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Catalogues of Efficient Minimal Weakly Balanced RMDs in Circular Periods of Three Different Sizes

Muhammad Riaz[1], *Hafiz Muhammad Arshad*[2], *Rida Jabeen*[1], *Muhammad Sajid Rashid*[3], *Adil Afzal*[1] *and Rashid Ahmed*[1,*]

[1]Department of Statistics, The Islamia University of Bahawalpur, Pakistan
[2]Department of Management Sciences, COMSATS Sahiwal, Pakistan
[3]Department of Computer Science, The Islamia University of Bahawalpur, Pakistan

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Abstract: In this article, efficient minimal circular weakly balanced RMDs are constructed for the situations where these designs cannot be constructed through generators. Catalogues of these efficient designs are compiled in periods of three different sizes. In the proposed designs, $v/2$ ordered pairs of distinct treatments do not appear as their preceded value while the remaining ones appear once, where v is number of treatments to be compared.

Keywords: Balanced RMDs; Efficiency of Separability; CBRMDs; CSBRMDs
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1 Introduction

In repeated measurements designs (RMDs) more than one measurements are observed on each subject. RMDs have application in many branches of scientific inquiry, therefore, choice of RMD must be made in a way that the treatments can be efficiently compared after allowing for the residual effects. Balanced RMDs (BRMDs) are called robust to residual effects. Experimenters always prefer the minimal BRMDs because these are most economical. If minimal BRMDs cannot be constructed then weakly BRMDs should be used to balance the residual effect.

- RMD is minimal balanced if each treatment is immediately preceded once by all other treatments (excluding itself).
- RMD is minimal weakly balanced if each treatment is immediately preceded once by most of the treatments (excluding itself) but not preceded with a few of remaining ones.
- Periods are considered as circular if periods are (i) divided in circular form, or (ii) not divided in circular form but pre period is used in which treatments applied are same as in the last period.

[1] constructed balanced RMDs for some cases. [2] introduced the idea of a circular BRMDs. [3] generated some classes of BRMDs. He also constructed extra balanced (strongly balanced) RMDs. [4] and [5] generated CSBRMDs and also discussed their optimality. [6] proposed balanced and strongly BRMDs in period of different sizes. [7] constructed (i) RMDs in periods of equal and different sizes through method of cyclic shifts for $3 \leq v \leq 10$, $3 \leq p \leq 10$, (ii) Strongly BRMD for $3 \leq v$, $p \leq 10$, and (iii) combinatorial balanced designs for two unequal period sizes.[8] constructed BRMDs when v is odd. Using cyclic shifts, [9] and [10] proposed CSBRMDs and CBRMDs which are balanced for first order as well as second order. These designs are constructed for $p < v$. [11] constructed SBRMDs in circular periods using integer programming. [12] presented some infinite series for CBRMDs in equal period sizes for different cases. [13] also presented construction procedures of CBRMDs for some cases.[14] developed some generators to obtain MCSBRMDs

* Corresponding author e-mail: rashid701@hotmail.com

for $p_1 < p_2 < p_3$ for different cases.

[15] presented some minimal circular strongly partially BRMDs (MCSPBRMDs) in periods of equal sizes using Rule I.[16] constructed some MCSPBRMDs in periods of two different sizes.[17] presented some minimal circular weakly BRMDs (MCWBRMDs) in periods of equal sizes. [18] presented the catalogues of MCWBRMDs in two different period sizes. [19] presented MCWBRMDs for p_1 even, in periods of two and three different sizes. MCWBRMDs have not been constructed for p_1 = r (odd) > 3, $p_2 = s$ (integer) and $p_3 = u$ (integer). In this article, therefore, efficient MCWBRMDs in which $v/2$ ordered pairs of distinct treatments do not appear as their preceded value while the remaining ones appear once, are constructed for p_1 odd and p_3 very small. Catalogues of the proposed designs are also compiled. Method of cyclic shifts (Rule I) is used to construct efficient MCWBRMDs-I in periods of three different sizes. This method is explained in Section 2. Model and efficiency measures are given in Section 3. Catalogues of efficient MCWBRMDs are presented in Section 4 for $v \leq 70, 5 \leq p_1(odd) \leq 11, 4 \leq p_2 \leq 10$ and $p_3 = 3$ and 4.

2 Method of Cyclic Shifts

Method of cyclic shifts introduced by [20] is used to construct several types of designs. Its Rule I is explained here only for the construction of MCBRMDs and MCWBRMDs in the periods of three different sizes.

• How a design is obtained from given set(s) of shifts using Rule I, is described here with the help of an example.Following arrays are obtained for $v = 20$ and $p = 8$ from $S = [2, 12, 3, 4, 5, 6, 7]$.

