}+(k-m_n-1)(a_n\hat{\beta})^{\alpha/\hat{\alpha}}/\beta)].$$

Make the change of variables from $(\hat{\alpha},\hat{\beta},a_1,...,a_{n-2})$ to $(z_1,z_2,a_1,...,a_{n-2})$, with noting that $\frac{g^{\alpha}(x_i)}{\beta}=\left(\frac{g^{\hat{\alpha}}(x_i)}{\hat{\beta}}\cdot\frac{\hat{\beta}}{\beta^{\hat{\alpha}/\alpha}}\right)^{\alpha/\hat{\alpha}}=(a_iz_2)^{z_1}$. The Jacobian of this transformation is $1/z_1z_2$. Thus, the joint pdf of z_1 and z_2 given $A=(a_1,a_2,...,a_{n-2})$ is in the form (5).

3 Confidence interval procedures

3.1 Conditional confidence intervals

The marginal density of Z_1 and the distribution function of Z_2 can be derived from (5) respectively as:

$$g_1^*(z_1|A) = D\Gamma(n) z_1^{n-2} \prod_{i=1}^{n} a_i^{z_1-1} a_i' U^{-n}, \tag{6}$$

$$G_{z_2}^*(t|A) = D\Gamma(n) \int_0^\infty z_1^{n-2} \prod_{i=1}^{n} a_i^{z_1-1} a_i' U^{-n} \left(1 - exp(-t^{z_1}U) \sum_{j=0}^{n-1} \frac{(t^{z_1}U)^j}{j!}\right) dz_1. \tag{7}$$

D is a normalizing constant does not depend on Z_1 and Z_2 and can be derived as:

$$D^{-1} = \Gamma(n) \int_0^\infty z_1^{n-2} \prod_{i=1}^{n} a_i^{z_1-1} a_i' U^{-n} \mathrm{d}\, z_1.$$

From (6) and (7) we can find the desired probabilities for Z_1 and Z_2 and convert them to the unknown parameters α and β fiducially.

3.2 Asymptotic confidence intervals

In this subsection, we obtained the Fisher information matrix to compute 95% asymptotic confidence intervals for the modified Weibull distribution parameters based on maximum likelihood estimators (MLEs). We have the first and second derivatives of the log likelihood function of (1) with respect to and can be derived as follows:

$$\frac{\partial lnL}{\partial \alpha} = \sum_{i=1}^{n} (ln(x_i) + \frac{1}{\alpha+\lambda x_i}) - \begin{bmatrix} \sum_{i=1}^{m} (1+m_i) x_i^\alpha ln(x_i) exp(\lambda x_i) \\ +(k-m_n-1) x_n^\alpha ln(x_n) exp(\lambda x_i) \end{bmatrix} / \beta,$$

$$\frac{\partial lnL}{\partial \beta} = -\frac{n}{\beta} + [\sum_{i=1}^{n} (1+m_i) x_i^\alpha exp(\lambda x_i) + (k-m_n-1) x_n^\alpha exp(\lambda x_n)] / \beta^2$$

$$I_{\alpha\alpha} = \frac{\partial^2 lnL}{\partial \alpha^2} = -\sum_{i=1}^{n} \frac{1}{(\alpha+\lambda x_i)^2} - \begin{bmatrix} \sum_{i=1}^{m} (1+m_i) x_i^\alpha (ln(x_i))^2 exp(\lambda x_i) \\ +(k-m_n-1) x_n^\alpha (ln(x_n))^2 exp(\lambda x_i) \end{bmatrix} / \beta,$$

$$I_{\beta\beta} = \frac{\partial^2 lnL}{\partial \beta^2} = \frac{n}{\beta^2} - 2[\sum_{i=1}^{n} (1+m_i) x_i^\alpha exp(\lambda x_i) + (k-m_n-1) x_n^\alpha exp(\lambda x_n)] / \beta^3$$

$$I_{\alpha\beta} = \frac{\partial^2 lnL}{\partial \beta \partial \alpha} = [\sum_{i=1}^{n} (1+m_i) x_i^\alpha ln(x_i) exp(\lambda x_i) + (k-m_n-1) x_n^\alpha ln(x_n) exp(\lambda x_n)] / \beta^2.$$

