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Some New Constructors of Circular Strongly Generalized Neighbor Designs

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Abstract: Minimal strongly balanced neighbor designs are well known designs to balance the neighbor effects at low cost as well as to estimate the direct effects and neighbor effects independently for v odd, where v is number of treatments to be compared. Minimal strongly generalized neighbor designs are used to minimize the bias due to neighbor effects for v even. In this article, constructors are developed to construct two useful classes of minimal circular strongly generalized neighbor designs.

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1 Introduction

Minimal strongly balanced neighbor designs (SBNDs) are well known designs to balance the neighbor effects at low cost as well as to estimate the direct effects and neighbor effects independently for v odd, where v is number of treatments to be compared. Minimal strongly generalized neighbor designs (SGNDs) are used to minimize the bias due to neighbor effects for v even. A circular design is minimal SBND (MCSBND) if each treatment has all other treatments (including itself) as its neighbors exactly once. It will be minimal circular SGND (MCSGND) if each treatment has most of the treatments once as its neighbors while a few twice (including itself).

[1] suggested neighbor balanced designs in non-circular blocks. [2] introduced neighbor designs in virus research. [3] and [4] showed that neighbor balanced designs (NBDs) minimize the bias due to neighbor effects. A detailed literature regarding NBDs since 1967 is reviewed by [5]. [6] derived the designs which are totally balanced to estimate direct and neighbor effects. [7] constructed the one sided right neighbors designs and concluded that the neighbor treatments followed the circularity property of the same order. For this purpose, [8] developed some methods to construct circular NBDs. In this article, constructors are developed to construct MCSGNDs-I and MCSGNDs-II. In MCSGNDs-I, only $v/2$ unordered pairs of distinct treatments appear twice as neighbors and in MCSGNDs-II, $3v/2$ unordered pairs of distinct treatments appear twice as neighbors, while the remaining ones appear once.

2 Method of Construction

[9] developed method of cyclic shifts which is used here for all constructions. Rule I of this method is explained here for the construction of minimal CSGNDs.

In this Section, construction procedures of MCSGNDs are described under the logic of Rule I. Let $m = (v-2)/2$ and complement of 'a'is '$v-a$'.

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- $A = [1, 2, ..., m, m+1]$ will produce MCSGND-I for $v = 2ik - 2$ if sum of elements in A is divisible by v. If not, replace one or more elements with their complements to make the sum divisible by v.
- Either $A = [1, 2, ..., m, m+1, m]$ or $A = [1, 2, ..., m, m+1, m-1]$ will produce MCSGND-II for $v = 2ik - 4$ if sum of elements in A is divisible by v. If not, replace one or more elements with their complements to make the sum divisible by v.

Divide the resultant elements of A in i groups of size k such that the sum of each group should be divisible of v. Then delete one element (any) from each group, resulting will be i sets of shifts to generate MCSGNDs for the following cases of equal blocks sizes.

3 Constructors to construct MCSGNDs-I

Using the logic behind the Rule I, following constructors are developed in this Section for the existence of MCSGNDs-I for $v = 2ik - 2$.

Constructor 3.1: If $m = 4u + 2$ then i sets of shifts derived from $A = [0, 1, 2, ..., m, m+1]$ will produce MCSGND-I for $v = 2ik - 2$.

Proof: Let $S = [0 + 1 + 2, ... + m + (m+1)]$
$= (m+1)(m+2)/2$
$= v(m+2)/4.$ Since $2(m+1) = v$.
$(m+2)/4$ will be integer for m (mod 4) = 2.
S (mod v) = 0 if $(m+2)/4$ is integer. Hence proved.

Constructor 3.2: If $m = 4u + 3$ then i sets of shifts derived from $A = [0, 1, 2, ..., (m-3)/4, (m+5)/4, (m+9)/4, ..., m, (m+1), 7(m+1)/4]$ will produce MCSGND-I for $v = 2ik - 2$.

