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Neutrosophic Non-linear Regression based on Kuhn-Tucker Necessary Conditions

Mahmoud A. Abo-Sinna[1] *and Nagwa G. Ragab*[2,*]

[1]Department of Basic Science, Faculty of Engineering, Badr University in Cairo BUC, Cairo, Egypt

[2] Department of Mathematics and Statistics, Future Academy Higher Institute of Specialized Technologies, Cairo, Egypt

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Abstract: Correlation coefficient and regression analysis are the most applied statistical approaches accessible in numerous disciplines due to its applicability and relevance. The neutrosophic sets found their place into recent research, whereas the world is full of indeterminacy. Kuhn-Tucker's necessary conditions are used to obtain the estimated parameters for non-linear regression models. This estimation procedure can use for any data set of non-linear regression models. We present, in this paper, the concepts of neutrosophic correlation and non-linear regression based on Kuhn-Tucker's necessary conditions. we provide some comparative studies between single-valued neutrosophic set and interval-valued neutrosophic set. Next, we apply scoring methods by different research. Numerical example is given to explain the result presented in this study. The results showed that the proposed approach can yield a fitting curve for any data set in neutrosophic environment.

Keywords: Correlation Coefficient, Fuzzy Set, Single-Valued Neutrosophic Set, Interval-Valued Neutrosophic Set, Neutrosophic Non-linear Regression, Kuhn-Tucker Necessary Conditions.

1 Introduction

The idea about fuzzy set was presented through Zadeh [1] as an extension of a standard set. In fuzzy set hypothesis, membership of a component to a fuzzy set is a single value somewhere in the range of zero and one. From that point forward, fuzzy sets and fuzzy logic are utilized closely in several implementation concerning vagueness. Yet, it has been seen that there stay a few circumstances that cannot be spread by fuzzy sets. Although the fuzzy set theory is a success in manipulating the uncertainties arising from the ambiguity of the element in a set, it is difficult to treat all kinds of uncertainties in the actual applications involving inadequate data. Consequently, the generalization of fuzzy set has been introduced by Atanassov [2] as an intuitionistic fuzzy set (IFS), that is handier in real-life problems. In IFS, as opposed to one membership degree, there is additionally a non-membership degree append to every component.

Moreover, there is a limitation that the total of these two degrees is less or to unity. IFS handles insufficient data, i.e., the degree of membership and non-membership functions, yet not the indeterminate and conflicting data that happens clearly in faith framework. Neutrosophic set (NS) was proposed by Smarandache [3-10], that is the generalization of intuitionistic fuzzy sets. NS is a good technique to contract with incomplete, indeterminate, and conflicting data that exists in real-life problems. NS is described by truth (T), indeterminacy (I), and falsity (F) membership functions.

Wang et al. [11] presented idea of a single-valued neutrosophic set (SVNS). SVNS can independently express truth, indeterminacy, and falsity membership degrees. All the variables that are characterized in SVNS are truly sensible for human reasoning because deficiency of information that human receives observes from reality concerning the world. Wang et al. [12] proposed idea about an interval-valued neutrosophic set (IVNS), and that is an extension of the SVNS. An IVNS, which represent the uncertain and incomplete information that exists in the real situation.

*Corresponding author e-mail: drnagwa.r@gmail.com, Nagwa.gamal@fa-hists.edu.eg

Thong et al. [13] developed some operators and a TOPSIS method to deal with the change of both criteria, alternatives, and decision-makers by time based on an extension of dynamic IVNS. Can et al. [14] presented a new method to classify harmful domain names using Neutrosophic Sets. Jha et al. [15] proposed a novel approach for image segmentation using neutrosophic sets, and they used Min-Max normalization to reduce uncertain noises. Dat et al. [16] introduced new concepts: single-valued linguistic complex neutrosophic set (SVLCNS-2) and interval linguistic complex neutrosophic set (ILCNS-2) that are more applicable and adjustable to real-world implementation than those of their previous counterparts. Son et al. [17] built metrics on the space of single-valued neutrosophic numbers induced from Hamming distance, and they defined some backgrounds on the limit, derivative and integral of single-valued neutrosophic functions with some applications. Basset et al. [18] proposed a novel framework based on computer supported diagnosis and Internet of Things to detect and monitor heart failure infected patients, where the data are attained from various other sources, and they validated the proposed model by numerical examples on real case studies. Dey et al. [19] presented a new genetic algorithm is to solve the minimum spanning tree with undirected connected weighted interval type 2 fuzzy graph (FMST-IT2FS) problem with used the addition, ranking and defuzzification of interval type 2 fuzzy set (IT2FSs) together with some Illustrative examples. Son et al. [20] studied the optimal control problem with operating a fractional differential equations and partial differential equations at minimum quadratic objective function in the framework of neutrosophic environment and granular computing, and they investigated with some key applications. Son et al. [21] introduced the concept of controllability and stabilizability of linear time invariant (LTI) systems containing neutrosophic uncertainty in the sense of both indeterminacy parameters and functional relationships with illustrated some numerical examples. Mohanta et al. [22] introduced a model of m-polar neutrosophic graph, which is applied in evaluating the teacher's performance of a college with illustrated numerical example.

Statistical regression analysis is notable for revaluation of the functional relationship between a dependent and one or a set of independent variables utilizing a set of observations based on normal distributions. The traditional methodology in regression analysis has relied upon the crisp information and the relationship between dependent and independent variables. Though, on account of loose information, or if the considered phenomenon has a vague variability rather than of stochastic variability, it is by all accounts a more normal approach to suppose a fuzzy relationship [23].

