Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/120618

# Turiyam Graphs and its Applications

*G. A. Ganati[1,*], V. N. S. Rao Repalle[1], M. A. Ashebo[1] and M. Amini[2]*

[1]Department of Mathematics, College of Natural and Computational Sciences, Wallaga University, Nekemte, Ethiopia
[2]Mathematical Sciences, Languages and General Studies Department, College of Arts and Science, Ahlia University, Bahrain

Received: 12 Nov. 2022, Revised: 31 Dec. 2022, Accepted: 31 Jan. 2023.
Published online: 1 Jun. 2023.

**Abstract:** The single valued neutrosophic set (SVNS) was developed to handle uncertainty in information depending on independent states called truth, indeterminacy and false. Recently, the Turiyam set was introduced for dealing with the uncertainty in data sets when those states are in silent mode based on human quantum cognition or awareness. In this way, this set gives a way to explore the uncertainty in data sets beyond the existing true, false, and indeterminacy regions. The precise analysis of data with the Turiyam set and its graphical representation is indeed a requirement for knowledge processing tasks. To achieve this goal, the current paper introduces Turiyam graphs with illustrations. In addition, we define a complete Turiyam graph, a strong Turiyam graph, and a constant Turiyam graph. Further, we apply a constant Turiyam graph to the Wi-Fi system.

**Keywords:** Single valued neutrosophic graph; Turiyam set; Uncertainty; Turiyam graphs; Four dimensions.

## 1 Introduction

Euler invented graph theory in 1736 as a recreational mathematics to solve a puzzle posed by the people of Konigsberg [1]. Since then, it has grown to solve relevant real-world problems in different fields like computer science, economics, sociology, chemistry, communications, etc. efficiently[2]. Also, fields in mathematics such as group theory, topology, matrix theory and operations research are helped by graph theory[3]. A graph consists of vertices linked by edges, where those vertices and edges represent the objects (systems, processes, etc.) and the relations between them, respectively [4]. Graph theory studies all those graphs in mathematics. The classical graph cannot represent a situation in which there is uncertainty regarding a vertex or edge or both [5]. To handle this situation, another aspect of graph theory called fuzzy graph (FG) [6] was introduced by applying fuzzy relations [7] to fuzzy set[8].This graph considers the membership value of both the vertex and the edge. The brief study on this graph is described in [9]. The intuitionistic fuzzy graph (IFG) [10] was developed based on intuitionistic fuzzy relations [11] to provide more flexibility to deal with uncertainty regarding any vertex or edge in the graph by considering membership and non-membership values. The comprehensive IFG study conducted in [$12-17$]. Although both the FG and IFG have been used to solve this problem, they fail when there is inconsistent and indeterminate uncertainty regarding a vertex or edge of the graph [18]. Therefore, another aspect of graph theory called single valued neutrosophic graph (SVNG) (or simply neutrosophic graph) was studied by many researchers [$18-20$] based on the neutrosophic set [21] as the generalization of IFG, FG and graph theory to efficiently handle this situation. In the context of SVNG, tree, planar and coloring are introduced and some of their properties are established[$22-25$]. The application areas of SVNG in real life situations like social networking, decision making, crime detection, Wi-Fi and so on are described precisely [26,27]. In SVNG, any vertex or edge is described by the truth value *(tv)*, indeterminacy value *(iv)* or falsity value *(fv)* [18].

There has been a problem when vertex sets and edge sets are beyond those three states, that is, truth value *(tv)*, indeterminacy value *(iv)* and falsity value *(fv)*. Consider the data on coronavirus (COVID-19) cases in three different regions of a given country [28]. In those regions, we have four types of patients. Those are recovered patients($tv$), active patients($iv$), death patients ($fv$), and vaccinated patients with a Turiyam or liberal dimension($lv$). In this case, the complement operator $1-(tv+iv+fv+lv)$ indicates the people who haven't been affected by COVID-19 in those regions. However, the graph visualization of this situation cannot be represented by SVNG since it is beyond truth value, indeterminacy value and falsity value. In 2021, P. K. Singh introduced Turiyam sets as the extension of neutrosophic sets at a research conference titled "Operations Research and Applications" held in Guilin, China [29]. The elements of this set are described by the truth value($tv$), the indeterminacy value($iv$), the falsity value($fv$), and the liberal (or Turiyam) value($lv$), all of which are in the range [0, 1] [28, 29]. The author also identified that this set is

*Corresponding author e-mail: gammeekoo@gmail.com

applicable in the voting system, sports data, medical diagnosis, identifying research paper quality and in the crime investigation system [29, 30]. However, the precise graphical representation of this situation is important for knowledge processing tasks [28, 30, 31]. In this regard, four-dimensional logic [32] and its algebra [33] are required. Also, the Turiyam context is introduced [34] for matrix representation [35] of the situation with the Turiyam set. At the same time, its other algebraic properties, like Turiyam rings, Turiyam spaces, Turiyam modules and Turiyam relations [36–39], are introduced as a generalization of the corresponding algebraic structures of neutrosophic sets [40 − 42] for dealing with these types of unknown non-Euclidean data sets [43]. In recent times, some of the authors have focused on the applications of the Turiyam set in self-driving intelligent vehicles in controlling accidents [44] as well as searching for a journal [28].

In this study, we develop the Turiyam graph as an extension of the SVNG and derive some of its properties. Then, by using this graph, we model Wi-Fi technology.

**Research Gap**: There is a research gap because no study on the Turiyam graph has been announced in the literature.

**Motivation**: To address the identified gap in the literature by describing the theory of the Turiyam graph.

The next parts of this document are arranged in the following order: in section 2, we give a survey of the concepts helpful in this study. In section 3, we present the Turiyam graph with some illustrations. In section 4, we apply the Turiyam graph to the Wi-Fi system. In section 5, we give the result and discussion. Finally, we give the conclusion and outline of the future work of the study in section 6.

## 2 Basics Concepts

In this section, we give a revision of some important existing concepts that are relevant to this work. Consider that $U$ is a nonempty universe set.