Proof: Let $S = 0 + 1 + 2 + ... + [(m-3)/4] + [(m+5)/4] + [(m+9)/4] + ... + m + (m+1) + [7(m+1)/4]$
$= 1 + 2 + ... + [(m-3)/4] + [(m+1)/4] + [(m+5)/4] + [(m+9)/4] + ... + m + (m+1) +$
$[7(m+1)/4[(m+1)/4]$
$= [1 + 2 + ... + m + (m+1)] + [7(m+1)/4] - [(m+1)/4]$
$= [(m+1)(m+2)/2] + 3(m+1)/2$
$= [(m+1)(m+2)/2] + 2(m+1) = (m+1)[(m+5)/2]$
$= 2(m+1)(m+5)/4 = v(m+5)/4$ Since $v = 2(m+1)$
$(m+5)/4$ will be integer for m (mod 4) = 3.
S (mod v) = 0 if $(m+5)/4$ is integer. Hence proved.

4 Constructors to construct MCSGNDs-II

Using the logic behind the method of cyclic shifts (Rule I), following theorems are developed in this Section for the existence of MCSGNDs-II for $v = 2ik - 4$.

Constructor 4.1: If $m = 4u + 1$ then i sets of shifts derived from $A = [0, 1, 2, ..., (m-5)/4, (m+3)/4, (m+7)/4, ..., m, (m+1), m, (7m+9)/4]$ produce MCSGND-II for $v = 2ik - 4$.

Proof: Let $S = 0 + 1 + 2+, ... + [(m-5)/4] + [(m+3)/4] + [(m+7)/4] + ... + m + (m+1) + m + [(7m+9)/4]$
$= 1 + 2 + ... + [(m-5)/4] + [(m-1)/4] + [(m+3)/4] + [(m+7)/4] + ... + m + (m+1) + m +$
$[(7m+9)/4] - [(m-1)/4]$
$= [1 + 2, +m + (m+1)] + m + [(7m+9)/4] - [(m-1)/4]$
$= [(m+1)(m+2)/2] + m + (3m+5)/2$
$= 2(m+1)(m+7)/4,$ Since $v = 2(m+1)$
$= v(m+7)/4.$
$(m+7)/4$ will be integer for m (mod 4) = 1.
S (mod v) = 0 if $(m+7)/4$ is integer. Hence proved.

Following are generators obtained from Constructor 4.1 to generate the MCSGND-II for $v \leq 50$.

- $[0,2,3,4,5,6,5,11]$ for $v = 12$ and $m = 5$.
- $[0,1,3,4,5,6,7,9,10,9,18]$ for $v = 20$ and $m = 9$.
- $[0,1,2,4,5,\ldots,13,14,13,25]$ for $v = 28$ and $m = 13$.
- $[0,1,2,3,5,6,17,18,17,32]$ for $v = 36$ and $m = 17$.
- $[0,1,2,3,4,6,\ldots,21,22,21,39]$ for $v = 44$ and $m = 21$.

Constructor 4.2: If $m = 4u+2$ then i sets of shifts derived from $A = [0,1,2,\ldots,(m-2)/2,(m+2)/2,(m+4)/2,\ldots,m,(m+1),m,(3m+4)/2]$ will produce MCSGND-II for $v = 2ik-4$.

Proof: Let $S = 0+1+2+,\ldots+[(m-2)/2]+[(m+2)/2]+[(m+4)/2]+\ldots+m+(m+1)+m+[(3m+4)/2]$

$$= 1+2+\ldots+[(m-2)/2]+[m/2]+[(m+2)/2]+[(m+4)/2]+\ldots+m+(m+1)+m+[(3m+4)/2]-[m/2]$$
$$= [1+2,\ldots+m+(m+1)]+m+[(3m+4)/2]-[m/2]$$
$$= [(m+1)(m+2)/2]+m+(m+2)$$
$$= [(m+1)(m+2)/2]+2(m+1) = (m+1)[(m+6)/2]$$
$$= 2(m+1)(m+6)/4, \qquad \text{Since } v = 2(m+1)$$
$$= v(m+6)/4$$

$(m+6)/4$ will be integer for m (mod 4) = 2.

S (mod v) = 0 if $(m+6)/4$ is integer. Hence proved.

Following are generators obtained from Constructor 4.2 to generate the MCSGND-II for $v \leq 50$.

- $[0,1,2,4,5,6,7,6,11]$ for $v = 14$ and $m = 6$.
- $[0,1,2,3,4,6,7,9,10,11,10,17]$ for $v = 22$ and $m = 10$.
- $[0,1,2,\ldots,6,8,9,\ldots,14,15,14,23]$ for $v = 30$ and $m = 14$.
- $[0,1,2,\ldots,8,10,11,\ldots,18,19,18,29]$ for $v = 38$ and $m = 18$.
- $[0,1,2,\ldots,10,12,13,\ldots,22,23,22,35]$ for $v = 46$ and $m = 22$.