Thus, the variance-covariance matrix is

$$AVC = \begin{bmatrix} var(\hat{\alpha}) & cov(\hat{\alpha}, \hat{\beta}) \\ cov(\hat{\beta}, \hat{\alpha}) & var(\hat{\beta}) \end{bmatrix} = \begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} \\ I_{\beta\alpha} & I_{\beta\beta} \end{bmatrix}^{-1}_{(\alpha,\beta)=(\hat{\alpha},\hat{\beta})}$$

The approximate $100(1-\gamma)\%$ two sided confidence intervals for α and β can be obtained respectively by $\hat{\alpha} \pm Z_{\gamma/2} \sigma_{\hat{\alpha}}$ and $\hat{\beta} \pm Z_{\gamma/2} \sigma_{\hat{\beta}}$, where $Z_{\gamma/2}$ is the upper $\gamma/2\_th$ percentile of a standard normal distribution, $\sigma_{\hat{\alpha}}$, $\sigma_{\hat{\beta}}$ are the standard deviations of the MLEs of the parameters α and β respectively.

4 Simulation studies

In this section, we mainly present some Monte Carlo simulation results, to measure the performance of the conditional inference comparing to the unconditional inference in terms of the following criteria:

1. Covering percentage (CP).

2. Mean length of intervals (MLIs).

3. The standard error of the covering percentage (SDE).

The comparative results, based on 1000 Monte Carlo simulation trials are given for sample sizes n=20, 40, 60, 80, and 100 with censoring levels 0.0%, 0.25%, and 0.50%, which have been generated using the rejection method from the Modfied Weibull distribution for shape parameter values $\alpha = 0.5, 1,\ 2,$ and 3. The scale parameter β was set equal 2 and λ was set equal 0.5 through, where all estimations are equivariant under scale changes of the data and the values of λ, so setting one value for each of β and λ involves no loss of generality. For the progressive type-II censoring sampling that are carried out with binomial random removals with probability $P = 0.5$, which means the number of units removed at each failure time follows a binomial distribution with probability P, where different values of P does not affect the calculations.

Table 1. The (MLIs), (CPs) and (SDEs) for the conditional and the AMLEs approaches based on the nominal level for the parameterα based on complete and censored samples with censored levels (50%, 25%, and 0.0%).

Approaches		Conditional CIs					AMLEs CIs				
		MLI,α					MLI,α				
n	m	0.5	1.0	2.0	CP	SDE	0.5	1.0	2.0	CP	SDE
20	10	0.6523	1.3045	2.6091	0.946	0.0071	0.6894	1.3788	2.7576	0.956	0.0065
	15	0.4789	0.9578	1.9156	0.949	0.0069	0.4926	0.9853	1.9705	0.957	0.0064
	20	0.3674	0.7347	1.4695	0.947	0.0071	0.3727	0.7455	1.4909	0.947	0.0071
40	20	0.4291	0.8582	1.7164	0.955	0.0066	0.4399	0.8797	1.7594	0.957	0.0064
	30	0.3255	0.6509	1.3018	0.955	0.0066	0.3298	0.6596	1.3192	0.958	0.0063
	40	0.2504	0.5008	1.0017	0.963	0.0059	0.2522	0.5044	1.0087	0.962	0.0060
60	30	0.3442	0.6883	1.3767	0.950	0.0069	0.3497	0.6994	1.3998	0.954	0.0066
	45	0.2621	0.5241	1.0482	0.944	0.0073	0.2644	0.5287	1.0575	0.947	0.0071
	60	0.2019	0.4037	0.8075	0.948	0.0070	0.2028	0.4056	0.8112	0.947	0.0071
80	40	0.2942	0.5884	1.1767	0.955	0.0066	0.2977	0.5954	1.1908	0.959	0.0063
	60	0.2249	0.4499	0.8998	0.955	0.0066	0.2264	0.4529	0.9057	0.954	0.0066
	80	0.1737	0.3475	0.6949	0.961	0.0061	0.1743	0.3487	0.6973	0.959	0.0063
100	50	0.2617	0.5088	1.0469	0.948	0.0070	0.2642	0.5285	1.0569	0.952	0.0068
	75	0.2009	0.4017	0.8034	0.955	0.0067	0.2019	0.4038	0.8075	0.955	0.0067
	100	0.1552	0.3104	0.6208	0.965	0.0056	0.1556	0.3112	0.6224	0.967	0.0056

Table 2. The (MLIs), (CPs) and (SDEs) for the conditional and the AMLEs approaches based on the nominal level 95% for the parameter α based on the progressive type-II censoring with binomial random removal with probability P = 0.5 for censored levels (50%, 75%).