**Definition 1** [27-28]: The form of a Turiyam set $B$ on a set $U$ is

$$B = \{<x, \mathrm{t}_B(x), i_B(x), f_B(x), l_B(x)>: x \in U\}$$

where $\mathrm{t}_B(x), i_B(x), f_B(x), l_B(x): U \to [0,1]$ is the truth value (*tv*), the indeterminacy value (*iv*), the falsity value (*fv*) and the Turiyam state (or liberal) value (*lv*) respectively, for each $x \in U$. Each $\mathrm{t}_B(x)$, $i_B(x), f_A(x)$ and $l(x)$ are independent and satisfies the condition

$0 \le \mathrm{t}_B(x) + i_B(x) + f_B(x) + l_B(x) \le 4, \forall x \in U.$

**Remark**: $1 - (t + i + f + l)$ is called the refusal degree of Turiyam sets.

**Definition 2** [27]: Let A and B be Turiyam sets on U. Then, A is a Turiyam subset of B if $\mathrm{t}_A(x) \le \mathrm{t}_B(x), i_B(x) \le i_B(x), f_A(x) \ge f_B(x)$ and $l_A(x) \le l_B(x), \forall x \in U.$

**Definition 3** [27]: Let A and B be Turiyam sets on U.

(a) The complement of A , denoted by $A^c$, is given as $A^c = \{t^c, i^c, f^c, l^c\}$ where $t^c = f, i^c = 1 - i, f^c = t, l^c = 1 - (t + i + f)$

(b) The union of A and B, denoted by A$\cup B$, given by

$A \cup B = \{t_A \vee t_B, i_A \wedge i_B, f_A \wedge f_B, l_A \vee l_B\}$ where $(t_A \vee t_B)(x) = t_A(x) \vee t_B(x)$,

$$(i_A \wedge i_B)(x) = i_A(x) \wedge i_B(x), (l_A \vee l_B)(x) = l_A(x) \vee l_B(x), \forall x \in U.$$

(c) The intersection A and B, denoted by $A \cap B$, defined as

$A \cap B = \{t_A \wedge t_B, i_A \vee i_B, f_A \vee f, l_A \wedge l_B\}.$

**Definition 4** [39] Let $A$ and $B$ be nonempty Turiyam sets on U. Then a Turiyam relation from A to B is a Turiyam subset of A×B of the form R = $\{t_R, i_R, f_R, l_R\}$ where $t_R, i_R, f_R, l_R: A \times B \to [0,1]$ represents the truth function, indeterminacy function, falsity function and liberal function respectively.

**Definition 5** [26]: A SVNG on $U$ is a pair G = (N, R), where N is a SVNS in $U$ and $R$ is (SVN) relation on $U$ such that

$$t_R(ab) \le \min\{t_N(a), t_N(b)\}$$

$$i_R(ab) \le \min\{i_N(a), i_N(b)\}$$

$$f_R(ab) \le \max\{f_N(a), f_N(b)\}, \forall a, b \in U.$$

In this case, N is a SVNS of G and R is a SVN edge set of G.

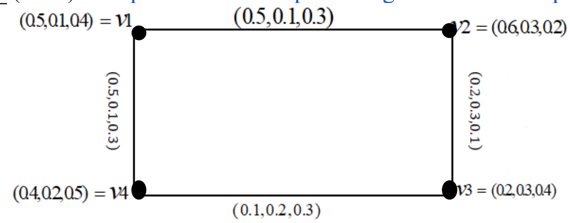


**Fig. 1:** The SVNG

**Definition 6** [27]: A SVNG is called a constant SVNG if all vertices of SVNG have the same degree $(k_1, k_2, k_3)$.

## 3 Turiyam Graphs

This section introduces the Turiyam graphs and some desired properties. Also, we clarify some concepts of Turiyam graphs with illustrative examples. Let U represent the universe set and $G = (V, E)$ the classical graph theory.

**Definition 1**: Consider a finite set of vertices and edges as $V = \{s_i : i = 1,2, \ldots, n\}$ and $E = \{(s_i, s_j) : i, j = 1,2, \ldots, n\}$ respectively. A Turiyam graph of a graph $G = (V, E)$ is defined by $T_G = (A, R)$, where

$(a) t_A, i_A, f_A, l_A : V \to [0,1]$ represents the truth value (*tv*) of $s_i$ , the indeterminacy value (*iv*) of $s_i$, the falsity value (*fv*) of $s_i$ and the turiyam state (or liberal) value (*lv*) of $s_i$ respectively, $\forall s_i \in V$ such that $0 \leq t_A(s_i) + i_A(s_i) + f_A(s_i) + l_A(s_i) \leq 4, \forall s_i \in V, i = 1,2, \ldots, n.$

$(b) t_R, i_R, f_R, l_R : E \subseteq V \times V \to [0,1]$ given by $t_R(a_i b_j) \leq \min\{t_A(a_i), t_A(b_j)\}, i_R(a_i b_j) \leq \min\{i_A(a_i), i_A(b_j)\}, f_R(a_i b_j) \leq \max\{f_A(a_i), f_A(b_j)\}$,and $l_R(a_i b_j) \leq \min\{t_A(a_i), t_A(b_j)\}, \forall a_i, b_j \in V$ represents the truth, indeterminacy, falsity and turiyam mappings from $E$ to $V \times V$ respectively, such that $0 \leq t_R(\{a_i b_j\}) + i_R(\{a_i b_j\}) + f_R(\{a_i b_j\}) + l_R(\{a_i b_j\}) \leq 4, \forall \{a_i b_j\} \in E, i, j = 1,2, \ldots, n.$

In this case, $A$ is the Turiyam vertex set of $T_G$ and $R$ is the Turiyam edge set of $T_G$.

Remark: The set $A$ and $R$ are Turiyam set over $V$ and $E$ respectively.

**Example 1**: Consider a Turiyam graph $T_G = (A, R)$ of $G = (V, E)$ where $V = \{v_1, v_2, v_3\}$ and $E = \{v_1 v_2, v_2 v_3, v_3 v_1\}$. Let $A$ and $R$ be Turiyam sets on $V$ and $E$ respectively as shown in table 1.