Get arrays from $S = [2, 12, 3, 4, 5, 6, 7]$ using v subjects. Consider $0, 1, ..., v-1$ respectively as the values of first period. Add 2 (mod 20) in these values to obtain the elements of second period, Similarly add $12, 3, 4, 5, 6$ and 7 in the values of $2nd, 3rd, ..., 7th$ period to obtain the values of $3rd, 4th, ..., 8th$ periods respectively, see Table 1.

Table 1: Arrays obtained from $S = [2, 12, 3, 4, 5, 6, 7]$

Periods	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	10	11
3	14	15	16	17	18	19	0	1	2	3
4	17	18	19	0	1	2	3	4	5	6
5	1	2	3	4	5	6	7	8	9	10
6	6	7	8	9	10	11	12	13	14	15
7	12	13	14	15	16	17	18	19	0	1
8	19	0	1	2	3	4	5	6	7	8

Periods	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	10	11	12	13	14	15	16	17	18	19
2	12	13	14	15	16	17	18	19	0	1
3	4	5	6	7	8	9	10	11	12	13
4	7	8	9	10	11	12	13	14	15	16
5	11	12	13	14	15	16	17	18	19	0
6	16	17	18	19	0	1	2	3	4	5
7	2	3	4	5	6	7	8	9	10	11
8	9	10	11	12	13	14	15	16	17	18

- **Construction of MCBRMDs and MCWBRMDs**

If $[q_{11}, q_{12}, ..., q_{1(p_1-1)}], [q_{21}, q_{22}, ..., q_{2(p_2-1)}]$ and $[q_{31}, q_{32}, ..., q_{3(p_3-1)}]$ are the sets with $1 \leq q_{ij} \leq v-1$ and each of $1, 2, ..., v-1$ appears once in S^*, where S^* contains (i) each element of all S_j, and (ii) v - [sum of all elements (mod v) in each S_j] then it will provide MCBRMD in p_1, p_2 and p_3. If most of $1, 2, ..., v-1$ appear once but a few do not appear then it will be MCWBRMD.

Example 2.1: $S_1 = [2, 4, 6, 13], S_2 = [1, 8, 10], S_3 = [5, 11]$ provide MCWBRMD for $v = 14, p_1 = 5, p_2 = 4, p_3 = 3$.

Proof: Hence $S^* = [2,4,6,13,3,1,8,10,9,5,11,12]$. Here $1,2,...,13$ appear once except 7 which does not appear. Therefore, it is MCWBRMD which is given below.

Table 2: Arrays obtained from $S_1 = [2,4,6,13]$

p	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	3	4	5	6	7	8	9	10	11	12	13	0	1
3	6	7	8	9	10	11	12	13	0	1	2	3	4	5
4	12	13	0	1	2	3	4	5	6	7	8	9	10	11
5	11	12	13	0	1	2	3	4	5	6	7	8	9	10

Table 3: Arrays obtained from $S_2 = [1,8,10]$

p	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	1	2	3	4	5	6	7	8	9	10	11	12	13	0
3	9	10	11	12	13	0	1	2	3	4	5	6	7	8
4	5	6	7	8	9	10	11	12	13	0	1	2	3	4

Table 4: Arrays obtained from $S_3 = [5,11]$

p	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	5	6	7	8	9	10	11	12	13	0	1	2	3	4
3	2	3	4	5	6	7	8	9	10	11	12	13	0	1

MCWBRMD is obtained for $v = 20$, $p_1 = 8$, $p_2 = 6$, $p_3 = 4$ using Table 1, 2 and 3 together.

3 Efficiency Measures

3.1 Efficiency of Residual Effects

Commonly used model for circular RMDs is, see [21].

$$Y = \mu E + D\delta + R\rho + U\nu + P\Pi + \varepsilon \qquad (1)$$

Canonical efficiency factors are non-zero Eigen values of information matrix C*, and efficiency factor for both direct and carryover effect can be regarded as the harmonic mean of non-zero Eigen values of their respective information matrix, see [22] and [23]. Design will be suitable to estimate residual effects when value of Er is high.

3.2 Efficiency of Separability

[24] developed following efficiency of Separability (Es) for BRMDs.

$$Es = [1 - 1/(v\sqrt{v-1})] * 100\%$$

4 Construction of efficient MCWBRMDs for p_1, p_2 and p_3

In this Section, efficient MCWBRMDs are constructed for p_1 odd and p_3 very small using the following sets of shifts with $p_1 = r > 3, p_2 = s, p_3 = u$.
$S_j = [q_{j1}, q_{j2}, ..., q_{j(r-1)}]$; $j = 1, 2, ..., i$.
$S_{i+1} = [q_{(i+1)1}, q_{(i+1)2}, ..., q_{(i+1)(s-1)}]$
$S_{i+2} = [q_{(i+2)1}, q_{(i+2)2}, ..., q_{(i+2)(u-1)}]$
Each of $1, 2, ..., v-1$ appears once in S* but $v/2$ does not appear.
Where S^* contains (i) Each element of all S_j, and (ii) v - [Sum of all elements (mod v) in each S_j].