Constructor 4.3: If $m = 4u+3$ then i sets of shifts derived from $A = [0,1,2,\ldots,(3m-5)/4,(3m+3)/4,(3m+7)/4,\ldots,m-1,m,m+1,m-1,(5m+9)/4]$ will produce MCSGND-II for $v = 2ik-4$.

Proof: Let $S = 0+1+2+,\ldots+[(3m-5)/4]+[(3m+3)/4]+[(3m+7)/4]+\ldots+m-1+m+m+1+m-1+[(5m+9)/4]$

$$= 1+2+,\ldots+[(3m-5)/4]+[(3m-1)/4]+[(3m+3)/4]+[(3m+7)/4]+\ldots+m-1+m+m+1+m-1+[(5m+9)/4]-[(3m-1)/4$$
$$= [1+2,\ldots+m+(m+1)]+m-1+[(5m+9)/4]-[(3m-1)/4]$$
$$= [(m+1)(m+2)/2]+m-1+(2m+10)/4$$
$$= [(m+1)(m+2)/2]+3(m+1)/2 = (m+1)[(m+5)/2]$$
$$= 2(m+1)(m+5)/4 \qquad = v(m+5)/4 \qquad \text{Since } v = 2(m+1)$$

$(m+5)/4$ will be integer for m (mod 4) = 3.

S (mod v) = 0 if $(m+5)/4$ is integer. Hence proved.

Following are generators obtained from Constructor 4.3 to generate the MCSGND-II for $v \leq 50$.

- $[0,1,2,3,4,6,7,6,7,8,6,11]$ for $v = 16$ and $m = 7$.
- $[0,1,2,\ldots,7,9,10,11,12,10,16]$ for $v = 24$ and $m = 11$.
- $[0,1,2,\ldots,10,12,13,14,15,16,14,21]$ for $v = 32$ and $m = 15$.
- $[0,1,2,\ldots,13,15,16,17,18,19,20,18,26]$ for $v = 40$ and $m = 19$.
- $[0,1,2,\ldots,16,18,19,,22,23,24,22,31]$ for $v = 48$ and $m = 23$.

Constructor 4.4: If $m = 4u$ then i sets of shifts derived from $A = [0,1,2,\ldots,m-1,m-1,m+1,m+2]$ will produce MCSGND-II for $v = 2ik-4$.

Proof: Let $S = [0+1+2+,\ldots+m-1]+[m-1+m+1+m+2]$

$$= [(m-1)m/2]+[3m+2]$$
$$= (m2+5m+4)2$$
$$= (m+1)(m+4)/2$$
$$= 2(m+1)(m+4)/4 = v(m+4)/4 \qquad \text{Since } v = 2(m+1)$$

$(m+4)/4$ will be integer for m (mod 4) = 0.

S (mod v) = 0 if $(m+4)/4$ is integer. Hence proved.

Following are generators obtained from Constructor 4.4 to generate the MCSGND-II for $v \leq 50$.
- $[0,1,2,3,3,5,6]$ for $v = 10$ and $m = 4$.
- $[0,1,2,3,4,5,6,7,7,9,10]$ for $v = 18$ and $m = 8$.
- $[0,1,2,3,4,5,6,7,8,9,10,11,11,13,14]$ for $v = 26$ and $m = 12$.
- $[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,15,17,18]$ for $v = 34$ and $m = 16$.
- $[0,1,2,...,18,19,19,21,22]$ for $v = 42$ and $m = 20$.
- $[0,1,2,...,22,23,23,25,26]$ for $v = 50$ and $m = 24$.

5 Conclusion

In this article, (i) two constructors are developed to obtain generators for MCSGNDs-I. In these designs, only $v/2$ unordered pairs of distinct treatments appear twice as neighbors while the remaining ones appear once. (ii) four constructors are developed to obtain generators for MCSGNDs-II. In these designs, $3v/2$ unordered pairs of distinct treatments appear twice as neighbors while the remaining ones appear once. Through these constructors, complete solution for MCSGNDs-I and MCSGNDs-II can be obtained in blocks of equal and unequal sizes.

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