**Table 1:** The tabulated form of A and R

| A | $v_1$ | $v_2$ | $v_3$ | R | $v_1 v_2$ | $v_1 v_3$ | $v_2 v_3$ |
|---|---|---|---|---|---|---|---|
| $t_A$ | 0.4 | 0.1 | 0.6 | $t_R$ | 0.1 | 0.2 | 0.1 |
| $i_A$ | 0.6 | 0.4 | 0.8 | $i_R$ | 0.2 | 0.5 | 0.4 |
| $f_A$ | 0.1 | 0.8 | 0.1 | $f_R$ | 0.7 | 0.1 | 0.8 |
| $l_A$ | 0.7 | 0.3 | 0.7 | $l_R$ | 0.2 | 0.4 | 0.2 |

Then, this Turiyam graph represented in figure 2 below

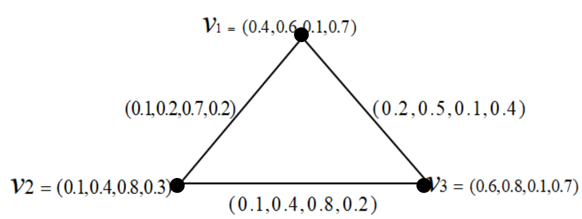


**Fig. 2:** Turiyam graph

In figure 2, $(i) v_1 = (0.4,0.6,0.1,0.7)$is a Turiyam vertex.

$(ii)(v_1 v_2) = (0.1,0.2,0.7,0.2)$ is a Turiyam edge.

**Definition 2**: Let $T_G = (A, R)$ be a Turiyam graph. The vertices $v_i$and $v_j$ are adjacent vertices if and only if $t_R(v_i v_j) = \min\{t_A(v_i), t_A(v_j)\}, i_R(v_i v_j) = \min\{i_A(v_i), i_A(v_j)\}, f_R(v_i v_j) = \max\{f_A(v_i), f_A(v_j)\}$ and $l_R(v_i v_j) =$

$\min\{t_A(v_i), t_A(v_j)\}, \forall v_i, v_j \in V$. In this case, $v_i$ and $v_j$ are called neighbor vertices and the edge $e = (v_i v_j)$ is incident at $v_i$ and $v_j$.

**Example 2**: Consider a Turiyam graph $T_G = (A, R)$ of $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_2v_3, v_3v_4\}$. Let $A$ and $R$ be Turiyam sets on $V$ and $E$ respectively as shown in table 2.

**Table 2:** The tabulated form of A and R

| A | $v_1$ | $v_2$ | $v_3$ | $v_4$ | R | $v_1v_2$ | $v_1v_4$ | $v_2v_3$ | $v_3v_4$ |
|---|---|---|---|---|---|---|---|---|---|
| *t* | 0.3 | 0.2 | 0.6 | 0.7 | *t* | 0.2 | 0.2 | 0.2 | 0.6 |
| *i* | 0.5 | 0.5 | 0.1 | 0.3 | *i* | 0.5 | 0.1 | 0.1 | 0.1 |
| *f* | 0.1 | 0.8 | 0.8 | 0.2 | *f* | 0.8 | 0.6 | 0.6 | 0.8 |
| *l* | 0.2 | 0.4 | 0.3 | 0.5 | *l* | 0.2 | 0.4 | 0.4 | 0.3 |

Then, this Turiyam graph represented in figure 3 below

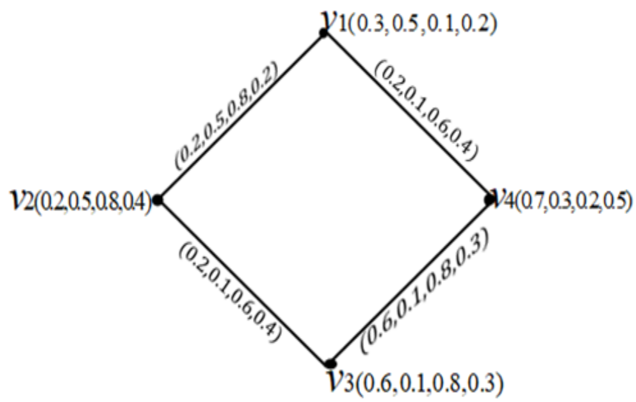


**Fig. 3:** Turiyam graph

In above graph, $(i)$ the vertices $v_1$and $v_3$ are adjacent vertices.

$(iii)$ the vertices $v_2$and $v_3$ are not adjacent vertices.$(iv)$ the edge $e = (v_1v_2)$ incident at $v_1$and $v_2$.

**Example 3**: Social network considered as Turiyam graph in a sense that connected people represent true (*tv*), partially connected or removed after some time represent uncertain (*iv*), not connected represent false (*fv*), and friend beyond three conditions represent liberal (*lv*).

Note: If at least one value of $t_R(v_iv_j), i_R(v_iv_j), f_R(v_iv_j)$ and $l_R(v_iv_j)$ is non-zero for some i and j, then there is an edge between $v_i$and $v_j$. Otherwise, there is no edge between $v_i$and $v_j$.

**Remark**: A Turiyam graph $T_G = (A, R)$ is empty if $t_R(v_iv_j) = i_R(v_iv_j) = f_R(v_iv_j) = l_R(v_iv_j) = 0, \forall (v_iv_j) \in E$.

**Definition 3**: A Turiyam graph $T_H = (A', R')$ is a Turiyam subgraph of Turiyam graph $T_G = (A, R)$ if $T_H$ is also a Turiyam graph such that $A' \subseteq A$ and $R' \subseteq R$ . That is,

$$(i)\ t_{A'}(u) \le t_A(u), i_{A'}(u) \le i_A(u), f_{A'}(u) \ge f_A(u), l_{A'}(u) \le l_A(u) \forall u \in V$$

$$(ii)\ t_{R'}(uv) \le t_R(uv), i_{R'}(uv) \le i_R(uv), f_{R'}(uv) \ge f_R(uv), l_{R'}(uv) \le l_R(uv), \forall uv \in E.$$

**Example 4**: Consider the Turiyam graph

$T' = (A', R')$ where $A' = \{v_1, v_2, v_3\}, R' = \{v_1v_2, v_1v_3, v_2v_3\}$shown in table 3

