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Monitor Reaction of Win Stay-Lose Shift Strategies in Iterated Three-Player Prisoner's Dilemma Game

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Abstract: In this paper, we present an extension model of Prisoner's Dilemma game, but with three players. We are interested in introducing this model and providing an analysis of competitions of some special types of strategies which have properties of Win Stay-Lose Shift. Therefore, we will show the best one among them and the largest one in the payoff through some graphs by using some numerical values. Also, we will discuss the effect of relatedness between players on the behavior of strategies.

Keywords: Iterated Prisoner's Dilemma Game (IPD), Payoff Matrix, Transition Matrix, Three-Player Game, Win Stay-Lose Shift Strategies (WSLS).

1 Introduction

Game theory is not a prescriptive way of how to play a game. Rather it is a set of ideas and techniques for analyzing these mathematical methods. It doesn't tell you how to play the game, but describes properties that certain ways of playing the game have, and which you might think desirable. Even when the analysis suggests a best way of playing the game, it only does it assuming that everyone is playing in the "best way" they can. It never allows for ways of punishing your opponent if he makes a mistake, which is the way most players do.

The theory of the game has applications in many fields of the social science, systems and computer science. This theory is commonly used in economics [1,2], physics, networks [3,4,5], business [6], mathematics [7,8], traffic engineering [9], philosophy, public health, biology [10,11,12] and political science [13,14,15,16].

The Prisoner's Dilemma is a standard example in the game theory, but in biology and economics there are many other iterated games such as: Hawk-Dove, location game, matching pennies, etc. It shows why two rational people would not cooperate, even if it seemed to be in their interest [17]. The Prisoner's Dilemma can be used to make a decision in a number of areas in personal life, such as competition between people, buying a car, salary negotiation skills, etc.

In the Prisoner's Dilemma, two-player have only two decisions, either cooperate or defect [18,19]. If two-player play the Prisoner's Dilemma more than once and remember their opponent's previous actions, the game is called two-player iterated Prisoner's Dilemma (2P-IPD). In this game, each player gets points based on his choice called a payoff. Players play simultaneously without knowing the other player's choice. The two-player get a reward, $\mathcal{R}$, if they cooperate. They get Punishment, $\mathcal{P}$, if both players defect. The defector gets a reward that is the temptation, $\mathcal{T}$, if the other player cooperates, while the player who cooperates is punished with a sucker's payoff, $\mathcal{S}$. Since defection results in a better payoff than cooperation of the other player's choice, it is a strictly dominant strategy for both players. The dominant strategies suggest the most suitable option for any player given each choice by the opponent player. Payoffs in the dominant strategy

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are in such a way that in spite of the choices other players make, no other strategy has a higher payoff. The payoff matrix of (2P-IPD) [19] is given by

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} \mathcal{R} & \mathcal{S} \\ \mathcal{T} & \mathcal{P} \end{bmatrix} \end{array}, \tag{1}$$

where

$$\mathcal{S} < \mathcal{P} < \mathcal{R} < \mathcal{T} \; and \; \mathcal{R} > \frac{\mathcal{T} + \mathcal{S}}{2}. \tag{2}$$

and Markov transition matrix in two-player [19] is given by

$$M = \begin{array}{c} \begin{array}{cccc} \quad\mathcal{R} & \qquad\mathcal{S} & \qquad\mathcal{T} & \qquad\mathcal{P} \end{array} \\ \begin{bmatrix} p_1q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\ p_2q_3 & p_2(1-q_3) & (1-p_2)q_3 & (1-p_2)(1-q_3) \\ p_3q_2 & p_3(1-q_2) & (1-p_3)q_2 & (1-p_3)(1-q_2) \\ p_4q_4 & p_4(1-q_4) & (1-p_4)q_4 & (1-p_4)(1-q_4) \end{bmatrix} \end{array}, \tag{3}$$

where player I plays with probability $\mathcal{P} = (p_1, p_2, p_3, p_4)$, player II with probability $\mathcal{Q} = (q_1, q_2, q_3, q_4)$.

The Prisoner's Dilemma game model is one of the most important models of game theory. Many researches have been interested in this model with two players (2P-IPD). While if we have three players playing the Prisoner's Dilemma game, this game will be called three-player Prisoner's Dilemma (3P-IPD). We consider an iterated game that consists of repeating the game infinitely with a probability of 1. The automata of each one of the three-player has two states C and D. We assume that the game is symmetric, so the outcomes decrease from eight to six combinations $[(C,(C,C))$, $(C,(C,D))$, $(C,(D,D))$, $(D,(C,C))$, $(D,(C,D))$, $(D,(D,D))]$ which are defined by the player's payoff $\mathcal{R}, \mathcal{K}, \mathcal{S}, \mathcal{T}, \mathcal{L}$ and $\mathcal{P}$. The payoff matrix of (3P-IPD) [20] is given by

$$\begin{array}{c c} & \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} \mathcal{R} & \mathcal{K} & \mathcal{S} \\ \mathcal{T} & \mathcal{L} & \mathcal{P} \end{bmatrix} \end{array}, \tag{4}$$

where

$$\mathcal{S} < \mathcal{P} < \mathcal{K} < \mathcal{L} < \mathcal{R} < \mathcal{T}. \tag{5}$$

Now for the Markov transfer matrix for three players, in our paper, we will generalize the matrix to three players as in section 2, and accordingly we will defined some strategies in three players which are called Win Stay-Lose Shift strategies as in section 3. While, we will study the competition of these strategies as in section 4. Furthermore, in section 5, we will discuss the effect of relationship between players on the behavior of the Win Stay-Lose Shift strategies.