Besides, there are numerous circumstances in the actual applications where the perceptions cannot be estimated as crisp amounts in order that the data is often imprecise, inadequate, or unclear. Along these lines, fuzzy designing approaches give suitable methods for managing those different kinds of uncertain data [24,25]. Fuzzy regression examination, as a non-statistical strategy, did not depend on probability hypothesis however possibility and fuzzy set hypotheses [1,26]. Thus, the fuzzy regression model does not have fault terms; rather, they are included in the fuzzy coefficients. Hose et al. [27] presented a novel methodology for fuzzy solution of linear least-squares based on concepts from possibility theory. Chen et al. [28] proposed the new operator called the fuzzy product core for creating fuzzy linear regression models with fuzzy parameters utilizing fuzzy perceptions and together with the fuzzy input and output variables.

Due to strength of nonlinearity in various real-world situations, consideration has been dedicated into non-linear regression models. The concept of non-linear regression is equivalent to that of linear regression. The simple fact characterizes non-linear regression that the prediction equation relies upon non-linearly on one or more obscure boundaries. An introduction to non-linear regression is provided by Ratkowsky and Bates & Watts [29,30].

In contrast to fuzzy models, there are some approaches for handling fuzzy non-linear regression problems, as Celmins [31] suggested a practical modeling method for non-linear fuzzy regression, using the sort of information and parameter fuzziness to conical membership functions. Guo et al. [32] presented upper and lower regression models (linear and non-linear dual possibilistic models) to information examination with crisp inputs and fuzzy outputs. Pourahmada et al. [33] used transformation of non-linear fuzzy regression model, i.e., the fuzzy strategic regression for clinical examinations dependent on least-squares approach. Rolan et al. [34] recommended another strategy where fuzzy regression was created based on a minimum criterion by utilizing diverse known distances between fuzzy numbers; the technique applied to a non-linear issue in biomass creation. Su et al. [35] utilized a technique of the Kernel into fuzzy regression to manage with non-linear regression issues where inputs and model structure boundaries were crisp, however, the output was fuzzy; they applied their methodology into observing unmeasured boundaries s in a power plant problem. Gaeta et al. [36] focused on fuzzy non-linear regression, which deals with real inputs and fuzzy output by adjusting the fuzzy functional network.

In a statistical study, the correlation coefficient plays a vital function in estimating the strong connection between two factors. As it is specified on the crisp sets have been examined, it is also popular in theory of fuzzy sets to discover the correlation between two variables in a fuzzy environment. Gerstenkorn et al. [37] characterized correlation coefficient of intuitionistic fuzzy numbers in limited set. Yu [38] explained the correlation coefficient of fuzzy numbers whose supports were included in a closed interval. Hong et al. [39] presented correlation coefficient of intuitionistic fuzzy numbers within a probability space. Salma et al. [40] introduced the thought of positively and negatively correlated. They utilized idea about centroid with giving meaning to a correlation coefficient of popularized intuitionistic fuzzy sets. Salma et al. [41] specified the correlation coefficient of neutrosophic fuzzy numbers in a finite set. Salma et al. [42] utilized idea of centroid to characterize the correlation coefficient of neutrosophic sets. Salma et al. [43] described the correlation coefficient of neutrosophic fuzzy numbers in a probability space. Likewise, N. Chukhrova et al. [44] presented a complete survey and bibliography about regression analysis in fuzzy conditions.

Kuhn-Tucker's necessary conditions were used to obtain the estimated parameters of quadratic and non-linear regression models. This estimation procedure can use for any data set of quadratic and non-linear regression models. Kasem et al. [45] introduced a new estimation method for the parameters in a non-linear regression model based on Kuhn-Tucker's necessary conditions. They offered some statistical studies such as autocorrelation of the error values.

Throughout this study, we present and discuss the concept of correlation coefficient in neutrosophic sets. Also, we consider the neutrosophic non-linear regression model based on Kuhn-Tucker's necessary conditions and possible application to data processing touched. The paper is arranged by the following procedure. The section 1 is an introduction, the section 2 formally introduces the brief fundamental concepts. The section 3 is built up the estimation of the parameters to quadratic regression using Kuhn-Tucker's necessary conditions. The section 4 is devoted to the neutrosophic quadratic regression model. In the section 5, numerical example is provided to explain the result of this study. Section 6 gives the summary of this study and intended future works.

2 Fundamental concept

In the following, we briefly describe some basic definitions and operational laws related to neutrosophic set.

Definition 2.1 [1] Let X be the universe set. A fuzzy set (FS) $\tilde{A}$ in X is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x\epsilon X\} \text{ where } \mu_{\tilde{A}}(x): X \longrightarrow [0,1]$$

characterizes grade of membership function of the component $x\epsilon X$ to the set $\tilde{A}$.

Definition 2.2 [2] Let X be the universe set. An intuitionistic fuzzy set (IFS) $\tilde{A}^i$ in X is a set having the structure

$$\tilde{A}^i = \left\{\left(x,\ \mu_{\tilde{A}^i}(x),\ \upsilon_{\tilde{A}^i}(x)\right), x\epsilon X\right\}$$

where: $\mu_{\tilde{A}^i}(x): X \longrightarrow [0,1]$, and $\upsilon_{\tilde{A}^i}(x): X \longrightarrow [0,1]$define the degree functions of membership and non-membership respectively of component to the set, that $0 \leq \mu_{\tilde{A}^i}(x) + \upsilon_{\tilde{A}^i}(x) \leq 1$ for each $x\epsilon X$.

Now for each element $x\epsilon X$, that value of $\pi_{\tilde{A}^i}(x) = 1 - \mu_{\tilde{A}^i}(x) - \upsilon_{\tilde{A}^i}(x)$ is called the degree of uncertainty of the element $x\epsilon X$ to the intuitionistic fuzzy set $\tilde{A}^i$.