**Table 3:** The tabulated form of $A'$ and $R'$

| $A'$ | $v_1$ | $v_2$ | $v_3$ | $R'$ | $v_1v_2$ | $v_1v_3$ | $v_2v_3$ |
|---|---|---|---|---|---|---|---|
| *t* | 0.3 | 0.1 | 0.4 | *t* | 0.1 | 0.2 | 0.0 |
| *i* | 0.5 | 0.3 | 0.6 | *i* | 0.2 | 0.4 | 0.4 |
| *f* | 0.3 | 0.9 | 0.2 | *f* | 0.8 | 0.3 | 0.8 |
| *l* | 0.6 | 0.2 | 0.7 | *l* | 0.2 | 0.3 | 0.1 |

This Turiyam graph is a Turiyam subgraph of a Turiyam graph of figure 2 and its representation is given in figure 4

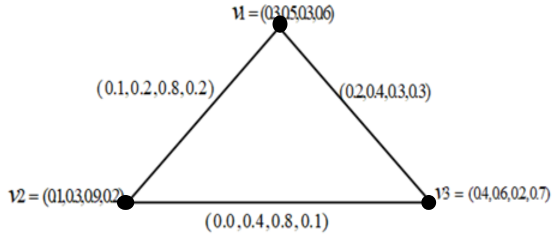


**Fig. 4:** Turiyam subgraph

**Definition 4:** Let $T_G = (A, R)$ be a Turiyam graph. Then, a path $p(v_0 v_n)$ in $T_G$ is a sequence of distinct vertices $v_0 v_1 v_2, \dots, v_n$ such that $t_R(v_{i-1} v_i) > 0, i_R(v_{i-1} v_i) > 0, f_R(v_{i-1} v_i) > 0$ and $l_R(v_{i-1} v_i) > 0$ for $0 \le i \le n$. In this case, $n \ge 0$ is called the length of path $p$. The successive pairs $(v_{i-1} v_i), 0 \le i \le n$, are edges of the path $p$. A single vertex $v_i$ also can be considered as path with length (0, 0, 0, 0). If $v_0 = v_n$, the path $p$ is a cycle where $n \ge 3$.

**Example 5**: In figure 4, a sequence of vertices $v_1 v_2 v_3$ is a path with length 2 in the given Turiyam graph.

**Definition 5:** A Turiyam graph $T_G = (A, R)$ is connected if each pair of vertices has at least one Turiyam path between them. If not, it is disconnected Turiyam graph.

**Example 6:** The Turiyam graph in figure 4 is connected while the Turiyam graph in figure 5 is not connected.

**Definition 6:** If there is no edge incident at vertex $v_i$ in the Turiyam graph$T_G = (A, R)$, then the vertex $v_i$ is an isolated vertex.

**Example 7**: Consider a Turiyam graph

$T_G = (A, R)$ of $G = (V, E)$ where

$V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1 v_2, v_1 v_3, v_2 v_3\}$. Let $A$ and $R$ be a Turiyam sets on $V$ and $E$ respectively as shown in table 4

**Table 4:** The tabulated form of A and R

| A | $v_1$ | $v_2$ | $v_3$ | $v_4$ | R | $v_1 v_2$ | $v_1 v_3$ | $v_2 v_3$ |
|---|---|---|---|---|---|---|---|---|
| *t* | 0.4 | 0.1 | 0.6 | 0.8 | *t* | 0.1 | 0.2 | 0.1 |
| *i* | 0.6 | 0.4 | 0.8 | 0.3 | *i* | 0.2 | 0.5 | 0.4 |
| *f* | 0.1 | 0.8 | 0.1 | 0.4 | *f* | 0.7 | 0.1 | 0.8 |
| *l* | 0.7 | 0.3 | 0.7 | 0.1 | *l* | 0.2 | 0.4 | 0.2 |

Then, this Turiyam graph is given in figure 5

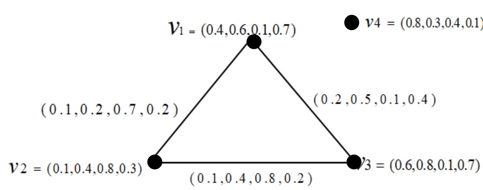


**Fig. 5:** Turiyam graph with isolated vertex

In figure 5, the Turiyam graph vertex $v_4$ is an isolated vertex.

**Definition 7**: A vertex in the Turiyam graph $T_G = (A, R)$ is referred to be a pendent vertex if it has precisely one neighbor. If not, then it is a non-pendent vertex.

**Remark:** (i) A pendent edge is an edge which has a pendent vertex in a Turiyam graph. If not, then it is said to be non-pendant edge.

(ii) A support of the pendent edge is a vertex that is connected to a pendent vertex.

**Example 8**: Given a Turiyam graph

$T_G = (A, R)$ of $G = (V, E)$ such that

$V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_3, v_2v_3, v_3v_4\}$. Let $A$ and $R$ be Turiyam sets on $V$ and $E$ respectively as shown in table 5

**Table 5:** The tabulated form of A and R

| $A$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $R$ | $v_1v_2$ | $v_1v_3$ | $v_2v_3$ | $v_3v_4$ |
|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0.4 | 0.1 | 0.6 | 0.8 | $t$ | 0.1 | 0.2 | 0.1 | 0.5 |
| $i$ | 0.6 | 0.4 | 0.8 | 0.3 | $i$ | 0.2 | 0.5 | 0.4 | 0.2 |
| $f$ | 0.1 | 0.8 | 0.1 | 0.4 | $f$ | 0.7 | 0.1 | 0.8 | 0.4 |
| $l$ | 0.7 | 0.3 | 0.7 | 0.1 | $l$ | 0.2 | 0.4 | 0.2 | 0.1 |

Then, this graph given in figure 6

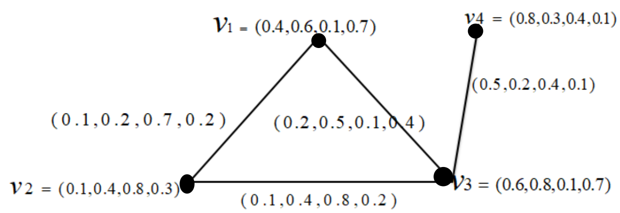


**Fig. 6:** Turiyam graph with a pendent vertex

Consider the above Turiyam graph. Then,

i) A vertex $v_4$is a pendent vertex, whereas the vertices $v_1, v_2$and$v_3$ are non-pendant vertices.

ii) While the edges $(v_2v_3)$ and $(v_1v_3)$ are non-pendent edges, the edge $(v_3v_4)$ is a pendent edge.

iii) The vertex $v_3$ is the pendent edge's support pendent edge $v_3v_4$, whereas $v_4$ is not pendent edge's support.