2 Transition Matrix for Three-Player

If we generalize Markov transition matrix for two players in (3) to three players, where player I plays with probability $\mathcal{P} = (p_1, p_2, p_3, p_4, p_5, p_6)$, player II with probability $\mathcal{Q} = (q_1, q_2, q_3, q_4, q_5, q_6)$ and player III with $\mathcal{W} = (w_1, w_2, w_3, w_4, w_5, w_6)$, we can defined it as

$$M = \begin{array}{c c} & \begin{array}{cccccc} \mathcal{R} & \mathcal{K} & \mathcal{S} & \mathcal{T} & \mathcal{L} & \mathcal{P} \end{array} \\ \begin{array}{c} \mathcal{R} \\ \mathcal{K} \\ \mathcal{S} \\ \mathcal{T} \\ \mathcal{L} \\ \mathcal{P} \end{array} & \begin{bmatrix} p_1q_1w_1 & p_1[q_1(1-w_1)+(1-q_1)w_1] & p_1(1-q_1)(1-w_1) & (1-p_1)q_1w_1 & (1-p_1)[q_1(1-w_1)+(1-q_1)w_1] & (1-p_1)(1-q_1)(1-w_1) \\ p_2q_2w_4 & p_2[q_2(1-w_4)+(1-q_2)w_4] & p_2(1-q_2)(1-w_4) & (1-p_2)q_2w_4 & (1-p_2)[q_2(1-w_4)+(1-q_2)w_4] & (1-p_2)(1-q_2)(1-w_4) \\ p_3q_5w_5 & p_3[q_5(1-w_5)+(1-q_5)w_5] & p_3(1-q_5)(1-w_5) & (1-p_3)q_5w_5 & (1-p_3)[q_5(1-w_5)+(1-q_5)w_5] & (1-p_3)(1-q_5)(1-w_5) \\ p_4q_2w_2 & p_4[q2(1-w_2)+(1-q_2)w_2] & p_4(1-q_2)(1-w_2) & (1-p_4)q_2w_2 & (1-p_4)[q_2(1-w_2)+(1-q_2)w_2] & (1-p_4)(1-q_2)(1-w_2) \\ p_5q_3w_5 & p_5[q_3(1-w_5)+(1-q_3)w_5] & p_5(1-q_3)(1-w_5) & (1-p_5)q_3w_5 & (1-p_5)[q_3(1-w_5)+(1-q_3)w_5] & (1-p_5)(1-q_3)(1-w_5) \\ p_6q_6w_6 & p_6[q_6(1-w_6)+(1-q_6)w_6] & p_6(1-q_6)(1-w_6) & (1-p_6)q_6w_6 & (1-p_6)[q_6(1-w_6)+(1-q_6)w_6] & (1-p_6)(1-q_6)(1-w_6) \end{bmatrix} \end{array} \tag{6}$$

For instance, if we want to transform state $\mathcal{R}$ to $\mathcal{K}$, it means we will transform from $(C,(C,C))$ to $[(C,(C,D))$ or $(C,(D,C))]$, so the player I will insist on his decision and one of the two other Players (II and III) will change his decision and the other one will insist on his decision. Therefore, the probability of the transformation from state $\mathcal{R}$ to $\mathcal{K}$ is equal to $p_1[q_1(1-w_1)+(1-q_1)w_1]$.

Assuming $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$ is the left eigenvector of M for eigenvalue 1 as shown in equation (7). Furthermore, this vector is the unique stationary distribution for infinitely repeated games. The following equation (8) calculates the payoff of player I which plays with probability $\mathcal{P}$ against player III which plays with probability $\mathcal{W}$

$$\Pi \; M = \Pi, \tag{7}$$

$$E(\mathcal{P}, \mathcal{Q}, \mathcal{W}) = \pi_1\mathcal{R} + \pi_2\mathcal{K} + \pi_3\mathcal{S} + \pi_4\mathcal{T} + \pi_5\mathcal{L} + \pi_6\mathcal{P}, \tag{8}$$

where

$$\sum_{i=1}^{6} \pi_i = 1. \tag{9}$$

Now, for example, if we take Player I which using the strategy $\mathcal{P} = (1, 1, 1, 0, 0, 0)$ against Player III which using $\mathcal{W} = (1, 1, 0, 0, 0, 0)$ with player II plays with $\mathcal{Q} = (1, 1, 1, 0, 0, 0)$ which is fixed player, then the transition matrix given by

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

It is clear that, the eigenvector $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$ is not uniquely defined. So, we will replace this method by direct approach which we will study it in the next section.