Definition 2.3 [3] Let X be the universe set. The neutrosophic set $\tilde{A}^N$ in X is characterized by three membership functions called a truth membership function $\mu_{\tilde{A}^N}(x)$, an indeterminacy membership function $\gamma_{\tilde{A}^N}(x)$, and a falsity membership function $\upsilon_{\tilde{A}^N}(x)$ and having of the form

$$\tilde{A}^N = \{\langle X: \mu_{\tilde{A}^N}(x),\ \gamma_{\tilde{A}^N}(x), \upsilon_{\tilde{A}^N}(x)\rangle, x\epsilon X\}$$

Where: $\mu_{\tilde{A}^N}(x)$, $\gamma_{\tilde{A}^N}(x), \upsilon_{\tilde{A}^N}(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ i.e. $\mu_{\tilde{A}^N}(x): X \longrightarrow]0^-, 1^+[$, $\gamma_{\tilde{A}^N}(x): X \longrightarrow]0^-, 1^+[$, and $\upsilon_{\tilde{A}^N}(x): X \longrightarrow]0^-, 1^+[$; there is no restriction on the $\mu_{\tilde{A}^N}(x)$, $\gamma_{\tilde{A}^N}(x)$, and$\upsilon_{\tilde{A}^N}(x)$, other than they are subsets of]- 0, 1+ [. So, $0^- \leq sup\ \mu_{\tilde{A}^N}(x) + sup\ \gamma_{\tilde{A}^N}(x) + sup\ \upsilon_{\tilde{A}^N}(x) \leq 3^+$.

Definition 2.4 [3,11] Let X be the universe set. A single-valued neutrosophic set (SVNS) $\tilde{A}^N$ in X is characterized by three membership functions called a truth membership function $\mu_{\tilde{A}^N}(x)$, an indeterminacy membership function $\gamma_{\tilde{A}^N}(x)$, and a falsity membership function $\upsilon_{\tilde{A}^N}(x)$ and having of the form

$$\tilde{A}^N = \{\langle X: \mu_{\tilde{A}^N}(x), \gamma_{\tilde{A}^N}(x), \upsilon_{\tilde{A}^N}(x)\rangle, x\epsilon X\}$$

where: $\mu_{\tilde{A}^N}(x): X \longrightarrow [0,1]$, $\gamma_{\tilde{A}^N}(x): X \longrightarrow [0,1]$, and $\upsilon_{\tilde{A}^N}(x): X \longrightarrow [0,1]$, satisfy the following condition:

$$0 \leq sup\ \mu_{\tilde{A}^N}(x) + sup\ \gamma_{\tilde{A}^N}(x) + sup\ \upsilon_{\tilde{A}^N}(x) \leq 3 \text{ for each } x\epsilon X.$$

Definition 2.5 [12] Let X be the universe set. An interval-valued neutrosophic set (IVNS) $\tilde{A}^N$in X is defined as an object of the form $\tilde{A}^N = \{\langle X:\ \mu_{\tilde{A}^N}(x),\ \gamma_{\tilde{A}^N}(x), \upsilon_{\tilde{A}^N}(x)\rangle, x\epsilon X\}$

where: $\mu_{\tilde{A}^N} = \left[\mu_{\tilde{A}^N}{}^L, \mu_{\tilde{A}^N}{}^U\right]$, $\gamma_A = \left[\gamma_{\tilde{A}^N}{}^L, \gamma_{\tilde{A}^N}{}^U\right]$, and $\upsilon_A = \left[\upsilon_{\tilde{A}^N}{}^L, \upsilon_{\tilde{A}^N}{}^U\right]$ define lower and upper membership functions (truth (T), indeterminacy (I), and falsity (F), respectively) for every component $x \in X$, $\mu_{\tilde{A}^N}{}^L, \mu_{\tilde{A}^N}{}^U, \gamma_{\tilde{A}^N}{}^L, \gamma_{\tilde{A}^N}{}^U, \upsilon_{\tilde{A}^N}{}^L, \upsilon_{\tilde{A}^N}{}^U: X \longrightarrow [0,1]$.

To rank the SVNS, Smarandache [11] described the following definition:

Definition 2.6 [10] Let $\tilde{A}^N = \langle \mu, \gamma, \upsilon \rangle$ be a single-valued neutrosophic set (SVNS). Then, the score function $s(\tilde{A}^N)$, accuracy function $a(\tilde{A}^N)$, and certainty function $c(\tilde{A}^N)$ of SVNS are defined as follows:

i. $s(\tilde{A}^N) = \frac{2+\mu-\gamma-\upsilon}{3}$.
ii. $a(\tilde{A}^N) = \mu - \upsilon$.
iii. $c(\tilde{A}^N) = \mu$.

Smarandache [10] gave an order relation between two SVNS, which is defined as follows:

Definition 2.7 [10] Let $\tilde{A}_1 = \langle \mu_1, \gamma_1, \upsilon_1 \rangle$ and $\tilde{A}_2 = \langle \mu_2, \gamma_2, \upsilon_2 \rangle$ are two SVNS. Then the ranking method is defined as follows:

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.
ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) \succ a(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.
iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, $c(\tilde{A}_1) \succ c(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.
iv. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, $c(\tilde{A}_1) = c(\tilde{A}_2)$, then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$.

To rank the IVNS, Smarandache [10] and Tang [47] and described the following definitions.

Definition 2.8 [10] Let $\tilde{A}^N = \langle \mu, \gamma, \upsilon \rangle$ be an interval-valued neutrosophic set (IVNS), where $\mu = [\mu^L, \mu^U]$, $\gamma = [\gamma^L, \gamma^U]$, $\upsilon = [\upsilon^L, \upsilon^U]$Then, the score function $s(\tilde{A}^N)$, and accuracy function $a(\tilde{A}^N)$ of IVNS are defined as follows:

i. $s(\tilde{A}^N) = \frac{(2+\mu^L+\mu^U-2\gamma^L-2\gamma^U-\upsilon^L-\upsilon^U)}{4}, s(\tilde{A}^N) \in [-1,1]$.
ii. $a(\tilde{A}^N) = \frac{(\mu^L+\mu^U-\gamma^U(1-\mu^U)-\gamma^L(1-\mu^L)-\upsilon^U(1-\gamma^U)-\upsilon^L(1-\gamma^L))}{2}, a(\tilde{A}^N) \in [-1,1]$.