**Definition 8:** If $G = (V, E)$ is a bipartite graph, then its Turiyam graph $T_G = (A, R)$ is called a bipartite Turiyam graph.

**Definition 9**: A Turiyam graph $T_G = (A, R)$ that has neither self-loops nor parallel edge is called a simple Turiyam graph.

**Example 9:** The Turiyam graph of figure 6 is a simple Turiyam graph.

**Definition 10**: Let $T_G = (A, R)$ be a Turiyam graph and let $u$ be any vertex of $T_G$. Then, the degree of $u$ is the sum of degree of $tv$, sum of degree of $iv$, sum of degree of $fv$ and sum of degree of $lv$ of all those edges which are incident on vertex $u$ and denoted by $d(u) = (\sum_{v\neq u} t_R(v,u), \sum_{v\neq u} i_R(v,u), \sum_{v\neq u} f_R(v,u), \sum_{v\neq u} l_R(v,u))$

where $\sum_{v\neq u} t_R(v,u), \sum_{v\neq u} i_R(v,u), \sum_{v\neq u} f_R(v,u), \sum_{v\neq u} l_R(v,u)$ denotes the sum of degree of $tv$, sum of degree of $iv$, sum of degree of $fv$ and sum of degree of $lv$ of all the arcs that are adjacent to $u$ respectively.

**Example 10**: Consider the above graph. Then, $d(v_1) = (0.3,0.7, 0.8, 0.6)$and $d(v_3) = (0.8,1.1,1.3,0.7)$.

**Definition 12**: A Turiyam graph $T_G = (A, R)$ is said to be constant if $d(v) = k = (k_1, k_2, k_3, k_4), \forall v \in V$.

**Example 11:** Consider a Turiyam graph

$T_G = (A, R)$ of $G = (V, E)$ where

$V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_2v_3, v_3v_4\}$. Let $A$ and $R$ be Turiyam sets on $V$ and $E$ respectively as shown in table 6

**Table 6:** The tabulated form of A and R

| $A$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $R$ | $v_1v_2$ | $v_1v_4$ | $v_2v_3$ | $v_3v_4$ |
|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0.3 | 0.4 | 0.5 | 0.7 | $t$ | 0.2 | 0.3 | 0.3 | 0.2 |
| $i$ | 0.5 | 0.8 | 0.4 | 0.6 | $i$ | 0.4 | 0.2 | 0.2 | 0.4 |
| $f$ | 0.7 | 0.1 | 0.9 | 0.2 | $f$ | 0.6 | 0.6 | 0.6 | 0.6 |
| $l$ | 0.9 | 1 | 0.8 | 0.9 | $l$ | 0.8 | 0.5 | 0.5 | 0.8 |

The graph of this example given in figure 7

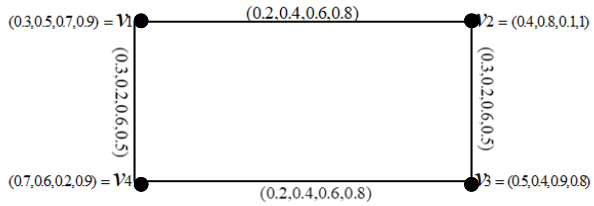


**Fig. 7:** Constant Turiyam graph

In this graph, $d(v_1) = d(v_2) = d(v_3) = d(v_4) = (0.5, 0.6, 1.2, 1.3)$. Then, the given graph is a constant Turiyam graph.

**Definition 13**: Let $T_G$ be Turiyam graph.

(a) the order of $T_G$, $O(T_G)$ , is the number of vertices of $T_G$.

(b) the size of $T_G$, $S(T_G)$, is the number of edges in$T_G$.

**Example 12:** Consider the Turiyam graph $T_G$ of above example. Then, $O(T_G) = (1.9,2.3,1.9,3.6)$ and $S(T_G) = (1,0.8,1.8,1.8)$.

**Definition 14**: A Turiyam graph $T_G = (A, R)$ of $G = (V, E)$ is a strong Turiyam graph if

$$t_R(a_i b_j) = \min\{t_A(a_i), t_A(b_j)\}$$
$$i_R(a_i b_j) = \min\{i_A(a_i), i_A(b_j)\}$$
$$f_R(a_i b_j) = \max\{f_A(a_i), f_A(b_j)\}$$
$$l_R(a_i b_j) = \min\{t_A(a_i), t_A(b_j)\}, \forall (a_i b_j) \in R$$

**Example 13**: Consider a Turiyam graph $T_G = (A, R)$ of $G = (V, E)$ where $V = \{v_1, v_2, v_3\}$ and $E = \{v_1 v_2, v_2 v_3, v_3 v_1\}$. Let $A$ and $R$ be Turiyam sets on $V$and $E$ respectively as shown in table 7.

**Table 7:** The tabulated form of A and R

| $A$ | $v_1$ | $v_2$ | $v_3$ | $R$ | $v_1 v_2$ | $v_2 v_3$ | $v_3 v_1$ |
|---|---|---|---|---|---|---|---|
| $t$ | 0.4 | 0.1 | 0.6 | $t$ | 0.1 | 0.1 | 0.4 |
| $i$ | 0.6 | 0.4 | 0.8 | $i$ | 0.4 | 0.4 | 0.8 |
| $f$ | 0.1 | 0.8 | 0.1 | $f$ | 0.8 | 0.8 | 0.1 |
| $l$ | 0.7 | 0.3 | 0.7 | $l$ | 0.3 | 0.3 | 0.7 |

The graphical representation of this strong Turiyam graph shown in figure 8 below

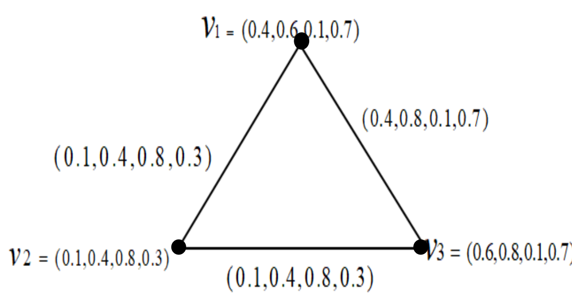


**Fig. 8:** Strong Turiyam graph

**Theorem 1**: A Turiyam graph is the extension of a neutrosophic graph.

**Proof**: Let $T_G = (A, R)$ be a Turiyam graph. Assume that the liberal value for both vertex and edge set of $T_G$ is zero. Thus, the Turiyam graph reduced to neutrosophic graph. Hence, the proof.

