Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/120110

Some New Constructors for Minimal Circular Partially Balanced Neighbor Designs

Qaisar Mehmood[1], *Hafiz Muhammad Arshad*[2], *Khadija Noreen*[3], *Imran Munir*[3], *Abdul Salam*[3], *and Rashid Ahmed*[3,*]

[1]Department of Quantitative Method, University of Management and Technology Lahore, Pakistan
[2]Department of Management Sciences, COMSATS Sahiwal, Pakistan
[3]Department of Statistics, The Islamia University of Bahawalpur, Pakistan

Received: 20 Oct. 2021, Revised: 21 Feb. 2022, Accepted: 8 Apr. 2022
Published online: 1 Jan. 2023

Abstract: Minimal circular neighbor designsa are economical to minimize the bias due to neighbor effects for v odd. For v even, minimal circular partially balanced neighbor designs (MCPBNDs) are used. Generators to obtain MCPBNDs-II in equal, two and three different blocks sizes are available in literature for $c = 0$ and 1, where c is remainder if m is divided by 4, $m = (v-2)/2$ and v is number of treatments. These designs have not been constructed for $c = 2$ and 3. To complete the construction of this class of neighbor designs, MCPBNDs-II are, therefore, constructed for the remaining cases. MCPNBDs-II are the neighbor designs in which $3v/2$ pairs of different treatments do not appear as neighbors.

Keywords: Neighbor effects; Direct effects; CBNDs; CSBNDs; CSGNDs.
Mathematics Subject Classification (2010): 05B05; 62K10; 62K05.

1 Introduction

Neighbor effect often becomes the major source of bias, which can be minimized with the use of neighbor balanced designs, see [1] and [2]. [3] used neighbor designs in virus research. A design where each pair of distinct treatments appears once as neighbors is called minimal neighbor balanced designs (NBD). [4] presented catalogue of NBDs using border plots. [5], [6], [7] and [8] are some more references for CNBDs. Minimal cicular NBDs can only be constructed for v odd, where v is the number of treatments. For v even, minimal circular partially balanced neighbor designs (MCPBNDs) are used. If each pair of distinct treatments appears as neighbors in circular design at most once, design is called MCPBND. [9] suggested that PNBDs should be used if minimal NBDs cannot be generated. [10] constructed generalized neighbor designs (GNDs) by relaxing balance property. [11], [12] and [13] constructed some classes of circular GNDs. [14] and [15] developed some infinite series to obtain the minimal circular GNDs. [16] presented list of CGNDs for blocks of sizes three.

[17] developed some generators to generate these designs in blocks of two and three different sizes using i sets of shifts for k_1 and two sets for k_2 for $c = 0$ and 1. These designs can also be constructed for $c = 2$ and 3, where c is reminder if m is divided by 4 and $m = (v-2)/2$. In this article, some generators are developed through method of cyclic shifts (Rule I) to obtain MCPBNDs-II for $c = 2$ and 3 in two different and three different block sizes. MCPBNDs-II are MCPBNDs in which $3v/2$ pairs of different treatments do not appear as neighbors while the remaining ones appear once.

2 Method of cyclic shifts

Method of cyclic shifts (Rule I) developed by [18] is explained here for the construction of MCPBNDs.

Let $S_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}]$ be i sets, where j = 1, 2,..., i, $1 \leq q_{ju} \leq v-1$, $u = 1, 2, ..., k-1$.

• If 1, 2,..., $v-1$ appears exactly once in S^* then designs will be MCBND.

* Corresponding author e-mail: rashid701@hotmail.com

• If each of 1, 2, ..., $v-1$ appears once except $v/2$ which does not appear in S^* then designs will be MCPBND-I.
• If each of 1, 2, ..., $v-1$ appears once except $v/2$ and two others which do not appear in S* then designs will be MCPBND-II.
Where S^* contains: Each element of all S_j.
Sum of all elements (mod v) in each S_j.
Complements of all elements in (i) and (ii). In Rule I, complement of 'a'is '$v-a$'.

Rule I expresses that:
• A = [1, 2, ..., $m-2$, m] will produce MCPBNDs-II for c = 2 and 3 if sum of A is divisible by v. Otherwise, replace one or more elements with their complements to make the sum divisible by v.

Example 2.1: S_1 = [2,3,4,5] and S_2 = [6,7,10] generate MCPBND-I for v = 22, k_1 = 5, k_2 = 4.

Take v blocks to get the blocks from S_1. Consider first unit elements as 0, 1, ..., $v-1$. Add 2 (mod v) to each first unit element, to obtain second unit elements. Add 3 (mod 22) to second unit elements to obtain third unit elements, then add 4 and 5, see Table 1.