Definition 2.9 [46] Let $\tilde{A}^N = \langle \mu, \gamma, \upsilon \rangle$ be an interval-valued neutrosophic set (IVNS), where $\mu = [\mu^L, \mu^U]$, $\gamma = [\gamma^L, \gamma^U]$, $\upsilon = [\upsilon^L, \upsilon^U]$Then, the score function $s(\tilde{A}^N)$, and accuracy function $a(\tilde{A}^N)$ of IVNS are defined as follows:

i. $s(\tilde{A}^N) = \frac{(2+\mu^L-\gamma^L-\upsilon^L)+(2+\mu^U-\gamma^U-\upsilon^U)}{6}, s(\tilde{A}^N) \in [0,1]$.
ii. $a(\tilde{A}^N) = \frac{(\mu^L-\upsilon^L)-(\mu^U-\upsilon^U)}{2}, a(\tilde{A}^N) \in [-1,1]$.

Smarandache [10] and Tang [46] gave an order relation between two IVNS, which is defined as follows:

Definition 2.10 [10], [46] Let $\tilde{A}_1 = \langle \mu_1, \gamma_1, \upsilon_1 \rangle$ and $\tilde{A}_2 = \langle \mu_2, \gamma_2, \upsilon_2 \rangle$ are two IVNS. Then the ranking method is defined as follows:

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.
ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) \succ a(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.
iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$.

Definition 2.11 In neutrosophic mathematics. An interval-valued neutrosophic number $\tilde{A}^N = \langle [\mu^L, \mu^U], [\gamma^L, \gamma^U], [\upsilon^L, \upsilon^U] \rangle$ is said to be empty if and only if: $\mu^L = \mu^U = 0$, $\gamma^L = \gamma^U = 1$, and $\upsilon^L = \upsilon^U = 1$ and is represented by: $\tilde{0}^N = \{\langle x, [0,0], [1,1], [1,1] \rangle : x \in X\}$.

Definition 2.12 Let $\tilde{X} = \{\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n\}$ and $\tilde{Y} = \{\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n\}$ be two neutrosophic sets defined within a limited space. At that point, the correlation coefficient of neutrosophic sets $\tilde{X}$ and $\tilde{Y}$ is characterized as:

$$\rho = \frac{C(\tilde{X}, \tilde{Y})}{\sqrt{C(\tilde{X}, \tilde{X})}\sqrt{C(\tilde{Y}, \tilde{Y})}}$$

Where: $C(\tilde{X}, \tilde{X}) = \sum_{i=1}^{n}[\mu^2(\tilde{x}_i) + \gamma^2(\tilde{x}_i) + \upsilon^2(\tilde{x}_i)]$, $C(\tilde{Y}, \tilde{Y}) = \sum_{i=1}^{n}[\mu^2(\tilde{y}_i) + \gamma^2(\tilde{y}_i) + \upsilon^2(\tilde{y}_i)]$,

and $C(\tilde{X}, \tilde{Y}) = \sum_{i=1}^{n}[\mu(\tilde{x}_i)\,\mu(\tilde{y}_i) + \gamma(\tilde{x}_i)\,\gamma(\tilde{y}_i) + \upsilon(\tilde{x}_i)\upsilon(\tilde{y}_i)]$.

Definition 2.13 [47] Let $f_i: R^n \longrightarrow R, g_i: R^n \longrightarrow R$, and $S = \{x \in R^n: g_i \leq 0\}$, a feasible solution $x \in S$ is said to satisfy Kuhn-Tucker's necessary conditions for optimality if:

i. All f_i and g_i are differentiable, and $S \neq \Phi$; and
ii. There exists $u_i \geq 0, i = 1,2, \dots, n$, which accurate inequality holding for at least one i and $v_j \geq 0,\ j = 1,2, \dots, m$, so that: $g_j(x) \leq 0,\ v_j g_j(x) = 0\ \ (j = 1, 2, \dots, m)$, and $\sum_{i=1}^{n} u_i \nabla f_i(x) + \sum_{j=1}^{m} v_j \nabla g_j(x) = 0$

3 Estimation the parameters of quadratic regression using Kuhn-Tucker necessary conditions (KTNC) [45]

A quadratic regression model is the type of non-linear regression model, which is the one including a quadratic term. A quadratic regression model which study by using KTNC take the following form:

$$Y_i = A_1 X_i + A_2\, {X_i}^2 + \varepsilon_i\,, \quad i = 1, 2, \dots, n \tag{3.1}$$

Where Y_i , X_i , and (A_1, A_2) are vectors of output variables, input variables, and regression parameters to be estimated respectively, also ε_i is an arbitrary error, thought to be normally distributed with expectation zero and variance $\sigma^2 : \varepsilon_i \approx N(0, \sigma^2\,)$. We assumed that the domain of input and output variables is universe of discourse. The objective of this part is to build up a new methodology that can always produce regression curve estimators for the quadratic model (3.1) using KTNC. Equation (3.1) written in the following form:

$$\min_{\substack{s.t.\\ \sum_{i=1}^{n}\left[Y_i-(A_1X_i+A_2\,{X_i}^2)\right]^2 \leq Q,\\ X_i \geq 0}} \qquad Q = \sum_{i=1}^{n} {\varepsilon_i}^2 \ \ , i = 1,2, \dots, n \tag{3.2}$$