Procedure 4.1: Efficient MCWBRMDs for $v = p_{1i}+p_2+p_3+2$; i odd, p_1 odd, p_2 even and p_3 = 3

v	p_1	p_2	p_3	sets	Er	Es
14	5	4	3	[13,2,4,6]+[1,8,10]+[11,5]	0.80	0.89
24	5	4	3	[1,2,4,14]+[6,7,9,18]+[16,17,11,15]+ [23,10,20]+[21,5]	0.80	0.94
34	5	4	3	[6,2,8,5]+[1,7,12,10]+[23,31,24,15]+ [16,26,20,21]+[11,14,25,30]+[27,29,18]+[3,33]	0.80	0.96
44	5	4	3	[1,12,13,4]+[6,11,8,9]+[7,2,3,17]+ [16,18,29,20]+[26,24,21,38]+[28,35,43,31]+ [32,33,36,41]+[37,30,40]+[19,27]	0.80	0.97
54	5	4	3	[1,17,18,5]+[52,20,11,16]+[10,4,25,8]+ [23,2,14,12]+[41,35,24,19]+[44,29,30,31]+ [32,34,21,36]+[37,45,49,47]+[42,33,40,48]+[51,46,50]+[26,22]	0.80	0.97
64	5	4	3	[1,8,13,39]+[27,18,61,15]+[12,58,5,10]+ [6,25,4,20]+[21,2,35,37]+[36,22,26,28]+ [31,23,47,57]+[19,44,45,46]+[42,29,50,41]+ [24,49,54,17]+[52,40,55,56]+[62,59,60]+[14,63]	0.80	0.98
16	7	4	3	[1,2,9,13,6,7]+[5,12,11]+[3,15]	0.83	0.91
30	7	4	3	[1,2,3,4,6,9]+[7,8,12,11,13,14]+[26,18,19,21,27,22]+ [23,28,29]+[20,16]	0.85	0.95
44	7	4	3	[2,1,21,42,12,7]+[8,9,10,11,13,14]+[16,15,5,18,20,19]+ [6,24,27,28,31,35]+[29,30,32,33,34,36]+[37,17,40]+ [41,43]	0.85	0.97
58	7	4	3	[1,2,39,56,10,3]+[8,48,15,12,13,14]+[38,16,28,19,20,21]+ [9,22,23,26,18,36]+[27,51,54,7,31,45]+[37,11,25,41,42,43]+ [44,46,47,24,49,50]+[35,53,34]+[55,57]	0.85	0.98
18	9	4	3	[10,7,8,12,5,3,6,1]+[11,13,14]+[15,17]	0.86	0.92
36	9	4	3	[1,2,3,14,6,7,8,9]+[4,12,13,15,17,16,27,19]+ [20,11,35,31,30,25,26,28]+[29,24,32]+[33,5]	0.88	0.96
54	9	4	3	[10,2,3,11,6,7,8,9]+[1,5,13,16,17,18,21,34]+ [22,23,25,24,26,19,28,20]+[30,32,31,33,14,35,12,36]+ [38,39,41,42,43,44,45,46]+[15,49,50]+[51,53]	0.88	0.97
20	11	4	3	[11,12,1,2,3,6,7,8,9,17]+[13,15,18]+[5,19]	0.87	0.93
42	11	4	3	[1,2,4,6,12,38,8,9,10,11]+[5,14,15,17,37,18,19,22,23,27]+ [24,7,26,20,30,28,31,32,33,34]+[35,36,39]+[3,41]	0.90	0.97
64	11	4	3	[50,12,30,4,6,7,8,29,36,11]+[17,18,19,20,21,22,39,13,14,15]+ [23,24,25,26,27,28,2,3,33,34]+[9,10,35,38,40,41,42,43,44,45]+ [46,47,48,49,1,51,52,53,55,56]+[57,16,60]+[61,5]	0.90	0.98
18	7	6	3	[7,6,3,5,2,1]+[8,10,11,13,14]+[15,17]	0.74	0.92
32	7	6	3	[1,2,3,5,6,11]+[8,9,12,13,14,30]+[25,18,19,20,27,22]+ [23,24,15,28,17]+[26,31]	0.86	0.95
46	7	6	3	[1,2,21,4,5,7]+[8,9,11,12,13,14]+[15,17,18,19,10,43]+ [22,20,24,27,26,29]+[37,31,32,33,35,28]+[30,39,40,41,42]+[3,45]	0.86	0.97
60	7	6	3	[18,7,42,39,12,13]+[8,9,11,5,14,3]+[15,17,1,38,6,27]+ [22,34,24,25,21,28]+[32,54,31,23,35,36]+[37,19,40,41,2,43]+ [44,45,46,47,48,20]+[52,53,33,55,56]+[57,59]	0.86	0.98
20	9	6	3	[13,2,12,4,6,7,8,9]+[1,3,14,15,16]+[17,18]	0.87	0.93