Table 1: Blocks generated from S_1.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4
9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8
14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8	9	10	11	12	13

Take v more blocks for S_2 and obtain the design, see Table 2.

Table 2: Blocks generated from S_2.

23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4	5
13	14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0

Table 1 and Table 2 jointly present the MCPBND-II for $v = 22, k_1 = 5$ and $k_2 = 4$.

In all next construction to obtain MCPBNDs-II, elements of A and B given in constructors 3.1 and 3.2 respectively, will be divided in (i) i groups of size k for $v = 2ik+4$, (ii) i groups of size k_1 and one of size k_2 for $v = 2ik_1+2k_2+4$, (iii) i groups of size k_1 and two of size k_2 for $v = 2ik_1+4k_2+4$, (iv) i groups of size k_1, one group each of size k_2 and k_3 for $v = 2ik_1+2k_2+2k_3+4$, (v) i groups of size k_1, one of size k_2 and two of size k_3 $v = 2ik_1+2k_2+4k_3+4$, such that the sum of each group should be divisible by v. Sets of shifts to produce MCPBNDs-II will be obtained by deleting any one element from each group.

3 Constructors to produce MCPBNDs-II

Here, MCPBNDs-II are constructed in two and three different block sizes for $m(mod 4) \equiv c$, where $m = (v-2)/2$ and c = 2, 3.

Construction 3.1: If c = 2 then sets of shifts obtained from A = [2, 3, ..., $m-2, m, 2m+1$] produce MCPBNDs-II.

Construction 3.2: If c = 3 then sets of shifts obtained from B = [1, 2, ..., $(m+1)/4, (m+9)/4, (m+13)/4, m-2, m, (7m+3)/4$] produce MCPBNDs-II.

4 MCPBNDs-II in two different blocks sizes

4.1 MCPBNDs-II in two different blocks sizes for c = 2

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4$ and $v = 2ik_1 + 4k_2 + 4$ in two different blocks sizes, using constructor 3.1.

Generator 4.1.1. MCPBNDs-II can be constructed from i sets for k_1 and one for k_2 for $v = 2ik_1 + 2k_2 + 4$ with c = 2 and:

$k_2 = 3$:
$k_1 = 6, 10, \ldots$, and i odd.
$k_1 = 5, 7, \ldots$, i = 2, 6, ...

Example 4.1.1. S_1 = [3,4,5,7,9,6] and S_2 = [7,10] produce MCPBND-II for $v = 18, k_1$ = 7 and k_2 = 3.

$k_2 = 4$:
$k_1 = 1, 5, \ldots$, and i = 1, 5, ...
$k_1 = 3, 7, \ldots$, and i = 3, 7, ...

$k_2 = 5$:
$k_1 = 8, 12, \ldots$, and i integer.
$k_1 = 6, 10, \ldots$, and i even.
$k_1 = 7, 9, \ldots$, and i = 4, 8, ...

Generator 4.1.2: MCPBNDs-II can be generated from i sets for k_1 and two for k_2 for $v = 2ik_1 + 4k_2 + 4$ with c = 2 and:

$k_2 = 3$:
$k_1 = 5, 9$, ..., i = 3, 7, ...
$k_1 = 7, 11$, ..., i = 1, 5, ...

Example 4.1.2. S_1 = [2,3,5,6,7,8], S_2 = [9,10] and S_3 = [4,12] produce MCPBND-II for $v = 30$, k_1 = 7, k_2 = 3.

$k_2 = 4$:
$k_1 = 5, 9$, ..., i = 1, 5, ...
$k_1 = 7, 11$, ..., i = 3, 7, ...

$k_2 = 5$:
$k_1 = 5, 9$, ..., i = 1, 5, ...
$k_1 = 7, 11$, ..., i = 3, 7, ...

4.2 MCPNBDs-II in two different blocks sizes for c = 3

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 4k_2 + 4$ and $v = 2ik_1 + 2k_2 + 4$ in two different blocks sizes, using constructor 3.2.

Generator 4.2.1. MCPBNDs-II can be constructed from i sets of shifts for k_1 and one for k_2 for $v = 2ik_1 + 2k_2 + 4$ with c = 3 and:

$k_2 = 3$:
$k_1 = 1, 5, \ldots$, and i = 3, 7, ...
$k_1 = 3, 7, \ldots$, i = 1, 5, ...

$k_2 = 4$:
$k_1 = 6, 10$, ..., i odd.
$k_1 = 5, 7$, ..., and i = 2, 6, ...

$k_2 = 5$:
$k_1 = 1, 5, ...$, and i = 1, 5, ...
$k_1 = 3, 7, ...$, and i = 3, 7, ...