**Definition 15**: The complement of Turiyam graph $T_G = (A, R)$ on $G$ is a Turiyam graph

$\overline{T_G} = (\bar{A}, \bar{R})$ on $G$ where $(i)\bar{A} = A$

$$(ii)\bar{t_A}(v) = t_A(v), \bar{\iota_A}(v) = i_A(v), \bar{f_A}(v) = f_A(v), \bar{l_A}(v) = l_A(v)\ \forall v \in V.$$

$$(iii)\bar{t_R}(uv) = \min[t_A(u), t_A(v)] - t_R(uv)$$

$$\bar{\iota_R}(uv) = \min[i_A(u), i_A(v)] - i_R(uv)$$

$$\bar{f_R}(uv) = \max[f_A(u), f_A(v)] - f_R(uv) \text{ and}$$

$$\bar{l_R}(uv) = \min[l_A(u), l_A(v)] - l_R(uv), \forall (uv) \in E.$$

**Example 14**: The complement of the Turiyam graph of example 10 is as follows


**Fig. 9:** Complement of Turiyam graph

**Definition 16**: A Turiyam graph $T_G = (A, R)$ of $G = (V, E)$ is said to be a complete Turiyam graph if

$$t_R(a_i b_j) = \min\{t_A(a_i), t_A(b_j)\}$$

$$i_R(a_i b_j) = \min\{i_A(a_i), i_A(b_j)\}$$

$$f_R(a_i b_j) = \max\{f_A(a_i), f_A(b_j)\}$$

$$l_R(a_i b_j) = \min\{t_A(a_i), t_A(b_j)\}, \forall a_i, b_j \in V.$$

**Example 15**: Consider a Turiyam graph

$T_G = (A, R)$ of $G = (V, E)$ where

$V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_1v_3, v_2v_3, v_2v_4, v_3v_4\}$. Let $A$ and $R$ be Turiyam sets on $V$ and $E$ as shown in table 8

**Table 8:** The tabulated form of A and R

| $A$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $R$ | $v_1v_2$ | $v_1v_3$ | $v_1v_4$ | $v_2v_3$ | $v_2v_4$ | $v_3v_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0.4 | 0.4 | 0.8 | 0.6 | $t$ | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.6 |
| $i$ | 0.8 | 0.5 | 0.2 | 0.7 | $i$ | 0.5 | 0.2 | 0.7 | 0.2 | 0.5 | 0.2 |
| $f$ | 0.2 | 0.7 | 0.3 | 0.3 | $f$ | 0.7 | 0.3 | 0.3 | 0.7 | 0.7 | 0.3 |
| $l$ | 0.1 | 0.3 | 0.4 | 0.8 | $l$ | 0.1 | 0.1 | 0.1 | 0.3 | 0.3 | 0.4 |

The graph of this example given in figure 10 below

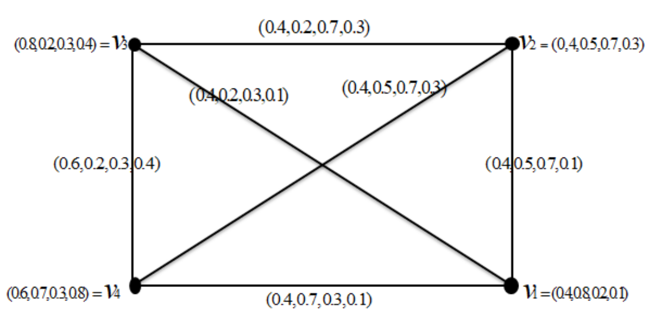


**Fig. 10:** Complete Turiyam graph

The complement of every Turiyam graph is not Turiyam graph. But we have the following theorem.

**Theorem 2**: The complement of complete Turiyam graph is a Turiyam graph.

**Proof**: Let $T_G = (A, R)$be complete Turiyam graph. That is, $t_R(uv) = \min\{t_A(u), t_A(v)\}$, $i_R(uv) = \min\{i_A(u), i_A(v)\}$,

$f_R(uv) = \max\{f_A(u), f_A(v)\}, l_R(uv) = \min\{t_A(u), t_A(v)\}, \forall u, v \in V$. Then in $\overline{T_G}$ ,

$\overline{t_R}(uv) = \min[t_A(u), t_A(v)] - t_R(uv) = \min[t_A(u), t_A(v)] - \min[t_A(u), t_A(v)] = 0.$ Similarly, $\overline{i_R}(uv)$= $\overline{f_R}(uv)$=$\overline{l_R}(uv) = 0, \forall (uv) \in E$. Thus, $(t_R(uv), i_R(uv), f_R(uv), l_R(uv)) = (0,0,0,0)$. Then, $\overline{T_G}$ is a Turiyam graph in which its edge set is null. Hence, the proof.

## 4 Application of Turiyam graph

In this section, we applied a constant Turiyam graph (CTG) to model the technology of Wi-Fi. The Wi-Fi system is a better way of sharing the internet with multiple devices among the customers at a given place. There is a problem the professionals face in making this technology attractive and suitable for users. One of this is to arrange for the access of this Wi-Fi internet between two or more Wi-Fi systems. Even though researchers conducted some research like [27] to overcome this situation, still there is a gap.

Recently, some of the authors paid attention to applications of the Turiyam set for self-driving intelligent cars [43] or searching for good journals [28]. The problem arises when the Wi-Fi system is beyond connectivity($tv$), uncertain ($iv$) and dis-connectivity($fv$). To handle this situation, we extend [27] to CTG and the novelty of using CTG to model such a system is clarified.