38	9	6	3	[27,14,34,5,4,7,8,9]+[10,11,12,13,16,2,17,18]+ [20,21,22,24,25,26,28,1]+[29,31,32,33,35]+[3,37]	0.88	0.96
56	9	6	3	[1,2,3,4,16,6,8,9]+[50,12,13,14,15,5,17,18]+ [19,20,21,23,11,55,26,27]+[49,29,32,53,34,35,36,37]+ [52,40,41,42,43,44,45,46]+[47,48,30,10,38]+[54,25]	0.88	0.97
22	11	6	3	[1,2,3,5,6,7,8,9,12,20]+[13,10,16,17,18]+[19,21]	0.89	0.93
44	11	6	3	[1,2,3,4,43,6,7,8,26,11]+[12,14,15,16,17,18,19,20,9,23]+ [24,25,27,10,28,35,33,34,31,32]+[30,37,38,39,40]+[42,5]	0.90	0.97
66	11	6	3	[45,30,63,4,5,6,8,9,10,11]+[34,13,15,28,27,18,19,20,21,55]+ [23,24,25,26,17,16,29,2,31,12]+[36,37,38,39,40,41,42,43,44,1]+ [46,47,48,49,50,53,54,52,22,56]+[57,32,60,61,62]+[64,65]	0.90	0.98
22	9	8	3	[1,2,3,5,20,8,7,14]+[10,13,12,15,9,16,18]+[19,21]	0.89	0.93
40	9	8	3	[2,1,3,4,6,7,10,8]+[9,11,28,14,35,16,17,18]+ [21,22,19,23,24,27,25,13]+[29,31,32,33,34,15,36]+[37,5]	0.89	0.96
58	9	8	3	[2,48,3,24,6,7,9,18]+[11,10,13,14,15,17,16,8]+ [19,20,21,28,23,43,26,27]+[22,30,32,33,34,35,36,37]+ [38,39,49,41,44,4,45,46]+[47,1,50,51,52,53,54]+[55,5]	0.89	0.98
24	11	8	3	[4,3,2,7,1,9,8,11,18,23]+[13,15,16,6,17,19,20]+[21,5]	0.90	0.94
46	11	8	3	[1,2,3,4,6,8,10,11,18,22]+[12,14,15,16,17,19,20,21,41,42]+ [25,24,26,27,28,29,30,32,37,33]+[35,36,34,38,39,40,9]+[44,5]	0.90	0.97
68	11	8	3	[1,2,3,4,67,6,7,8,9,42]+[13,14,15,16,17,18,19,20,21,22]+ [23,24,25,26,28,27,12,31,33,32]+[35,37,38,39,40,41,10,43,44,45]+ [46,47,48,49,50,51,52,53,62,56]+[57,59,60,61,54,63,64]+[65,5]	0.91	0.98
26	11	10	3	[19,2,3,4,25,6,7,8,10,11]+[15,16,12,17,1,18,21,22,20]+[23,5]	0.91	0.94
48	11	10	3	[45,33,1,5,6,7,8,9,17,11]+[12,14,15,16,10,18,19,20,21,34]+ [23,22,32,31,30,28,29,25,26,38]+[35,37,27,39,40,41,42,43,44]+ [3,47]	0.91	0.97
70	11	10	3	[25,2,3,4,69,6,7,8,65,12]+[11,14,16,15,17,18,19,20,23,21]+ [22,24,44,46,27,28,29,34,32,33]+[30,37,38,39,40,41,43,1,42,26]+ [45,47,49,48,50,51,53,54,55,56]+[57,58,59,60,61,64,63,10,66]+ [67,5]	0.91	0.98

Procedure 4.2: Efficient MCWBRMDs for $v = p_{1i}+p_2+p_3+2$; i even, p_1, p_2 odd and $p_3 = 3$.