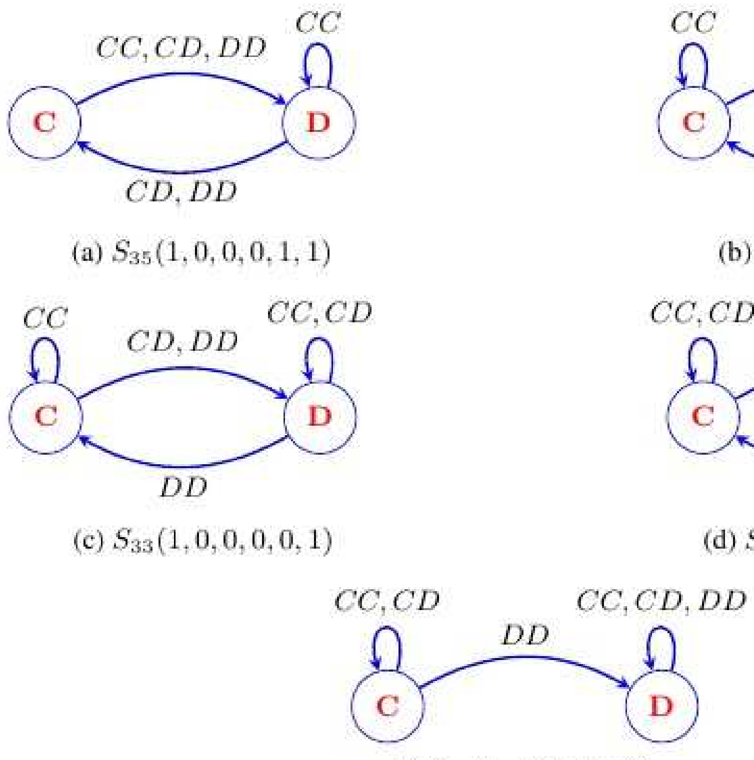
3 Win Stay-Lose Shift Strategies (WSLS)

There are 64 strategies for three-player where some of them have some special features such as Tit-For-Tat strategies which have been studied before in [20]. In this section, we will study some of these strategies, namely "Win Stay-Lose Shift" strategies. The Win Stay-Lose Shift strategies do not accept the loss. If it obtains the biggest payoff, it will repeat the same move in the next round, this is win-stay. If it obtains small payoff, it will switch to the opposite move in the next round, this is the lose-shift. Win Stay-Lose Shift strategies are also known as Pavlov strategies in (2P-IPD) [21]. In Table 1, we will discover which strategies we can call Win Stay-Lose Shift with different payoff conditions (Keeping $\mathcal{P} < \mathcal{R}$) as the following in Table 1.

Table 1: The WSLS strategies with different conditions

Condition	WSLS
$\mathcal{S} < \mathcal{P} < \mathcal{K} < \mathcal{L} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{33}, S_{49}, S_{48}$
$\mathcal{S} < \mathcal{K} < \mathcal{P} < \mathcal{L} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{33}, S_{32}, S_{48}$
$\mathcal{S} < \mathcal{P} < \mathcal{L} < \mathcal{K} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{51}, S_{49}, S_{48}$
$\mathcal{S} < \mathcal{L} < \mathcal{P} < \mathcal{K} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{51}, S_{50}, S_{48}$
$\mathcal{S} < \mathcal{K} < \mathcal{L} < \mathcal{P} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{34}, S_{32}, S_{48}$
$\mathcal{S} < \mathcal{L} < \mathcal{K} < \mathcal{P} < \mathcal{R} < \mathcal{T}$	$S_3, S_{35}, S_{34}, S_{50}, S_{48}$

In order to do this, we will introduce five different strategies to discuss the concept of Win Stay-Lose Shift strategies in details according to the inequality (5). These strategies are called WSLS 1 (S_3), WSLS 2 (S_{35}), WSLS 3 (S_{33}), WSLS 4 (S_{49}) and WSLS 5 (S_{48}) and presented by the automatons in Fig. 1



(a) $S_{35}(1, 0, 0, 0, 1, 1)$ (b) $S_3(0, 0, 0, 0, 1, 1)$ (c) $S_{33}(1, 0, 0, 0, 0, 1)$ (d) $S_{49}(1, 1, 0, 0, 0, 1)$ (e) $S_{48}(1, 1, 0, 0, 0, 0)$

Fig. 1: Automates of WSLS strategies

Our choice is based on their attitude which we observe in Fig. 1 and now we realize the following:

1. WSLS 1 (S_3): it always shifts from the state C to D. Moreover, in state D, it plays D only if the two other players play C together otherwise it shifts to C. i.e. it stays if gets $\mathcal{T}$ otherwise it makes a shift.

2. WSLS 2 (S_{35}): it still plays C if the two other players play C together otherwise it shifts to D. Moreover, in state D, it plays D if two other players play C together otherwise it shifts to C. i.e. it stays if gets $\mathcal{T}$ or $\mathcal{R}$ otherwise it makes a shift.

3. WSLS 3 (S_{33}): it plays C if the two other players play C together while, it shifts from the state C to D if only one of the other players plays D. Moreover, in state D, it stays if at least one of the other players plays C and shifts to C if the two other players play D together. i.e. it stays if it gets $\mathcal{T}$, $\mathcal{R}$ or $\mathcal{L}$ otherwise it makes a shift.

4. WSLS 4 (S_{49}): it still plays C if at least one of the other players plays C while, it shifts to the state D if the two players play D together. Moreover, in state D, it plays D only if at least one of the other players plays C otherwise it shifts to C. i.e. it stays if gets $\mathcal{T}$, $\mathcal{R}$, $\mathcal{L}$ or $\mathcal{K}$ otherwise it makes a shift.

5. WSLS 5 (S_{48}): it still plays C if at least one of the other players plays C while, it shifts to the state D if the two players play D together. Moreover, in state D, it always plays D. i.e. it stays if gets $\mathcal{T}$, $\mathcal{R}$, $\mathcal{L}$, $\mathcal{K}$ or $\mathcal{P}$ otherwise it make a shift.