Wi-Fi technology is precisely described in [27]. In [27], the authors modeled the Wi-Fi system by using a constant single valued neutrosophic graph (CSVNG). This CSVNG only helps us to model three conditions i.e., connectivity, uncertainty and dis-connectivity of a Wi-Fi system [27]. On the contrary, this system cannot be modeled with CSVNG if the situation is beyond three conditions due to non-supporting software facilities, non-supporting towers like Jungles or sea areas that need exploration. This fourth dimension is named Turiyam or liberal in CTG. Thus, CTG is useful to describe this Wi-Fi system precisely since it has four components. The first component shows connectivity with Wi-Fi, the second one shows uncertainty about the connectivity of the Wi-Fi due to some issues, the third component shows the Wi-Fi is not connected and the fourth component shows the Wi-Fi unknown i.e. beyond connectivity, uncertainty and dis-connectivity.

An outside home Wi-Fi management contains four nodes, and those nodes denotes the Wi-Fi tools such that there exists a block between any two routers. Both routers yield signals to the block between them as shown by the following figure 11. This signal is expressed in terms of CTG. Based on the definition of CTG, we use the degree of each vertex. This degree shows that each router is giving identical signal; hence the degree of each router is the same. Also, this means that each router gives identical to the block. Therefore, the idea of CTG plays its role in operating this system effectively.

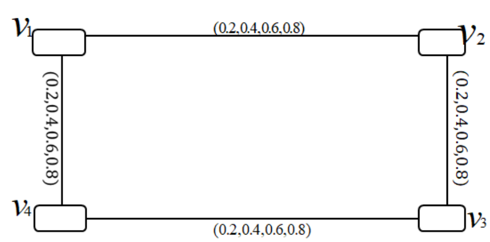


**Fig. 11:** Turiyam graph

From this fig. 11, we have four different routers, and the edge that indicates the signal strength of routers between two routers. In this case, edge and node are given in terms of a Turiyam numbers such that the first number denotes connectivity, the second number shows uncertainty of connectivity, the third number denotes dis-connectivity, and the fourth number shows the unknown Wi-Fi. By the definition of CTG, the degree of every node is the same. This shows that the routers have been giving identical signals. Thus, the concept of CTG has its role in the practical operations of this Wi-Fi. The degree of $v_1, v_2, v_3, and\ v_4$ given in the following table.

**Table 9:** vertex and its degree

| Vertex | Degree |
|---|---|
| $v_1$ | (0.4,0.8,1.2,1.6) |
| $v_2$ | (0.4,0.8,1.2,1.6) |
| $v_3$ | (0.4,0.8,1.2,1.6) |
| $v_4$ | (0.4,0.8,1.2,1.6) |

The benefits of Turiyam graphs rather than the concept of SVNGs are due to the liberal value. If this value is not considered, it is impossible to handle the conditions of this Wi-Fi system.

## 5 Results and Discussion

Graph theory has several versions for modeling uncertainty in a situation. In this study, a new version of graph theory based on the Turiyam sets was developed. This new graph is called the Turiyam graph, and it helps us model the uncertainty that is beyond membership, indeterminate, and non-membership in a given situation. In this paper, we applied this graph to the Wi-Fi technology arrangement of the smart city and its novelty has been precisely described.

## 6 Conclusions

In this study, the Turiyam graphs are introduced to represent data based on human cognition and their properties are derived. Moreover, we explained the degree, order and size of a Turiyam graph and identified their properties. Some types Turiyam graph like complete Turiyam graphs, constant Turiyam graphs and strong Turiyam graphs are discussed. Further, some examples included to make those definitions and results more understandable. It is discussed that the CTG can be useful to describe Wi-Fi system precisely than CSVNG when Wi-Fi situation unknown. It is expected that the concept discussed in this paper will open a new aspect of graph theory. In the near future, we extend the concept of Turiyam graph to regular and irregular Turiyam graph, bipolar Turiyam graph, interval valued Turiyam graph, Turiyam trees, Turiyam planar graphs and Turiyam coloring. The Turiyam graph will be applied in computer processing, data science, machine learning, telecommunication and control theory.

**Funding**: This research conducted without funding.

**Conflicts of Interest:** There is no conflict of interest between authors.

**Data Availability:** All data included in the manuscript.

## References

[1] L. R. Foulds, *Graph theory applications*, Springer Science & Business Media, New York USA, 3, (2012).

[2] J. L. Gross, J. Yellen, and M. Anderson, *Graph theory and its applications*, CRC Press, Boca Raton, Florida, United States, $xi$, (2018).

[3] Frank Harary, *Graph theory*, Addison-Wesley Publishing company, United States, 4-7, (1969).

[4] A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications, Academic Press, New York, 77- 95, (1975).

[5] S. Das, R.Das, & S.Pramanik, Single Valued Pentapartitioned Neutrosophic Graphs, *Neutrosophic Sets and Systems*, ***50***, 225-238(2022).

[6] L.A. Zadeh, Similarity relations and fuzzy orderings, Information Sciences, 3, 17-200(1971).

[7] L.A. Zadeh, Fuzzy sets, *Information and Control,* ***8***, 338-353(1965).

[8] S. Mathew, J. N. Mordeson & D. S. Malik, *Fuzzy graph theory*, Springer International Publishing, Berlin Germany, 1-2, (2018).

[9] A. Shannon & K. Atanassov. A first step to a theory of the intuitionistic fuzzy graphs, *Proc. of the First Workshop on Fuzzy Based Expert Systems,* D. Lakov, Ed., Sofia, Sep 28, 59-61(1994).

[10] Z. Shao, S. Kosari, H. Rashmanlou, & M. Shoaib, New concepts in intuitionistic fuzzy graph with application in water supplier systems, *Mathematics*, ***8***, 1241(2020).

[11] A. Shannon & K. Atanassov, Intuitionistic Fuzzy Graphs from a, ß and (a; ß)- levels, *Notes on Intuitionistic Fuzzy Sets,* ***1***, 32-35(1995).