From definition 2.13, the KTNC for this problem takes the form:

$$u_i\left[-2\sum_{i=1}^{n}\left[Y_i - (\hat{A}_1 X_i + \hat{A}_2\, {X_i}^2)\right]\ (\hat{A}_1 + 2\,\hat{A}_2\, X_i)\right] - \sum_{j=1}^{m} v_j\ X_j = 0, \tag{3.3}$$

$$\sum_{i=1}^{n} u_i = 1, \tag{3.4}$$

$$\sum_{i=1}^{n}\left[Y_i - (\hat{A}_1 X_i + \hat{A}_2\, {X_i}^2)\right]^2 \leq 0, \tag{3.5}$$

$$-\sum_{j=1}^{m} v_j\ X_j = 0, \tag{3.6}$$

$$X_j \geq 0, \tag{3.7}$$

$$u_i\left[\sum_{i=1}^{n}\left[Y_i - (\hat{A}_1 X_i + \hat{A}_2\, {X_i}^2)\right]^2\right] = 0, \tag{3.8}$$

$$u_i \geq 0, \qquad i = 1,2, \dots, n, \tag{3.9}$$

$$v_j \geq 0, \qquad j = 1, 2, \dots, m. \tag{3.10}$$

Parameters $\hat{A}_1, \hat{A}_2$ can be determined depending on the obtained values of u_i which give as follows:
From equation (3.5), we get:

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} (\hat{A}_1 X_i + \hat{A}_2\, {X_i}^2). \tag{3.11}$$

Therefore,

$$\hat{A}_1 = \frac{\bar{Y}}{\bar{X}} - \frac{\hat{A}_2}{n\bar{X}} \sum_{i=1}^{n} X_i^{\,2} . \quad (3.12)$$

All possible probabilities of u_i and v_j values are rejected except the following case, which represents the optimal solution. If $u_i > 0, v_j = 0$ from (3.3), we obtain:

$$\sum_{i=1}^{n} \left[Y_i \left(\hat{A}_1 + 2\, \hat{A}_2\, X_i \right) - \left(\hat{A}_1 X_i + \hat{A}_2\, X_i^{\,2} \right) \left(\hat{A}_1 + 2\, \hat{A}_2\, X_i \right) \right] = 0. \quad (3.13)$$

By using equations (3.12) and (3.13) and after simplifying, we get:

$$\hat{A}_2 = \frac{\sum_{i=1}^{n} X_i\, Y_i - \frac{\bar{Y}}{\bar{X}} \sum_{i=1}^{n} X_i^{\,2}}{\sum_{i=1}^{n} X_i^{\,3} - \frac{\left(\sum_{i=1}^{n} X_i^{\,2}\right)^2}{n\,\bar{X}}} . \quad (3.14)$$

Hence, we conclude that the parameters $\hat{A}_1, \hat{A}_2$ gave in equations (3.12), (3.14).

4 Neutrosophic Quadratic Regression

In this part, we consider quadratic regression model in the neutrosophic set, which denoted by:

$$\hat{\tilde{Y}}_i = \hat{A}_1 \tilde{X}_i + \hat{A}_2\, \tilde{X}_i^{\,2} , \quad i = 1, 2, \dots, n \quad (4.1)$$

In this model, it assumed that input and output variables are neutrosophic sets with domain universe of discourse. Let $\tilde{X}$ be space of elements with regular components denoted $\tilde{x}$. A neutrosophic set is expressed by the degree functions of truth $\mu(\tilde{x})$, indeterminacy $\gamma(\tilde{x})$, and falsity $\upsilon(\tilde{x})$. To each point $\tilde{x}$ in $\tilde{X}$, $\mu(\tilde{x}),\ \gamma(\tilde{x}),\ \upsilon(\tilde{x}) \in [0,1]$. A neutrosophic set $\tilde{X}$ can write as $\{\langle \mu(\tilde{x}),\ \gamma(\tilde{x}),\ \upsilon(\tilde{x})\rangle, \tilde{x} \in \tilde{X}\}$.

Also, let $\tilde{Y}$ be the space of elements with nonexclusive components denoted $\tilde{y}$. A neutrosophic set is expressed by the degree functions of truth $\mu(\tilde{y})$, indeterminacy $\gamma(\tilde{y})$, and falsity $\upsilon(\tilde{y})$. to each point $\tilde{y}$ in $\tilde{Y}$, $\mu(\tilde{y}),\ \gamma(\tilde{y}),\ \upsilon(\tilde{y}) \in [0,1]$. A neutrosophic set $\tilde{Y}$ can write as $\{\langle \mu(\tilde{y}),\ \gamma(\tilde{y}),\ \upsilon(\tilde{y})\rangle, \tilde{y} \in \tilde{Y}\}$. The parameter estimation method, so-called Kuhn-tucker necessary conditions define the neutrosophic coefficients $(\hat{A}_1, \hat{A}_2)$ as equations (3.12) and (3.14) given by:

$$\hat{A}_1 = \frac{\bar{\tilde{Y}}}{\bar{\tilde{X}}} - \frac{\hat{A}_2}{n\bar{\tilde{X}}} \sum_{i=1}^{n} \tilde{X}_i^{\,2} , \qquad \hat{A}_2 = \frac{\sum_{i=1}^{n} \tilde{X}_i\, \tilde{Y}_i - \frac{\bar{\tilde{Y}}}{\bar{\tilde{X}}} \sum_{i=1}^{n} \tilde{X}_i^{\,2}}{\sum_{i=1}^{n} \tilde{X}_i^{\,3} - \frac{\left(\sum_{i=1}^{n} \tilde{X}_i^{\,2}\right)^2}{n\,\bar{\tilde{X}}}} .$$

Where: $\bar{\tilde{X}} = \frac{1}{3}\left[\bar{\mu}(\tilde{x}_i) + \bar{\gamma}(\tilde{x}_i) + \bar{\upsilon}(\tilde{x}_i)\right]$, $\bar{\tilde{Y}} = \frac{1}{3}\left[\bar{\mu}(\tilde{y}_i) + \bar{\gamma}(\tilde{y}_i) + \bar{\upsilon}(\tilde{y}_i)\right]$,

$\bar{\mu}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \mu(\tilde{x}_i)$, $\bar{\gamma}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma(\tilde{x}_i)$, $\bar{\upsilon}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \upsilon(\tilde{x}_i)$,

$\bar{\mu}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \mu(\tilde{y}_i)$, $\bar{\gamma}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma(\tilde{y}_i)$, $\bar{\upsilon}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \upsilon(\tilde{y}_i)$.