4 Competition Between WSLS Strategies

In this section, we study the competition between WSLS strategies using an example for the three players S_{48}, S_{33}, S_{49} such as player I (S_{48}) against player III (S_{49}) with player II (S_{33}). We will get the following eight sequences as follows:

- Case 1: If the three-player (S_{48}, S_{33}, S_{49}) start with C

$$
\begin{array}{lccccccccccccc}
S_{48} & : & C & C & C & C & C & C & C & C & C & C & \ldots & \\
S_{33} & : & C & C & C & C & C & C & C & C & C & C & \ldots & \mathcal{R} \\
S_{49} & : & C & C & C & C & C & C & C & C & C & C & \ldots &
\end{array}
$$

- Case 2: If players I and III (S_{48}, S_{49}) start with C but player II (S_{33}) starts with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & C & C & C & C & C & C & C & C & C & C & \ldots & \\
S_{33} & : & D & D & D & D & D & D & D & D & D & D & \ldots & \mathcal{K} \\
S_{49} & : & C & C & C & C & C & C & C & C & C & C & \ldots &
\end{array}
$$

- Case 3: If players I and II (S_{48}, S_{33}) start with C but player III (S_{49}) starts with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & C & C & D & D & D & D & D & D & D & D & \ldots & \\
S_{33} & : & C & D & D & C & D & D & C & D & D & C & \ldots & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\
S_{49} & : & D & D & D & C & C & D & C & C & D & C & \ldots &
\end{array}
$$

- Case 4: If player I (S_{48}) starts with C but players II and III (S_{33}, S_{49}) start with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & C & D & D & D & D & D & D & D & D & D & \ldots & \\
S_{33} & : & D & D & C & D & D & C & D & D & C & D & \ldots & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\
S_{49} & : & D & D & C & C & D & C & C & D & C & C & \ldots &
\end{array}
$$

- Case 5: If players II and III (S_{33}, S_{49}) start with C but player I (S_{48}) starts with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & D & D & D & D & D & D & D & D & D & D & \ldots & \\
S_{33} & : & C & D & D & C & D & D & C & D & D & C & \ldots & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\
S_{49} & : & C & C & D & C & C & D & C & C & D & C & \ldots &
\end{array}
$$

- Case 6: If player III (S_{49}) starts with C but players I and II (S_{48}, S_{33}) start with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & D & D & D & D & D & D & D & D & D & D & \ldots & \\
S_{33} & : & D & D & C & D & D & C & D & D & C & D & \ldots & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\
S_{49} & : & C & D & C & C & D & C & C & D & C & C & \ldots &
\end{array}
$$

- Case 7: If player II (S_{33}) starts with C but players I and III (S_{48}, S_{49}) start with D

$$
\begin{array}{lccccccccccccc}
S_{48} & : & D & D & D & D & D & D & D & D & D & D & \ldots & \\
S_{33} & : & C & D & C & D & D & C & D & D & C & D & \ldots & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\
S_{49} & : & D & D & C & C & D & C & C & D & C & C & \ldots &
\end{array}
$$

• Case 8: If the three players (S_{48}, S_{33}, S_{49}) start with D

$$\begin{array}{llllllllllllll} S_{48} & : & D & D & D & D & D & D & D & D & D & D & ... & \\ S_{33} & : & D & C & D & D & C & D & D & C & D & D & ... & \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} \\ S_{49} & : & D & C & C & D & C & C & D & C & C & D & ... & \end{array}$$

From the previous cases, we have three regimes $R_1 = \mathcal{R}$, $R_2 = \mathcal{K}$ and $R_3 = \frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3}$. When we do the perturbation, we get:

In regime R_1, if S_{48} and S_{49} play C instead of D then regime R_1 will transfer to R_3, and if S_{33} plays C instead of D then regime R_1 will transfer to R_2. In regime R_2, if S_{48} and S_{49} play D instead of C then regime R_2 will transfer to R_3, and if S_{33} plays C instead of D then regime R_2 will transfer to R_1. In regime R_3, if S_{48}, S_{33} and S_{49} play C instead of D in column 1, S_{33} and S_{49} play D instead of C in column 2, S_{33} plays C instead of D in column 3 and if S_{49} plays D instead of C in column 3 then in all these cases, regime R_3 will not change, but if S_{48} plays C instead of D in column 2 then regime R_3 will transfer to R_1 and also if S_{48} plays C instead of D in column 3 then regime R_3 will transfer to R_2. Therefore, the corresponding transition matrix becomes

$$\begin{array}{c c} & \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1/3 & 0 & 2/3 \\ 1/9 & 1/9 & 7/9 \end{bmatrix} \end{array}.$$

By calculate the left eigenvectors for the eigenvalue 1 according to equation (7), we get the following equations:

$$-v_1 + \frac{1}{3}v_2 + \frac{1}{9}v_3 = 0,$$

$$\frac{1}{3}v_1 - v_2 + \frac{1}{9}v_3 = 0,$$

$$\frac{2}{3}v_1 + \frac{2}{3}v_2 - \frac{2}{9}v_3 = 0.$$

By solving the linear system of the previous equations with the equation $v_1 + v_2 + v_3 = 1$, then we obtain the eigenvector V as

$$V = (v_1, v_2, v_3) = (\frac{1}{8}, \frac{1}{8}, \frac{3}{4}).$$

Now, we can get the payoff values by

$$\begin{aligned} E(S_{48}, S_{33}, S_{49}) &= v_1.R_1 + v_2.R_2 + v_3.R_3 \\ &= \frac{1}{8}\mathcal{R} + \frac{1}{8}\mathcal{K} + \frac{3}{4}\frac{\mathcal{P}+\mathcal{T}+\mathcal{L}}{3} = \frac{\mathcal{R}+\mathcal{K}+2\mathcal{T}+2\mathcal{L}+2\mathcal{P}}{8}. \end{aligned}$$