[12] R. Parvathi & M. G. Karunambigai, Intuitionistic fuzzy graphs, In *Computational intelligence, theory and applications,* Springer, Berlin, Heidelberg, 139-150(2006).

[13] A. Nagoor Gani, & S. Shajitha Begum, Degree, order and size in intuitionistic fuzzy graphs, International Journal of Algorithms, Computing and Mathematics, **3**, 11-16 (2010).

[14] M. Akram & B. Davvaz, Strong intuitionistic fuzzy graphs. Filomat, **26**, 177-196(2012).

[15] S. Sahoo & M. Pal, Different types of products on intuitionistic fuzzy graphs, Pacific Science Review A: Natural Science and Engineering, **17**, 87-96(2015).

[16] S.K. Dhavudh & R. Srinivasan, Intuitionistic fuzzy graphs of second type, Advances in Fuzzy Mathematics, **12**,

197-204(2017).

[17] M. Akram & R. Akmal, Intuitionistic fuzzy graph structures, Kragujevac Journal of Mathematics, 41, 219-237(2017).

[18] S. Broumi, F.Smarandache, M. Talea & A.Bakali, Single valued neutrosophic graphs: degree, order and size, *IEEE International Conference on Fuzzy Systems (FUZZ)*, 2444-2451(2016).

[19] S. Broumi, M.Talea, A. Bakali & F.Smarandache, Single valued neutrosophic graphs. *Journal of New Theory, **10***, 86-101(2016).

[20] S. Broumi, M. Talea, A. Bakali& F. Smarandache, Single valued neutrosophic graphs, *New trends in neutrosophic theory and applications,* Brussels: Pons Editions, 187-202(2016).

[21] F. Smarandache, *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*, Rehoboth: American Research Press, (1998).

[22] A. Hassan & M. A. Malik, Single valued neutrosophic trees, *TWMS Journal of Applied and Engineering Mathematics*, ***8***, 255-266(2018).

[23] M. Akram, Single-valued neutrosophic planar graphs, International Journal of Algebra and Statistics, **5**, 157-167, (2016).

[24] A. Rohini, M. Venkatachalam, S. Broumi; and F. Smarandache, Single Valued Neutrosophic Coloring, Neutrosophic Sets and Systems, **28**, 1 (2019).

[25] R. K. Mahapatra, S. Samanta, M .Pal ,Edge Colouring of Neutrosophic Graphs and Its Application in Detection of Phishing Website, Discrete Dynamics in Nature and Society, **2022**, (2022).

[26] M. Akram, *Single-valued neutrosophic graphs*, Springer Singapore, 48-127, (2018).

[27] N. Jan, L. Zedam, T. Mahmood, K. Ullah, S. Broumi & F. Smarandache, *Constant single valued neutrosophic graphs with applications, Infinite study,(2019).*

[28] P. K. Singh, Four-Way Turiyam set-based human quantum cognition analysis, *Journal of Artificial Intelligence and Technology*, ***2***, 144-151,(2022).

[29] P. K. Singh, Turiyam set a fourth dimension data representation, *Journal of Applied Mathematics and Physics*, ***9***, 1821-1828, (2021).

[30] P. K. Singh, Data with Turiyam set for fourth dimension quantum information processing, *Journal of Neutrosophic and Fuzzy Systems*, ***1***, 9-23, (2021).

[31] X. Deng and C. H. Papadimitriou, Exploring an unknown graph, In*: Proceedings of 31st Annual Symposium on Foundations of Computer Science,* **1**, 355-361, (1990).

[32] N. D. Belnap, A useful four-valued logic, In *Modern uses of multiple-valued logic*, Springer, Dordrecht, 5-37, (1977).

[33] T. Chen, Four-Valued Logic and Its Applications, In: Fault Diagnosis and Fault Tolerance, Springer, Berlin, Heidelberg, 1-64, (1992).

[34] P. K. Singh, Fourth dimension data representation and its analysis using Turiyam Context, Journal of Computer and Communications, **9**, 222-229(2021).

[35] M. Bal, P. K.Singh, K. D. Ahmad , A Short Introduction to The Concept Of Symbolic Turiyam Matrix, Journal of Neutrosophic and Fuzzy Systems, **2**, 88-99, (2022).

[36] M. Bal, P. K.Singh, K. D. Ahmad , A Short Introduction to The Symbolic Turiyam Vector Spaces and Complex Numbers, Journal of Neutrosophic and Fuzzy Systems, **2**, 76-87, (2022).

[37] M. Bal, P. K.Singh, K. D. Ahmad , An Introduction To The Symbolic Turiyam R-Modules and Turiyam Modulo Integers, Journal of Neutrosophic and Fuzzy Systems, **2**, 8-19, (2022).

[38] P. K. Singh, K. D.Ahmad, M.Bal, M.Aswad ,On The Symbolic Turiyam Rings, Journal of Neutrosophic and Fuzzy Systems, **1**, 80-88, (2022).

[39] G. A. Ganati, V. N.Srinivasa Rao Repalle & M. A. Ashebo, Relations in the context of Turiyam sets. *BMC Research Notes*, ***16***, 1-6, (2023).

[40] A. A. A. Agboola, A. D. Akinola & O. Y. Oyebola, Neutrosophic rings I, *Smarandache Multispace & Multistructure*, ***28***, 114, (2013).

[41] A. A. A. Agboola, & S. A. Akinleye, Neutrosophic vector spaces, *Neutrosophic Sets and Systems*, ***4***, 9-18, (2014).

[42] N. Olgun & M. Bal, Neutrosophic modules, *Neutrosophic Oper. Res*, ***2***, 181-192, (2017).

[43] P.K. Singh, Data with Non-Euclidean Geometry and its Characterization, Journal of Artificial Intelligence and Technology, **2**, 3-8, (2022).

[44] B.Said, M .Lathamaheswari, P.K. Singh, A.A. Ouallane, A .Bakhouyi, A . Bakali, M . Talea, A . Dhital , M . Deivanayagampillai, An Intelligent Traffic Control System Using Neutrosophic Sets, Rough sets, Graph Theory, Fuzzy sets and its Extended Approach: A Literature Review, Neutrosophic Sets and Systems, **50**, 11-46, (2022).