v	p_1	p_2	p_3	**sets**	**Er**	**Es**
24	7	5	3	[1,7,4,14,20,23]+[8,11,9,13,6,15]+[16,18,19,2]+[21,5]	0.85	0.94
38	7	5	3	[2,1,18,4,6,8]+[7,9,11,12,13,14]+[15,17,3,20,22,21]+ [23,24,25,34,28,29]+[30,32,33,26]+[35,5]	0.85	0.96
52	7	5	3	[2,1,33,4,6,7]+[28,3,11,19,13,21]+[15,17,18,12,20,14]+ [42,23,24,47,27,29]+[30,31,10,34,35,36]+[37,38,40,22,41,43]+ [44,45,46,48]+[49,5]	0.85	0.97
66	7	5	3	[1,2,3,45,65,10]+[24,56,11,18,13,14]+[27,17,12,19,20,21]+ [46,39,36,25,15,28]+[32,31,29,34,35,7]+[37,23,40,41,42,43]+ [44,4,22,47,49,50]+[51,53,54,55,8,57]+[58,60,61,26]+[63,64]	0.85	0.98
28	9	5	3	[24,2,3,4,20,6,7,10]+[12,11,13,9,15,16,27,19]+[17,21,23,1]+[25,5]	0.87	0.95
46	9	5	3	[1,42,17,3,45,6,8,9]+[11,10,35,13,19,16,4,18]+ [15,21,22,20,24,25,2,28]+[29,30,32,33,34,14,36,37]+ [38,40,41,26]+[43,5]	0.88	0.97
64	9	5	3	[1,3,4,63,6,60,8,9]+[13,12,10,20,15,16,17,18]+ [14,19,21,29,22,24,11,27]+[28,30,31,2,34,35,36,37]+ [33,39,40,41,42,44,45,46]+[47,49,50,51,52,53,43,55]+ [56,58,59,26]+[61,5]	0.88	0.98
32	11	5	3	[1,2,3,31,6,7,12,9,10,11]+[14,8,15,18,17,19,20,21,24,23]+ [22,26,27,28]+[29,5]	0.89	0.95
54	11	5	3	[12,3,4,53,6,7,8,9,33,11]+[2,14,15,17,1,18,19,20,21,22]+ [23,24,25,28,29,38,31,34,10,50]+[30,35,39,36,40,41,42,43,44,45]+ [46,47,48,26]+[52,5]	0.90	0.97
30	9	7	3	[12,2,13,6,29,7,8,9]+[10,1,3,25,16,17,19,18]+[20,22,23,24,14,26]+ [27,5]	0.86	0.95

48	9	7	3	[1,44,47,6,4,7,8,25]+[10,12,13,14,15,16,35,18]+ [19,21,22,23,9,26,27,28]+[31,30,32,39,34,20,36,37]+ [38,33,40,41,43,3]+[46,5]	0.89	0.97
66	9	7	3	[1,42,53,4,65,6,8,12]+[10,9,13,14,15,16,17,27]+ [18,19,21,20,22,23,24,25]+[29,31,32,2,34,35,36,37]+ [38,39,57,54,30,43,44,46]+[47,49,50,51,52,3,41,55]+ [56,40,58,59,61,62]+[64,5]	0.89	0.98
34	11	7	3	[1,2,3,15,18,33,7,9,29,11]+[12,14,4,6,16,19,20,21,22,23]+ [24,25,26,10,27,30]+[31,5]	0.90	0.96
56	11	7	3	[33,47,3,4,55,36,7,8,9,12]+[11,14,15,17,16,18,19,20,23,2]+ [25,21,26,27,24,29,30,31,32,34]+[35,37,38,39,22,41,42,43,44,45]+ [46,40,48,49,51,52]+[53,5]	0.90	0.97
36	11	9	3	[1,2,4,33,6,5,7,9,14,17]+[12,8,13,16,15,19,20,21,22,23]+ [24,25,26,27,28,30,31,32]+[3,35]	0.91	0.96
58	11	9	3	[1,2,3,4,57,6,20,8,54,45]+[12,14,15,16,17,18,19,7,21,22]+ [23,24,25,26,27,28,9,30,33,34]+[35,36,38,39,40,41,42,43,44,11]+ [46,47,48,49,50,51,53,10]+[56,5]	0.91	0.98

Procedure 4.3: Efficient MCWBRMDs for $v = p_{1i}+p_2+p_3+2$; i odd, p_1, p_2 odd and $p_3 = 4$