According to equation (8), the payoff vector Π=(π_1, π_2, π_3, π_4, π_5, π_6) is equal to $(\frac{1}{8}, \frac{1}{8}, 0, \frac{2}{8}, \frac{2}{8}, \frac{2}{8})$, but we can use

$$\begin{aligned} \Pi &= (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \\ &= (\frac{n_1}{N}, \frac{n_2}{N}, \frac{n_3}{N}, \frac{n_4}{N}, \frac{n_5}{N}, \frac{n_6}{N}) = \frac{1}{N}(n_1, n_2, n_3, n_4, n_5, n_6) \end{aligned}$$

where

$$N = \sum_{i=1}^{6} n_i.$$

Finally, we can express the payoff as $(n_1, n_2, n_3, n_4, n_5, n_6)$.

Using the same approach, we examined all possibililites for studying three strategies from the five WSLS strategies, which are 125 possibililites times 8 sequences. Therefore, we were able to obtain the results in Tables 2:6 which represent the payoff of player I against III when player II fixed as shown

Table 2: The payoff vectors for player I against player III when player II fixed with the strategy WSLS 1 (S_3)

Player II (S_3) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	$(1,0,0,0,0,1)$	$(1,1,0,0,1,1)$	$(3,3,2,0,5,3)$	$(3,3,2,0,5,3)$	$(0,3,2,0,2,3)$
WSLS 2 S_{35}	$(1,0,1,1,0,1)$	$(1,1,0,0,0,1)$	$(2,2,1,0,1,2)$	$(6,12,5,0,11,6)$	$(0,3,2,0,2,3)$
WSLS 3 S_{33}	$(3,0,3,3,4,3)$	$(1,1,0,0,1,1)$	$(1,1,0,0,0,1)$	$(1,1,0,0,2,1)$	$(0,1,0,0,0,1)$
WSLS 4 S_{49}	$(3,0,3,3,4,3)$	$(3,3,3,3,5,3)$	$(1,1,1,0,1,1)$	$(0,1,0,0,0,0)$	$(0,2,1,0,1,1)$
WSLS 5 S_{48}	$(0,0,0,3,4,3)$	$(0,0,0,3,4,3)$	$(0,0,0,1,0,1)$	$(0,1,0,1,2,1)$	$(0,0,0,0,1,1)$

Table 3: The payoff vectors for player I against player III when player II fixed with the strategy WSLS 2 (S_{35})

Player II (S_{35}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	$(1,1,0,0,1,1)$	$(1,0,0,1,0,1)$	$(2,0,1,2,1,2)$	$(6,6,5,6,11,6)$	$(0,3,2,0,2,3)$
WSLS 2 S_{35}	$(1,1,0,0,0,1)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,3,2,0,2,3)$
WSLS 3 S_{33}	$(1,1,0,0,1,1)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,3,0,0,0,3)$
WSLS 4 S_{49}	$(3,3,3,3,5,3)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(3,1,0,0,0,0)$	$(1,3,2,0,2,2)$
WSLS 5 S_{48}	$(0,0,0,3,4,3)$	$(1,0,0,3,4,3)$	$(1,0,0,3,0,3)$	$(1,1,0,2,4,2)$	$(0,0,0,0,1,1)$

Table 4: The payoff vectors for player I against player III when player II fixed with the strategy WSLS 3 (S_{33})

Player II (S_{33}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	$(3,3,2,0,5,3)$	$(2,0,1,2,1,2)$	$(1,0,0,1,0,1)$	$(1,0,1,1,1,1)$	$(0,1,0,0,0,1)$
WSLS 2 S_{35}	$(2,2,1,0,1,2)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,3,0,0,0,3)$
WSLS 3 S_{33}	$(1,1,0,0,0,1)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(1,3,0,0,0,3)$
WSLS 4 S_{49}	$(1,1,1,0,1,1)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(3,1,0,0,0,0)$	$(1,3,2,0,0,2)$
WSLS 5 S_{48}	$(0,0,0,1,0,1)$	$(1,0,0,3,0,3)$	$(1,0,0,3,0,3)$	$(1,1,0,2,2,2)$	$(0,0,0,0,1,1)$

Table 5: The payoff vectors for player I against player III when player II fixed with the strategy WSLS 4 (S_{49})

Player II (S_{49}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	$(3,3,2,0,5,3)$	$(6,6,5,6,11,6)$	$(1,0,1,1,1,1)$	$(0,0,0,1,0,0)$	$(0,1,1,1,1,1)$
WSLS 2 S_{35}	$(6,12,5,0,11,5)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(3,0,0,1,0,0)$	$(1,2,2,1,2,2)$
WSLS 3 S_{33}	$(1,1,0,0,2,1)$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$(3,0,0,1,0,0)$	$(1,2,0,1,2,2)$
WSLS 4 S_{49}	$(0,1,0,0,0,0)$	$(3,1,0,0,0,0)$	$(3,1,0,0,0,0)$	$(3,2,0,1,0,0)$	$(3,2,0,1,0,0)$
WSLS 5 S_{48}	$(0,1,0,1,2,1)$	$(1,1,0,2,4,2)$	$(1,1,0,2,2,2)$	$(3,2,0,7,0,0)$	$(3,5,0,4,9,9)$

Table 6: The payoff vectors for player I against player III when player II fixed with the strategy WSLS 5 (S_{48})