Example 4.2.1. S_1 = [3,4,6,7,11] and S_2 = [6,8] produce MCPBND-II for $v = 16, k_1$ = 6 and k_2 = 3.

Generator 4.2.2: MCPBNDs-II can be generated from i sets for k_1 and two for k_2 for $v = 2ik_1 + 4k_2 + 4$ with c = 3 and:

$k_2 = 3$:
$k_1 = 4, 8$, ..., i integer.
$k_1 = 6, 10$, ..., i even.
$k_1 = 5, 7$, ..., i = 4, 8, ...

Example 4.2.2. S_1 = [1,8,11], S_2 = [6,7,9], S_3 = [4,13] and S_4 = [2,3] produce MCPBND-II for $v = 32, k_1$ = 4 and k_2 = 3.

$k_2 = 4$:
$k_1 = 6, 10$, ..., i odd.
$k_1 = 5, 7$, ..., i = 2, 6, ...

$k_2 = 5$:
$k_1 = 8, 12$, ..., i integer.
$k_1 = 6, 10$, ..., i even.
$k_1 = 7, 9$, ..., i = 4, 8, ...

5 MCPBNDs-II in three different blocks sizes

5.1 MCPBNDs-II in three different blocks sizes for c = 2

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ and $v = 2ik_1 + 2k_2 + 4k_3 + 4$ in three different blocks sizes, using constructor 3.1.

Generator 5.1.1: MCPBNDs-II can be constructed from i sets for k_1, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 2k_3 + 4$ with c = 2 and:

$k_3 = 3$:
$k_1 = 8, 12, ..., k_2 = k1 - 1$, and i integer.
$k_1 = 6, 10, ..., k_2 = k_1 - 1$, and i even.
$k_1 = 5, 7, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 2$, and i integer.
$k_1 = 6, 10, ..., k_2 = k_1 - 2$, and i odd.
$k_1 = 7, 9, ..., k_2 = k1 - 2$, and i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 3$, and i = 4, 8, ...
$k - 1 = 7, 11, ..., k_2 = k_1 - 3$, and i = 2, 6, ...

Example 5.1.1. $S_1 = [5, 9, 10, 11, 13], S_2 = [3, 6, 7, 8]$ and $S_3 = [11, 14]$ produce MCPBND-II for $v = 26, k_1 = 6, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 1, 5, ..., k_2 = k_1 - 1$, and i = 1, 5, ...
$k_1 = 3, 7, ..., k_2 = k_1 - 1$, and i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, and i = 4, 8, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 3$, i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 3$, and i odd.
$k_1 = 9, 11, ..., k_2 = k_1 - 3$, and i = 3, 7, ...

$k_3 = 5$:
$k_1 = 1, 5, ..., k_2 = k_1 - 1$, and i = 4, 8, ...
$k_1 = 3, 7, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, i even.
$k_1 = 9, 11, ..., k_2 = k1 - 2$, and i = 1, 5, ...

Generator 5.1.2: MCPBNDs-II can be constructed from i sets for $_1$, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ with c = 2 and:

$k_3 = 3$:
$k_1 = 8, 12, ..., k_2 = k_1 - 1$, i integer.
$k_1 = 6, 10, ..., k_2 = k_1 - 1$, i odd.
$k_1 = 5, 7, ..., k_2 = k_1 - 1$, i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, i = 2, 6, ...

Example 5.1.2. $S_1 = [3, 7, 8, 10, 11], S_2 = [2, 5, 6, 12], S_3 = [9, 14]$ and $S_4 = [4, 16]$ produce MCPBND-II for $v = 38, k_1 = 6, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 6, 10, ..., k_2 = k_1 - 1$, i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 1$, i = 1, 5, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, i = 2, 6, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, i = 4, 8, ...

$k_3 = 5$:
$k_1 = 8, 12, ..., k_2 = k_1 - 1$, i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 1$, i odd.
$k_1 = 7, 9, ..., k_2 = k_1 - 1$, i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, i = 4, 8, ...
$k_1 = 11, 15, ..., k_2 = k_1 - 2$, i = 2, 6, ...

5.2 MCPBNDs-II in three different blocks sizes for c = 3

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ and $v = 2ik_1 + 2k_2 + 4k_3 + 4$ in three different blocks sizes, using constructor 3.2.

Generator 5.2.1: MCPBNDs-II can be constructed from i sets for k_1, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 2k_3 + 4$ with c = 3 and:

$k_3 = 3$:
$k_1 = 5, 9, ..., k_2 = k_1 - 1$, and i = 3, 7, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, and i = 1, 5, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, and i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, and i even.
$k_1 = 10, 14, ..., k_2 = k_1 - 3$, and i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 3$, and i = 1, 5, ...