v	p_1	p_2	p_3	**sets**	**Er**	**Es**
18	7	5	4	[1,2,3,5,6,15]+[8,11,10,13]+[14,7,17]	0.86	0.92
32	7	5	4	[27,10,4,5,7,9]+[30,6,15,12,13,17]+[11,18,19,23,21,22]+ [20,25,26,1]+[28,29,31]]	0.86	0.95
46	7	5	4	[2,4,5,6,30,44]+[10,9,11,12,13,17]+[14,15,18,19,3,7]+ [22,27,24,37,21,28]+[31,33,32,34,35,36]+[38,39,40,41]+[43,8,45]	0.86	0.97
60	7	5	4	[49,28,24,5,9,7]+[6,10,11,22,15,14]+[16,17,18,34,1,21]+ [23,4,25,26,27,3]+[31,32,33,19,35,36]+[39,38,40,41,2,43]+ [45,46,47,48,20,50]+[52,53,29,55]+[56,57,59]	0.86	0.98
20	9	5	4	[1,2,3,4,5,16,7,14]+[12,11,13,15]+[6,17,18]	0.87	0.93
38	9	5	4	[27,14,3,4,5,7,1,9]+[10,12,13,2,15,16,17,18]+ [21,22,24,23,25,36,29,28]+[26,30,32,33]+[35,8,37]	0.88	0.96
56	9	5	4	[51,27,3,4,5,6,54,11]+[9,12,1,14,15,16,17,18]+ [20,21,22,23,24,25,26,44]+[13,29,31,32,33,34,35,37]+ [40,39,41,42,43,2,45,46]+[47,49,50,30]+[52,53,55]	0.88	0.97
22	11	5	4	[1,2,4,3,5,15,8,12,7,21]+[13,14,6,17]+[18,19,9]	0.89	0.93
44	11	5	4	[1,2,3,5,16,40,7,8,10,11]+[12,14,15,4,17,18,19,21,20,23]+ [24,25,26,27,28,9,39,32,33,34]+[36,37,38,30]+[41,42,43]	0.90	0.97
66	11	5	4	[12,61,3,4,5,6,13,64,10,11]+[45,14,15,16,17,19,18,53,21,22]+ [7,23,25,26,27,28,29,32,2,31]+[36,37,38,39,40,41,42,43,44,1]+ [46,47,48,49,50,52,51,20,54,56]+[58,59,60,30]+[63,8,65]	0.90	0.98
22	9	7	4	[10,2,4,3,5,6,7,21]+[1,13,14,15,16,17]+[19,20,9]	0.89	0.93
40	9	7	4	[1,2,3,4,5,12,38,9]+[34,7,14,13,15,16,17,18]+ [22,19,23,24,25,11,27,28]+[29,30,31,32,33,35]+[37,8,39]	0.89	0.96
58	9	7	4	[20,12,3,4,25,38,7,9]+[11,13,2,14,15,16,17,18]+ [1,21,28,23,5,24,26,27]+[30,31,32,33,34,35,36,37]+ [6,39,41,40,42,44,47,46]+[48,49,50,51,52,53]+[55,8,57]	0.89	0.98
24	11	7	4	[1,4,19,2,5,6,7,22,9,11]+[15,14,16,17,18,3]+[21,8,23]	0.82	0.94
46	11	7	4	[1,2,40,28,3,6,7,44,13,11]+[10,14,15,16,17,18,19,20,21,22]+ [25,24,26,27,5,9,30,31,33,34]+[36,37,38,39,4,41]+[43,8,45]	0.90	0.97
68	11	7	4	[23,2,3,30,5,6,7,66,9,11]+[1,14,15,16,17,19,18,49,21,22]+ [13,25,24,26,27,28,29,4,32,33]+[36,37,38,39,40,41,10,43,44,45]+ [58,47,48,50,20,51,52,53,55,56]+[46,59,60,61,62,63]+[65,8,67]	0.91	0.98
26	11	9	4	[14,1,4,8,7,10,11,6,15,25]+[12,5,16,18,17,19,20,21]+[23,24,9]	0.91	0.94
48	11	9	4	[26,17,3,4,5,6,7,46,9,11]+[13,14,15,16,2,38,19,20,21,22]+ [25,18,1,27,28,31,29,32,33,34]+[36,23,39,37,40,41,42,43]+[45,8,47]	0.91	0.97
70	11	9	4	[22,2,3,5,4,65,67,12,10,11]+[13,14,16,15,17,18,19,23,21,46]+ [20,24,25,28,1,29,30,31,32,33]+[37,36,38,39,40,44,41,26,42,43]+ [47,45,49,48,51,50,53,52,54,56]+[58,59,60,61,62,6,63,64]+[7,68,69]	0.91	0.98