Player II (S_{48}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	$(0,3,2,0,2,3)$	$(0,3,2,0,2,3)$	$(0,1,0,0,0,1)$	$(0,1,1,1,1,1)$	$(0,0,1,0,0,1)$
WSLS 2 S_{35}	$(0,3,2,0,2,3)$	$(1,3,2,0,2,3)$	$(1,3,0,0,0,3)$	$(1,2,2,1,2,2)$	$(0,0,1,0,0,1)$
WSLS 3 S_{33}	$(0,1,0,0,0,1)$	$(1,3,0,0,0,3)$	$(1,3,0,0,0,3)$	$(1,2,0,1,2,2)$	$(0,1,0,0,0,1)$
WSLS 4 S_{49}	$(0,2,1,0,1,1)$	$(1,3,2,0,2,2)$	$(1,3,2,0,0,2)$	$(3,8,0,1,0,0)$	$(3,8,9,1,0,9)$
WSLS 5 S_{48}	$(0,0,0,0,1,1)$	$(0,0,0,0,1,1)$	$(0,0,0,0,1,1)$	$(3,5,0,4,9,9)$	$(0,0,0,0,0,1)$

4.1 The Domination Between WSLS Strategies

In this subsection, we will discuss the domination between the Win Stay-Lose Shift strategies for (3P-IPD). We note that, S_i outcompeted by S_j if both $a_{ji} > a_{ii}$ and $a_{jj} > a_{ij}$, where a_{ii}, a_{ij}, a_{ji} and a_{jj} are elements of payoff matrix. If the strategy S_i outcompeted by S_j, we can write $S_i << S_j$. Therefore, from Tables 2:6, we can get

Table 7: A list of strategies outcompeting S_i

	Player II: WSLS 1 S_3 fixed	Player II: WSLS 2 S_{35} fixed	Player II: WSLS 3 S_{33} fixed	Player II: WSLS 4 S_{49} fixed	Player II: WSLS 5 S_{48} fixed
WSLS 1 $S_3 <<$	S_{48}	S_{49}, S_{48}	S_{33}, S_{48}	S_{33}, S_{48}	S_{48}, S_{49}
WSLS 2 $S_{35} <<$	S_{33}, S_{48}	—	—	—	S_{48}, S_{49}
WSLS 3 $S_{33} <<$	S_3	—	—	—	—
WSLS 4 $S_{49} <<$	S_{33}, S_{48}	—	—	S_{35}, S_{33}	—
WSLS 5 $S_{48} <<$	S_{33}	—	—	S_{35}, S_{33}	—

4.2 The payoff Using Numerical Values

Now, we use some numerical values $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ then the expectation payoff for player I against player III is given as in the following Tables 8:12

Table 8: The payoff with $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ when Player II (S_3)

Player II (S_3) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	4	4	3.625	3.625	2.2
WSLS 2 S_{35}	4.25	3.666667	3.375	3.904762	2.2
WSLS 3 S_{33}	4.4375	4	3.666667	3.475	4.4375
WSLS 4 S_{49}	4.4375	4.25	3.2	3	2.4
WSLS 5 S_{48}	5	5	5	4.6	3

Table 9: The payoff with $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ when Player II (S_{35})

Player II (S_{35}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	4	5.666667	4.875	4.375	2.2
WSLS 2 S_{35}	3.666667	7	7	7	2.636364
WSLS 3 S_{33}	4	7	7	7	2.714286
WSLS 4 S_{49}	4.25	7	7	6	2.8
WSLS 5 S_{48}	5	5.181818	5	5	3

Table 10: The payoff with $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ when Player II (S_{33})

Player II (S_{33}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	3.625	4.875	5.666667	4.4	2
WSLS 2 S_{35}	3.375	7	7	7	2.714286
WSLS 3 S_{33}	3.666667	7	7	7	2.714286
WSLS 4 S_{49}	3.2	7	7	6	2.25
WSLS 5 S_{48}	5	5.285714	5.285714	5	3

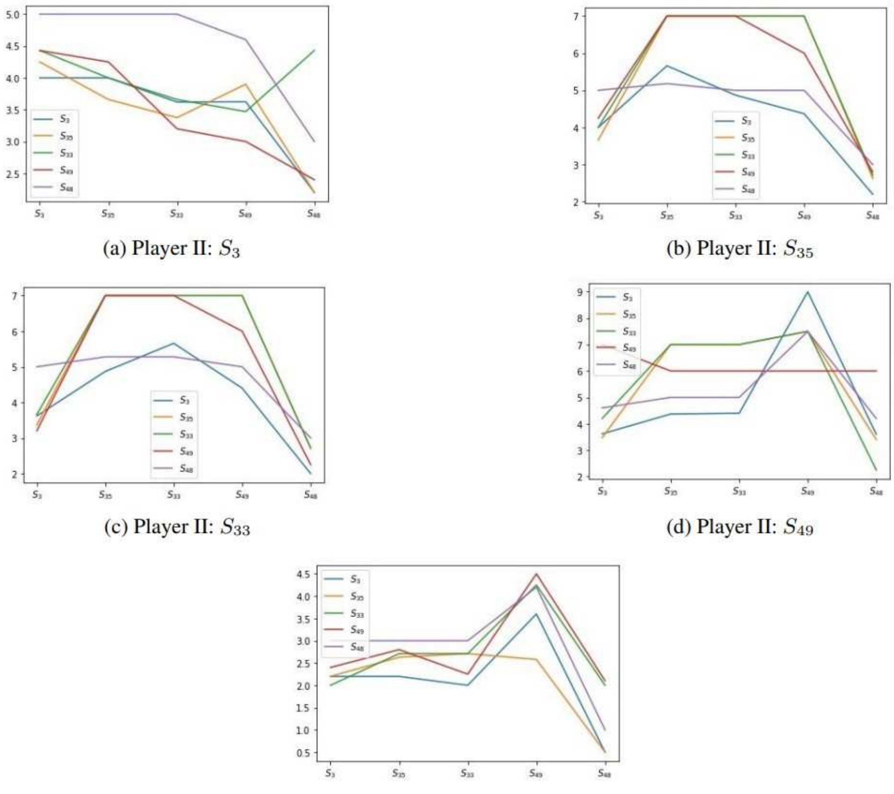
Table 11: The payoff with $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ when Player II (S_{49})