$k_3 = 4$:
$k_1 = 7, 9, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 2$, and i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, and i odd.
$k_1 = 7, 9, ..., k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 3$, and i = 4, 8, ...
$k_1 = 11, 15, ..., k_2 = k_1 - 3$, and i = 2, 6, ...

$k_3 = 5$:
$k_1 = 10, 14, ..., k_2 = k_1 - 1$, and i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 1, k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 11, 15, ..., k_2 = k_1 - 2$, and i = 4, 8, ...

Generator 5.2.2: MCPBNDs-II can be generated from i sets for k_1, one set for k_2 and two for k_3 obtained from for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ with c = 3 and:

$k_3 = 3$:
$k_1 = 5, 9, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 6, 10, ..., k_2 = k_1 - 2$, i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 2$, i = 1, 5, ...

Example 5.2.2. $S_1 = [8, 9, 10, 11, 13, 14], S_2 = [2, 3, 7, 12], S_3 = [4, 17], S_4 = [1, 5]$ produce MCPBND-II for $v = 40, k_1 = 7, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 9, 13, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 2$, i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, i odd.
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, i = 3, 7, ...

$k_3 = 5$:
$k_1 = 9, 13, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, i even.
$k_1 = 9, 11, ..., k_2 = k_1 - 2$, i = 1, 5, ...

Acknowledgement

Authors are thankful to the reviewers for their valuable corrections and suggestions which improved the readability of the article.

Conflicts of interest The authors declare that there is no conflict of interest regarding the publication of this article.

References

[1] J. M. Azais, Design of experiments for studying intergenotypic competition. Journal of the Royal Statistical Society: Series B, **49**, 334-345 (1987).
[2] J. Kunert, Randomization of neighbour balanced designs. Biometrical Journal, **42(1)**, 111-118 (2000).
[3] D. H. Rees, Some designs of use in serology. Biometrics, **23**, 779-791 (1967).
[4] J. M. Azais, R. A. Bailey, and H. Monod, A catalogue of efficient neighbor designs with border plots. Biometrics, **49**(4), 1252-61 (1993).
[5] I. Iqbal, M. H. Tahir, and S. S. A. Ghazali, Circular neighbor-balanced designs using cyclic shifts. Science in China Series A: Mathematics, **52**(10), 2243-2256 (2009).
[6] M. Akhtar, R. Ahmed, and F. Yasmin, A catalogue of nearest neighbor balanced- designs in circular blocks of size five. Pakistan Journal of Statistics, **26**(2), 397-405 (2010).
[7] R. Ahmed, and M. Akhtar, Designs balanced for neighbor effects in circular blocks of size six. Journal of Statistical Planning and Inference, **141**, 687-691 (2011).
[8] F. Shehzad, M. Zafaryab, and R. Ahmed, Minimal neighbor designs in circular blocks of unequal sizes. Journal of Statistical Planning and Inference, **141**, 3681-3685 (2011a).
[9] G. N. Wilkinson, S. R. Eckert, T. W. Hancock, and O. Mayo, Nearest neighbor (nn) analysis of field experiments (with discussion). Journal of Royal Statistical Society Series B, **45**, 151- 211 (1983).

[10] B. L. Misra, Bhagwandas and S. M. Nutan, Families of neighbor designs and their analysis, Communications in Statistics-Simulation and Computation, **20**, 427-436 (1991).
[11] N. K. Chaure, and B. L. Misra, On construction of generalized neighbor design. Sankhya Series B, **58**, 45-253 (1996).
[12] S. M. Nutan, Families of proper generalized neighbor designs. Journal of Statistical Planning and Inference, **137**, 1681-1686 (2007).
[13] R. G. Kedia, and B. L. Misra, On construction of generalized neighbor design of use in serology. Statistics and Probability Letters, **18**, 254-256 (2008).
[14] R. Ahmed, M. Akhtar, and M. H. Tahir, Economical generalized neighbor designs of use in Serology. Computational Statistics and Data Analysis, **53**, 4584-4589. (2009).
[15] F. Shehzad, M. Zafaryab, and R. Ahmed, Some series of proper generalized neighbor designs. Journal of Statistical Planning and Inference, **141**, 3808-3818 (2011b).
[16] I. Iqbal, M. H. Tahir, M. L. Aggarwal, A. Ali, and I. Ahmed, Generalized neighbor designs with block size 3. Journal of Statistical Planning and Inference, **142**, 626-632 (2012).
[17] M. Nadeem, R. Ahmed, M. Qaisar, and R. A Berihan, Some new constructions of minimal circular partially balanced neighbor designs. Journal of Statistics Application and Probability, In press (2022).
[18] I. Iqbal, Construction of experimental design using cyclic shifts, Ph.D. Thesis, University of Kent at Canterbury, U.K (1991).