Procedure 4.4: Efficient MCWBRMDs for $v = p_{1i}+p_2+p_3+2$; i odd, p_1, p_2 even and $p_3 = 4$

v	p_1	p_2	p_3	sets	Er	Es
26	7	6	4	[25,2,4,5,6,7]+[8,1,11,17,12,15]+[16,18,19,20,21]+[23,24,9]	0.78	0.94
40	7	6	4	[35,2,8,3,19,7]+[4,28,10,12,13,14]+[16,17,18,11,21,22]+ [24,23,25,26,5,30]+[31,32,33,34,1]+[37,38,9]	0.86	0.96
54	7	6	4	[23,9,4,10,50,7]+[20,3,11,12,13,14]+[17,16,18,19,2,21]+ [22,8,25,24,28,29]+[31,32,33,34,1,36]+[38,39,30,41,42,43]+ [44,45,46,47,40]+[51,52,53]	0.86	0.97
68	7	6	4	[48,43,3,23,25,56]+[9,10,11,12,13,15]+[16,29,17,19,20,21]+ [22,4,24,5,27,28]+[7,32,33,31,35,36]+[38,39,63,41,42,2]+ [44,45,46,37,1,50]+[51,53,54,55,18,57]+[59,60,61,62,40]+[65,8,67]	0.86	0.98
30	9	6	4	[2,1,3,4,5,7,9,8]+[11,13,14,12,16,17,23,19]+[26,22,18,10,24]+[6,27,29]	0.88	0.95
48	9	6	4	[32,13,14,5,44,7,8,9]+[11,2,3,15,4,16,17,18]+[19,1,22,23,21,26,28,25]+ [31,30,20,33,34,38,36,37]+[35,39,40,41,42]+[6,46,47]	0.88	0.97
66	9	6	4	[54,2,3,4,5,11,48,9]+[8,12,13,15,14,16,17,18]+ [10,21,20,22,23,24,26,27]+[29,30,32,31,34,7,36,37]+ [35,39,61,41,43,42,44,45]+[38,49,50,51,52,53,1,55]+ [56,57,58,59,40]+[6,64,65]	0.89	0.98
34	11	6	4	[1,2,3,4,19,28,7,8,10,11]+[12,15,18,14,16,30,20,21,22,23]+ [24,25,5,26,29]+[6,32,33]	0.90	0.96
56	11	6	4	[1,29,3,4,5,23,7,8,12,11]+[52,14,15,16,17,18,19,20,21,22]+ [13,23,25,26,27,2,30,31,36,34]+[37,33,38,39,40,41,42,43,44,45]+ [35,47,48,49,50]+[53,54,55]	0.90	0.95
38	13	6	4	[1,33,3,4,5,34,8,7,10,11,9,13]+[16,12,18,15,20,17,22,25,21,27,24,26]+ [29,30,31,32,2]+[6,35,36]	0.91	0.96
64	13	6	4	[1,14,3,5,60,7,8,9,10,47,11,13]+[2,15,17,18,19,28,21,22,23,24,25,26]+ [27,30,31,20,33,34,35,36,38,37,39,59]+ [41,43,45,44,46,12,48,49,50,52,51,53]+[54,56,57,58,40]+[6,62,63]	0.92	0.98
32	9	8	4	[2,3,4,5,28,7,26,9]+[11,15,1,14,13,17,18,19]+ [10,21,23,24,22,8,27]+[6,30,31]	0.89	0.95
50	9	8	4	[1,42,29,3,5,46,7,8]+[31,22,13,14,15,16,17,18]+ [20,19,21,12,23,24,27,28]+[30,11,32,33,34,35,36,37]+ [38,39,40,41,10,43,44]+[47,48,49]	0.89	0.97
68	9	8	4	[13,2,54,42,5,6,7,66]+[11,31,1,14,15,19,17,18]+ [20,16,21,22,23,24,26,27]+[29,30,33,12,35,32,37,36]+ [38,39,40,4,41,43,44,45]+[48,49,50,51,52,53,3,55]+ [56,57,58,59,60,61,62]+[65,8,67]	0.89	0.98
36	11	8	4	[17,2,3,5,4,7,32,8,9,11]+[13,14,16,15,1,19,24,21,22,23]+ [20,25,26,27,28,29,30]+[6,34,35]	0.91	0.96
58	11	8	4	[35,50,26,17,5,6,7,56,9,11]+[13,14,15,18,4,16,19,20,21,22]+ [23,24,3,25,28,27,30,32,31,34]+[1,37,38,39,40,42,41,43,44,45]+ [46,47,48,49,2,51,52]+[55,8,57]	0.91	0.98
38	11	10	4	[1,2,3,4,5,6,7,9,36,29]+[13,15,17,14,18,16,20,21,22,23]+ [24,25,26,28,27,10,30,32,33]+[35,8,37]	0.91	0.96
60	11	10	4	[1,22,35,4,5,6,7,8,59,10]+[14,3,13,17,16,18,20,46,19,2]+ [11,24,26,25,27,29,28,31,33,34]+[36,37,39,38,40,41,42,43,45,44]+ [21,47,48,49,50,51,52,53,54]+[57,58,9]	0.91	0.98