Approaches		Conditional CIs					AMLEs CIs				
		MLI,α					MLI,α				
n	m	0.5	1.0	2.0	CP	SDE	0.5	1.0	2.0	CP	SDE
20	10	0.5560	1.1120	2.2241	0.953	0.0067	0.5745	1.1490	2.2980	0.952	0.0068
	15	0.4338	0.8675	1.7350	0.937	0.0077	0.4425	0.8850	1.7701	0.944	0.0073
40	20	0.3697	0.7394	1.4787	0.947	0.0071	0.3751	0.7502	1.5004	0.946	0.0072
	30	0.2929	0.5859	1.1719	0.952	0.0068	0.2958	0.5915	1.1831	0.958	0.0063
60	30	0.2925	0.5849	1.1699	0.951	0.0068	0.2953	0.5905	1.1810	0.954	0.0066
	45	0.2355	0.4711	0.9421	0.951	0.0068	0.2370	0.4740	0.9480	0.948	0.0070
80	40	0.2496	0.4993	0.9986	0.956	0.0065	0.2514	0.5028	1.0056	0.956	0.0065
	60	0.2019	0.4037	0.8074	0.954	0.0066	0.2028	0.4056	0.8111	0.955	0.0066
100	50	0.2277	0.4455	0.8911	0.946	0.0071	0.2240	0.4480	0.8961	0.944	0.0073
	75	0.1801	0.3601	0.7203	0.948	0.0070	0.1807	0.3615	0.7229	0.948	0.0070

Table 3. The conditional and the AMLEs, (MLIs), (CPs) and (SDEs) based on the nominal level 95% for the parameter β based on the type-II censored and type-II progressively censoring with binomial random removal with probability $P = 0.5$ with censored levels (50%, 25%, and 0.0%).

Approaches			Conditional CIs			AMLEs CIs		
	n	m	MLI	CP	SDE	MLI	CP	SDE
Type-II Censored Samples	20	10	0.7084	0.961	0.0061	2.9042	0.888	0.0099
		15	2.7690	0.946	0.0071	2.8148	0.928	0.0082
		20	2.0518	0.949	0.0069	2.7950	0.924	0.0084
	40	20	2.8842	0.952	0.0068	1.8568	0.919	0.0086
		30	1.6417	0.951	0.0068	1.7812	0.933	0.0079
		40	1.3643	0.940	0.0075	1.7808	0.935	0.0078
	60	30	2.0470	0.950	0.0069	1.4962	0.926	0.0083
		45	1.2799	0.947	0.0071	1.4207	0.945	0.0072
		60	1.0974	0.942	0.0074	1.4163	0.939	0.0076
	80	40	1.6559	0.956	0.0065	1.2717	0.941	0.0065
		60	1.0802	0.957	0.0064	1.2006	0.942	0.0064
		80	0.9392	0.950	0.0069	1.1974	0.946	0.0071
	100	50	1.4324	0.957	0.0064	1.1389	0.957	0.0064
		75	0.9545	0.952	0.0068	1.0732	0.948	0.0070
		100	0.8233	0.943	0.0062	1.0702	0.945	0.0072
Type-II Progressive Censored Samples	20	10	3.3582	0.959	0.0063	5.0298	0.909	0.0090
		15	2.4757	0.948	0.0070	3.4734	0.916	0.0088
	40	20	2.0319	0.951	0.0068	2.7683	0.925	0.0083
		30	1.5959	0.943	0.0073	2.1209	0.934	0.0079
	60	30	1.6119	0.936	0.0077	2.1495	0.942	0.0074
		45	1.2813	0.949	0.0069	1.6741	0.952	0.0068
	80	40	1.6559	0.956	0.0065	1.2717	0.941	0.0065
		60	1.0922	0.952	0.0068	1.4048	0.947	0.0071
	100	50	1.2044	0.939	0.0076	1.5604	0.942	0.0074
		75	0.9694	0.940	0.0075	1.2428	0.937	0.0077

From the simulation results in Tables 1, 2, and 3 using 95% confidence intervals for the parameters α and β based on the conditional and the AMLEs approaches we can summarize the following main points:

1. The values of MLI decrease and the CPs get almost increase and the values of SDEs get almost decrease as the sample size increases for both parameters α and β.
2. The values of MLI for α increase with the same average of increasing α as expected, however the values of MLI for β are still constant for increasing α as expected.
3. The values of MLI for α and β based on the conditional inference are smaller than those based on the AMLEs, in spite of they have almost higher CPs based on complete and censored samples. However, both approaches have greater MLIs values for $n = 10$, based on the complete and censored samples.
4. Both approaches are almost conservative for estimating α and β, however the AML approach is anti-conservative when the sample size is less than or equal to 20.
5. The results based on the type-II progressive samples are better than those based on the censored samples, in which they have smaller *MLIs* and higher *CPs*.
6. Finally, both approaches are adequate because their *SDEs* are less than $\pm 2\%$ for the nominal level 95%.

Thus, the simulation results indicated that the conditional intervals possess good statistical properties and they can perform quite well even when the sample size is extremely small. However, the MLEs turn out to be impercise or even unreliable for small or highly censored samples.

5 Numerical example

Consider the data in Aarset [13] that represent the lifetime of 50 industrial devices, which fit the Modified Weibull model. 0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67,72,

Table 4. The Lower (LL) and the Upper limits (UL) and the lengths of the 90% and 95% confidence intervals (CI) for the parameters α , β based on the Conditional and the AMLEs approaches for complete, Type-II censored and Type-II progressive censored samples for the ball bearings data.

Approaches		Conditional CIs				AMLEs CIs			
	Par.	LL	UL	LL	UL	LL	UL	LL	UL
Complete	α	0.7471	1.1367	0.7152	1.1797	0.7517	1.1462	0.7146	1.1832
		(0.3896)		(0.4645)		(0.3945)		(0.4686)	
	β	28.7669	47.5181	27.3029	49.9186	5.6238	68.3371	-0.2674	74.2284
		(18.7513)		(22.6157)		(62.7133)		(74.4958)	
Censored 50%	α	0.3966	0.7425	0.3705	0.7829	0.4023	0.7562	0.3691	0.7895
		(0.3459)		(0.4125)		(0.3539)		(0.7895)	
	β	10.0939	22.8488	9.5441	25.7292	3.8928	23.2841	2.0712	25.1057
		(12.7549)		(16.1851)		(19.3914)		(23.0346)	
Censored 25%	α	0.5320	0.8744	0.5048	0.9130	0.5369	0.8847	0.5043	0.9174
		(0.3478)		(0.4083)		(0.3478)		(0.4131)	
	β	14.3145	25.4639	13.5917	27.2521	4.6577	32.3259	2.0585	34.9251
		(11.1495)		(13.6605)		(27.6683)		(32.8666)	
Prog. Cen. 50%	α	0.5494	0.9668	0.5167	1.0143	0.5582	0.9829	0.5183	1.0228
		(0.4173)		(0.4976)		(0.4247)		(0.5045)	
	β	5.3184	11.1527	4.9059	12.0187	1.9643	13.4483	0.8845	14.5272
		(5.8343)		(7.1128)		(11.4843)		(13.6426)	
Prog. Cen. 25%	α	0.6285	1.0118	0.5977	1.0547	0.6343	1.0232	0.5978	1.0597
		(0.3833)		(0.4569)		(0.3888)		(0.4619)	
	β	12.3461	22.3457	11.5929	23.6957	3.5112	29.7271	1.0485	32.1898
		(9.9997)		(12.1028)		(26.2159)		(31.1413)	

(The values in parentheses are the length of intervals)

75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, 86. Thus for the purpose of comparison, the 90% and 95% confidence intervals for the parameters α and β are derived based on the conditional and the AMLEs approaches. The results in Table 4 have indicated that the length of intervals for the parameters α and β based on the conditional approach are smaller than those based on the AMLE approach which ensures the simulation results.

6 Conclusion

In this work, the conditional inference has been applied for deriving the confidence intervals (CCI) for the modified Weibull distribution parameters compared with the asymptotic confidence intervals (ACI) based on the GOS. We found the mean length and covering percentage of the intervals based on the conditional inference are more efficient than the corresponding ones based on the asymptotic confidence intervals, where the mean length of intervals is less than the ones for the ACI and the covering percentages are close to the nominal levels for the CCI than the ACI. Therefore, the conditional inference method is more effective than the asymptotic maximum likelihood method.

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