5 Numerical Example

In this part, the numerical example is used to clarify the suggested approach. This model adopts the data set of an investment company includes two variables $\tilde{X}$(cost) and $\tilde{Y}$ (revenue). This data set includes ten views (to develop): Vehicles, FMCGs, Electronics, Army supplies, medical supplies, pharmaceutical products, Retail, Oil, Constructions, and IT solutions. It takes into consideration three requirements: risk control capacity, growth potential, and the environment impact.

5.1 First kind of observation

Each point in these observations is a single-valued neutrosophic set (SVNS) (see Table 1). In this example, we compute that:

$\bar{\mu}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \mu(\tilde{x}_i) = 0.42$, $\bar{\gamma}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma(\tilde{x}_i) = 0.26$, $\bar{\upsilon}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n} \upsilon(\tilde{x}_i) = 0.27$,

$\bar{\mu}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \mu(\tilde{y}_i) = 0.47$, $\bar{\gamma}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma(\tilde{y}_i) = 0.33$, $\bar{\upsilon}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n} \upsilon(\tilde{y}_i) = 0.22$,

$\bar{\tilde{X}} = \frac{1}{3}\left[\bar{\mu}(\tilde{x}_i) + \bar{\gamma}(\tilde{x}_i) + \bar{\upsilon}(\tilde{x}_i)\right] = 0.317$, $\bar{\tilde{Y}} = \frac{1}{3}\left[\bar{\mu}(\tilde{y}_i) + \bar{\gamma}(\tilde{y}_i) + \bar{\upsilon}(\tilde{y}_i)\right] = 0.34$.

Then the quadratic regression model in SVNS obtained using Kuhn-Tucker's necessary conditions is:

$$\hat{\tilde{Y}} = 0.91\,\tilde{X}_i + 0.14\,\tilde{X}_i^{\ 2}, \quad i = 1,2,\dots,10.$$

In this example, as definition 2.12, the correlation coefficient of SVNS $\tilde{X}$ and $\tilde{Y}$ is:

$\rho = \frac{C(\tilde{X},\tilde{Y})}{\sqrt{C(\tilde{X},\tilde{X})}\,\sqrt{C(\tilde{Y},\tilde{Y})}}$ where:

$C(\tilde{X},\tilde{X}) = \sum_{i=1}^{n}[\mu^2(\tilde{x}_i) + \gamma^2(\tilde{x}_i) + \upsilon^2(\tilde{x}_i)] = 3.71$, $C(\tilde{Y},\tilde{Y}) = \sum_{i=1}^{n}[\mu^2(\tilde{y}_i) + \gamma^2(\tilde{y}_i) + \upsilon^2(\tilde{y}_i)] = 4.34$,
and $C(\tilde{X},\tilde{Y}) = \sum_{i=1}^{n}[\mu(\tilde{x}_i)\,\mu(\tilde{y}_i) + \gamma(\tilde{x}_i)\,\gamma(\tilde{y}_i) + \upsilon(\tilde{x}_i)\upsilon(\tilde{y}_i)] = 3.61$.

Then we get the correlation coefficient is $\rho(\tilde{X},\tilde{Y}) = 0.89$, this value gives us the information which the SVNS $\tilde{X},\tilde{Y}$ positively and strictly related to strength 0.89.

Table 1. (Data set for first kind observation)

Obs.	$\tilde{X}$	$\tilde{Y}$	$\hat{\tilde{Y}}$
Vehicles	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.4, 0.3, 0.3\rangle$	$\langle 0.3864, 0.1876, 0.2856\rangle$
FMCGs	$\langle 0.2, 0.2, 0.5\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.1876, 0.1876, 0.49\rangle$
Electronics	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.5964, 0.1876, 0.1876\rangle$
Army	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.2856, 0.1876, 0.2856\rangle$
Medical	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.49, 0.2856, 0.1876\rangle$
Pharma.	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.5964, 0.1876, 0.2856\rangle$
Retail	$\langle 0.3, 0.5, 0.2\rangle$	$\langle 0.2, 0.6, 0.4\rangle$	$\langle 0.2856, 0.49, 0.1876\rangle$
Oil	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.49, 0.1876, 0.3864\rangle$
Const.	$\langle 0.7, 0.3, 0.1\rangle$	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.7056, 0.2856, 0.0924\rangle$
IT	$\langle 0.1, 0.3, 0.2\rangle$	$\langle 0.4, 0.5, 0.2\rangle$	$\langle 0.0924, 0.2856, 0.1876\rangle$

To rank the estimated revenue of SVNS, as definition 2.6 (see table 2).

Table 2. (The estimated revenue ranking for first kind observation)

Obs.	$\hat{\tilde{Y}}$	$s(\hat{\tilde{Y}})$	$a(\hat{\tilde{Y}})$	$c(\hat{\tilde{Y}})$
Vehicles	$\langle 0.3864, 0.1876, 0.2856\rangle$	0.637733	0.1008	0.3864
FMCGs	$\langle 0.1876, 0.1876, 0.49\rangle$	0.503333	−0.3024	0.1876
Electronics	$\langle 0.5964, 0.1876, 0.1876\rangle$	0.7404	0.4088	0.5964
Army	$\langle 0.2856, 0.1876, 0.2856\rangle$	0.604133	0	0.2856
Medical	$\langle 0.49, 0.2856, 0.1876\rangle$	0.672267	0.3024	0.49
Pharma.	$\langle 0.5964, 0.1876, 0.2856\rangle$	0.707733	0.3108	0.5964
Retail	$\langle 0.2856, 0.49, 0.1876\rangle$	0.536	0.098	0.2856
Oil	$\langle 0.49, 0.1876, 0.3864\rangle$	0.638667	0.1036	0.49
Const.	$\langle 0.7056, 0.2856, 0.0924\rangle$	0.775867	0.6132	0.7056
IT	$\langle 0.0924, 0.2856, 0.1876\rangle$	0.539733	−0.0952	0.0924