Player II (S_{49}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	3.625	4.375	4.4	9	3.6
WSLS 2 S_{35}	3.473	7	7	7.5	3.4
WSLS 3 S_{33}	4.2	7	7	7.5	2.25
WSLS 4 S_{49}	7	6	6	6	6
WSLS 5 S_{48}	4.6	5	5	7.5	4.2

Table 12: The payoff with $\mathcal{S} = 0, \mathcal{P} = 1, \mathcal{K} = 3, \mathcal{L} = 5, \mathcal{R} = 7$ and $\mathcal{T} = 9$ when Player II (S_{48})

Player II (S_{48}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	2.2	2.2	2	3.6	0.5
WSLS 2 S_{35}	2.2	2.636364	2.714286	2.588235	0.5
WSLS 3 S_{33}	2	2.714286	2.714286	4.25	2
WSLS 4 S_{49}	2.4	2.8	2.25	4.5	2.1
WSLS 5 S_{48}	3	3	3	4.2	1

Therefore, we show the best one among the WSLS strategies through some graphs as shown in Fig. 2



(a) Player II: S_3

(b) Player II: S_{35}

(c) Player II: S_{33}

(d) Player II: S_{49}

(e) Player II: S_{48}

Fig. 2: The payoffs of player I against other strategies

We note that, in 2(a), when S_3 is fixed, S_{48} is the best against S_3, S_{35}, S_{33} and S_{49}, but S_{33} is the best against S_{48}. Strategies S_{33} and S_{49} have the same payoff against S_3 also S_3 and S_{35} have the same payoff against S_{48} and also S_3, S_{33} have the same payoff against S_{35}. And in 2(b) and in 2(c), when S_{35} and S_{33} are fixed, S_{35}, S_{33} and S_{49} are the best strategies against S_{35} and S_{33}, also S_{35} and S_{33} are the best strategies against S_{49}, but S_{48} is the best with S_3 and S_{48}. The strategies S_3 and S_{33} have the same payoff with S_3. Also in 2(d), when S_{49} is fixed, S_3 is the best strategy against S_{49} and vice versa, also S_{35} and S_{33} are the best against themselves, but S_{49} is the best with S_3 and S_{48}. The strategies S_{35}, S_{33} and S_{48} have the same payoff with S_{49} . But in 2(e), when S_{48} is fixed, S_{49} is the best strategy against S_{49} and S_{48}, but S_{48} is the best against S_3, S_{35} and S_{33}. The strategies S_3 and S_{35} have the same payoff with S_3 and S_{48}. The strategies S_{33} and S_{48} have the same payoff with S_{49}.

5 Relatedness Between WSLS Strategies

A relationship is a numerical value denoting how much one player cares for another player's payoff. We will use the inclusive fitness method as in [10,22,23] for two-player. Since the payoff matrix of two-player Prisoner's Dilemma with relatedness is

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} \mathcal{R}(1+r) & \mathcal{S}+\mathcal{T}r \\ \mathcal{T}+\mathcal{S}r & \mathcal{P}(1+r) \end{bmatrix} \end{array}. \tag{10}$$

where

$$0 \leq r \leq 1 \tag{11}$$

In this section, we introduce the concepts of relationship in (3P-IPD). Therefore, the payoff matrix of three-player Prisoner's Dilemma with relatedness will be given by

$$\begin{array}{c c} & \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} \mathcal{R}(1+r) & \dfrac{2\mathcal{K}(1+r)+\mathcal{T}r}{2} & \mathcal{S}+\mathcal{T}r \\ \dfrac{2\mathcal{T}(1+r)+\mathcal{K}r}{2} & \mathcal{L}+\mathcal{S}r & \mathcal{P}(1+r) \end{bmatrix} \end{array}. \tag{12}$$

Now, by applying the effect of relatedness on Tables 2:6 with $\mathcal{S}=0, \mathcal{P}=1, \mathcal{K}=3, \mathcal{L}=5, \mathcal{R}=7$ and $\mathcal{T}=9$ through four cases of the value of r according to (12).