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References

[1] Hedayat, A. and Afsarinejad, K. (1975). Repeated measurements designs, I. In a Survey of Statistical Design and Linear Models (J. N. Srivastava, Ed.), pp. 229–242 (North-Holland, Amsterdam).
[2] Magda, C. G. (1980). Circular balanced repeated measurements designs. Communications in Statistics-Theory and Methods, 9, 1901-1918.
[3] Afsarinejad, K. (1983). Balanced repeated measurements designs. Biometrika, 70, 199-204.
[4] Roy, B. K. (1988). Construction of strongly balanced uniform repeated measurements designs. Journal of Statistical Planning and Inference, 19, 341–348.
[5] Sen, M. and R. Mukerjee (1987). Optimal repeated measurements designs under interaction. Journal of Statistical Planning and Inference, 17, 81-91.
[6] Afsarinejad, K. (1994). Repeated measurements designs with unequal periods sizes. Journal of the Italian Statistical Society, 2, 161-168.
[7] Iqbal, I. and Jones, B. (1994). Efficient repeated measurements designs with equal and unequal period sizes. Journal of Statistical Planning and Inference, 42, 79-88.
[8] Sharma, V. K., Jaggi, S. and Varghese, C. (2003). Minimal balanced repeated measurements designs. Journal of Applied Statistics, 30, 867-872.
[9] Iqbal, I. and Tahir, M. H. (2009). Circular strongly balanced repeated measurements designs. Communications in Statistics - Theory and Methods, 38, 3686-3696.
[10] Iqbal, I., Tahir, M. H. and Ghazali, S. S. A. (2010). Circular first-and second-order balanced repeated measurements designs. Communications in Statistics - Theory and Methods, 39, 228-240.
[11] Mendal, B. N., Parsad, R. and Gupta, V. K. (2016). Cyclic circular balanced and strongly balanced crossover designs through integer programming. Communications In Statistics - Theory and Methods, 45(4), 859-871.
[12] Rajab, M., Ahmed, R., Shehzad, F. and Tahir, M. H. (2018). Some new constructions of circular balanced repeated measurements designs. Communications in Statistics -Theory and Methods, 47(17), 4142-4152.
[13] Bashir, Z, Ahmed, R., Tahir, M. H., Ghazali, S. S. A. and Shehzad, F. (2018). Some extensions of circular balanced and circular strongly balanced repeated measurements designs. Communications in Statistics - Theory and Methods, 47(9), 2183-2194.
[14] Rasheed, U., Rasheed, H. M. K., Rasheed, M. and Ahmed, R. (2018). Minimal circular strongly balanced repeated measurements designs in periods of three different sizes. Communications in Statistics - Theory and Methods, 47(16), 4088-4094.
[15] Jabeen, R., Rasheed, H. M. K., Ahmed, R. and Shehzad, F. (2019). Construction of circular strongly partially-balanced repeated measurements designs. Journal of King Saud University- Science. 31, 345-351.
[16] Nazeer, Y., Ahmed, R., Jabeen, R., Daniyal, M. and Tahir, M. H. (2019). Circular strongly partially-balanced repeated measurements designs in periods of two different sizes. Journal of Probability and Statistical Science, 17(1), 85-95.
[17] Khan, A., Ahmed, R., Shehzad, F., Tahir, M. H. and Ghazali, S. S. A. (2019). Construction of circular partially balanced-repeated measurement designs using cyclic shifts. Communications in Statistics - Simulation and Computation, 48(2), 506-515.
[18] Rasheed, H. M. K., Jabeen, R., Hussain, S., Shah, A. and Ahmed, R. (2019). Catalogues of efficient circular weakly balanced repeated measurements designs in periods of two different sizes. Aligarh Journal of Statistics, 39, 93-116.
[19] Hussain, S., Ahmed, R., Aslam, M., Shah, A. and Rasheed, H. M. K. (2020). Some new construction of circular weakly balanced repeated measurements designs in periods of two different sizes. Communications in Statistics-Theory and Methods, 49(9), 2253-2263.
[20] Iqbal, I. (1991). Construction of experimental designs using cyclic shifts. Ph.D. thesis, University of Kent at Canterbury, UK.
[21] Davis, A. W. and Hall, W. B. (1969). Cyclic change over designs. Biometrika, 56, 283-293.
[22] James, A. T. and Wilkinson, G.N. (1971). Factorization of the residual operator and canonical decomposition of non-orthogonal factors in the analysis of variance. Biometrika, 58, 258-294.
[23] Pearce, S.C., Calinski, T. and Marshall, T. F. de C. (1974). The basic contrasts of an experimental design with special reference to the analysis of data. Biometrika, 61, 449-460.
[24] Divecha, J. and Gondaliya, J. (2014). Construction of minimal balanced crossover designs having good efficiency of separability. Electronic Journal of Statistics, 8, 2923-2936.