5.2 Second kind of observation

Each point in these observations is an interval-valued neutrosophic set (IVNS) (see Table 3). In this example, we compute that:

$\mu(\tilde{x}_i) = \frac{[\mu^L(\tilde{x}_i)+\mu^U(\tilde{x}_i)]}{2}$, $\gamma(\tilde{x}_i) = \frac{[\gamma^L(\tilde{x}_i)+\gamma^U(\tilde{x}_i)]}{2}$, $\upsilon(\tilde{x}_i) = \frac{[\upsilon^L(\tilde{x}_i)+\upsilon^U(\tilde{x}_i)]}{2}$,

$\bar{\mu}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n}\mu(\tilde{x}_i) = 0.405$, $\bar{\gamma}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n}\gamma(\tilde{x}_i) = 0.31$, $\bar{\upsilon}(\tilde{x}_i) = \frac{1}{n}\sum_{i=1}^{n}\upsilon(\tilde{x}_i) = 0.315$,

$\mu(\tilde{y}_i) = \frac{[\mu^L(\tilde{y}_i)+\mu^U(\tilde{y}_i)]}{2}$, $\gamma(\tilde{y}_i) = \frac{[\gamma^L(\tilde{y}_i)+\gamma^U(\tilde{y}_i)]}{2}$, $\upsilon(\tilde{y}_i) = \frac{[\upsilon^L(\tilde{y}_i)+\upsilon^U(\tilde{y}_i)]}{2}$,

$\bar{\mu}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n}\mu(\tilde{y}_i) = 0.45$, $\bar{\gamma}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n}\gamma(\tilde{y}_i) = 0.345$, $\bar{\upsilon}(\tilde{y}_i) = \frac{1}{n}\sum_{i=1}^{n}\upsilon(\tilde{y}_i) = 0.245$,

$\bar{\tilde{X}} = \frac{1}{3}\,[\bar{\mu}(\tilde{x}_i) + \bar{\gamma}(\tilde{x}_i) + \bar{\upsilon}(\tilde{x}_i)] = 0.343$, $\bar{\tilde{Y}} = \frac{1}{3}\,[\bar{\mu}(\tilde{y}_i) + \bar{\gamma}(\tilde{y}_i) + \bar{\upsilon}(\tilde{y}_i)] = 0.3467$.

Then the quadratic regression model in IVNS obtained using Kuhn-Tucker's necessary conditions is:

$\hat{\tilde{Y}}_i = 0.94\,\tilde{X}_i + 0.058\,\tilde{X}_i^{\,2}, \quad i = 1,2,\dots,10.$

In this example, as definition 2.12, the correlation coefficient of IVNS $\tilde{X}$ and $\tilde{Y}$ is:

$\rho = \frac{C(\tilde{X},\tilde{Y})}{\sqrt{C(\tilde{X},\tilde{X})}\,\sqrt{C(\tilde{Y},\tilde{Y})}}$ where:

$C(\tilde{X},\tilde{X}) = \sum_{i=1}^{n}[\mu^2(\tilde{x}_i) + \gamma^2(\tilde{x}_i) + \upsilon^2(\tilde{x}_i)] = 3.985,$ $C(\tilde{Y},\tilde{Y}) = \sum_{i=1}^{n}[\mu^2(\tilde{y}_i) + \gamma^2(\tilde{y}_i) + \upsilon^2(\tilde{y}_i)] = 4.295,$ and $C(\tilde{X},\tilde{Y}) = \sum_{i=1}^{n}[\mu(\tilde{x}_i)\,\mu(\tilde{y}_i) + \gamma(\tilde{x}_i)\,\gamma(\tilde{y}_i) + \upsilon(\tilde{x}_i)\,\upsilon(\tilde{y}_i)] = 3.86.$

Then we get the correlation coefficient is $\rho(\tilde{X},\tilde{Y}) = 0.93$, this worth gives us the data which IVNS $\tilde{X},\tilde{Y}$ positively and strictly related to strength 0.93.

Table 3. (Data set for second kind observation)

Obs.	$\tilde{X}$	$\tilde{Y}$	$\hat{\tilde{Y}}$
Vehicles	⟨[0.5, 0.3], [0.1, 0.6], [0.4, 0.2]⟩	⟨[0.4, 0.3], [0.3, 0.5], [0.3, 0.2]⟩	⟨[0.48, 0.29], [0.09, 0.58], [0.39, 0.19]⟩
FMCGs	⟨[0.3, 0.2], [0.4, 0.3], [0.4, 0.5]⟩	⟨[0.6, 0.4], [0.1, 0.1], [0.2, 0.3]⟩	⟨[0.29, 0.19], [0.39, 0.29], [0.39, 0.48]⟩
Electronics	⟨[0.6, 0.3], [0.4, 0.1], [0.5, 0.4]⟩	⟨[0.5, 0.4], [0.2, 0.3], [0.1, 0.4]⟩	⟨[0.58, 0.29], [0.39, 0.09], [0.48, 0.39]⟩
Army	⟨[0.3, 0.2], [0.4, 0.2], [0.3, 0.2]⟩	⟨[0.2, 0.3], [0.2, 0.1], [0.3, 0.2]⟩	⟨[0.29, 0.19], [0.39, 0.19], [0.29, 0.19]⟩
Medical	⟨[0.5, 0.7], [0.3, 0.2], [0.2, 0.3]⟩	⟨[0.7, 0.7], [0.2, 0.3], [0.1, 0.3]⟩	⟨[0.48, 0.69], [0.29, 0.19], [0.19, 0.29]⟩
Pharma.	⟨[0.6, 0.5], [0.2, 0.4], [0.3, 0.4]⟩	⟨[0.4, 0.5], [0.3, 0.2], [0.2, 0.3]⟩	⟨[0.58, 0.48], [0.19, 0.39], [0.29, 0.39]⟩
Retail	⟨[0.3, 0.2], [0.5, 0.6], [0.2, 0.4]⟩	⟨[0.2, 0.3], [0.6, 0.7], [0.4, 0.3]⟩	⟨[0.29, 0.19], [0.48, 0.58], [0.19, 0.39]⟩
Oil	⟨[0.5, 0.4], [0.3, 0.2], [0.4, 0.3]⟩	⟨[0.4, 0.6], [0.6, 0.5], [0.3, 0.2]⟩	⟨[0.48, 0.39], [0.29, 0.19], [0.39, 0.29]⟩
Const.	⟨[0.7, 0.6], [0.3, 0.2], [0.1, 0.2]⟩	⟨[0.6, 0.5], [0.3, 0.4], [0.1, 0.2]⟩	⟨[0.69, 0.58], [0.29, 0.19], [0.09, 0.19]⟩
IT	⟨[0.1, 0.3], [0.3, 0.2], [0.2, 0.4]⟩	⟨[0.4, 0.6], [0.5, 0.5], [0.2, 0.3]⟩	⟨[0.09, 0.29], [0.29, 0.19], [0.19, 0.39]⟩