• Case 1: For $r=1$ "Perfect Relationship", we get the following Tables 13:17

Table 13: The payoff for player I against player III with Perfect Relationship when Player II (S_3)

Player II (S_3) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	8	9.125	7.15625	7.15625	5.75
WSLS 2 S_{35}	10.125	8.8333	7.75	8.05	5.75
WSLS 3 S_{33}	8.84375	7.875	8.8333	7.3	8.84375
WSLS 4 S_{49}	8.84375	8.15	7.3	10.5	6.6
WSLS 5 S_{48}	8.45	8.45	10.75	8.4	3.5

Table 14: The payoff for player I against player III with Perfect Relationship when Player II (S_{35})

Player II (S_{35}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	7.875	11.833333	10.125	8.9	5.75
WSLS 2 S_{35}	8.833333	14	14	14	6.5
WSLS 3 S_{33}	7.875	14	14	14	7.357143
WSLS 4 S_{49}	8.9	14	14	13.125	6.95
WSLS 5 S_{48}	8.45	8.954545	11.21429	8.75	3.5

Table 15: The payoff for player I against player III with Perfect Relationship when Player II (S_{33})

Player II (S_{33}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	7.15625	10.125	11.833333	9.1	6.25
WSLS 2 S_{35}	9.6875	14	14	14	7.357143
WSLS 3 S_{33}	8.833333	14	14	14	7.357143
WSLS 4 S_{49}	7.3	14	14	13.125	8.5
WSLS 5 S_{48}	10.75	9.516667	9.516667	9.6875	3.5

Table 16: The payoff for player I against player III with Perfect Relationship when Player II (S_{49})

Player II (S_{49}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	7.15625	8.9	9.1	19.5	8.4
WSLS 2 S_{35}	8.05	14	14	15.375	7.85
WSLS 3 S_{33}	7.3	14	14	15.375	8.5625
WSLS 4 S_{49}	10.5	13.125	13.125	13.75	13.75
WSLS 5 S_{48}	8.4	8.75	9.6875	16.625	7.85

Table 17: The payoff for player I against player III with Perfect Relationship when Player II (S_{48})

Player II (S_{48}) is fixed	WSLS 1 S_3	WSLS 2 S_{35}	WSLS 3 S_{33}	WSLS 4 S_{49}	WSLS 5 S_{48}
WSLS 1 S_3	5.75	5.75	6.25	8.4	3.5
WSLS 2 S_{35}	5.75	5.590909	7.357143	6.279412	3.5
WSLS 3 S_{33}	6.25	7.351743	7.357143	8.5625	6.25
WSLS 4 S_{49}	6.6	5.95	6.1875	12.125	6.95
WSLS 5 S_{48}	3.5	3.5	3.5	7.85	2

In this case and according to the Tables 13:17, we see that

1. When player II is WSLS 1 (S_3), there is equilibrium between the pair $< S_{33}, S_{49} >$.
2. When player II is WSLS 2 (S_{35}), there is equilibrium between the pairs $< S_3, S_{49} >$, $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
3. When player II is WSLS 3 (S_{33}), the equilibrium is between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
4. When player II is WSLS 4 (S_{49}), the equilibrium is only between the pairs $< S_3, S_{48} >$ and $< S_{35}, S_{33} >$.
5. When player II is WSLS 5 (S_{48}), the equilibrium is between the pairs $< S_3, S_{35} >$, $< S_3, S_{33} >$ and $< S_{35}, S_{33} >$.

Using the same approach, we can discuss the following cases:

• Case 2: For $r = 0.9$ “Strong Relationship”, we note that

1. There is no equilibrium between any pairs when player II WSLS 1 (S_3).
2. When player II WSLS 2 (S_{35}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
3. When player II WSLS 3 (S_{33}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
4. When player II WSLS 4 (S_{49}), the equilibrium is only between the pair $< S_{35}, S_{33} >$.
5. When player II WSLS 4 (S_{49}), the equilibrium is between the pairs $< S_3, S_{35} >$, $< S_3, S_{33} >$ and $< S_{35}, S_{33} >$.

• Case 3: For $r = 0.1$ “Weak Relationship”, we note that

1. There is no equilibrium between any pairs, when player II WSLS 1 (S_3).
2. When player II WSLS 2 (S_{35}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
3. When player II WSLS 3 (S_{33}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
4. When player II WSLS 4 (S_{49}), the equilibrium is only between the pair $< S_{35}, S_{33} >$.
5. When player II WSLS 5 (S_{48}), the equilibrium is between the pairs $< S_3, S_{35} >$, $< S_3, S_{33} >$ and $< S_{35}, S_{33} >$.

• Case 4: For $r = 0$ ”No Relationship”, we get the normal payoff given in tables 8:12 and we see that

1. There is no equilibrium between any pairs, when player II (S_3).
2. When player II WSLS 2 (S_{35}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
3.When player II WSLS 3 (S_{33}), there is equilibrium between the pairs $< S_{35}, S_{33} >$, $< S_{35}, S_{49} >$ and $< S_{33}, S_{49} >$.
4.When player II WSLS 4 (S_{49}), the equilibrium is only between the pair $< S_{35}, S_{33} >$.
5.When player II WSLS 5 (S_{48}), the equilibrium is between the pairs $< S_3, S_{35} >$, $< S_3, S_{33} >$ and $< S_{35}, S_{33} >$.

6 Conclusion

We conclude that WSLS 1 S_3 is the weakest strategy because it is attacked from other different strategies (at all possibilities of player II). Nevertheless, WSLS 1 (S_3) is stronger in only one case that when it plays against WSLS 3 (S_{33}) with player II WSLS 1 (S_3).

We can also note that, when the second player plays by WSLS 1 (S_3), there is a cycle $S_3 \to S_{33} \to S_{48} \to S_3$.

Whatever the strength of the relationship between players and whether the second player is WSLS $2 : 5$, there is a common equilibrium pair $< S_{35}, S_{33} >$.

Future Work

1. Using the same approach for different strategies and study the effect of their behavior during competition with other strategies.
2. Studying the effect of the length of the memory on competition between strategies.
3. Studying the possibility of applying the idea of this paper on the model of the alternating game (EG).
4. Discuss the dynamics for all equilibrium pairs that we have obtained.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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