To rank the estimated revenue of IVNS, we compare it with scoring methods used by different research, as definitions 2.8, and 2.9 (see table 4)

Table 4. (The estimated revenue ranking for second kind observation by Smarandache and Tang)

		Smarandache		Tang	
Obs.	$\hat{\tilde{Y}}$	$s(\hat{\tilde{Y}})$	$a(\hat{\tilde{Y}})$	$s(\hat{\tilde{Y}})$	$a(\hat{\tilde{Y}})$
Vehicles	⟨[0.48, 0.29], [0.09, 0.58], [0.39, 0.19]⟩	0.2125	−0.06165	0.586667	−0.005
FMCGs	⟨[0.29, 0.19], [0.39, 0.29], [0.39, 0.48]⟩	0.0625	−0.30525	0.488333	0.095
Electronics	⟨[0.58, 0.29], [0.39, 0.09], [0.48, 0.39]⟩	0.26	−0.0027	0.586667	0.1
Army	⟨[0.29, 0.19], [0.39, 0.19], [0.29, 0.19]⟩	0.21	−0.1408	0.57	0
Medical	⟨[0.48, 0.69], [0.29, 0.19], [0.19, 0.29]⟩	0.4325	0.29525	0.701667	−0.055
Pharma.	⟨[0.58, 0.48], [0.19, 0.39], [0.29, 0.39]⟩	0.305	0.1523	0.633333	0.1
Retail	⟨[0.29, 0.19], [0.48, 0.58], [0.19, 0.39]⟩	−0.055	−0.2966	0.473333	0.15
Oil	⟨[0.48, 0.39], [0.29, 0.19], [0.39, 0.29]⟩	0.3075	0.04575	0.618333	−0.005
Const.	⟨[0.69, 0.58], [0.29, 0.19], [0.09, 0.19]⟩	0.5075	0.44125	0.751667	0.105
IT	⟨[0.09, 0.29], [0.29, 0.19], [0.19, 0.39]⟩	0.21	−0.2348	0.553333	0

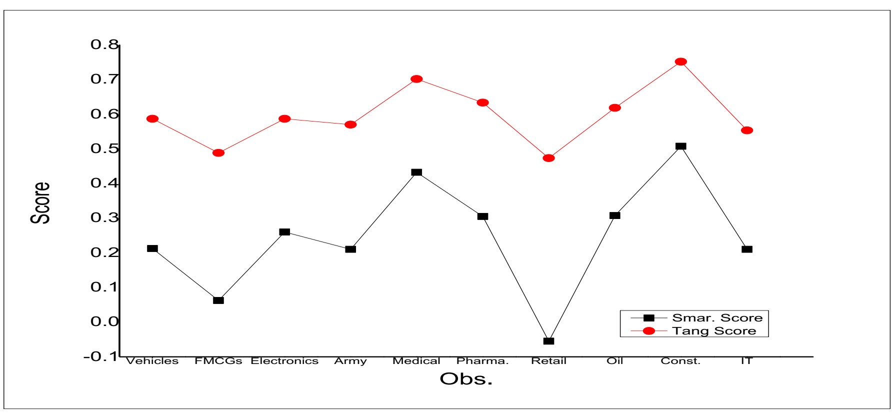

Figure 1. The estimated revenue ranking for second kind observation by Smarandache and Tang

From figure 1., Tang score of ranking is showing same pattern as Smarandache score of ranking in case of interval-valued neutrosophic set.

6 Conclusions

The major objective of this paper is to propose a methodology for determining the correlation coefficient of neutrosophic non-linear regression based on Kuhn-Tucker's necessary conditions. The numerical example is given to clarify the intended result in this study. Also, scoring methods by different research were provided in this example. An illustrative example has been provided to demonstrate the validity and applicability of the proposed approach. Evidently, the proposed approach gives the optimum solution for any optimization problems under the single-valued and interval-valued neutrosophic environment. The effectiveness of the ranking method is shown by comparative example in the case of single-valued and interval-valued neutrosophic sets.

Kuhn-Tucker conditions provided powerful means to verify solutions, but they are limitations as: sufficiency conditions are difficult to verify in some problems, some of practical problems do not have required nice properties (for example, we will have a problem if we do not know the explicit constraint equations), and if we have a multi-objective formulation then we would suggest testing each priority level separately.

Thus, the proposed method may be employed for neutrosophic non-linear regression as we may encounter in many real-world applications such as economy, artificial intelligence, or social studies. Further research would be necessary to focus various kinds of neutrosophic non-linear regression and their hypothetical development. Some comparisons with previous approaches can also be considered for further research.

Conflicts of interest:

The authors declare that there is no conflict of interest regarding the publication of this article.

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